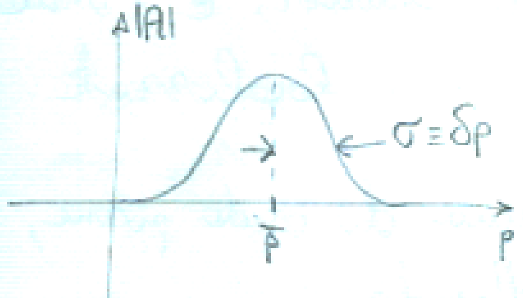
Analizziamo anche il caso importante di pacchetto gaussiano:

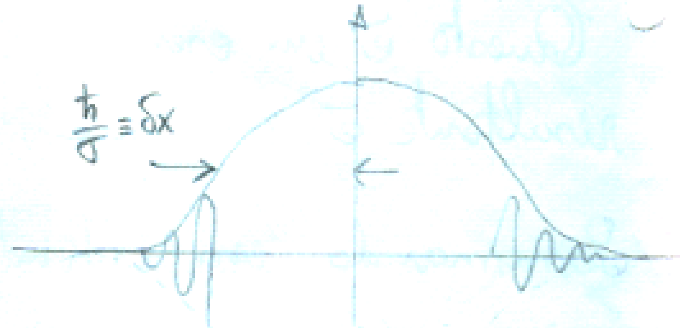
$$\psi(x) = \int_{-\infty}^{+\infty} \frac{e^{-\frac{1}{2}\left(\frac{p-\bar{p}}{\sigma}\right)^2}}{\sigma}\, e^{\frac{ipx}{\hbar}} = \frac{1}{\sigma}\int \exp\left\{\frac{-1}{2\sigma^2}\left[p^2+\bar{p}^2-2p\bar{p}-2\sigma^2 i\frac{px}{\hbar}\right]\right\} dp$$

$$= \frac{1}{\sigma}\int \exp\left\{-\frac{1}{2\sigma^2}\Big[\underbrace{p^2-2p\left(\bar{p}+\frac{i\sigma^2 x}{\hbar}\right)+\left(\bar{p}+\frac{i\sigma^2 x}{\hbar}\right)^2}_{\pi^2 \equiv \left(p-\left(\bar{p}+\frac{i\sigma^2 x}{\hbar}\right)\right)^2}\ \underbrace{-\left(\bar{p}+\frac{i\sigma^2 x}{\hbar}\right)^2+\bar{p}^2}_{\text{porto fuori}}\Big]\right\} dp$$

$$= \frac{1}{\sigma}\exp\left\{+\frac{1}{2\sigma^2}\left[\cancel{\bar{p}^2}+2\bar{p}\frac{i\sigma^2 x}{\hbar}-\frac{\sigma^4 x^2}{\hbar^2}-\cancel{\bar{p}^2}\right]\right\}\underbrace{\int_{-\infty}^{+\infty} e^{-\frac{\pi^2}{2\sigma^2}}\, d\pi}_{\sqrt{2\pi}\,\sigma}$$

$$= \sqrt{2\pi}\, e^{i\frac{\bar{p}x}{\hbar}}\, e^{-\frac{1}{2}\left(\frac{\sigma x}{\hbar}\right)^2}$$





$\delta p \sim \sigma \qquad \delta x \sim \frac{\hbar}{\sigma} \qquad \delta p \cdot \delta x \sim \hbar$

Tutto questo vale anche per onde classiche:

$$\delta k \cdot \delta x \gtrsim 1 \quad \Rightarrow \quad \delta p \cdot \delta x \gtrsim \hbar$$

$$\delta \omega \cdot \delta t \gtrsim 1 \quad \Rightarrow \quad \delta E \cdot \delta t \gtrsim \hbar$$

$$\delta L \cdot \delta \varphi \gtrsim \hbar$$