

Fotonica

Proprietà generali 2

Equazioni fotonica

$$\left\{ \begin{array}{l} \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \vec{H} \\ \vec{E}(\vec{r}) = \frac{i}{\omega \epsilon_0 \epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}) \end{array} \right. \quad \vec{\nabla} \cdot \vec{H} = 0 \quad \left(\vec{H}_i(\vec{r}), \vec{H}_j(\vec{r}) \right) = A_j \delta_{i,j}$$

$$\left\{ \begin{array}{l} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E} \\ \vec{H}(\vec{r}) = \frac{-i}{\omega \mu} \vec{\nabla} \times \vec{E}(\vec{r}) \end{array} \right. \quad \vec{\nabla} \cdot \epsilon(\vec{r}) \vec{E} = 0 \quad \left(\vec{E}_i(\vec{r}), \epsilon(\vec{r}) \vec{E}_j(\vec{r}) \right) = A_i (\mu_0 c)^2 \frac{\omega_2}{\omega_1} \delta_{i,j}$$

Spettro discreto vs continuo

$\vec{H}_\omega(\vec{r}) \quad \omega$ ha spettro continuo

$$(\vec{H}_\omega(\vec{r}), \vec{H}_\omega(\vec{r})) = 1 \quad \forall \omega$$

$$(\vec{H}_\omega(\vec{r}), \vec{H}_{\omega+\delta\omega}(\vec{r})) = 0 \quad \forall \delta\omega$$

$$\vec{H}_{\omega+\delta\omega}(\vec{r}) = \vec{H}_\omega(\vec{r}) + \delta\vec{H}$$

$$(\vec{H}_\omega(\vec{r}), \vec{H}_{\omega+\delta\omega}(\vec{r})) = (\vec{H}_\omega(\vec{r}), \vec{H}_\omega(\vec{r}) + \delta\vec{H}(\vec{r})) = \\ (\vec{H}_\omega(\vec{r}), \vec{H}_\omega(\vec{r})) + (\vec{H}_\omega(\vec{r}), \delta\vec{H}(\vec{r}))$$

Se e solo se $V = \infty$

$$(\vec{H}_\omega(\vec{r}), \delta\vec{H}(\vec{r})) = \int_V d^3r \vec{H}_\omega^*(\vec{r}) \cdot \delta\vec{H}(\vec{r}) \not\approx O(\delta\omega)$$

Stati localizzati \rightarrow spettro discreto

Stati delocalizzati \rightarrow spettro continuo

Metodi perturbativi

Perturbazione dielettrica

$$\tilde{\epsilon}(\vec{r}) = \epsilon(\vec{r}) + \delta\epsilon(\vec{r})$$

Perturbazione magnetica

$$\tilde{\mu}(\vec{r}) = \mu(\vec{r}) + \delta\mu(\vec{r})$$

Perturbazione dielettrica $\tilde{\epsilon}(\vec{r}) = \epsilon(\vec{r}) + \delta\epsilon(\vec{r})$

Mezzo dielettrico non magnetico ($\mu=1$, $\epsilon=\epsilon(r)$))

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \vec{H}$$

Autostati e autovalori del problema iniziale

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}_i(\vec{r})) = \frac{\omega_i^2}{c^2} \epsilon(\vec{r}) \vec{E}_i(\vec{r})$$

Perturbazione dielettrica

$$\tilde{\epsilon}(\vec{r}) = \epsilon(\vec{r}) + \delta\epsilon(\vec{r})$$

Metodo perturbativo

Soluzione generale

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}(\vec{r})) = \frac{\omega^2}{c^2} \tilde{\epsilon}(\vec{r}) \vec{E}(\vec{r})$$

Stima variazione n-esimo autostato (hp piccola perturbazione)

$$\vec{E}(\vec{r}) = c_n \vec{E}_n(\vec{r}) + \sum_{i \neq n} \delta c_i \vec{E}_i(\vec{r}) \quad \delta c_i \ll c_n$$

Stima variazione n-esimo autovalore

$$\frac{\omega_n^2}{c^2} \epsilon(\vec{r}) c_n \vec{E}_n(\vec{r}) + \frac{\omega_i^2}{c^2} \epsilon(\vec{r}) \sum_{i \neq n} \delta c_i \vec{E}_i(\vec{r}) =$$

$$= \frac{\omega^2}{c^2} (\epsilon(\vec{r}) + \delta\epsilon(\vec{r})) \left[c_n \vec{E}_n(\vec{r}) + \sum_{i \neq n} \delta c_i \vec{E}_i(\vec{r}) \right]$$

Proiezione sullo stato n-esimo

Metodo perturbativo

$$\begin{aligned} \frac{\omega_n^2}{c^2} c_n (\vec{E}_n(\vec{r}), \mathcal{E}(\vec{r}) \vec{E}_n(\vec{r})) + \frac{\omega_i^2}{c^2} \sum_{i \neq n} \delta c_i (\vec{E}_n(\vec{r}), \mathcal{E}(\vec{r}) \vec{E}_i(\vec{r})) = \\ = \frac{\omega^2}{c^2} \left[c_n (\vec{E}_n(\vec{r}), (\mathcal{E}(r) + \delta \mathcal{E}(\vec{r})) \vec{E}_n(\vec{r})) + \sum_{i \neq n} \delta c_i (\vec{E}_n(\vec{r}), \mathcal{E}(r) \vec{E}_i(\vec{r})) \right] \end{aligned}$$

Ricordando che per autostati vale:

$$(\vec{E}_n(\vec{r}), \mathcal{E}(\vec{r}) \vec{E}_i(\vec{r})) = (\vec{E}_n(\vec{r}), \mathcal{E}(\vec{r}) \vec{E}_n(\vec{r})) \delta_{n,i} \equiv A_n \delta_{n,i}$$

Si ha:

$$\frac{\omega_n^2}{c^2} c_n A_n = \frac{\omega^2}{c^2} c_n (A_n + (\vec{E}_n(\vec{r}), \delta \mathcal{E}(\vec{r}) \vec{E}_n(\vec{r})))$$

Quindi

Metodo perturbativo

$$\frac{(\omega_n^2 - \omega^2)}{\omega^2} = \frac{(\vec{E}_n(\vec{r}), \delta\epsilon(\vec{r})\vec{E}_n(\vec{r}))}{(\vec{E}_n(\vec{r}), \epsilon(\vec{r})\vec{E}_n(\vec{r}))}$$

Ponendo: $\Delta\omega_n = \omega - \omega_n$ e se $|\Delta\omega_n| \ll \omega_n$

Si ottiene:

$$\Delta\omega_n = -\frac{\omega_n}{2} \frac{\int d^3r \vec{E}_n^*(\vec{r}) \delta\epsilon(\vec{r}) \vec{E}_n(\vec{r})}{\int d^3r \vec{E}_n^*(\vec{r}) \epsilon(\vec{r}) \vec{E}_n(\vec{r})}$$

Un aumento di dielettrico produce un red shift

Una diminuzione di dielettrico produce un blue shift

Mezzo dielettrico emagnetico

$$\vec{\nabla} \times \left(\frac{1}{\mu(\vec{r})} \vec{\nabla} \times \vec{E} \right) = - \left(\frac{\partial}{\partial t} \vec{\nabla} \times \mu_o \vec{H} \right) = \\ = -\mu_o \left(\frac{\partial^2}{\partial t^2} \epsilon_o \epsilon(\vec{r}) \vec{E} \right) = \frac{\omega^2}{c^2} \epsilon(\vec{r}) \vec{E}$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \epsilon_o \frac{\partial}{\partial t} \vec{\nabla} \times \vec{E} = \\ = -\epsilon_o \frac{\partial^2}{\partial t^2} \mu_o \mu(\vec{r}) \vec{H} = \frac{\omega^2}{c^2} \mu(\vec{r}) \vec{H}$$

Mezzo dielettrico e magnetico

$$\vec{\nabla} \times \left(\frac{1}{\mu(\vec{r})} \vec{\nabla} \times \vec{E} \right) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \mu(\vec{r}) \vec{H}$$

Perturbazione magnetica $\tilde{\mu}(\vec{r}) = \mu(\vec{r}) + \delta\mu(\vec{r})$

Mezzo dielettrico non magnetico ($\mu=1$, $\epsilon=\epsilon(r)$)

$$\vec{\nabla} \times \left(\frac{1}{1 + \delta\mu(\vec{r})} \vec{\nabla} \times \vec{E} \right) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} (1 + \delta\mu(\vec{r})) \vec{H}$$

Autostati e autovalori del problema iniziale

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H}_i(\vec{r}) \right) = \frac{\omega_i^2}{c^2} \vec{H}_i(\vec{r})$$

Perturbazione magnetica

$$\tilde{\mu}(\vec{r}) = \mu(\vec{r}) + \delta\mu(\vec{r}) \quad \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}) \right) = \frac{\omega^2}{c^2} \tilde{\mu}(\vec{r}) \vec{H}(\vec{r})$$

Stima variazione n-esimo autostato (hp piccola perturbazione)

$$\vec{H}(\vec{r}) = c_n \vec{H}_n(\vec{r}) + \sum_{i \neq n} \delta c_i \vec{H}_i(\vec{r}) \quad \delta c_i \ll c_n$$

Stima variazione n-esimo autovalore

Metodo perturbativo

$$\mu(\vec{r}) = 1$$

$$\frac{\omega_n^2}{c^2} c_n \vec{H}_n(\vec{r}) + \frac{\omega_i^2}{c^2} \sum_{i \neq n} \delta c_i \vec{H}_i(\vec{r}) =$$

$$= \frac{\omega^2}{c^2} (1 + \delta\mu(\vec{r})) \left[c_n \vec{H}_n(\vec{r}) + \sum_{i \neq n} \delta c_i \vec{H}_i(\vec{r}) \right]$$

Proiezione sullo stato n-esimo

Metodo perturbativo

$$\begin{aligned} \frac{\omega_n^2}{c^2} c_n \left(\vec{H}_n(\vec{r}), \vec{H}_n(\vec{r}) \right) + \frac{\omega_i^2}{c^2} \sum_{i \neq n} \delta c_i \left(\vec{H}_n(\vec{r}), \vec{H}_i(\vec{r}) \right) &= \\ = \frac{\omega^2}{c^2} \left[c_n \left(\vec{H}_n(\vec{r}), (1 + \delta\mu(\vec{r})) \vec{H}_n(\vec{r}) \right) + \sum_{i \neq n} \delta c_i \left(\vec{H}_n(\vec{r}), \vec{H}_i(\vec{r}) \right) \right] \end{aligned}$$

Ricordando che per autostati vale:

$$\left(\vec{H}_n(\vec{r}), \vec{H}_i(\vec{r}) \right) = \left(\vec{H}_n(\vec{r}), \vec{H}_n(\vec{r}) \right) \delta_{n,i} \equiv A_n \delta_{n,i}$$

Si ha:

$$\frac{\omega_n^2}{c^2} c_n A_n = \frac{\omega^2}{c^2} c_n \left(A_n + \left(\vec{H}_n(\vec{r}), \delta\mu(\vec{r}) \vec{H}_n(\vec{r}) \right) \right)$$

Quindi

Metodo perturbativo

$$\frac{(\omega_n^2 - \omega^2)}{\omega^2} = \frac{(\vec{H}_n(\vec{r}), \delta\mu(\vec{r})\vec{H}_n(\vec{r}))}{(\vec{H}_n(\vec{r}), \vec{H}_n(\vec{r}))}$$

Ponendo: $\Delta\omega_n = \omega - \omega_n$ e se $|\Delta\omega_n| \ll \omega_n$

Si ottiene:

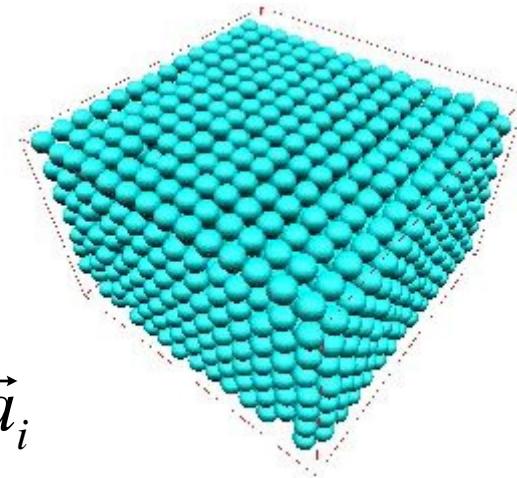
$$\Delta\omega_n = -\frac{\omega_n}{2} \frac{\int d^3r \vec{H}_n^*(\vec{r}) \cdot \delta\mu(\vec{r}) \vec{H}_n(\vec{r})}{\int d^3r \vec{H}_n^*(\vec{r}) \cdot \vec{H}_n(\vec{r})}$$

Un aumento di μ produce un red shift

Una diminuzione di μ produce un blue shift

Teorema di Bloch

$$\hat{T}_{\vec{R}} \mathcal{E}(\vec{r}) = \mathcal{E}(\vec{r} + \vec{R}) = \mathcal{E}(\vec{r}) \quad \vec{R} = \sum \ell_i \vec{a}_i$$



$$\hat{\Theta} \vec{H}(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r}) \quad \hat{T}_{\vec{R}} \vec{H}(\vec{r}) = e^{i\varphi(\vec{R})} \vec{H}(\vec{r})$$

$$[\hat{T}_{\vec{R}}, \hat{\Theta}] \vec{H}(\vec{r}) = \hat{T}_{\vec{R}} \hat{\Theta} \vec{H}(\vec{r}) - \hat{\Theta} \hat{T}_{\vec{R}} \vec{H}(\vec{r}) = 0$$

$$\vec{H}(\vec{r}) \equiv \vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \vec{u}_{n,\vec{k}}(\vec{r}) \quad \vec{u}_{n,\vec{k}}(\vec{r} + \vec{R}) = \vec{u}_{n,\vec{k}}(\vec{r})$$

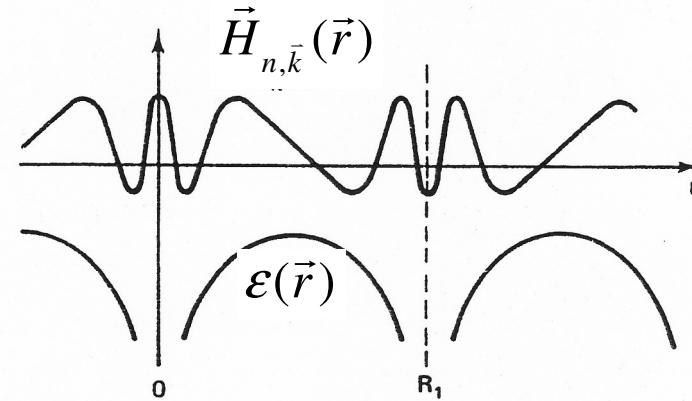
n=indice di banda (numera i modi in ordine di ω crescente)

k=vettore onda modo fotonico

Prima zona di Brillouin (FBZ)

Th. Bloch

$$\vec{H}_{n,\vec{k}}(\vec{r} + \vec{R}) = \vec{H}_{n,\vec{k}}(\vec{r}) e^{i\vec{k} \cdot \vec{R}}$$



Periodicità $\vec{K} \cdot \vec{R} = 2n\pi$

$$\vec{H}_{n,\vec{k}+\vec{K}}(\vec{r} + \vec{R}) = e^{i(\vec{k}+\vec{K}) \cdot \vec{r}} e^{i\vec{k} \cdot \vec{R}} \vec{u}_{n,\vec{k}+\vec{K}}(\vec{r}) =$$

$$= \vec{H}_{n,\vec{k}+\vec{K}}(\vec{r}) e^{i\vec{k} \cdot \vec{R}} = \vec{H}_{n,\vec{k}}(\vec{r} + \vec{R})$$

$$\boxed{\vec{k} + \vec{K} \equiv \vec{k}}$$

First Brillouin Zone

- Il vettore d'onda definisce univocamente un modo solo nella FBZ
- Tutti i modi sono rappresentati nella FBZ

Time-reversal

$$\hat{\Theta} \vec{H}_{n,\vec{k}}(\vec{r}) = \frac{\omega_n^2(\vec{k})}{c^2} \vec{H}_{n,\vec{k}}(\vec{r})$$

$$[\hat{\Theta} \vec{H}_{n,\vec{k}}(\vec{r})]^* = \hat{\Theta} \vec{H}_{n,\vec{k}}^*(\vec{r}) = \frac{\omega_n^2(\vec{k})}{c^2} \vec{H}_{n,\vec{k}}^*(\vec{r})$$

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \vec{u}_{n,\vec{k}}(\vec{r})$$

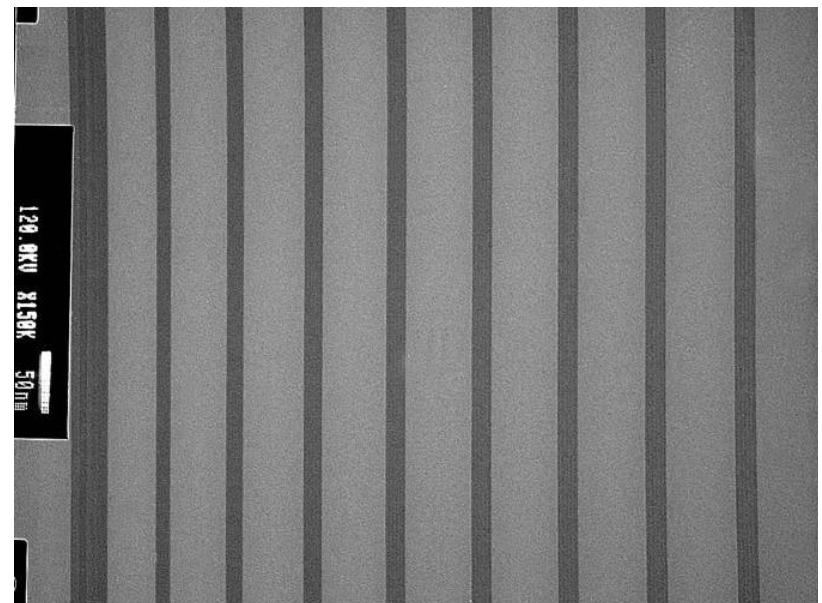
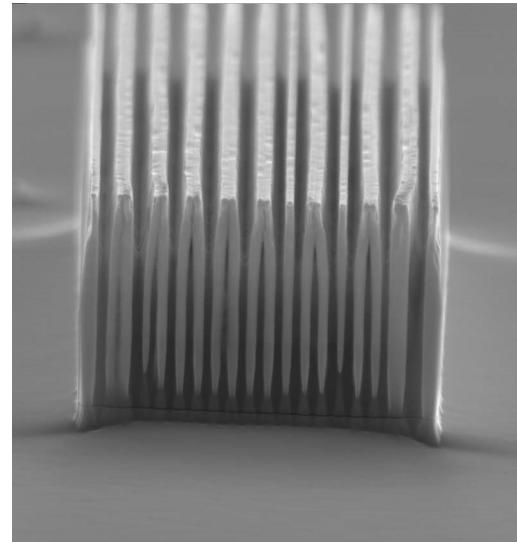
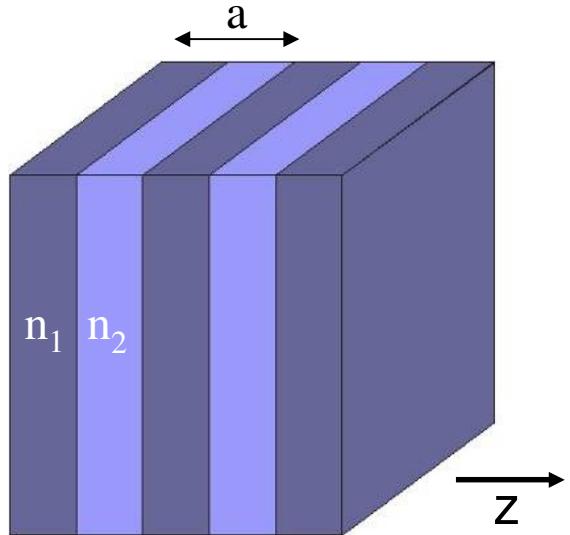
$$\vec{H}_{n,\vec{k}}^*(\vec{r}) = e^{-i\vec{k}\cdot\vec{r}} \vec{u}_{n,\vec{k}}^*(\vec{r}) \equiv \vec{H}_{n,-\vec{k}}(\vec{r})$$

$$\omega_n(\vec{k}) = \omega_n(-\vec{k})$$

Le bande sono simmetriche in k

PhC in 1D

1D



$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{||} \cdot \vec{\rho}} \vec{u}_{n,k_z}(z)$$

$$\vec{k} = \vec{k}_{||} + \vec{k}_z \quad \vec{\rho} = x\hat{x} + y\hat{y}$$

$$FBZ \quad -\frac{\pi}{a} \leq |\vec{k}_z| \leq \frac{\pi}{a}$$

Infinite 1D systems at $\vec{k}_{\parallel} = 0$

Legge di scala

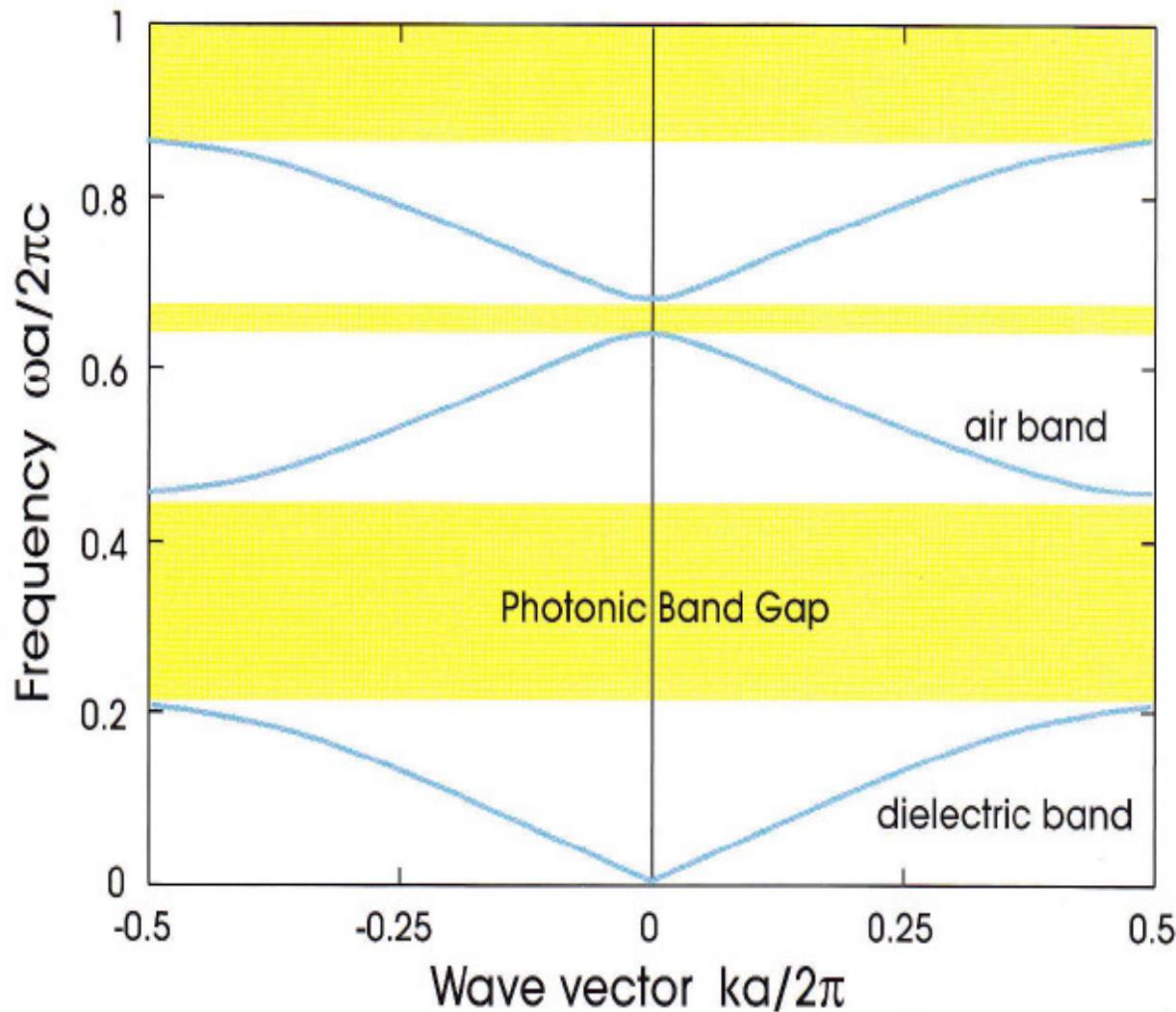
$$a' = a / s$$

$$\omega' = s \omega$$

$$\omega' a' = \omega a$$

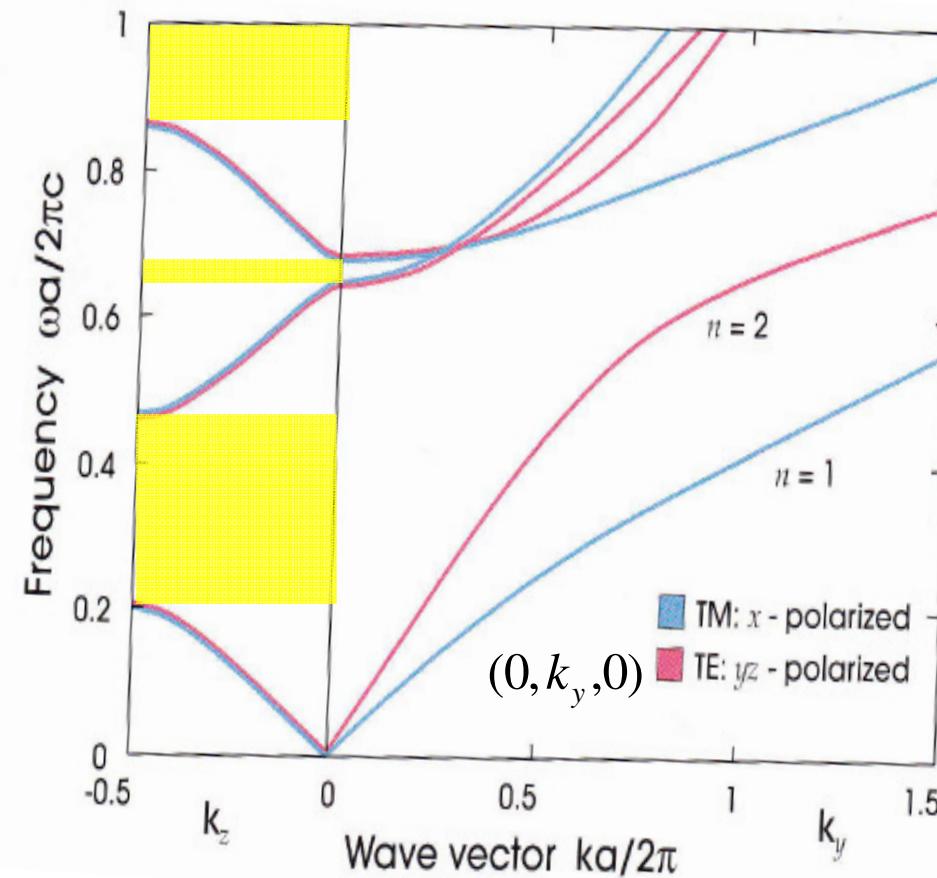
FBZ

$$-\frac{\pi}{a} \leq |\vec{k}_z| \leq \frac{\pi}{a}$$

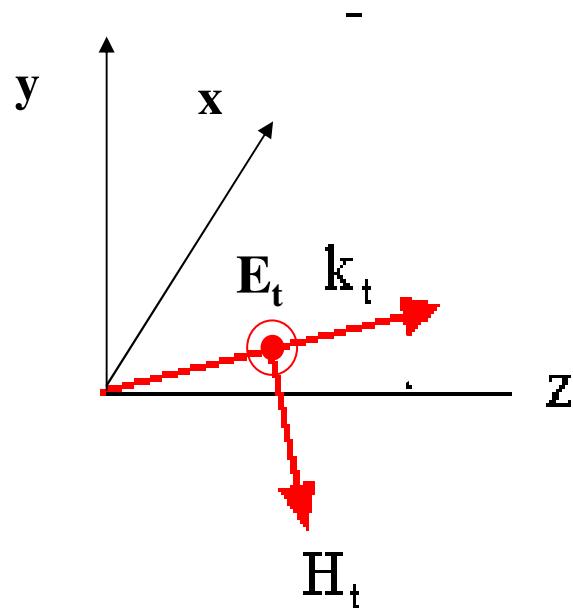


Struttura a bande per propagazione nel piano

Assenza band gap completo sia in TM e TE



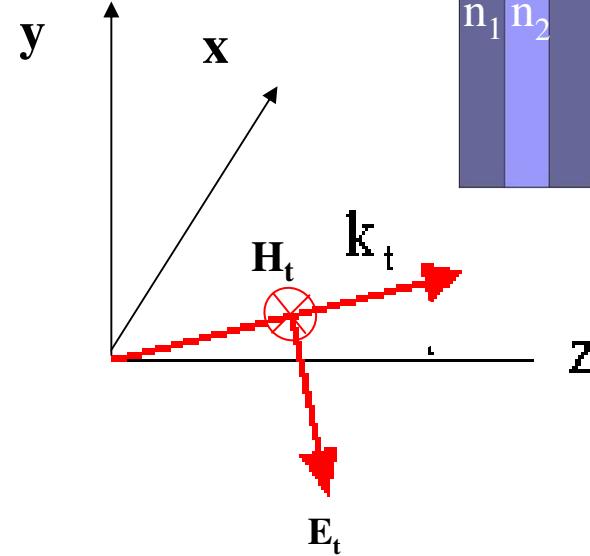
Nota su onde TE e TM (attenzione!)



Ottica classica

Onda s (senkrecht)

Polarizzazione TE

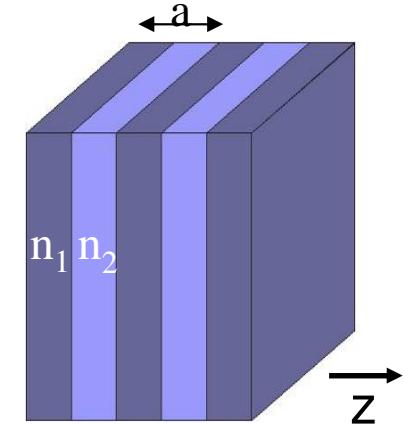


Onda s (parallel)

Polarizzazione TM

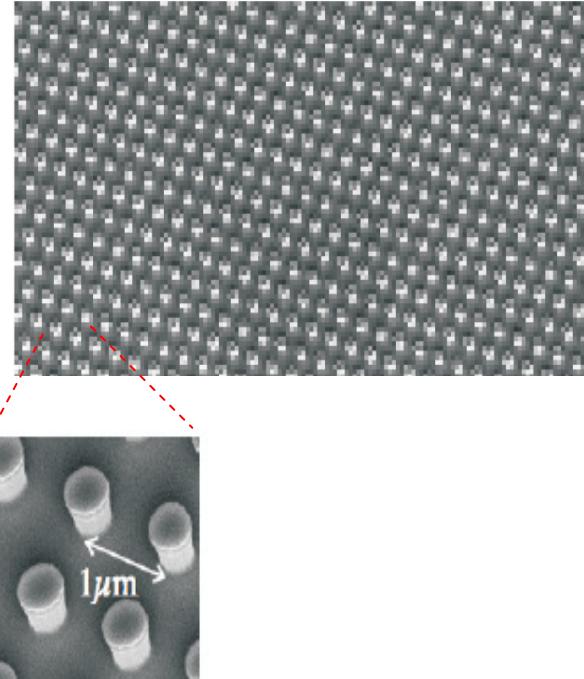
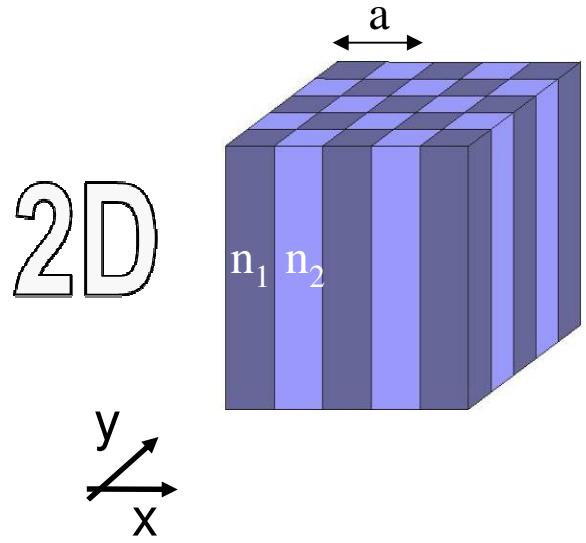
Fotonica (Joannopoulos)

Polarizzazione TM



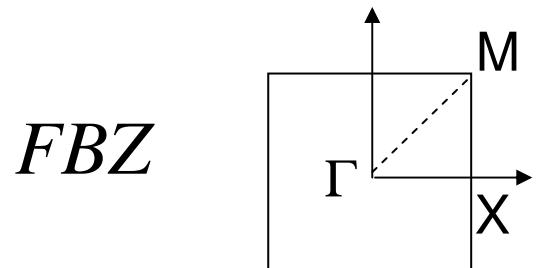
Polarizzazione TE

PhC in 2D

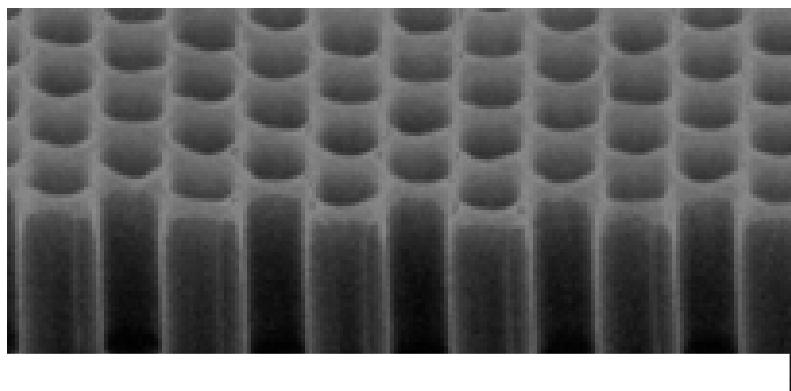


$$\vec{H}_{n,\bar{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{||} \cdot \vec{\rho}} \vec{u}_{n,\bar{k}_{||}}(\vec{\rho})$$

$$\vec{k} = \vec{k}_{||} + \vec{k}_z \quad \vec{\rho} = x\hat{x} + y\hat{y}$$



Γ -point $k=0$
 X-point $k=(\pi/a, 0)$, etc.
 M-point $k=(\pi/a, \pi/a)$, etc.



Struttura a bande

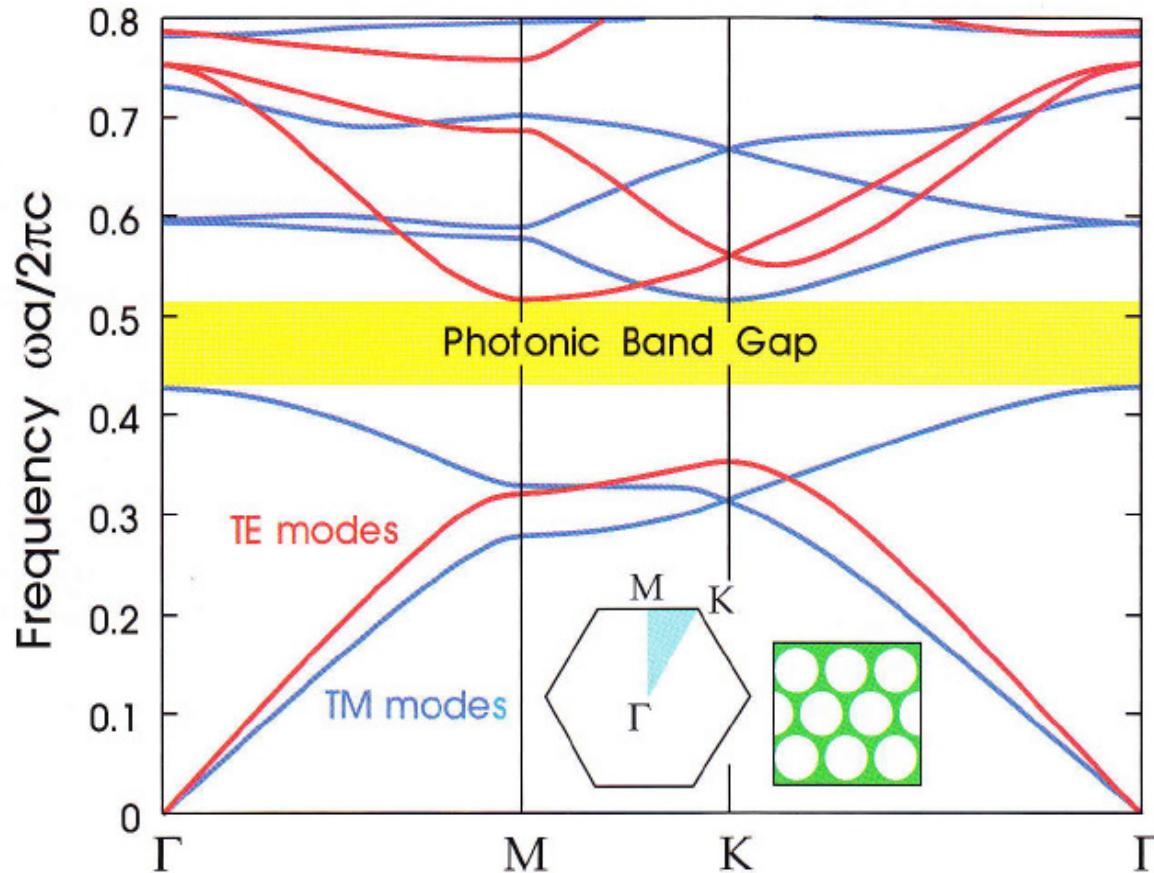


Figure 10: The photonic band structure for the modes of a triangular array of air columns drilled in a dielectric substrate ($\epsilon = 13$). The blue lines represent TM bands and the red lines represent TE bands. The inset shows the high-symmetry points at the corners of the irreducible Brillouin zone (shaded light blue). Note the complete photonic band gap.

Propagazione lungo z

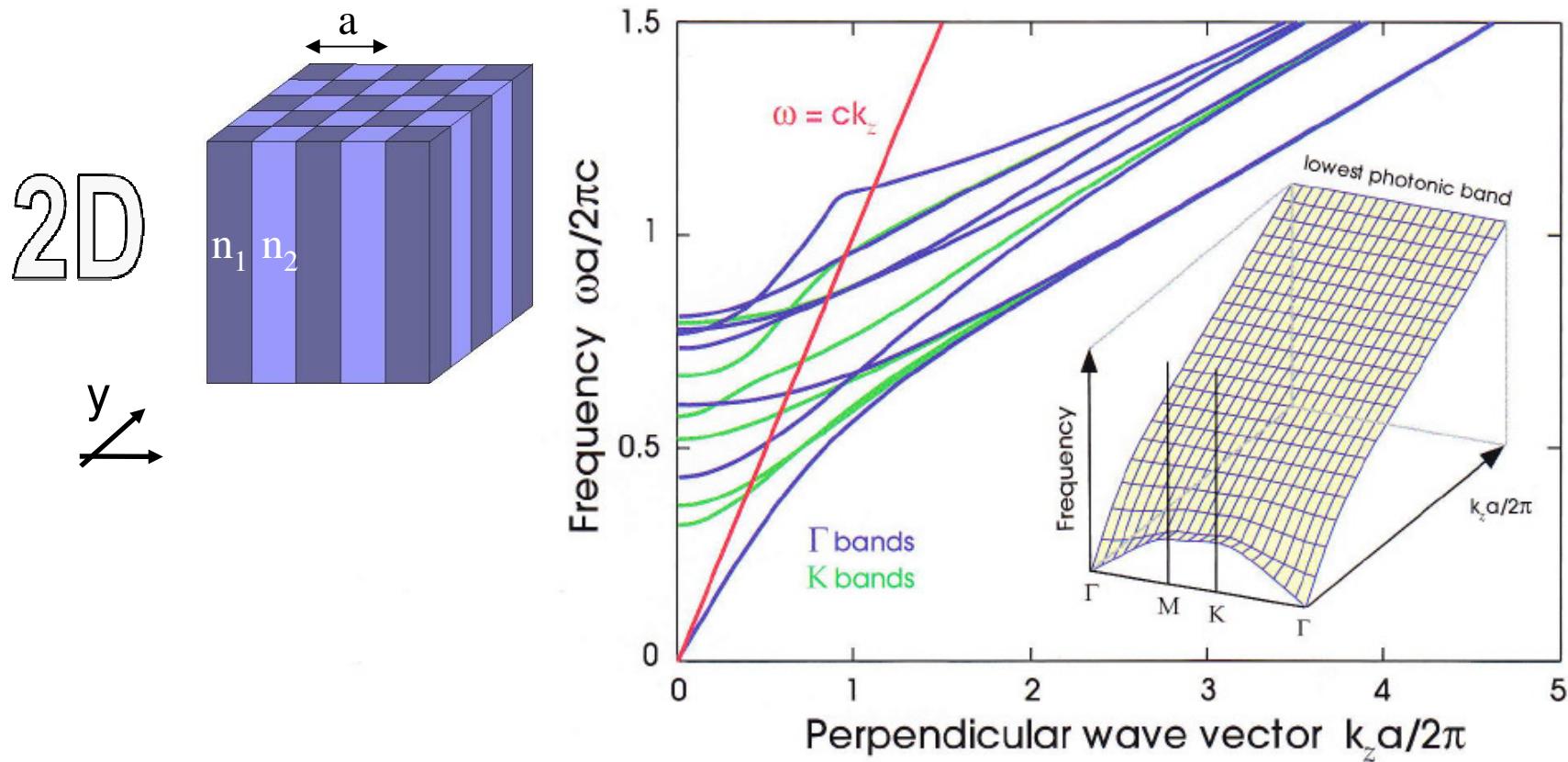
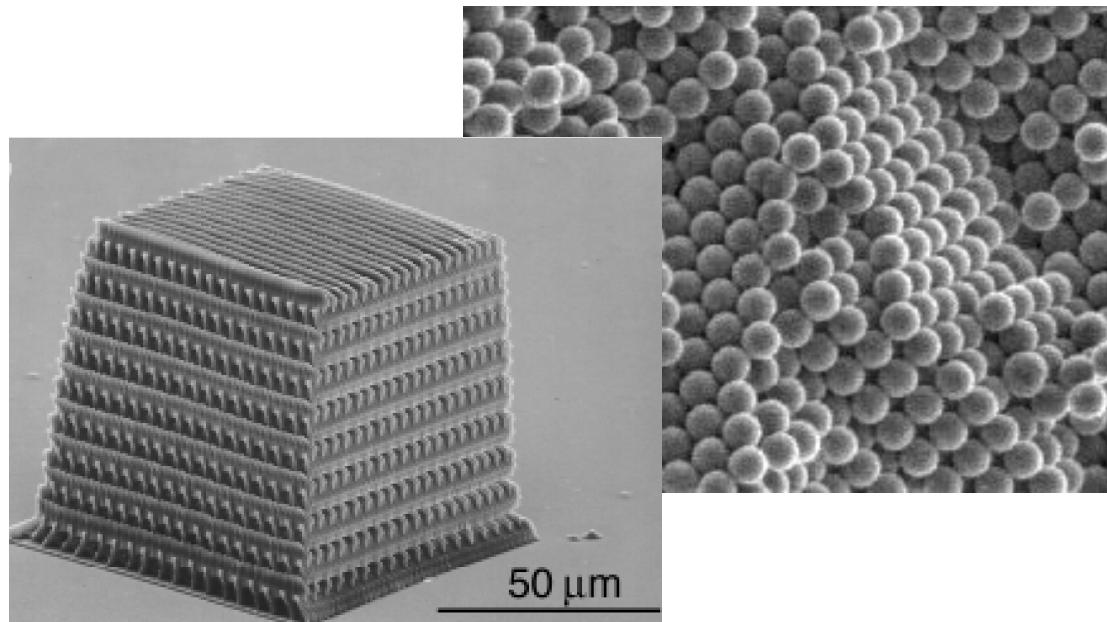
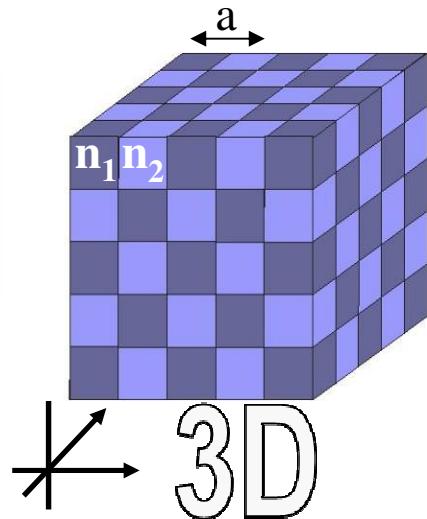


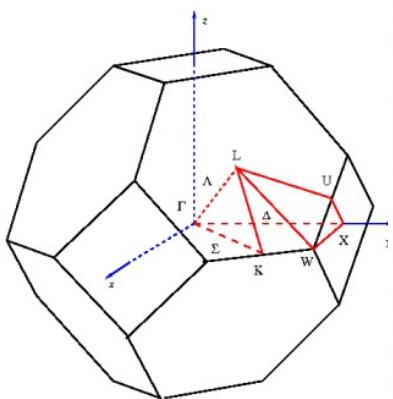
Figure 11: The out-of-plane band structure of the triangular lattice of air columns for the first few bands. The bands that start at Γ , $\omega(\Gamma, k_z)$, are plotted with blue lines, whereas the bands that start at K , $\omega(K, k_z)$, are plotted with green lines. The light line $\omega = ck_z$ (red) separates the modes that are oscillatory ($\omega \geq ck_z$) in the air regions from those that are evanescent ($\omega < ck_z$) in the air regions. The inset shows the frequency dependence of the lowest band as k_z varies. Note that as k_z increases, the lowest band flattens.

PhC in 3D



$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \vec{u}_{n,\vec{k}}(\vec{r})$$

FBZ



Γ -point: $\mathbf{k} = 0$

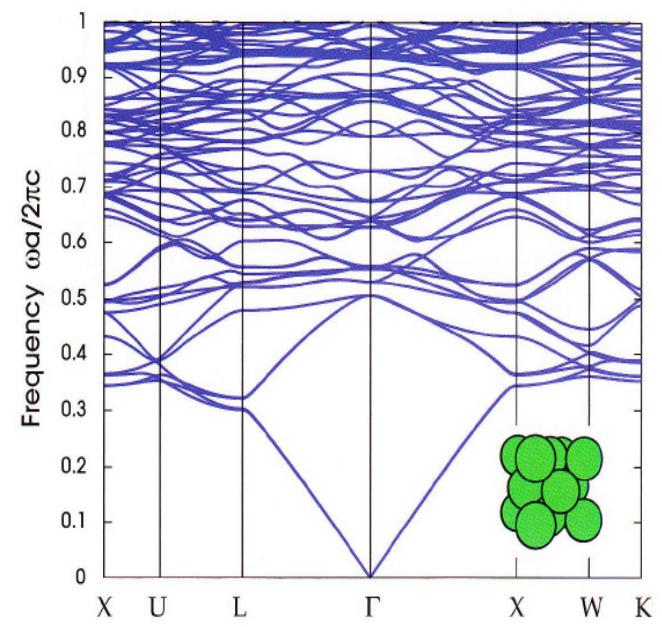
X -points: $\mathbf{k} = \pm \frac{2\pi}{a} \hat{x}, \pm \frac{2\pi}{a} \hat{y}$, etc.

L -points: $\mathbf{k} = \pm \frac{\pi}{a} (\hat{x} + \hat{y} + \hat{z}), \dots$

$\pm \frac{\pi}{a} (\hat{x} - \hat{y} + \hat{z}),$ etc.

K -points: $\mathbf{k} = \pm \frac{3\pi}{2} \frac{\pi}{a} (\hat{x} + \hat{y}),$ etc.

W -points: $\mathbf{k} = \pm \frac{\pi}{a} (2\hat{x} + \hat{y}),$ etc.



Fotonica

Campo

$$\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{-i\omega t}$$

Problema
autovalori

$$\hat{\Theta}\vec{H}(\vec{r}) = \frac{\omega^2}{c^2}\vec{H}(\vec{r})$$

Operatore
Hermitiano

$$\hat{\Theta} = \vec{\nabla} \times \frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times$$

Principio
variazionale

$$U_f(\vec{H}) \equiv \frac{(\vec{H}, \hat{\Theta}\vec{H})}{(\vec{H}, \vec{H})}$$

Elettronica

$$\psi(\vec{r},t) = \psi(\vec{r})e^{-i\frac{E}{\hbar}t}$$

$$\hat{H}\psi = E\psi$$

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$$

$$\langle \hat{H} \rangle \equiv \frac{(\psi, \hat{H}\psi)}{(\psi, \psi)}$$

Fotonica

Energia

$$U_H(\vec{H}) \equiv (\mu_0 \vec{H}, \vec{H})$$
$$\vec{S} \equiv (u_E + u_H) \vec{v}_g$$

Legge di
scala

$$\epsilon'(\vec{r}) = \epsilon(\vec{r}/s)$$

$$\omega' = \frac{\omega}{sc}$$

Teorema
Bloch

$$\epsilon(\vec{r} + \vec{R}) = \epsilon(\vec{r})$$

$$\vec{H}(\vec{r}) \equiv \vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \vec{u}_{n,\vec{k}}(\vec{r})$$

Elettronica

$$\langle \hat{H} \rangle \equiv \frac{(\psi, \hat{H} \psi)}{(\psi, \psi)}$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e c^2}$$

$$V(\vec{r} + \vec{R}) = V(\vec{r})$$

$$\psi(\vec{r} + \vec{R}) = e^{i\vec{k}\cdot\vec{r}} u_{n,\vec{k}}(\vec{r})$$

Fotonica

Bande

$$\omega = \omega_n(\vec{k})$$

FBZ

Contiene tutti e soli i
valori non ridondanti di k

Correzione
perturbativa

$$\Delta\omega_n = -\frac{\omega_n}{2} \frac{\int d^3r \vec{E}_n^*(\vec{r}) \delta\epsilon(\vec{r}) \vec{E}_n(\vec{r})}{\int d^3r \vec{E}_n^*(\vec{r}) \epsilon(\vec{r}) \vec{E}_n(\vec{r})}$$

Elettronica

$$E = E_n(\vec{k})$$

Contiene tutti e soli i
valori non ridondanti di k

$$\Delta E_n = \frac{\int d^3r \psi_n^*(\vec{r}) \delta V(\vec{r}) \psi_n(\vec{r})}{\int d^3r \psi_n^*(\vec{r}) \psi_n(\vec{r})}$$

$$\vec{S}(\vec{r}, t) = \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \quad \text{Vettore di Poynting}$$

$$\vec{S}(\vec{r}, t) = \frac{1}{4} \left(\vec{E}(\vec{r}, t) + \vec{E}^*(\vec{r}, t) \right) \times \left(\vec{H}(\vec{r}, t) + \vec{H}^*(\vec{r}, t) \right)$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{4} \left(\langle \vec{E}(\vec{r}, t) \times \vec{H}^*(\vec{r}, t) \rangle + \langle \vec{E}^*(\vec{r}, t) \times \vec{H}(\vec{r}, t) \rangle \right)$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{2} \Re e \left[\langle \vec{E}(\vec{r}, t) \times \vec{H}^*(\vec{r}, t) \rangle \right]$$

$$\boxed{\vec{S} \equiv (\boldsymbol{u}_E + \boldsymbol{u}_H) \vec{v}_e \quad \vec{v}_e = \vec{v}_g = \vec{\nabla}_{\vec{k}} \omega(\vec{k})}$$

Equazione per parte Bloch (forse non serve)

$$\vec{H}_{n,\bar{k}}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \vec{u}_{n,\bar{k}}(\vec{r})$$

$$\hat{\Theta} \vec{H} = \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \vec{H}$$

$$(i\vec{k} + \vec{\nabla}) \times \left(\frac{1}{\epsilon(\vec{r})} (i\vec{k} + \vec{\nabla}) \times \vec{u}_{n,\bar{k}}(\vec{r}) \right) = \frac{\omega^2}{c^2} \vec{u}_{n,\bar{k}}(\vec{r})$$

$$\hat{\Theta}_{\bar{k}} \vec{u}_{n,\bar{k}}(\vec{r}) = \frac{\omega^2}{c^2} \vec{u}_{n,\bar{k}}(\vec{r})$$