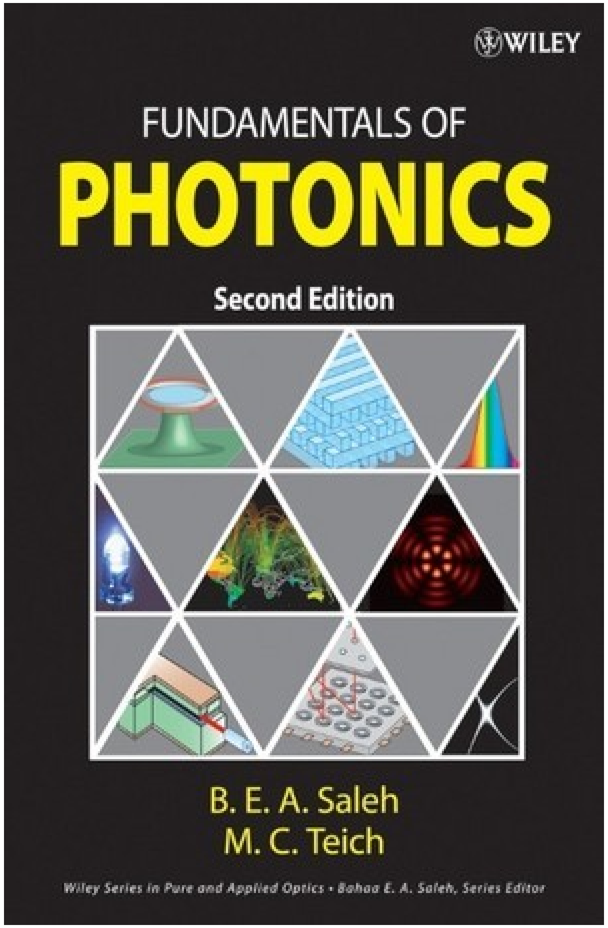
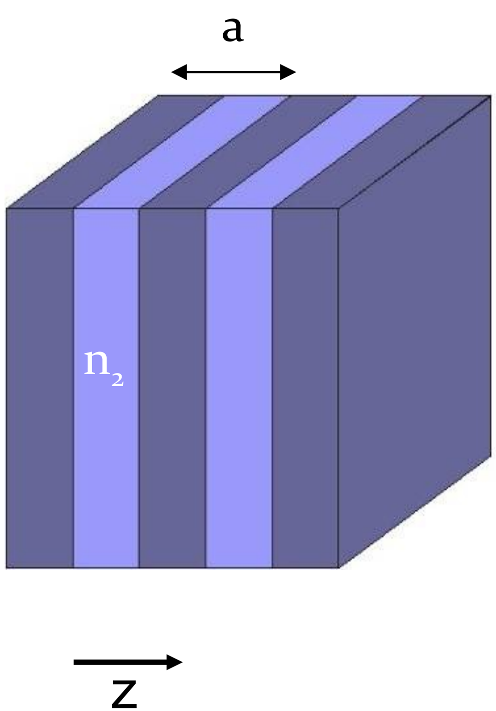
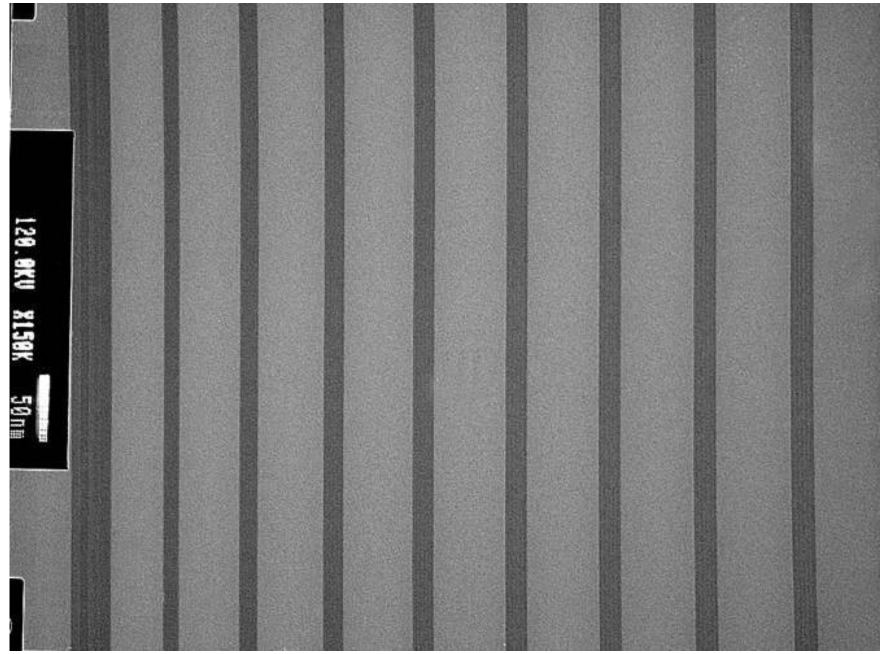
# Fotonica 1D
# Metodo matrici



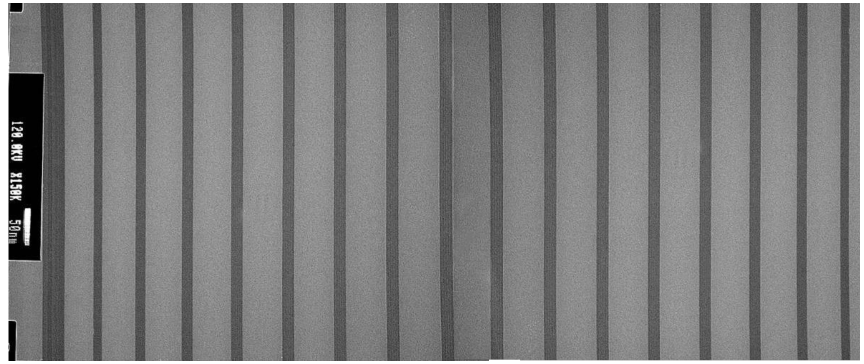
# PhC in 1D

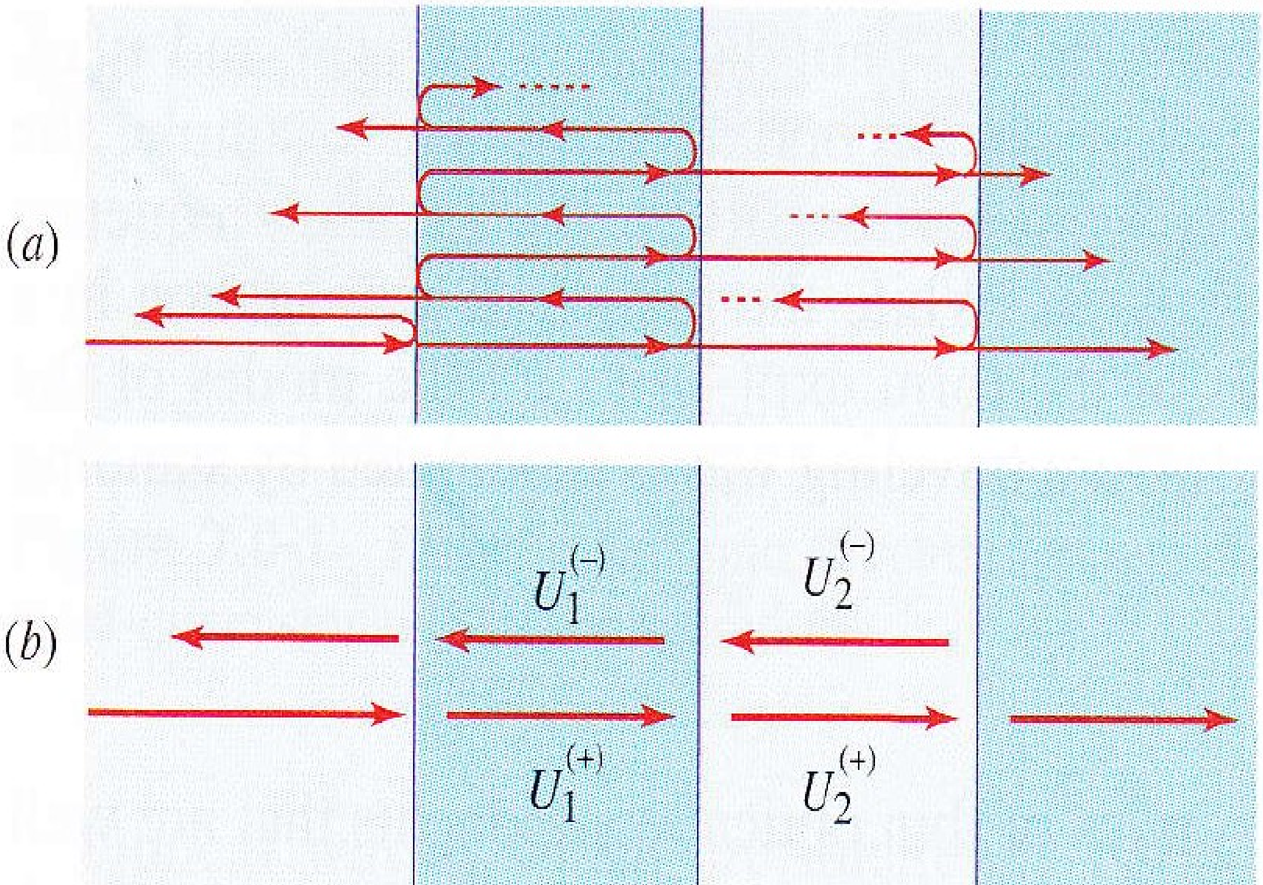
1D



# Sistema 1D periodico



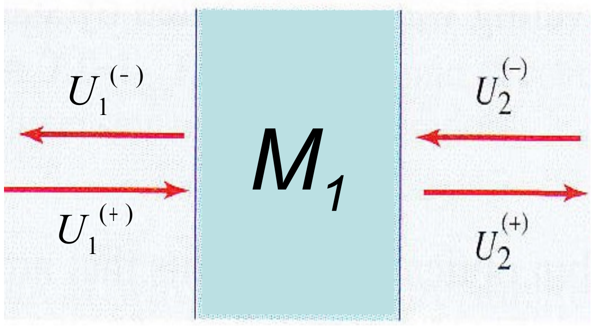


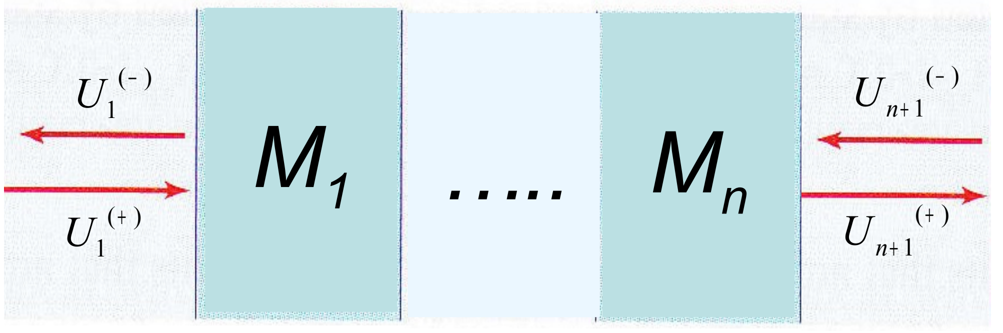

Visione cinetica

Visione statica

# Metodo matrici M

$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$



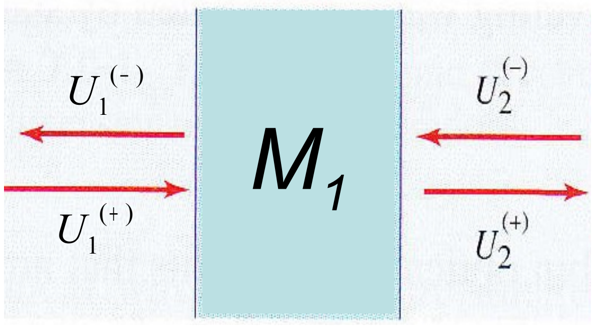




$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix} \quad ; \quad \begin{bmatrix} U_3^{(+)} \\ U_3^{(-)} \end{bmatrix} = M_2 \begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} \quad ; \ldots\ldots \quad \begin{bmatrix} U_{n+1}^{(+)} \\ U_{n+1}^{(-)} \end{bmatrix} = M_n \begin{bmatrix} U_n^{(+)} \\ U_n^{(-)} \end{bmatrix}$$

$$\begin{bmatrix} U_{n+1}^{(+)} \\ U_{n+1}^{(-)} \end{bmatrix} = M_n \ldots M_2 M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix} = M \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

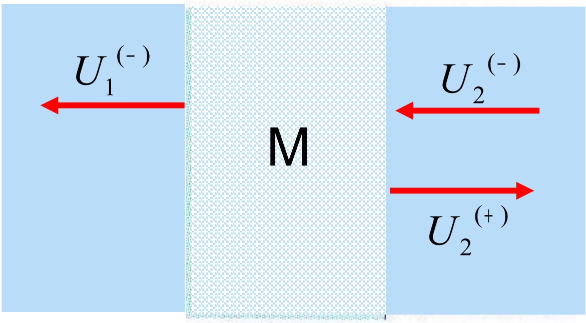
# Definizione M

$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$




$$M_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

# Calcolo dx verso sx




$$U_1^{(+)} = 0$$

$$U_2^{(+)} = r_{2,1} U_2^{(-)}$$

$$U_1^{(-)} = t_{2,1} U_2^{(-)}$$

$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

$$U_2^{(+)} = b U_1^{(-)}$$

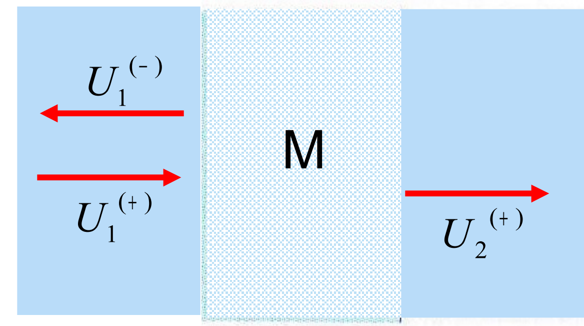
$$U_2^{(-)} = d U_1^{(-)}$$

$$M_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$d = \frac{1}{t_{2,1}}$$

$$b = \frac{r_{2,1}}{t_{2,1}}$$

# Calcolo sx verso dx




$$U_2^{(-)} = 0 \qquad U_1^{(-)} = r_{1,2} U_1^{(+)}$$

$$U_2^{(+)} = t_{1,2} U_1^{(+)}$$

$$d = \frac{1}{t_{2,1}} \, ; \, b = \frac{r_{2,1}}{t_{2,1}}$$

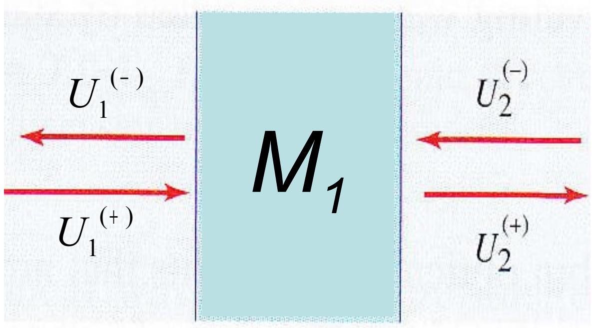
$$0 = cU_1^{(+)} + dU_1^{(-)} \quad \Rightarrow \quad U_1^{(-)} = -\frac{c}{d} U_1^{(+)}$$

$$U_2^{(+)} = aU_1^{(+)} + bU_1^{(-)} \quad \Rightarrow \quad U_2^{(+)} = \left( a - \frac{bc}{d} \right) U_1^{(+)}$$

$$r_{1,2} = -\frac{c}{d} = -ct_{2,1} \quad \rightarrow \quad c = -\frac{r_{1,2}}{t_{2,1}}$$

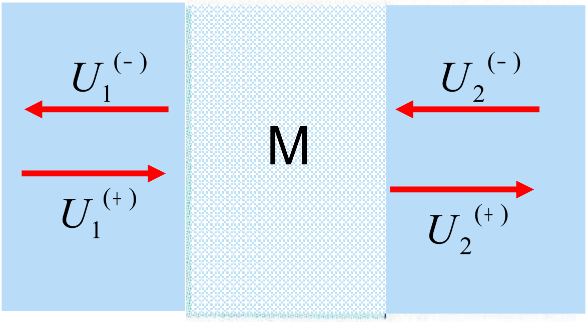
$$t_{1,2} = a - \frac{bc}{d} = a + \frac{r_{2,1}}{t_{2,1}} r_{1,2} \quad \rightarrow \quad a = \frac{t_{1,2} t_{2,1} - r_{2,1} r_{1,2}}{t_{2,1}}$$

In generale




$$M_1 = \begin{bmatrix} \dfrac{t_{1,2}t_{2,1} - r_{2,1}r_{1,2}}{t_{2,1}} & \dfrac{r_{2,1}}{t_{2,1}} \\ -\dfrac{r_{1,2}}{t_{2,1}} & \dfrac{1}{t_{2,1}} \end{bmatrix}$$

Mezzo simmetrico e senza perdite



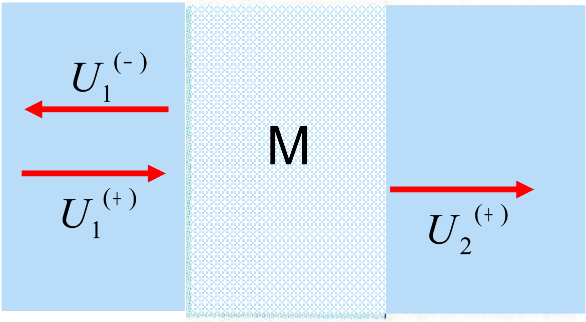

$$r_{1,2} = r_{2,1} = r$$

$$t_{1,2} = t_{2,1} = t$$

$$conservazione \quad energia$$

$$\left|U_1^{(+)}\right|^2 + \left|U_2^{(-)}\right|^2 = \left|U_1^{(-)}\right|^2 + \left|U_2^{(+)}\right|^2$$

# Sistema lossless e con stesso n




## Calcolo sx verso dx

$$U_2^{(-)} = 0 \qquad U_1^{(-)} = r$$

$$U_1^{(+)} = 1 \qquad U_2^{(+)} = t$$

*conservazione energia*

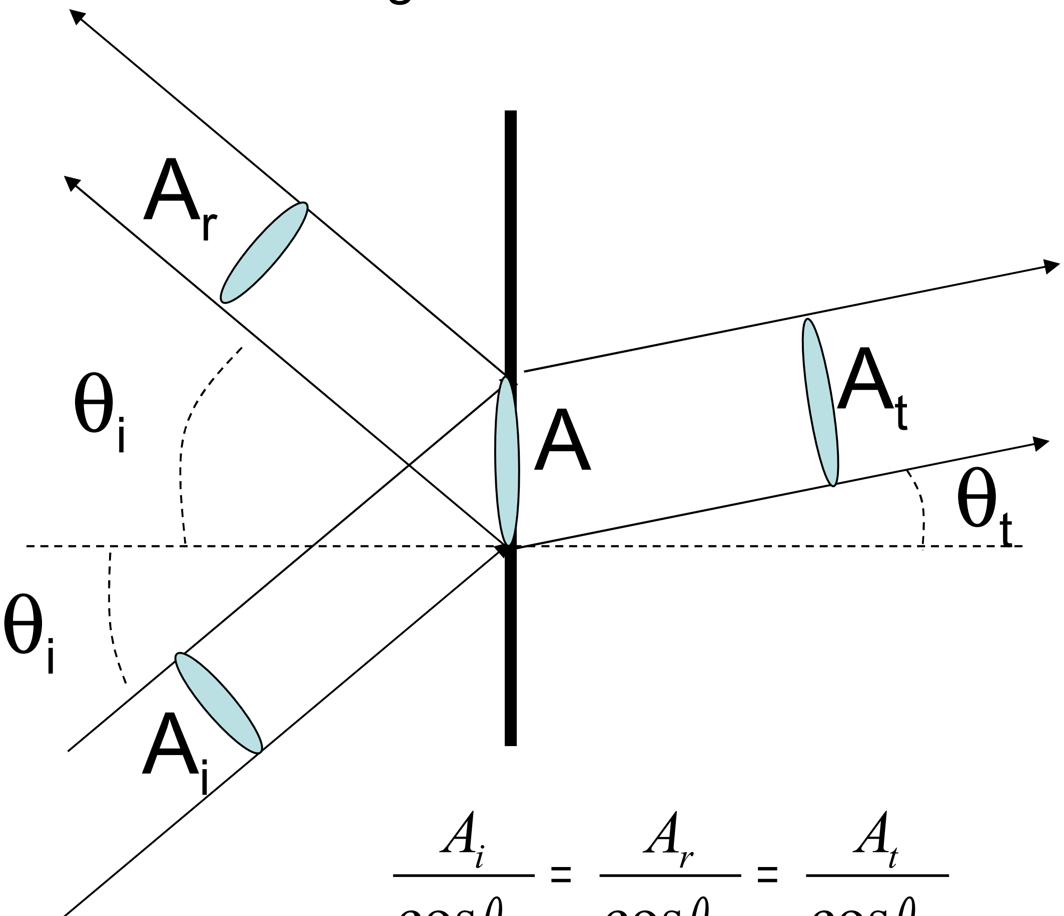
$$\left|U_1^{(+)}\right|^2 + \left|U_2^{(-)}\right|^2 = \left|U_1^{(-)}\right|^2 + \left|U_2^{(+)}\right|^2$$

$$1 = |t|^2 + |r|^2$$

A proposito ricordiamo che la conservazione in generale è:

$$\left|\vec{S}\right| = \left|\vec{E} \times \vec{H}^*\right| = \left|\vec{E} \times \left(\frac{\vec{k} \times \vec{E}^*}{\mu\,\omega}\right)\right|$$

$$= \frac{n}{\mu\, c}\left|\vec{E}\right|^2 = \frac{\varepsilon\, c}{n}\left|\vec{E}\right|^2 = nc\left|\vec{E}\right|^2$$




Conservazione energia

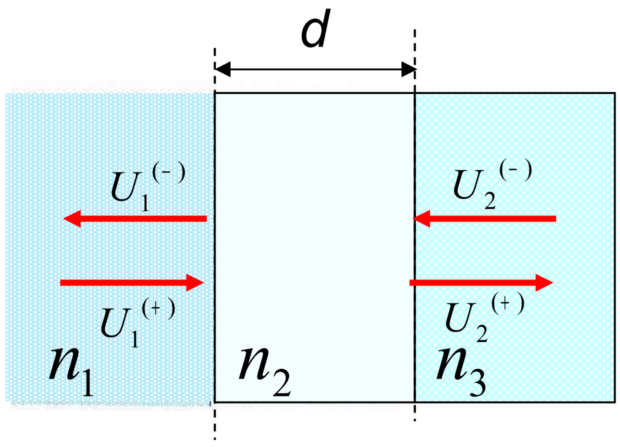
$$\left|\vec{S}_i\right| A_i = \left|\vec{S}_r\right| A_r + \left|\vec{S}_t\right| A_t$$

$$\frac{A_i}{\cos\theta_i} = \frac{A_r}{\cos\theta_i} = \frac{A_t}{\cos\theta_t}$$

$$n_1\left|\vec{E}_i\right|^2 A_i = n_1\left|\vec{E}_r\right|^2 A_r + n_2\left|\vec{E}_t\right|^2 A_t$$

$$1 = \left|r\right|^2 + \left|t\right|^2 \frac{n_2}{n_1}\frac{\cos\theta_i}{\cos\theta_t}$$

# Conservazione energia in sistema asimmetrico




$$1-|r|^2=\frac{n_3}{n_1}|t|^2$$

Sistema lossless e con stesso n

Calcolo generale

$$U_2^{(-)} = 1 \qquad U_1^{(-)} = r + t$$

$$U_1^{(+)} = 1 \qquad U_2^{(+)} = r + t$$

*conservazione energia*

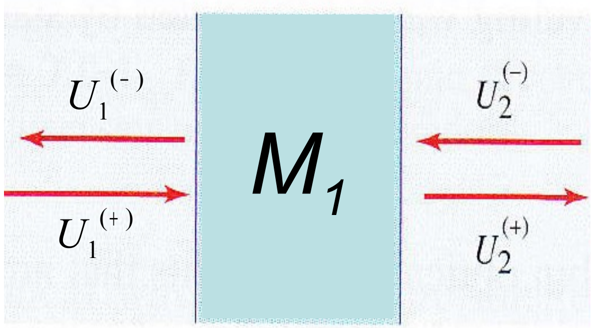
$$\left|U_1^{(+)}\right|^2 + \left|U_2^{(-)}\right|^2 = \left|U_1^{(-)}\right|^2 + \left|U_2^{(+)}\right|^2$$

$$1 = \left|r + t\right|^2 =$$

$$= \left|r\right|^2 + \left|t\right|^2 + r^* t + r t^*$$

$$r^* t + r t^* = 0 \quad \Rightarrow \quad \frac{r}{t} = -\frac{r^*}{t^*}$$

# Definizione M

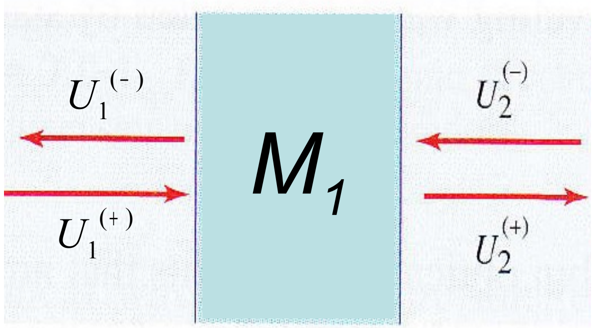



$$M_1 = \begin{bmatrix} \dfrac{t_{1,2}t_{2,1} - r_{2,1}r_{1,2}}{t_{2,1}} & \dfrac{r_{2,1}}{t_{2,1}} \\ -\dfrac{r_{1,2}}{t_{2,1}} & \dfrac{1}{t_{2,1}} \end{bmatrix} = \begin{bmatrix} t - \dfrac{rr}{t} & \dfrac{r}{t} \\ -\dfrac{r}{t} & \dfrac{1}{t} \end{bmatrix} =$$

$$= \begin{bmatrix} t + \dfrac{rr^*}{t^*} & \dfrac{r}{t} \\ \dfrac{r^*}{t^*} & \dfrac{1}{t} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{t^*} & \dfrac{r}{t} \\ \dfrac{r^*}{t^*} & \dfrac{1}{t} \end{bmatrix}$$

# Mezzo simmetrico e privo di perdite

$$M_1 = \begin{bmatrix} \frac{1}{t^*} & \frac{r}{t} \\ \frac{r^*}{t^*} & \frac{1}{t} \end{bmatrix}$$

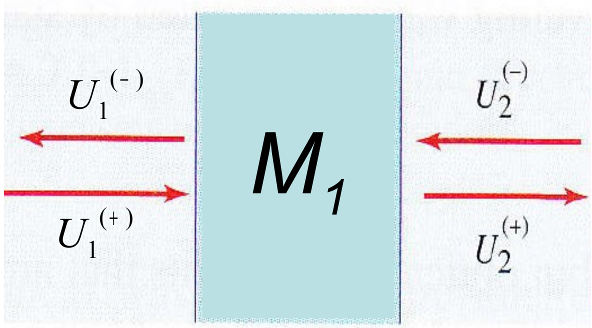



$$\frac{r}{t} = -\frac{r^*}{t^*} \qquad \arg\{t\} - \arg\{r\} = \pm\frac{\pi}{2}$$

Fase associata a *r* e *t* differisce di π/2

# Mezzo simmetrico e privo di perdite

$$M_1 = \begin{bmatrix} \frac{1}{t^*} & \frac{r}{t} \\ \frac{r^*}{t^*} & \frac{1}{t} \end{bmatrix}$$




$$\det M_1 = \frac{1}{|t|^2} - \frac{|r|^2}{|t|^2} = 1$$

Trasformazione unitaria

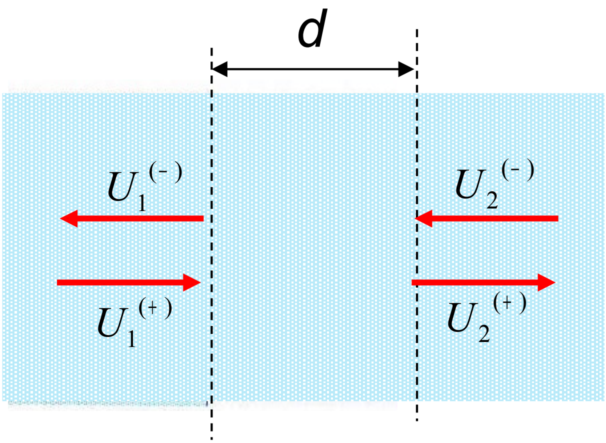
# Propagazione attraverso un mezzo omogeneo

$$U^{(+)}(z) = E^{(+)}e^{i(kz-\omega t)} \qquad U^{(-)}(z) = E^{(-)}e^{i(-kz-\omega t)}$$

$$U_2^{(+)} = E^{(+)}(z+d) \qquad U_2^{(-)} = E^{(-)}(z+d)$$

$$U_1^{(+)} = E^{(+)}(z) \qquad U_1^{(-)} = E^{(-)}(z)$$

$$U_2^{(+)} = U_1^{(+)}e^{ikd} \qquad U_2^{(-)} = U_1^{(-)}e^{-ikd}$$




Ricordando

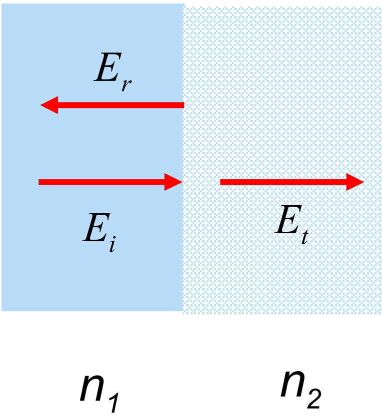
$$U_2^{(+)} = tU_1^{(+)} \rightarrow t = e^{ikd} \rightarrow 1/t^* = e^{ikd}$$

$$M = \begin{bmatrix} \exp(j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix} \qquad \varphi = \frac{2\pi}{\lambda}nd$$

# Singola interfaccia dielettrica sistema asimmetrico

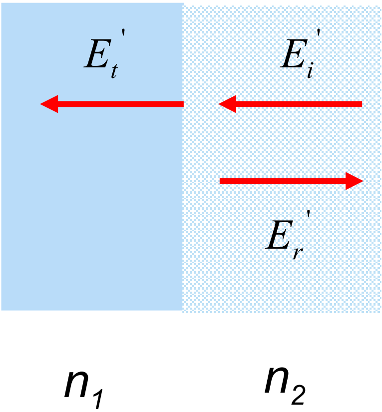
*Relazioni di Fresnel da sx verso dx*

$$\begin{cases} E_r = \dfrac{n_1 - n_2}{n_1 + n_2} E_i = r_{1,2} E_i \\ E_t = \dfrac{2n_1}{n_1 + n_2} E_i = t_{1,2} E_i \end{cases}$$




*Relazioni di Fresnel da dx verso sx*

$$\begin{cases} E_r = \dfrac{n_2 - n_1}{n_1 + n_2} E_i = r_{2,1} E_i \\ E_t = \dfrac{2n_2}{n_1 + n_2} E_i = t_{2,1} E_i \end{cases}$$

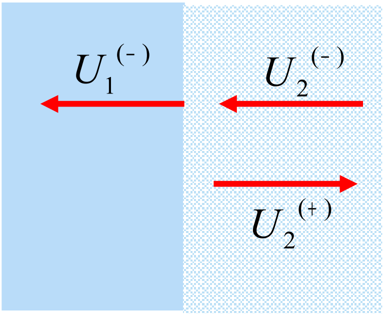


$$\frac{2n_1}{n_1 + n_2} - \frac{n_1 - n_2}{n_1 + n_2}\frac{n_2 - n_1}{n_1 + n_2}\frac{n_1 + n_2}{2n_2} =$$

$$= \frac{4n_1 n_2}{2n_2\left(n_1 + n_2\right)} + \frac{\left(n_1 - n_2\right)^2}{2n_2\left(n_1 + n_2\right)} =$$

$$= \frac{\left(n_1 + n_2\right)^2}{2n_2\left(n_1 + n_2\right)} = \frac{\left(n_1 + n_2\right)}{2n_2}$$

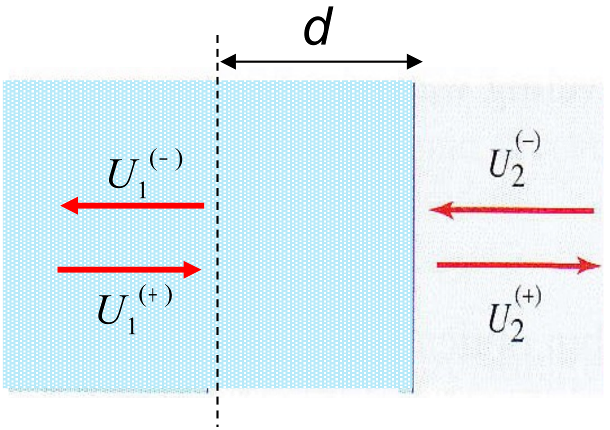
Singola interfaccia dielettrica
Sistema asimmetrico, vale




M

$$M_1 = \begin{bmatrix} \dfrac{t_{1,2}t_{2,1} - r_{2,1}r_{1,2}}{t_{2,1}} & \dfrac{r_{2,1}}{t_{2,1}} \\ -\dfrac{r_{1,2}}{t_{2,1}} & \dfrac{1}{t_{2,1}} \end{bmatrix} = = \begin{bmatrix} \dfrac{n_2 + n_1}{2n_2} & \dfrac{n_2 - n_1}{2n_2} \\ \dfrac{n_2 - n_1}{2n_2} & \dfrac{n_2 + n_1}{2n_2} \end{bmatrix}$$

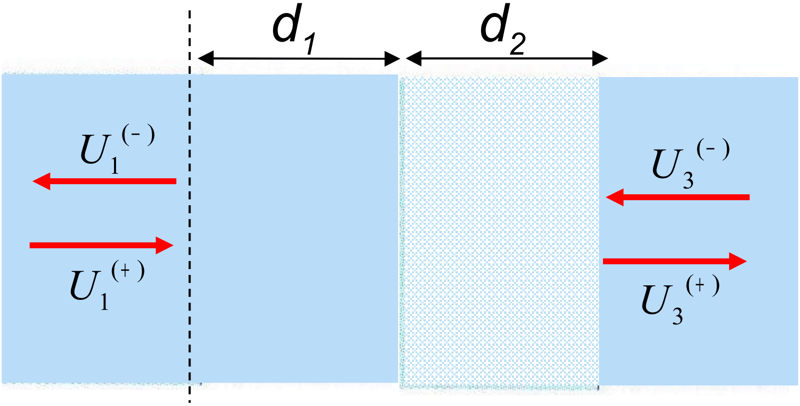
Propagazione attraverso un mezzo omogeneo seguita da una interfaccia dielettrica




$$M = \begin{bmatrix} \frac{n_2 + n_1}{2n_2} & \frac{n_2 - n_1}{2n_2} \\ \frac{n_2 - n_1}{2n_2} & \frac{n_2 + n_1}{2n_2} \end{bmatrix} \begin{bmatrix} \exp(j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{n_2 + n_1}{2n_2}\exp(j\varphi) & \frac{n_2 - n_1}{2n_2}\exp(-j\varphi) \\ \frac{n_2 - n_1}{2n_2}\exp(j\varphi) & \frac{n_2 + n_1}{2n_2}\exp(-j\varphi) \end{bmatrix} \qquad \varphi = \frac{2\pi}{\lambda} nd$$

Propagazione attraverso un mezzo omogeneo seguita da una slab dielettrica




$$M = \begin{bmatrix} \frac{n_2 + n_1}{2n_2}\exp(j\varphi_2) & \frac{n_1 - n_2}{2n_2}\exp(-j\varphi_2) \\ \frac{n_2 - n_2}{2n_2}\exp(j\varphi_2) & \frac{n_2 + n_1}{2n_2}\exp(-j\varphi_2) \end{bmatrix} \quad ; \quad \varphi_i = \frac{2\pi}{\lambda} n_i d_i$$

$$\times \begin{bmatrix} \frac{n_2 + n_1}{2n_1}\exp(j\varphi_1) & \frac{n_2 - n_1}{2n_1}\exp(-j\varphi_1) \\ \frac{n_2 - n_1}{2n_1}\exp(j\varphi_1) & \frac{n_2 + n_1}{2n_1}\exp(-j\varphi_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{t^*} & \frac{r}{t} \\ \frac{r^*}{t^*} & \frac{1}{t} \end{bmatrix}$$

Propagazione attraverso un mezzo omogeneo
seguita da una slab dielettrica

$$\frac{1}{t^*} = \frac{(n_2 + n_1)^2}{4n_2n_1} \exp j(\varphi_2 + \varphi_1) - \frac{(n_2 - n_1)^2}{4n_2n_1} \exp j(-\varphi_2 + \varphi_1) =$$

$$= \frac{\exp(j\varphi_1)}{4n_2n_1}\left[(n_2 + n_1)^2 \exp(j\varphi_2) - (n_2 - n_1)^2 \exp(-j\varphi_2)\right]$$

$$t = \frac{4n_2n_1 \exp(j\varphi_1)}{\left[(n_2 + n_1)^2 \exp(-j\varphi_2) - (n_2 - n_1)^2 \exp(j\varphi_2)\right]} =$$

$$= \frac{4n_2n_1 \exp(j\varphi_1)}{\left[-2j\left(n_2^{\,2} + n_1^{\,2}\right)\sin(\varphi_2) + 4n_2n_1 \cos(\varphi_2)\right]}$$

Propagazione attraverso un mezzo omogeneo seguita da una slab dielettrica

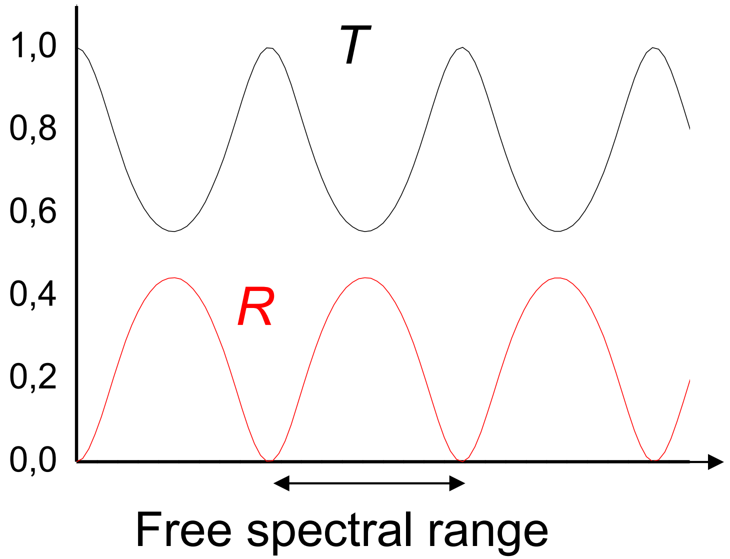
$$t = \exp(j\varphi_1)\frac{2n_2n_1}{-j\left(n_2^2+n_1^2\right)\sin(\varphi_2)+2(n_2n_1)\cos(\varphi_2)}$$

$$T = |t|^2 = \frac{1}{1+\frac{\left(n_2^2-n_1^2\right)^2}{4n_2^2n_1^2}\sin^2(\varphi_2)}$$

$$\varphi_2 = \frac{2\pi}{\lambda}n_2d_2$$

$$R = 1 - T$$

Formula di Airy




$$\frac{n_2}{n_1} = 2.2$$

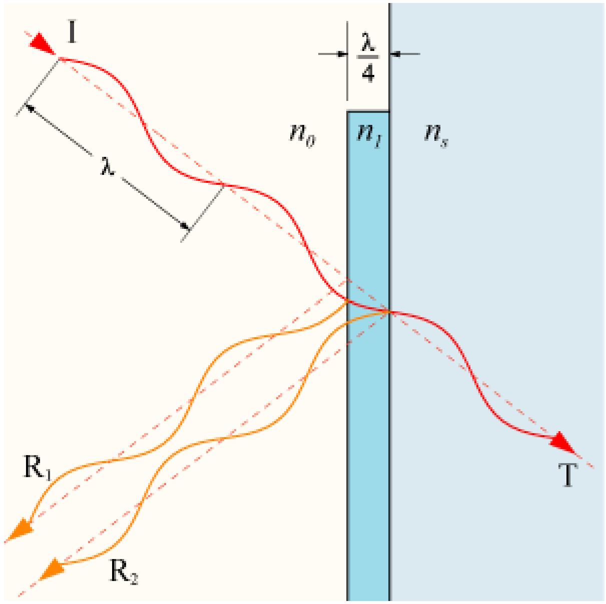
Interferenza pellicole

# Antireflection coating (in realtà sono multilayer)

$$T = |t|^2 = \frac{1}{1 + \frac{\left(n_2^{\,2} - n_1^{\,2}\right)^2}{4 n_2^{\,2} n_1^{\,2}} \sin^2(\varphi_2)}$$

$$\varphi_2 = \frac{2\pi}{\lambda} n_2 d_2$$

$$\varphi_2 = \frac{\pi}{2} \qquad d_2 = \frac{\lambda}{4 n_2} \qquad T = 1$$




Without Anti reflection

With Anti reflection