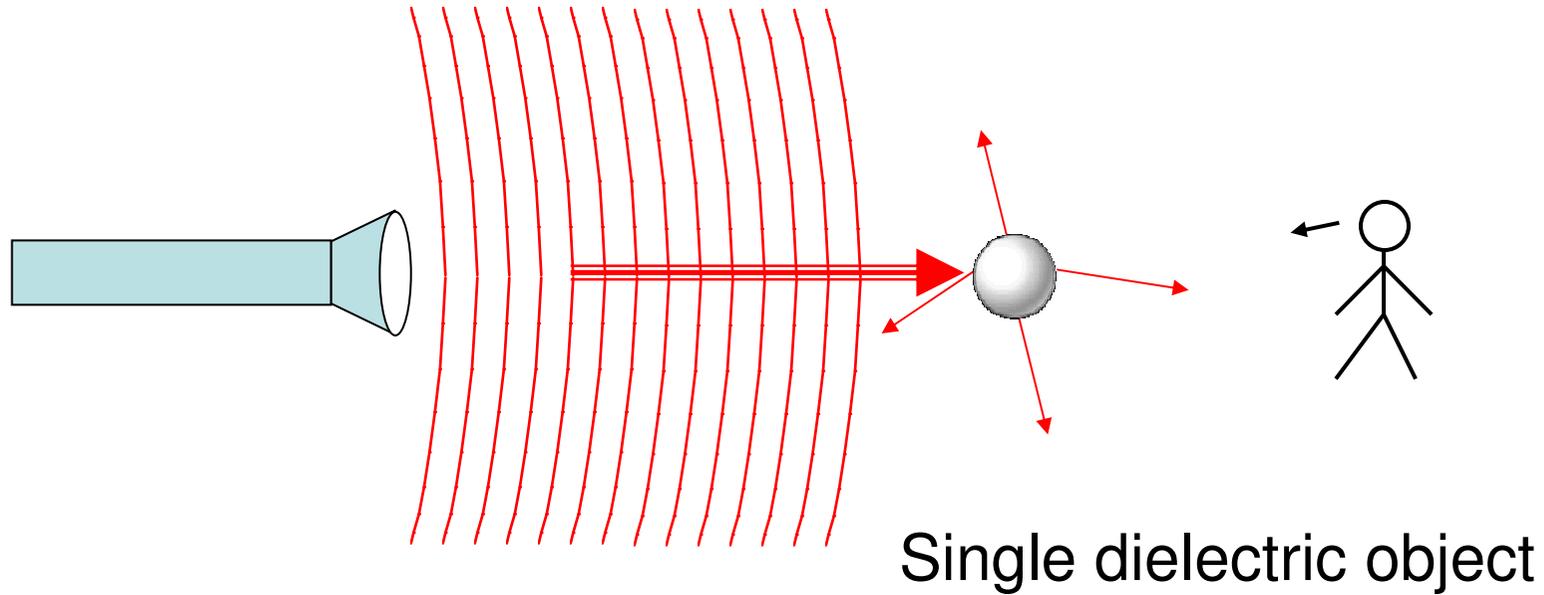


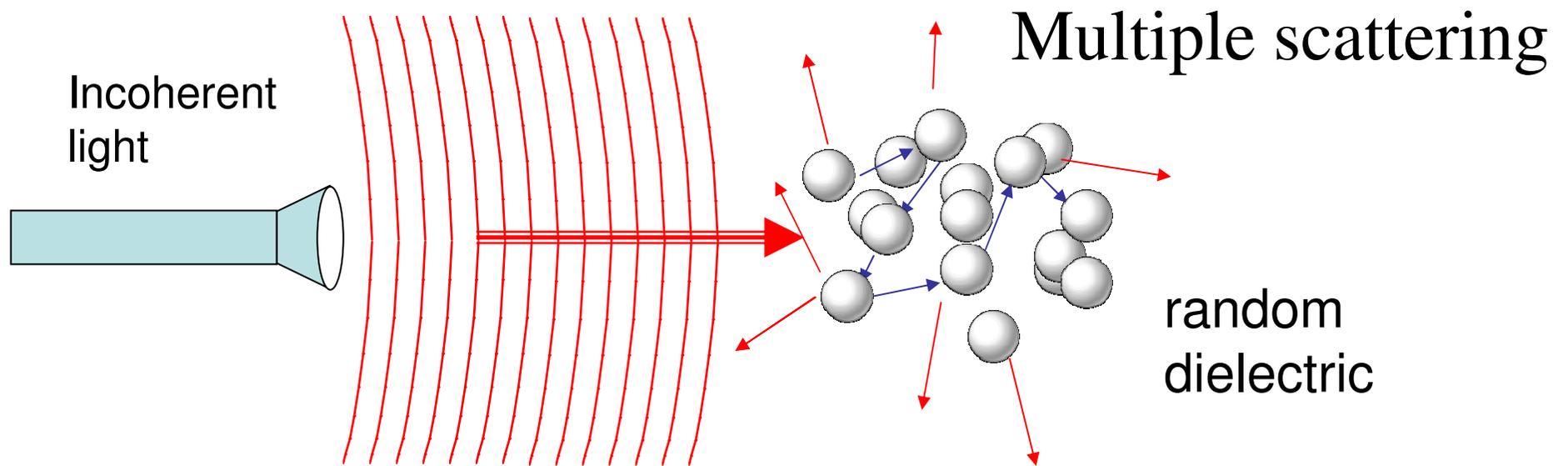
Fotonica

Equazione autovalori

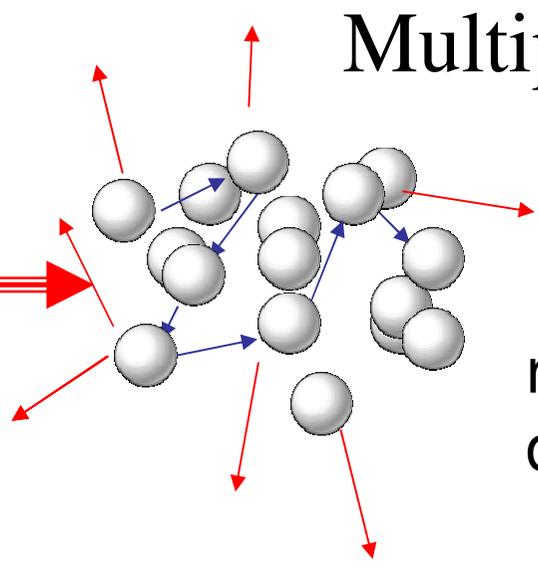
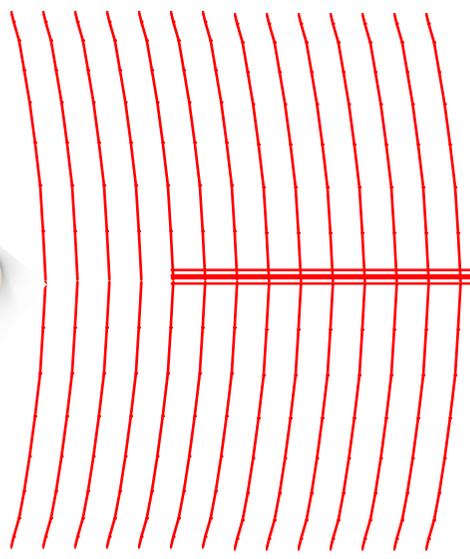
Single scattering



Frequency and phase are maintained

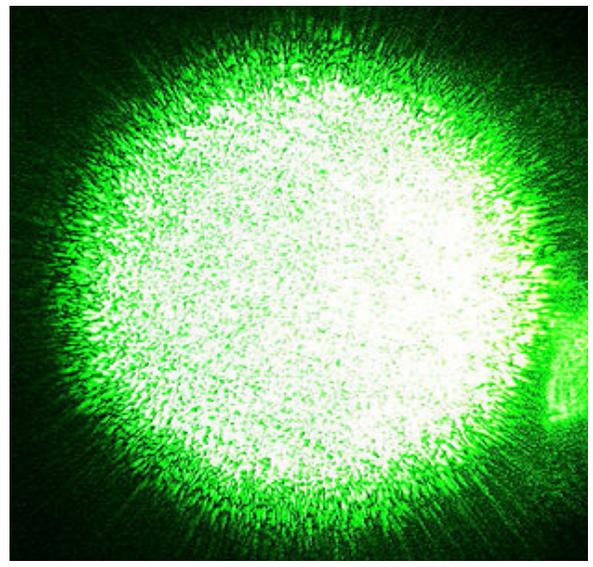


Coherent
light



Multiple scattering

random
dielectric

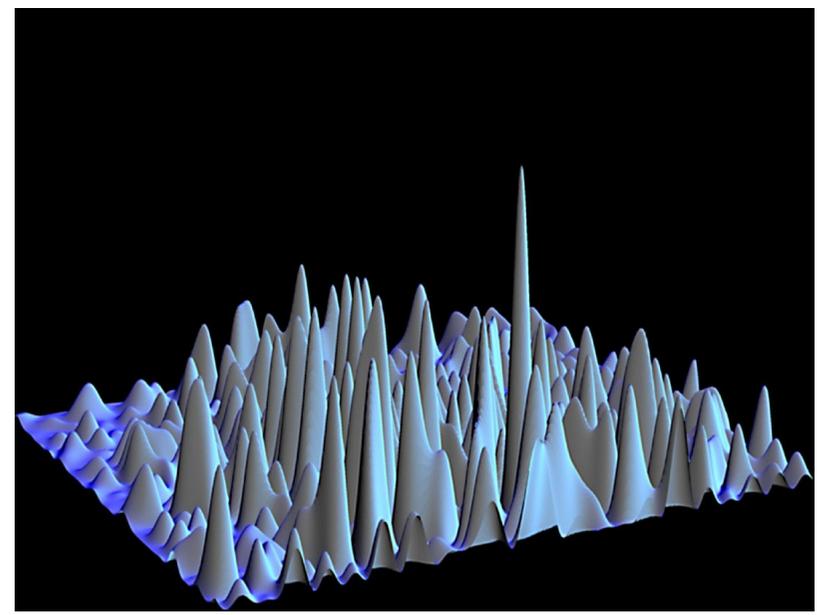


*phase is
maintained*

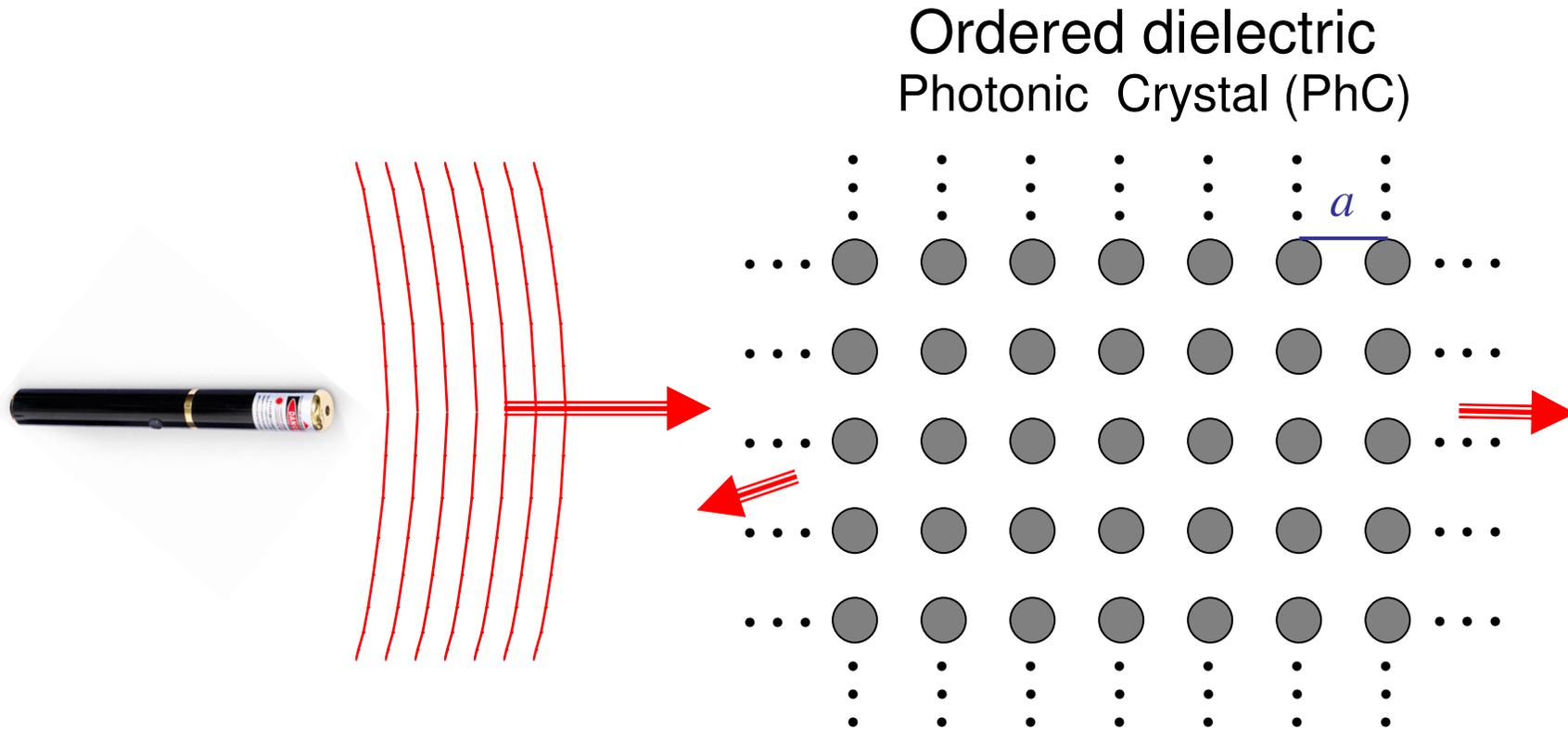


interference

Speckle
pattern



Bragg scattering



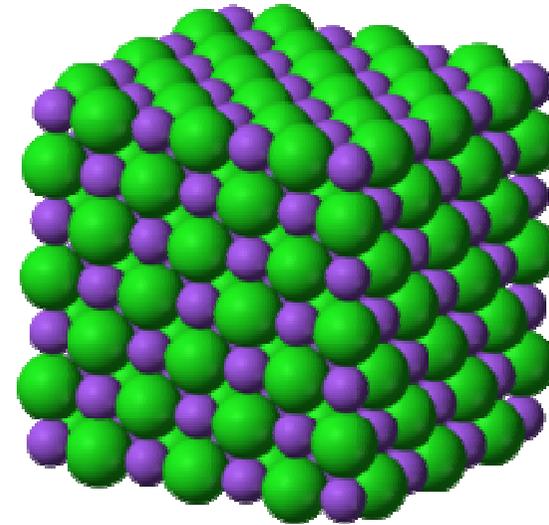
Usually only reflection and refraction

...but for some λ ($\sim 2a$), no light could propagate: a PhC band gap

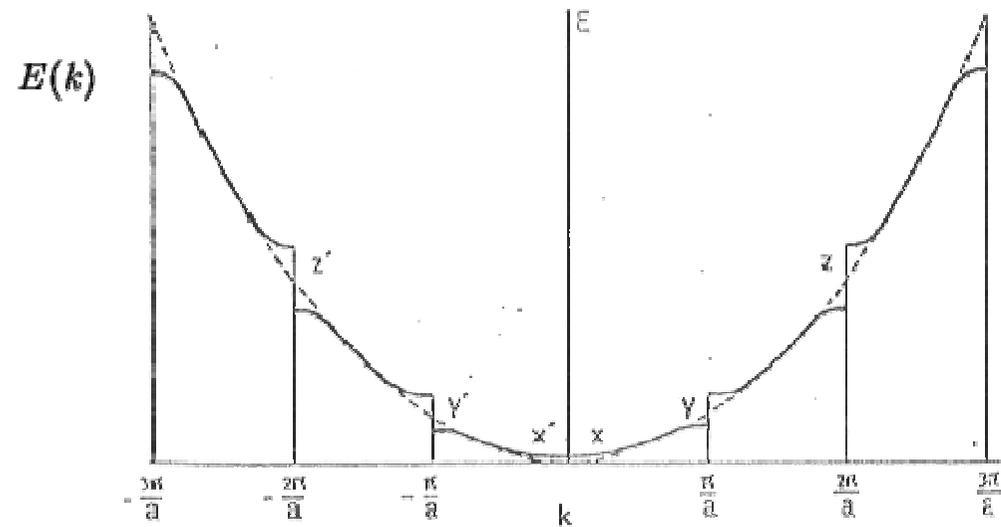
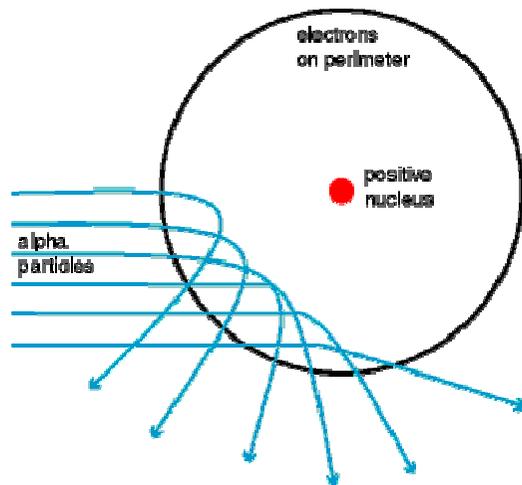
for **most** λ , beams propagate through crystal **without scattering** (scattering cancels **coherently**)

Quantum mechanics

Multiple (Bragg) scattering



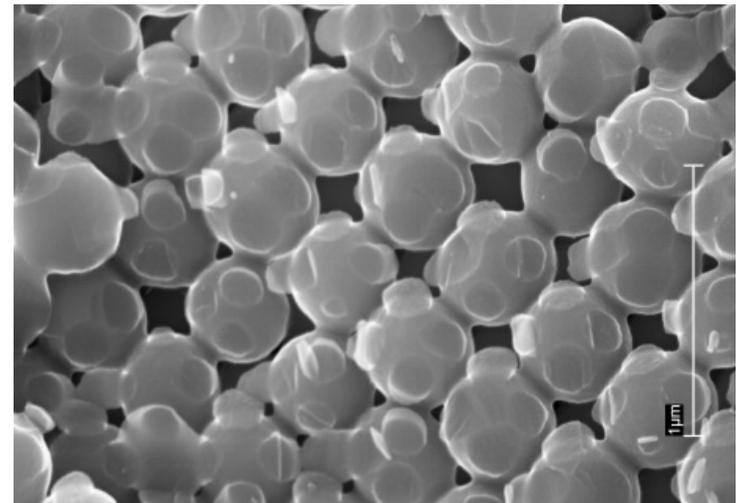
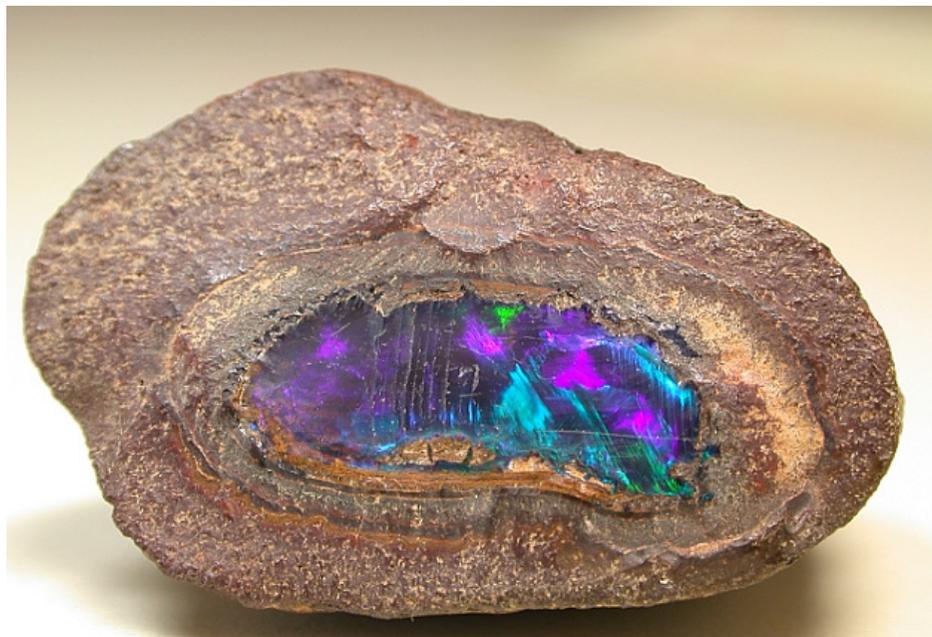
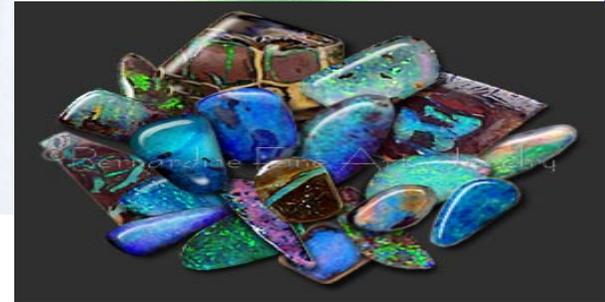
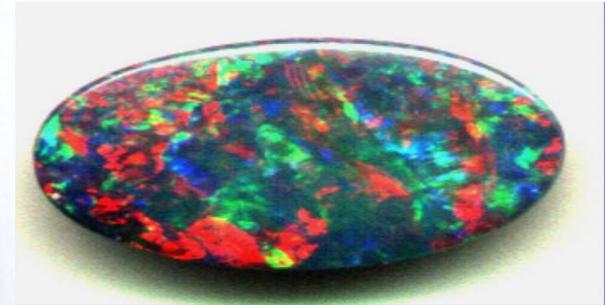
Single (Rutherford) scattering



PhC in natura



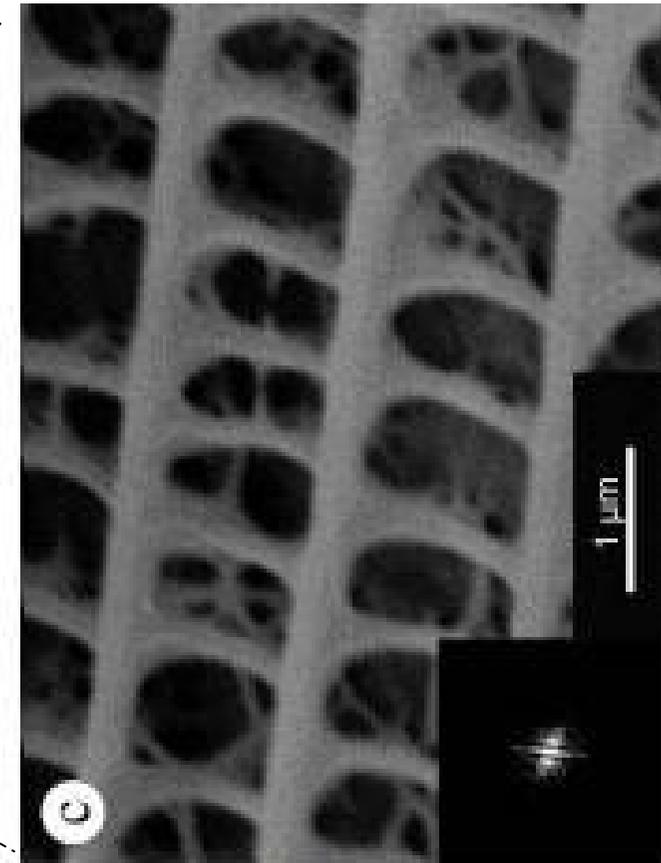
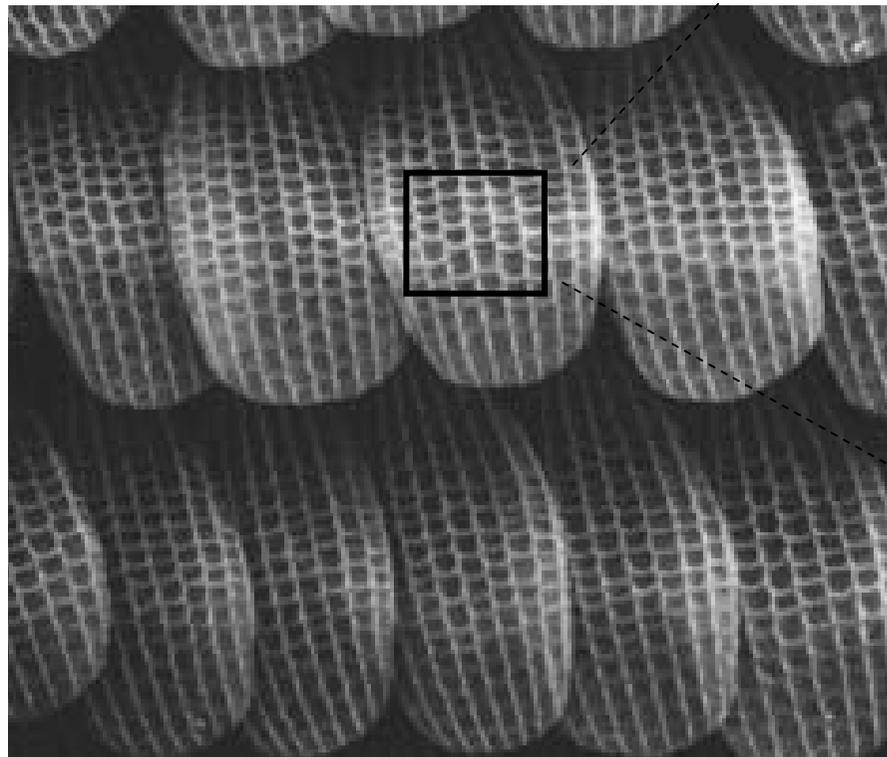
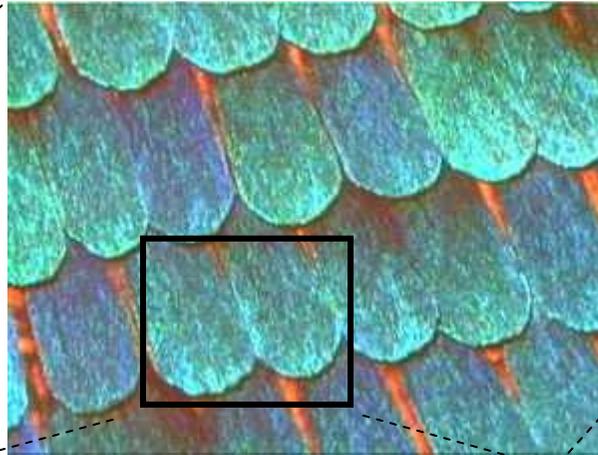
OPALI



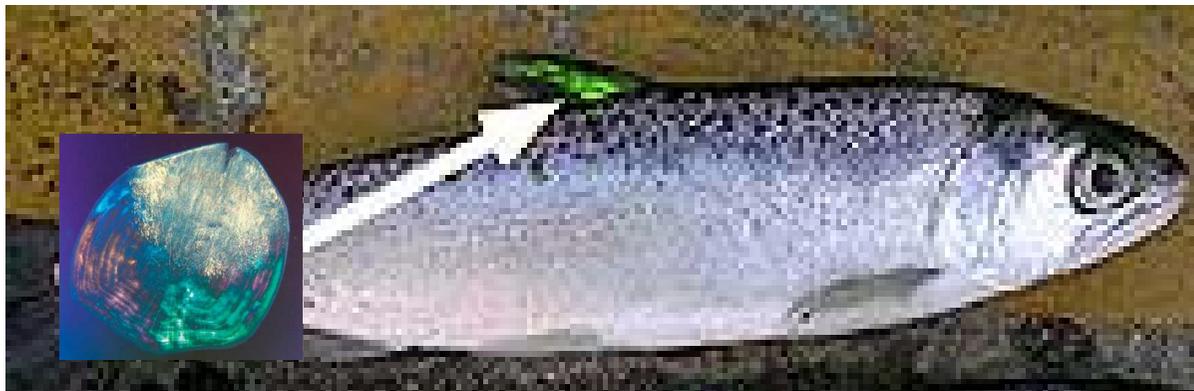
PhC in biologia



PhC in biologia



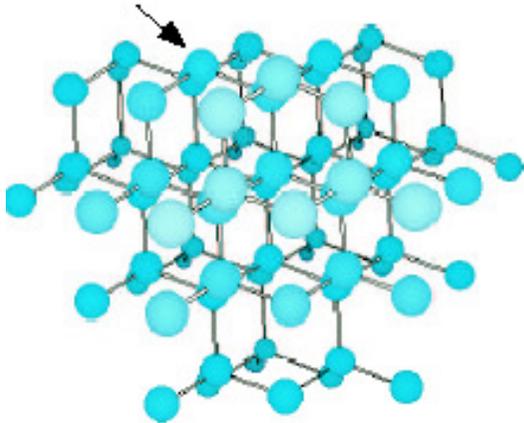
PhC in biologia



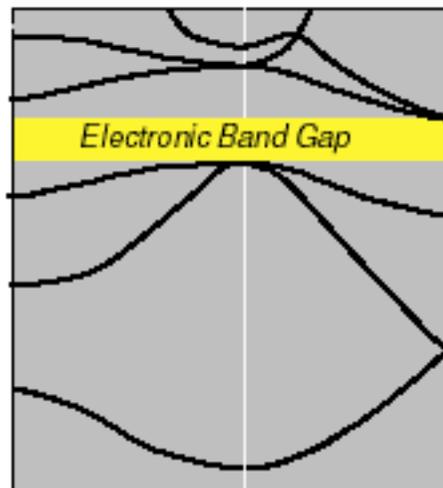
Idea di base: analogia con cristalli

Periodic Medium
Bloch waves:
Band Diagram

atoms in diamond structure



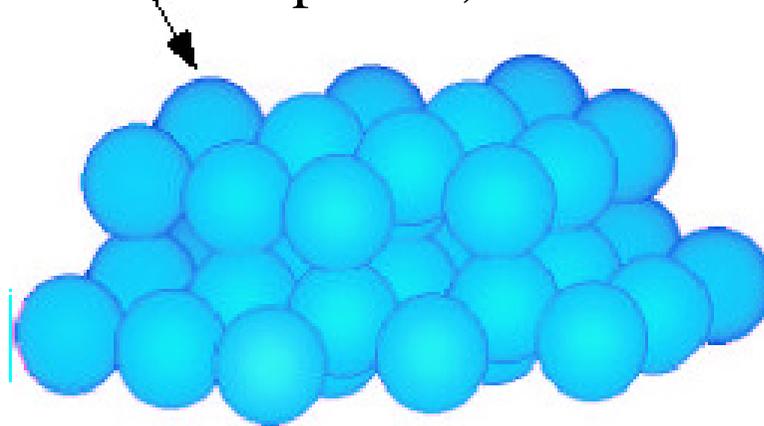
electron energy



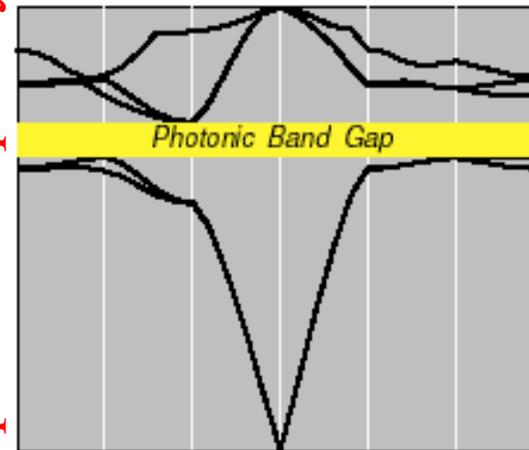
wavevector

strongly interacting fermions

dielectric spheres, diamond lattice



photon frequency



wavevector

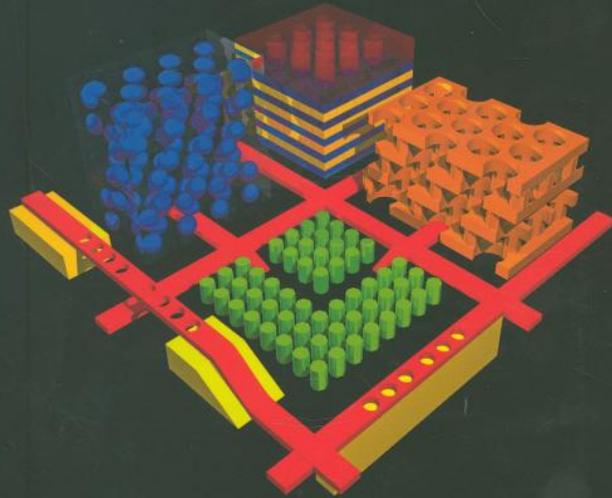
weakly-interacting bosons

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Photonic Crystals

Molding the Flow of Light

SECOND EDITION



John D. Joannopoulos
Steven G. Johnson
Joshua N. Winn
Robert D. Meade

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<http://ab-initio.mit.edu/book/photonic-crystals-book.pdf>

Equazioni Maxwell semplificate nella materia

$$\vec{\nabla} \cdot \varepsilon_0 \varepsilon \vec{E} = 0 \quad \vec{\nabla} \times \vec{E} = -\mu_0 \mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot \mu_0 \mu \vec{H} = 0 \quad \vec{\nabla} \times \vec{H} = \varepsilon_0 \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon \vec{E}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} = \mu_0 \mu \vec{H}$$

Mezzo dielettrico non magnetico ($\mu=1$, $\varepsilon= \varepsilon(r)$)

$$\vec{\nabla} \cdot \varepsilon(\vec{r})\vec{E} = 0 \quad \vec{\nabla} \times \vec{E} = -\mu_0\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \quad \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{D} = \varepsilon_0 \varepsilon(\vec{r})\vec{E} \quad \vec{B} = \mu_0 \vec{H}$$

Equazioni onde monocromatiche ($\mu=1$, $\varepsilon= \varepsilon(r)$)

$$\begin{aligned}\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) &= - \left(\frac{\partial}{\partial t} \vec{\nabla} \times \mu_o \mu \vec{H} \right) = \\ &= -\mu_o \mu \left(\frac{\partial^2}{\partial t^2} \varepsilon_o \varepsilon(r) \vec{E} \right) = \frac{\omega^2}{c^2} \varepsilon(r) \mu \vec{E} \quad \vec{\nabla} \cdot \varepsilon(\vec{r}) \vec{E} = 0\end{aligned}$$

$$\begin{aligned}\vec{\nabla} \times \left(\frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) &= \varepsilon_o \frac{\partial}{\partial t} \vec{\nabla} \times \vec{E} = \\ &= -\varepsilon_o \frac{\partial^2}{\partial t^2} \mu_o \mu \vec{H} = \frac{\omega^2}{c^2} \vec{H} \quad \vec{\nabla} \cdot \vec{H} = 0\end{aligned}$$

Mezzo dielettrico non magnetico ($\mu=1$, $\epsilon = \epsilon(\mathbf{r})$)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(\mathbf{r}) \vec{E} \qquad \vec{\nabla} \cdot \epsilon(\vec{r}) \vec{E} = 0$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \vec{H} \qquad \vec{\nabla} \cdot \vec{H} = 0$$

Problema autovalori con vincolo

$$\vec{E}(\vec{r}) = \frac{i}{\omega \epsilon_0 \epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r})$$

Equazioni onde

$$\hat{\Theta} \vec{H}(\vec{r}) = \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}) \right)$$

Master equation

Schroedinger equation

$$\hat{\Theta} \vec{H}(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r})$$

$$\hat{H} \psi = E \psi$$

autovalore

Linearità

$$\hat{\Theta}(\alpha \vec{H}_1(\vec{r}) + \beta \vec{H}_2(\vec{r})) = \alpha \hat{\Theta} \vec{H}_1(\vec{r}) + \beta \hat{\Theta} \vec{H}_2(\vec{r})$$

Hermitianicità (ci riconduce alle proprietà della MQ)

Norma:

$$(\vec{F}, \vec{G}) \equiv \int d^3 r \vec{F}^*(\vec{r}) \cdot \vec{G}(\vec{r})$$

$$(\vec{F}, \vec{F}) \geq 0$$

$$\forall \vec{F}' \neq 0 \quad \exists \vec{F} = \frac{\vec{F}'}{\sqrt{(\vec{F}', \vec{F}')}} \quad e \quad (\vec{F}, \vec{F}) = 1$$

$\hat{\Xi}$ è Hermitiano se

$$(\vec{F}, \hat{\Xi} \vec{G}) = (\hat{\Xi} \vec{F}, \vec{G})$$

Dimostriamo che $\hat{\Theta}$ è Hermitiano

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3 r \underbrace{\vec{F}^*(\vec{r})}_{\vec{B}} \cdot \left[\vec{\nabla} \times \underbrace{\left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right)}_{\vec{A}} \right]$$

Usando $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\begin{aligned} & \vec{F}^*(\vec{r}) \cdot \left[\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \right] = \\ & = \vec{\nabla} \cdot \left[\left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \times \vec{F}^*(\vec{r}) \right] + \left[\left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \cdot \vec{\nabla} \times \vec{F}^*(\vec{r}) \right] \end{aligned}$$

Trascurando il termine di superficie

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3 r \vec{\nabla} \times \vec{F}^*(\vec{r}) \cdot \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right)$$

Dimostriamo che $\hat{\Theta}$ è Hermitiano

$$(\vec{F}, \hat{\Theta}\vec{G}) = \int d^3r \frac{1}{\epsilon(\vec{r})} \underbrace{\vec{\nabla} \times \vec{F}^*(\vec{r})}_{\vec{B}} \cdot \underbrace{(\vec{\nabla} \times \vec{G}(\vec{r}))}_{\vec{A}}$$

Usando $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \cdot (\vec{\nabla} \times \vec{G}(\vec{r})) =$$

$$\vec{\nabla} \cdot \left(\vec{G}(\vec{r}) \times \frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \right) + \vec{G}(\vec{r}) \cdot \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \right)$$

Trascurando il termine di superficie e per ϵ reale

$$(\vec{F}, \hat{\Theta}\vec{G}) = \int d^3r \left[\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}(\vec{r}) \right) \right]^* \cdot \vec{G}(\vec{r}) = (\hat{\Theta}\vec{F}, \vec{G}) \quad \therefore$$

Perché H e non E ?

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\frac{1}{\epsilon(r)} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \vec{E}$$

$$\hat{H} = \frac{1}{\epsilon(r)} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) \quad \hat{H} \text{ non è Hermitiano}$$

$$\vec{\nabla} \cdot \vec{E} \neq 0 \quad \vec{E} \text{ non è trasverso}$$

Let's demonstrate that $\hat{\mathbf{E}}$ is not Hermitian

$$(\vec{F}, \hat{\mathbf{E}}\vec{G}) = \int d^3r \underbrace{\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r})}_{\vec{B}} \cdot \underbrace{[\vec{\nabla} \times (\vec{\nabla} \times \vec{G}(\vec{r}))]}_{\vec{A}}$$

Being $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\begin{aligned} & \frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \cdot [\vec{\nabla} \times (\vec{\nabla} \times \vec{G}(\vec{r}))] = \\ & = \vec{\nabla} \cdot \left[(\vec{\nabla} \times \vec{G}(\vec{r})) \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \right] + \left[(\vec{\nabla} \times \vec{G}(\vec{r})) \cdot \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \right] \end{aligned}$$

The surface integral is negligible

$$(\vec{F}, \hat{\mathbf{E}}\vec{G}) = \int d^3r \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \cdot \vec{\nabla} \times \vec{G}(\vec{r})$$

Let's demonstrate that $\hat{\mathbf{E}}$ is not Hermitian

$$(\vec{F}, \hat{\mathbf{E}}\vec{G}) = \int d^3r \underbrace{\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right)}_{\vec{B}} \cdot \underbrace{\vec{\nabla} \times \vec{G}(\vec{r})}_{\vec{A}}$$

Being $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \cdot \vec{\nabla} \times \vec{G}(\vec{r}) =$$

$$\vec{\nabla} \cdot \left(\vec{G}(\vec{r}) \times \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \right) + \vec{G}(\vec{r}) \cdot \vec{\nabla} \times \left(\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}^*(\vec{r}) \right) \right)$$

The surface integral is negligible

$$(\vec{F}, \hat{\mathbf{E}}\vec{G}) = \int d^3r \left[\vec{\nabla} \times \left(\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{F}(\vec{r}) \right) \right) \right]^* \cdot \vec{G}(\vec{r}) \neq (\hat{\mathbf{E}}\vec{F}, \vec{G}) \quad \therefore$$

Mezzo magnetico non dielettrico ($\epsilon=1$, $\mu=\mu(r)$)

$$\begin{aligned}\vec{\nabla} \times \left(\frac{1}{\mu(r)} \vec{\nabla} \times \vec{E} \right) &= - \left(\frac{\partial}{\partial t} \vec{\nabla} \times \mu_o \vec{H} \right) = \\ &= -\mu_o \left(\frac{\partial^2}{\partial t^2} \epsilon_o \epsilon \vec{E} \right) = \frac{\omega^2}{c^2} \vec{E} \quad \vec{\nabla} \cdot \vec{E} = 0\end{aligned}$$

$$\begin{aligned}\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) &= \epsilon_o \epsilon \frac{\partial}{\partial t} \vec{\nabla} \times \vec{E} = \\ &= -\epsilon_o \frac{\partial^2}{\partial t^2} \mu_o \mu(r) \vec{H} = \frac{\omega^2}{c^2} \mu(r) \vec{H} \quad \vec{\nabla} \cdot \mu(r) \vec{H} = 0\end{aligned}$$

Mezzo magnetico non dielettrico ($\epsilon = 1$, $\mu = \mu(\vec{r})$)

$$\vec{\nabla} \times \left(\frac{1}{\mu(\vec{r})} \vec{\nabla} \times \vec{E} \right) = \frac{\omega^2}{c^2} \vec{E} \quad \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \frac{\omega^2}{c^2} \mu(\vec{r}) \vec{H} \quad \vec{\nabla} \cdot \mu(\vec{r}) \vec{H} = 0$$

Problema autovalori con
vincolo

$$\vec{H}(\vec{r}) = \frac{-i}{\omega \mu_0 \mu(\vec{r})} \vec{\nabla} \times \vec{E}(\vec{r})$$

Proprietà generali

$$\hat{\Theta}\vec{H}(\vec{r}) = \frac{\omega^2}{c^2}\vec{H}(\vec{r})$$

$$\left(\vec{H}(\vec{r}), \hat{\Theta}\vec{H}(\vec{r})\right) = \frac{\omega^2}{c^2}\left(\vec{H}(\vec{r}), \vec{H}(\vec{r})\right)$$

$$\left(\vec{H}(\vec{r}), \hat{\Theta}\vec{H}(\vec{r})\right)^* = \left(\frac{\omega^2}{c^2}\right)^* \left(\vec{H}(\vec{r}), \vec{H}(\vec{r})\right)^*$$


Autovalori reali

Proprietà generali $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\frac{\omega^2}{c^2} (\vec{H}(\vec{r}), \vec{H}(\vec{r})) = (\vec{H}(\vec{r}), \hat{\Theta} \vec{H}(\vec{r})) =$$

$$= \int d^3 r \vec{H}^*(\vec{r}) \hat{\Theta} \vec{H}(\vec{r}) =$$

$$= \int d^3 r \vec{H}^*(\vec{r}) \cdot \vec{\nabla} \times \left(\frac{1}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \right) =$$

$$= \int d^3 r \frac{1}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \cdot \vec{\nabla} \times \vec{H}^*(\vec{r}) = \int d^3 r \frac{1}{\epsilon} |\vec{\nabla} \times \vec{H}(\vec{r})|^2$$

Autovalori non negativi
 ω reale per ϵ positivo

Proprietà generali

$$\begin{aligned}\omega_1^2 \left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) &= c^2 \left(\vec{H}_2(\vec{r}), \hat{\Theta} \vec{H}_1(\vec{r}) \right) = \\ &= c^2 \left(\hat{\Theta} \vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) = \omega_2^2 \left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right)\end{aligned}$$

$$\left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) = 0$$

Ortogonalità autovettori