

Onde evanescenti  
Near field optics

# Helmholtz's equations

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = -k^2 \vec{E}$$

$$\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = -k^2 \vec{B}$$

$$k^2 = \frac{\omega^2 n^2}{c^2}$$

# Plane waves

$$\vec{k} \cdot \vec{E} = 0 \quad \vec{k} \times \vec{H} = \omega \epsilon \vec{E}$$

$$\vec{k} = (k_x, 0, k_z) \quad k^2 = \frac{\omega^2}{c^2} n^2 = \left( \frac{2\pi n}{\lambda} \right)^2$$

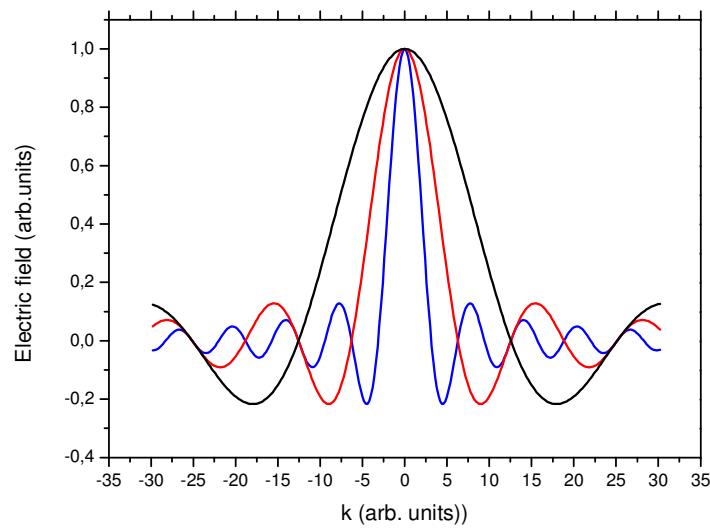
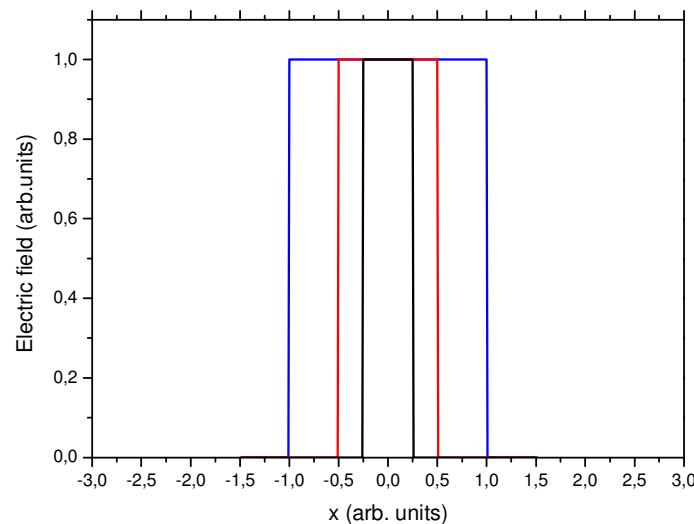
$$k_z^2 = k^2 - k_x^2$$

$$k_z^2 > 0 \quad \vec{E} = \vec{E}_o e^{i(k_x x + k_z z - \omega t)}$$

$$|k_z|, |k_x| < \frac{2\pi n}{\lambda}$$

# Angular decomposition

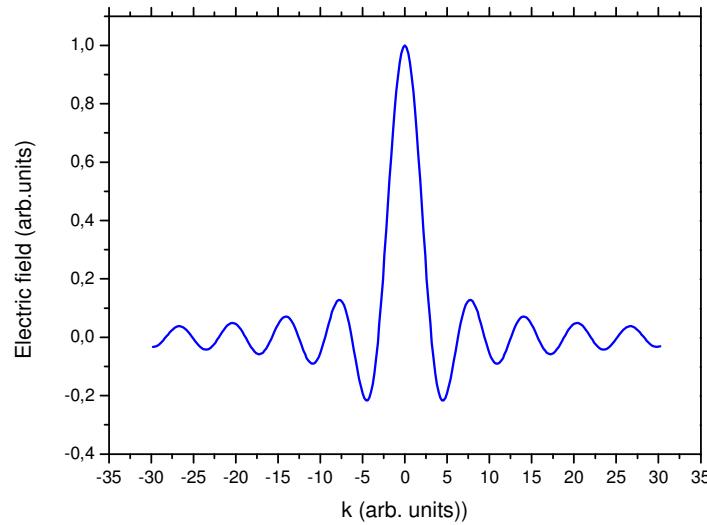
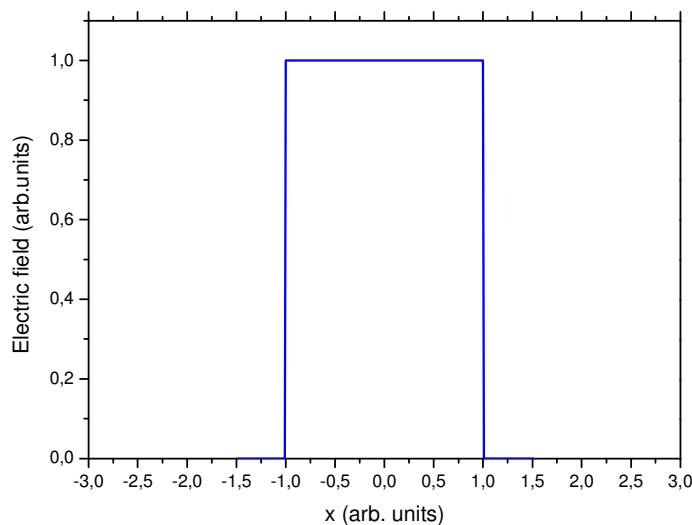
$$\vec{E}(x, y, z) = \int \int \vec{\tilde{E}}(k_x, k_y, z = 0) e^{i[k_x x + k_y y + k_z z]} dk_x dk_y$$



$$\Delta k_x \Delta x \geq 0.5$$

# Angular decomposition

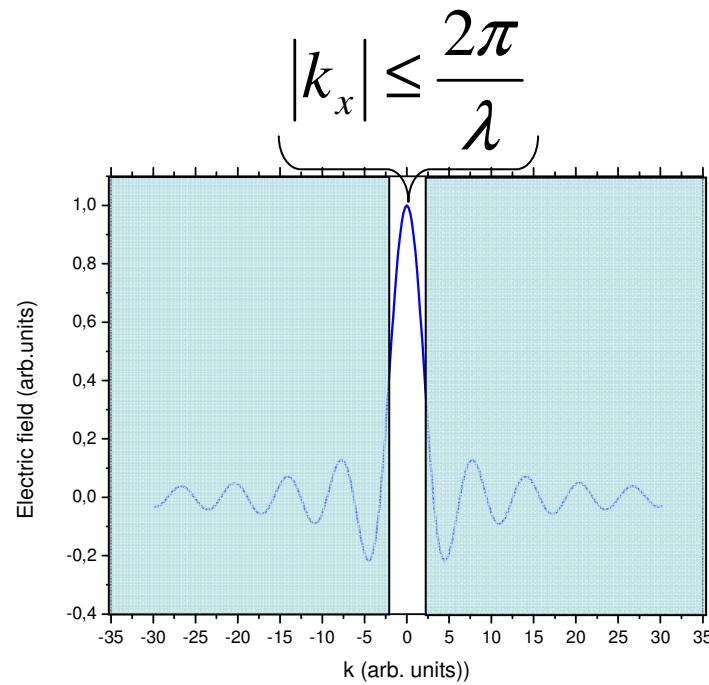
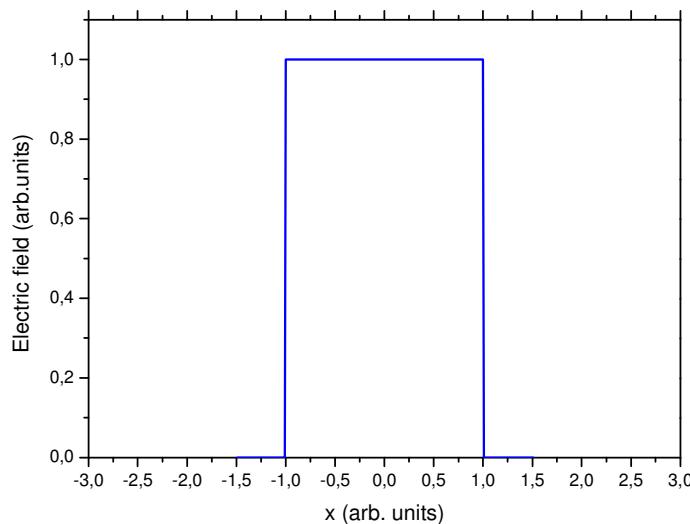
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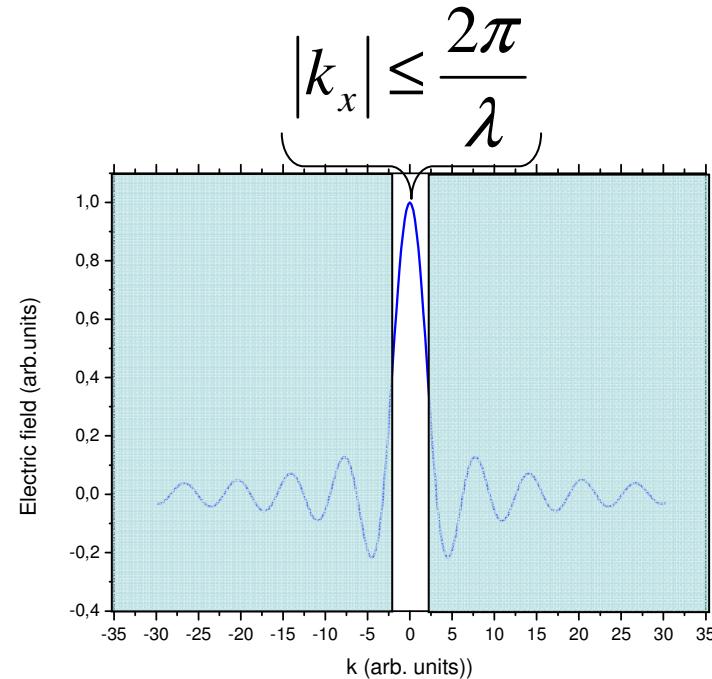
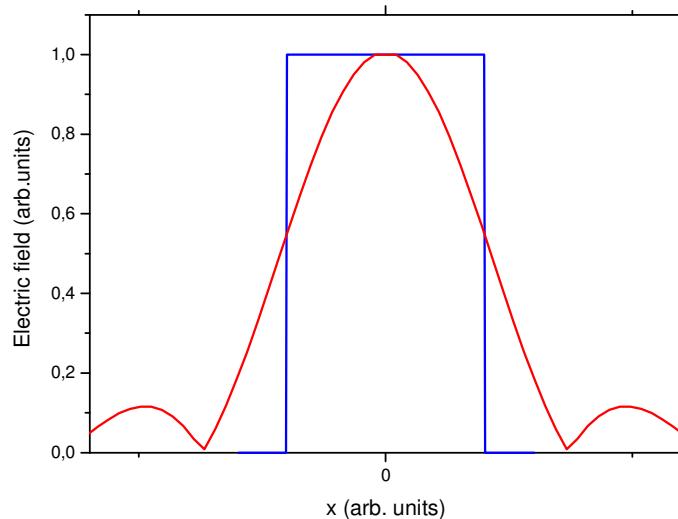
$$\vec{E}(x, y, z) = \int \int_{|k_x| \leq \frac{2\pi}{\lambda}} \vec{\tilde{E}}(k_x, k_y, z = 0) e^{i[k_x x + k_y y + k_z z]} dk_x dk_y$$



$$\Delta k_x \Delta x \geq 0.5 \quad \Rightarrow \Delta x \geq \frac{0.5\lambda}{2\pi}$$

# Angular decomposition

$$\vec{E}(x, y, z) = \int_{|k_x| \leq \frac{2\pi}{\lambda}} \int \vec{\tilde{E}}(k_x, k_y, z = 0) e^{i[k_x x + k_y y + k_z z]} dk_x dk_y$$



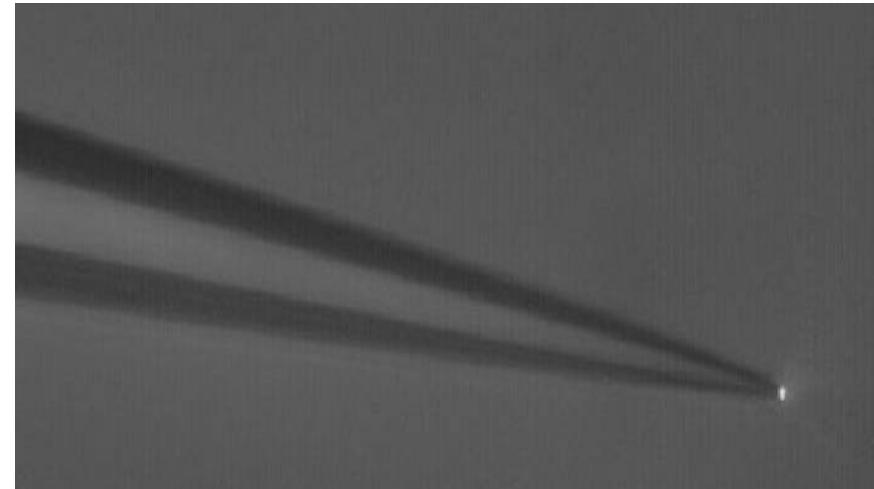
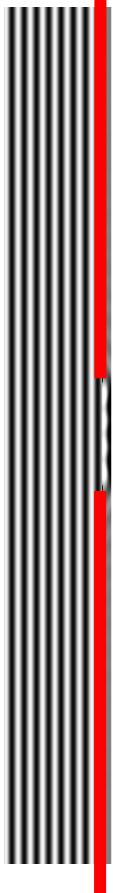
$$\Delta k_x \Delta x \geq 0.5 \quad \Rightarrow \Delta x \geq \frac{0.5\lambda}{2\pi}$$

It seems impossible to confine light much below  $\lambda$ .

## Angular decomposition

$$\vec{E}(x, y, z) = \int \int \vec{\tilde{E}}(k_x, k_y, z = 0) e^{i[k_x x + k_y y + k_z z]} dk_x dk_y$$

However it is simple to confine light much below  $\lambda$ .



$$\Delta x \geq 0.5 \quad \Rightarrow \Delta x \geq \frac{0.5\lambda}{2\pi}$$

## Plane waves

$$\vec{k} \cdot \vec{E} = 0 \quad \vec{k} \times \vec{H} = \omega \epsilon \vec{E}$$

$$\vec{k} = (k_x, 0, k_z) \quad k^2 = \frac{\omega^2}{c^2} n^2 = \left( \frac{2\pi n}{\lambda} \right)^2$$

$$k_z^2 = k^2 - k_x^2$$

$$k_z^2 > 0 \quad \vec{E} = \vec{E}_o e^{i(k_x x + k_z z - \omega t)}$$

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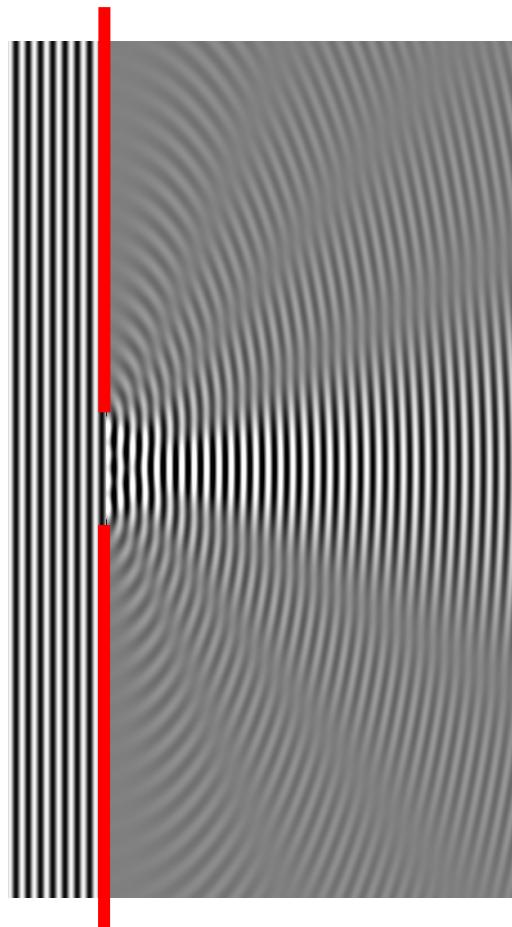
Evanescent  
waves

$$k_z^2 < 0 \quad \vec{E} = \vec{E}_o e^{i(k_x x - \omega t)} e^{\mp |k_z| z}$$

$$k_x^2 = \left( \frac{2\pi n}{\lambda} \right)^2 + |k_z|^2 > \left( \frac{2\pi n}{\lambda} \right)^2$$

## Angular decomposition

$$\vec{E}(x, y, z) = \int \int \vec{\tilde{E}}(k_x, k_y, z = 0) e^{i[k_x x + k_y y + k_z z]} dk_x dk_y$$



$$k_z^2 > 0 \quad k_x^2 < \left( \frac{2\pi}{\lambda} \right)^2$$
$$k_z^2 < 0 \quad k_x^2 > \left( \frac{2\pi}{\lambda} \right)^2$$

Fields localized below  $\lambda$  need  
evanescent waves

SNOM



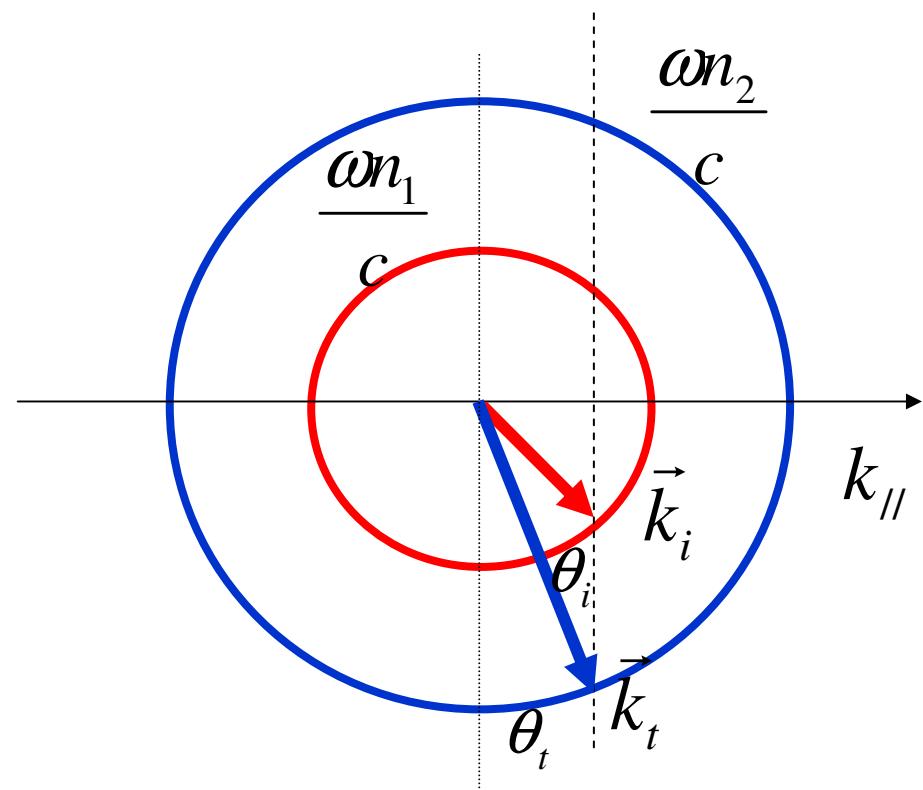
# Riflessione totale

Metodo grafico per rifrazione  $n_2 > n_1$

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

$$\vec{k}_{i,\parallel} = \vec{k}_{r,\parallel} = \vec{k}_{t,\parallel}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

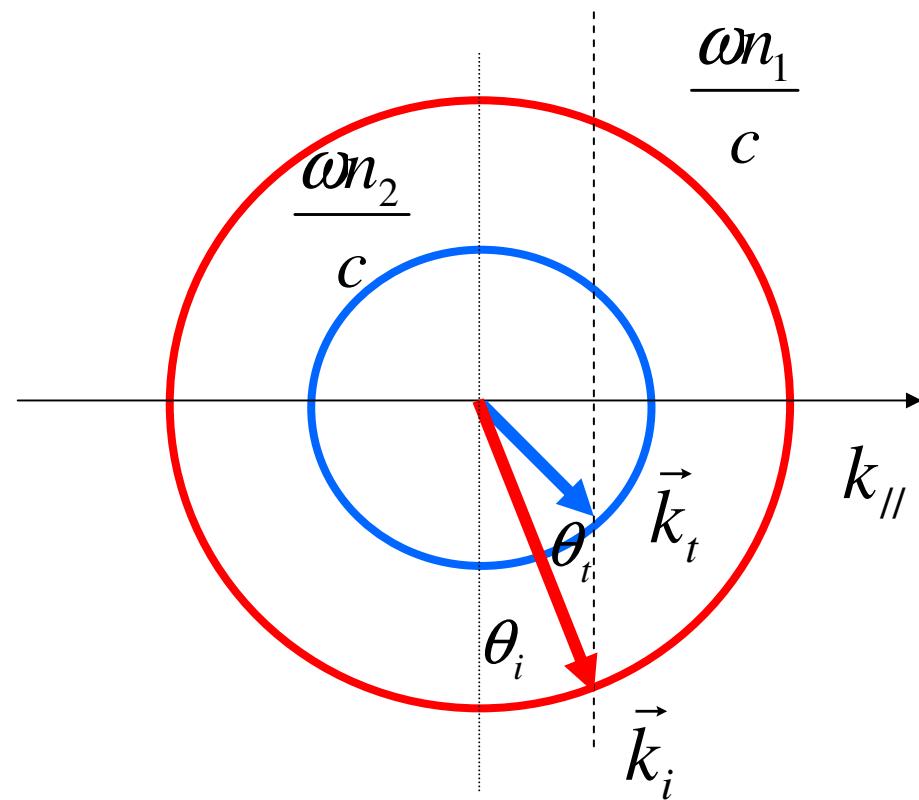


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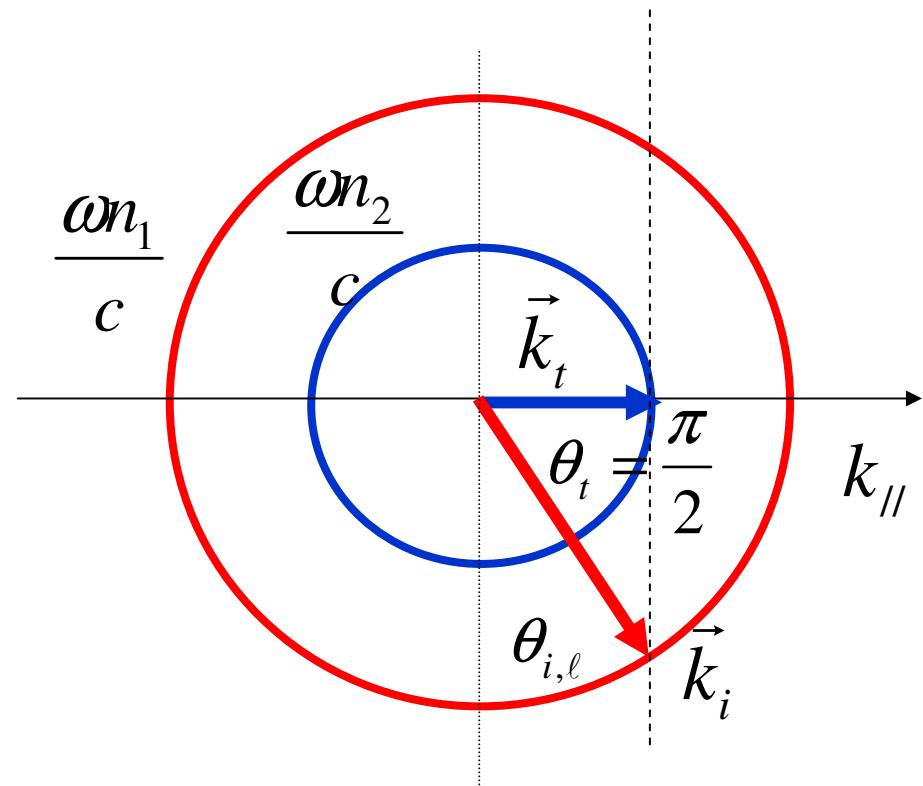
Metodo grafico per rifrazione  $n_1 > n_2$

$$\vec{k}_{i,\parallel} = \vec{k}_{r,\parallel} = \vec{k}_{t,\parallel}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

Angolo limite

$$\sin \theta_{i,\ell} = \frac{n_2}{n_1}$$



Metodo grafico per rifrazione  $n_1 > n_2$

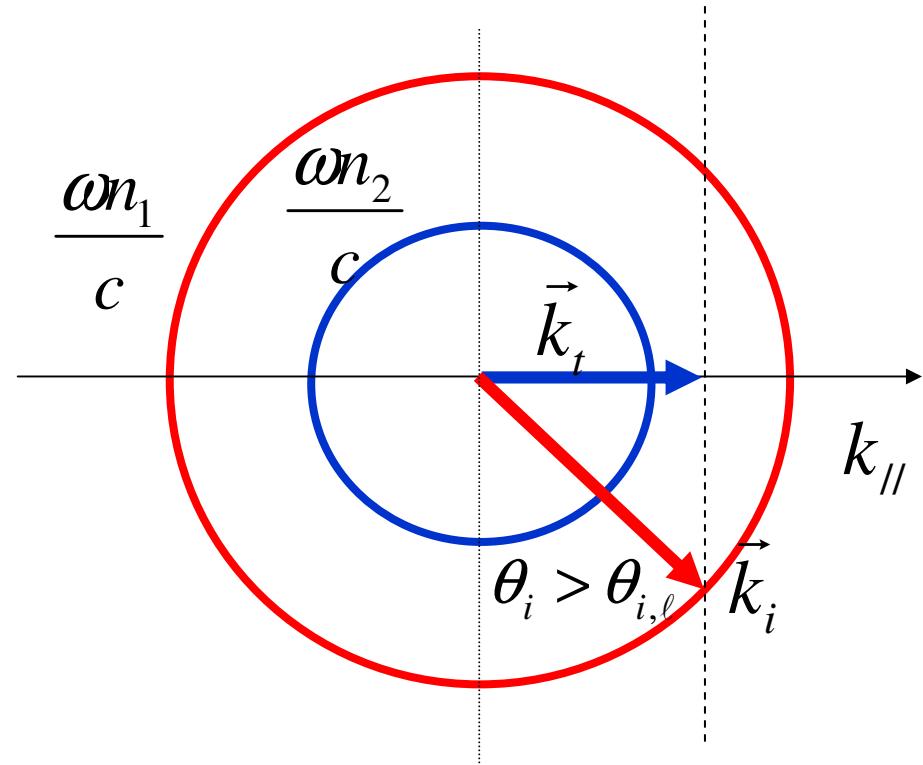
$$\vec{k}_{i,\parallel} = \vec{k}_{r,\parallel} = \vec{k}_{t,\parallel}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

Oltre angolo limite

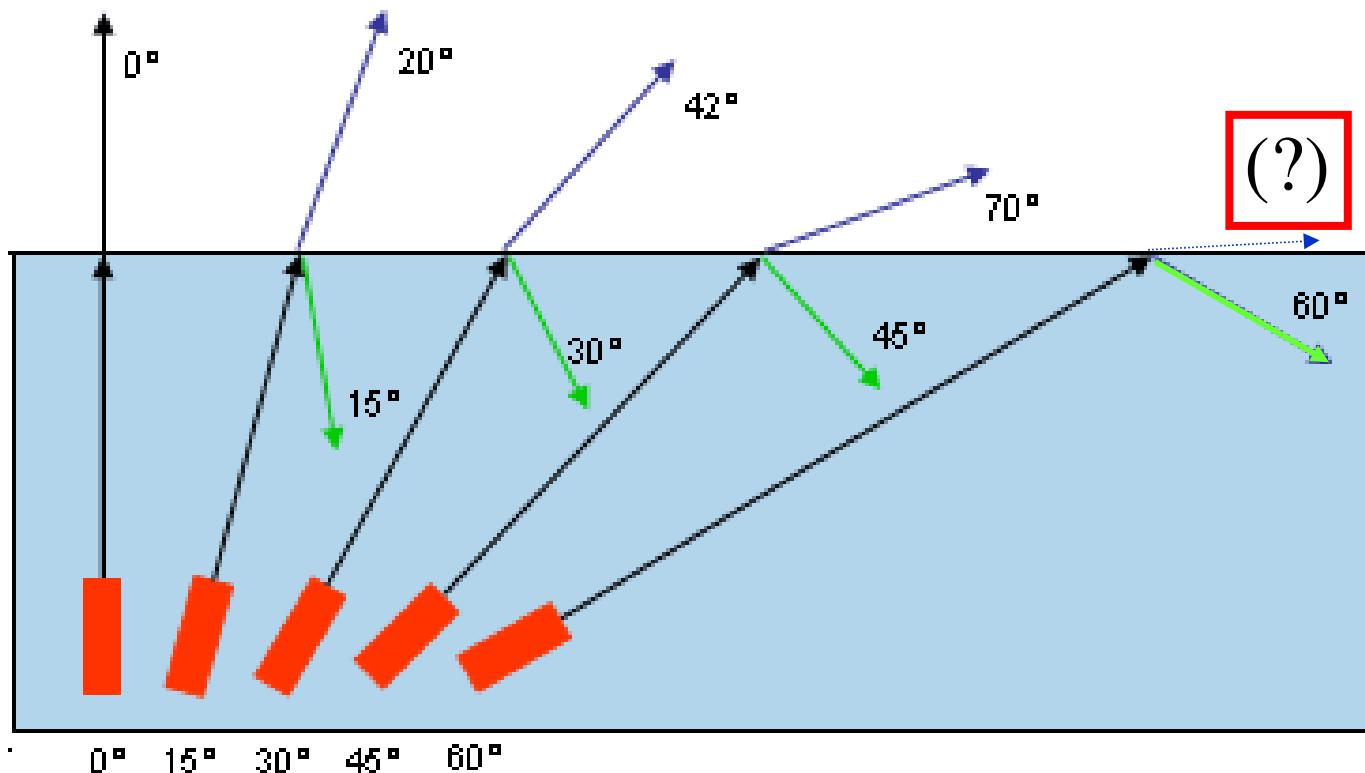
$$\sin \theta_i > \frac{n_2}{n_1}$$

$$|\vec{k}_{t,\parallel}| > \frac{\omega n_2}{c} \quad (?)$$



# Rifrazione oltre angolo limite

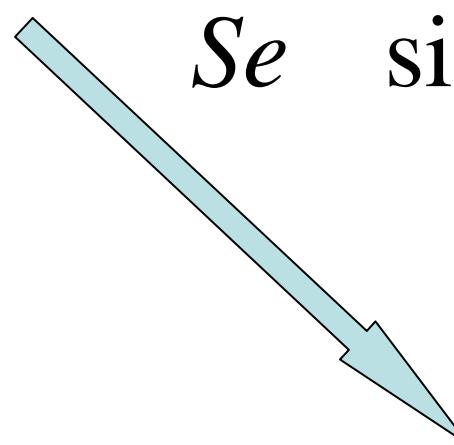
$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad n_1 > n_2$$



# Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\left\{ \begin{array}{l} \left| \vec{k}_{t,\parallel} \right| = \left| \vec{k}_{i,\parallel} \right| = \frac{\omega n_1}{c} \sin \theta_i \\ \vec{k}_t = \vec{k}_{t,\parallel} + \vec{k}_{t,z} \\ \left| \vec{k}_t \right| = \frac{\omega n_2}{c} \end{array} \right.$$

$$Se \quad \sin \theta_i > \frac{n_2}{n_1}$$


$$\left| \vec{k}_{t,z} \right|^2 = \left| \vec{k}_t \right|^2 - \left| \vec{k}_{i,\parallel} \right|^2 < 0$$

# Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\left| \vec{k}_{t,z} \right|^2 = \left| \vec{k}_t \right|^2 - \left| \vec{k}_{i,\parallel} \right|^2 < 0 \quad \text{Se} \quad \sin \theta_i > \frac{n_2}{n_1}$$

$$\left| \vec{k}_{t,z} \right| = j\beta = \frac{\omega n_2}{c} \cos \theta_t \quad \boxed{\cos \theta_t = ja}$$

$$\boxed{\vec{k}_t = \vec{k}_{i,\parallel} + j \frac{\omega n_2}{c} a \hat{e}_z = \vec{k}_{i,\parallel} + j \beta \hat{e}_z}$$

Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\vec{k}_t = \vec{k}_{i,\parallel} + j \frac{\omega n_2}{c} a \hat{e}_z = \vec{k}_{i,\parallel} + j \beta \hat{e}_z$$

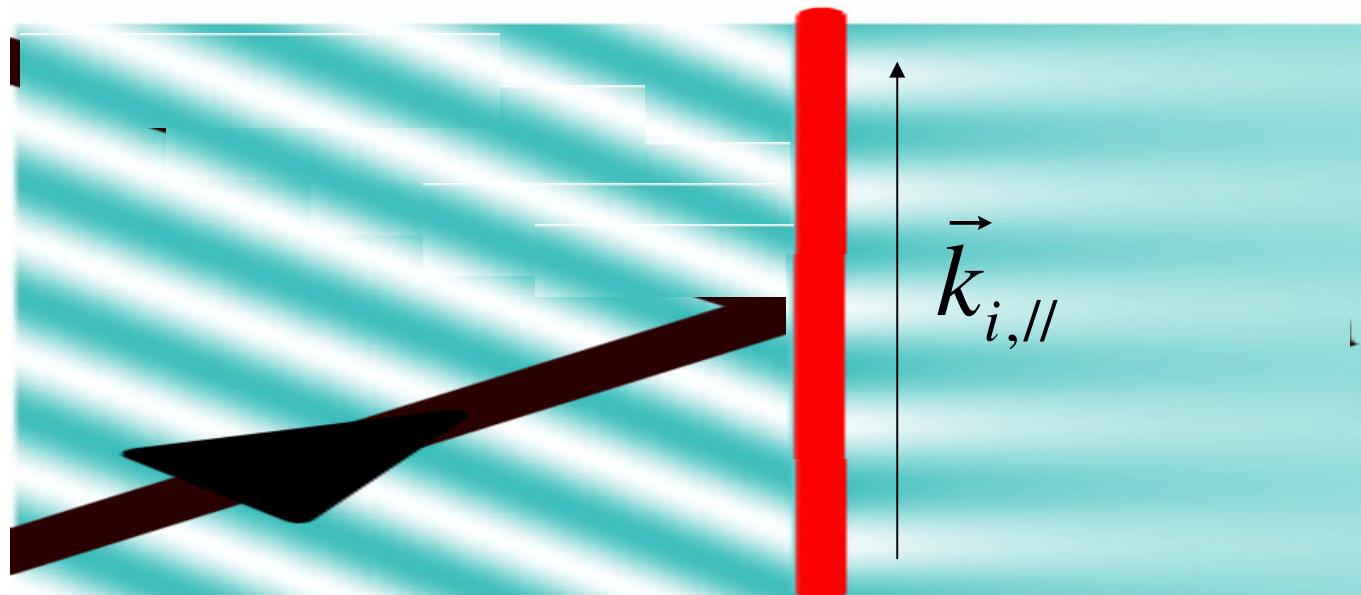
$$\left( \frac{\omega n_2}{c} \right)^2 = \left( \frac{\omega n_1}{c} \sin \theta_i \right)^2 - \left( \frac{\omega n_2}{c} \right)^2 a^2$$

$$a^2 = \left( \frac{n_1}{n_2} \sin \theta_i \right)^2 - 1$$

$$\cos \theta_t = ja$$

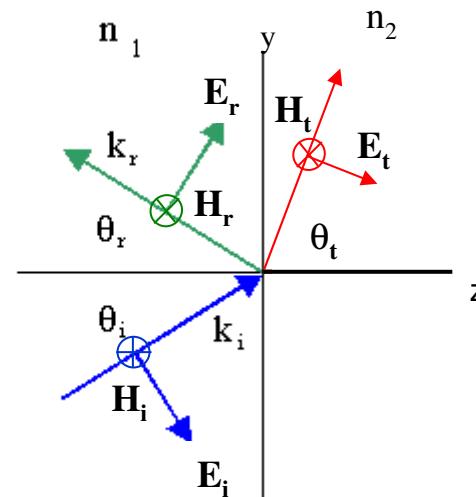
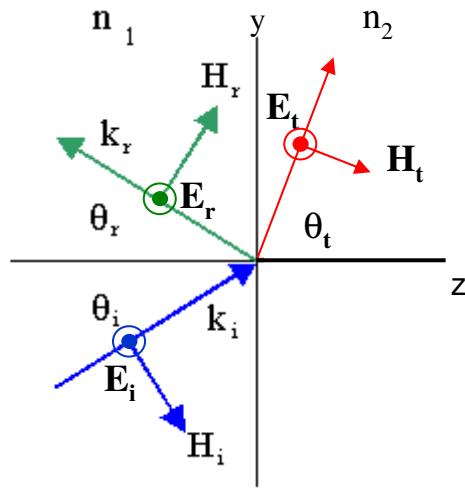
# Onda evanescente

$$\vec{E}_t(\vec{r}, t) = E_t e^{j(\vec{k}_{i,\parallel} \cdot \vec{r}_{\parallel} + j\beta z - \omega_i t)} \hat{e}$$
$$= E_t e^{j(\vec{k}_{i,\parallel} \cdot \vec{r}_{\parallel} - \omega_i t)} e^{-\beta z} \hat{e}$$



# Relazioni di Fresnel

$$n_1 > n_2$$

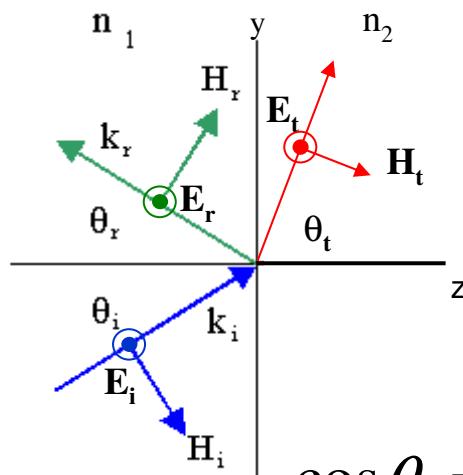


$$\begin{cases} E_r = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_i \\ E_t = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_i \end{cases}$$

$$\begin{cases} E_r = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} E_i \\ E_t = \frac{2n_1 \cos \theta_i}{n_2 \cos \theta_i + n_1 \cos \theta_t} E_i \end{cases}$$

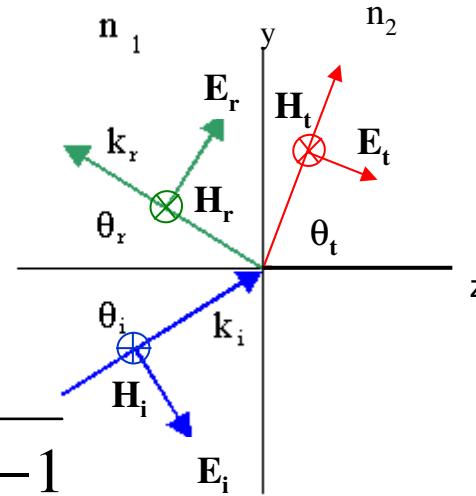
$$H_i = \frac{n_1}{\mu_1 c} E_i \quad H_r = \frac{n_1}{\mu_1 c} E_r \quad H_t = \frac{n_2}{\mu_2 c} E_t$$

# Relazioni di Fresnel



$$n = \frac{n_2}{n_1} < 1$$

$$\cos \theta_t = ja = j\sqrt{(n \sin \theta_i)^2 - 1}$$

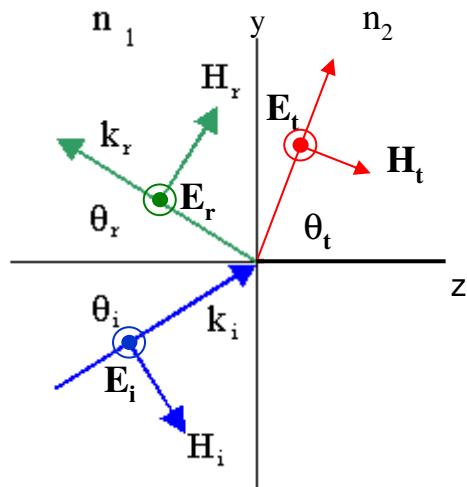


$$\begin{cases} E_r = \frac{\cos \theta_i - jna}{\cos \theta_i + jna} E_i = r_{\perp} E_i \\ E_t = \frac{2 \cos \theta_i}{\cos \theta_i + jna} E_i = t_{\perp} E_i \end{cases}$$

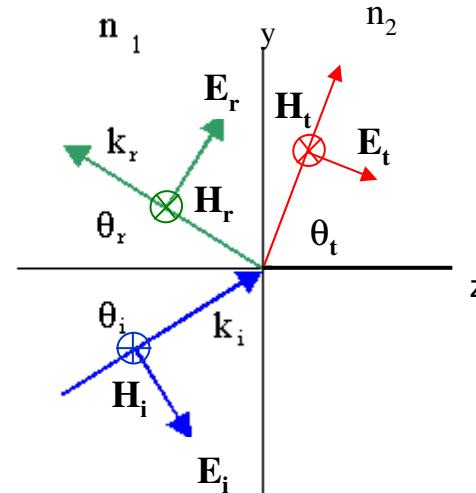
$$\begin{cases} E_r = \frac{n \cos \theta_i - ja}{n \cos \theta_i + ja} E_i = r_{\parallel} E_i \\ E_t = \frac{2n \cos \theta_i}{n \cos \theta_i + ja} E_i = t_{\parallel} E_i \end{cases}$$

$$H_i = \frac{n_1}{\mu_1 c} E_i \quad H_r = \frac{n_1}{\mu_1 c} E_r \quad H_t = \frac{n_2}{\mu_2 c} E_t$$

# Riflessione e rifrazione



$$n = \frac{n_2}{n_1} < 1$$



$$r_{\perp} = e^{j\phi_{\perp}}$$

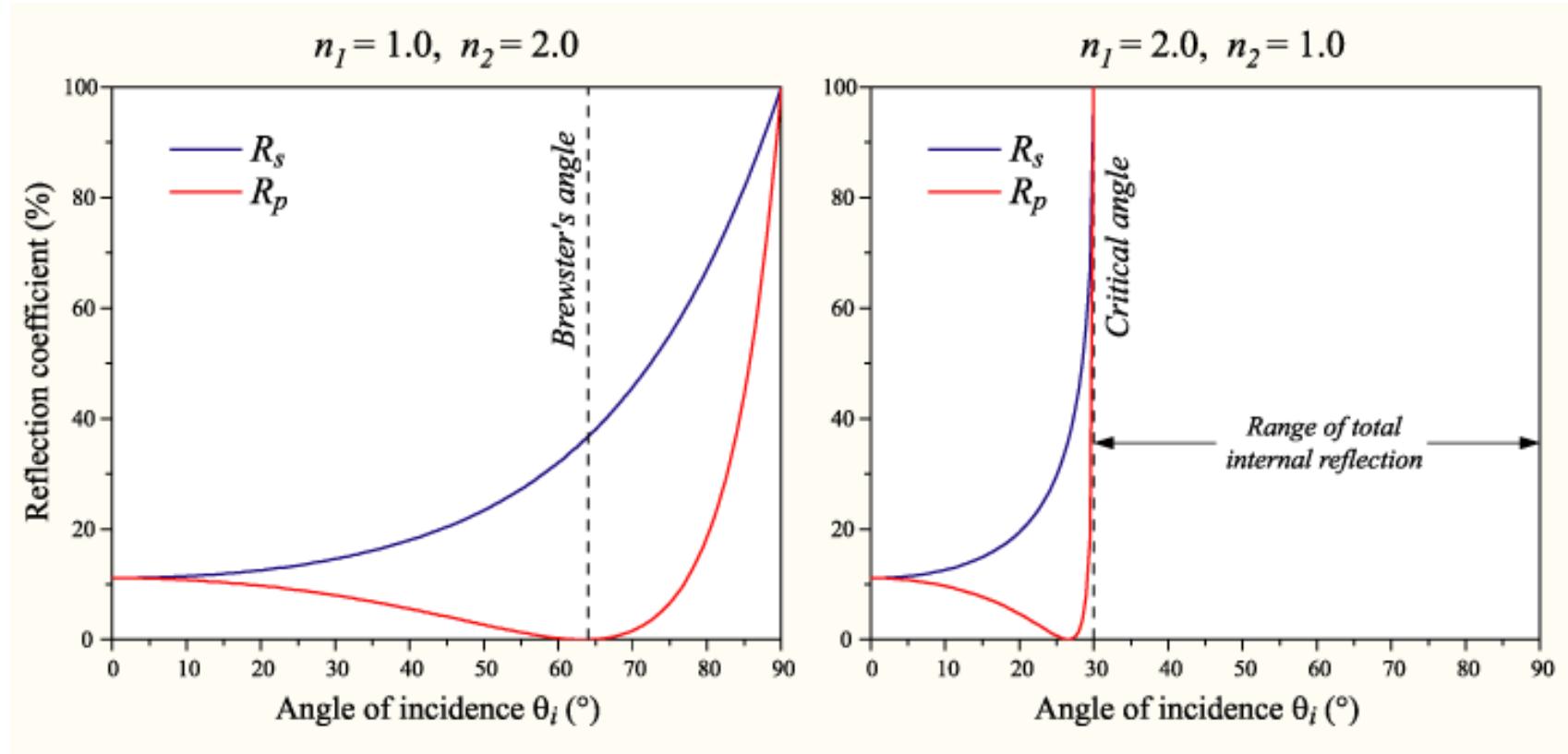
$$r_{\parallel} = e^{j\phi_{\parallel}}$$

$$\tan \frac{\phi_{\perp}}{2} = \frac{-\sqrt{\sin^2 \theta_i - n^2}}{\cos \theta_i}$$

$$\tan \frac{\phi_{\parallel}}{2} = \frac{-\sqrt{\sin^2 \theta_i - n^2}}{n^2 \cos \theta_i}$$

$$|\vec{S}_r| = |\vec{S}_i|$$

# Riflessione e rifrazione

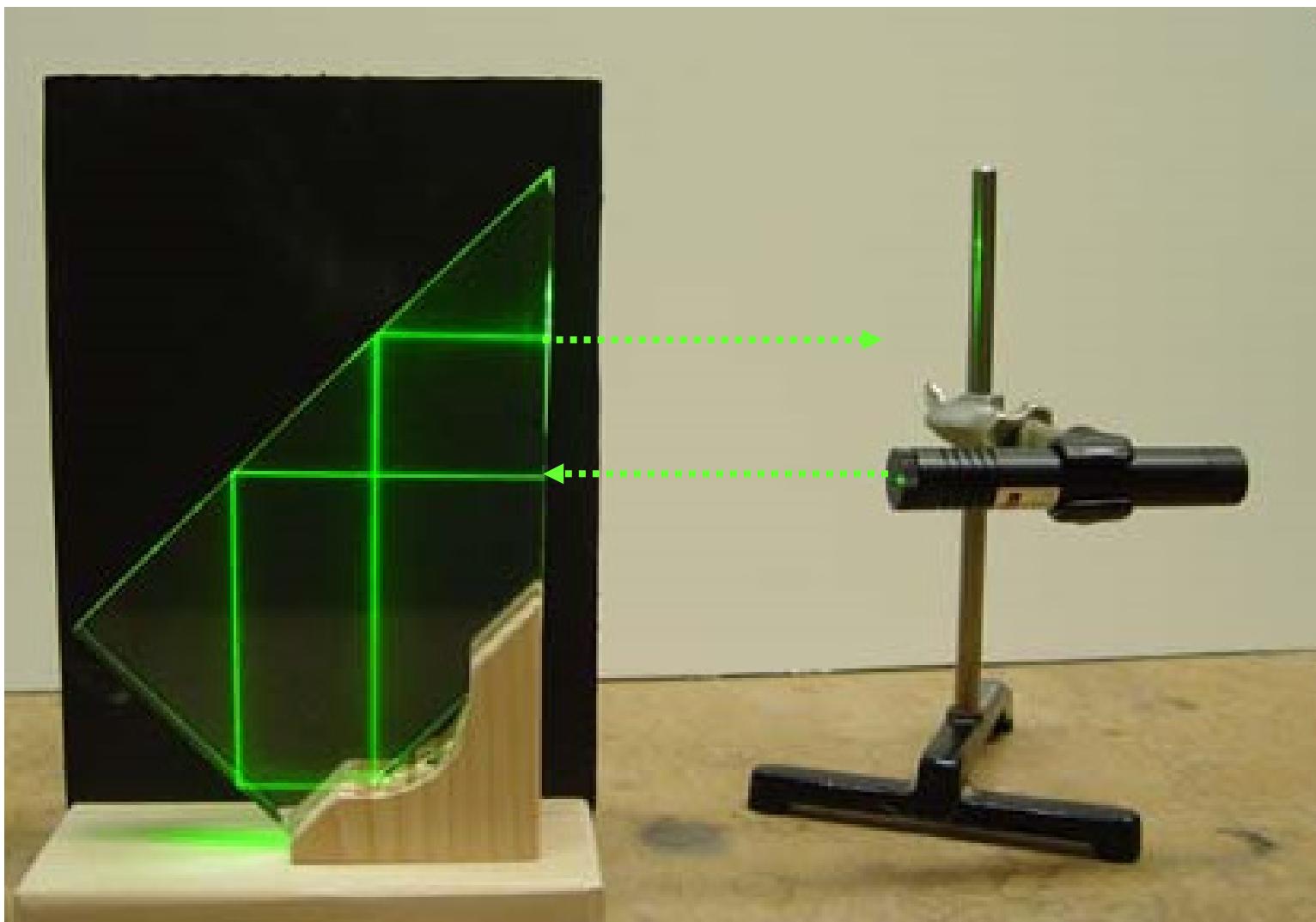




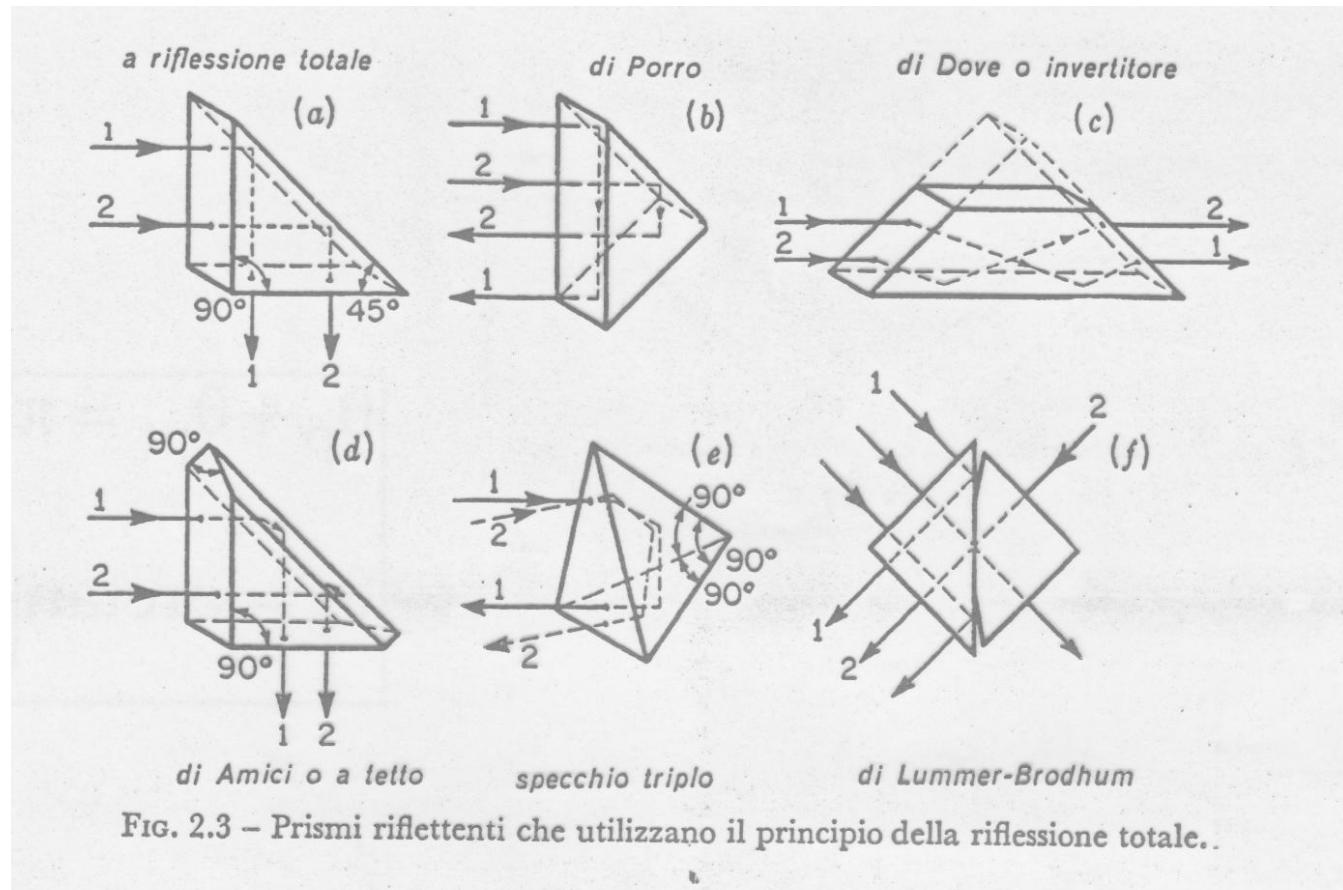
Riflessione totale



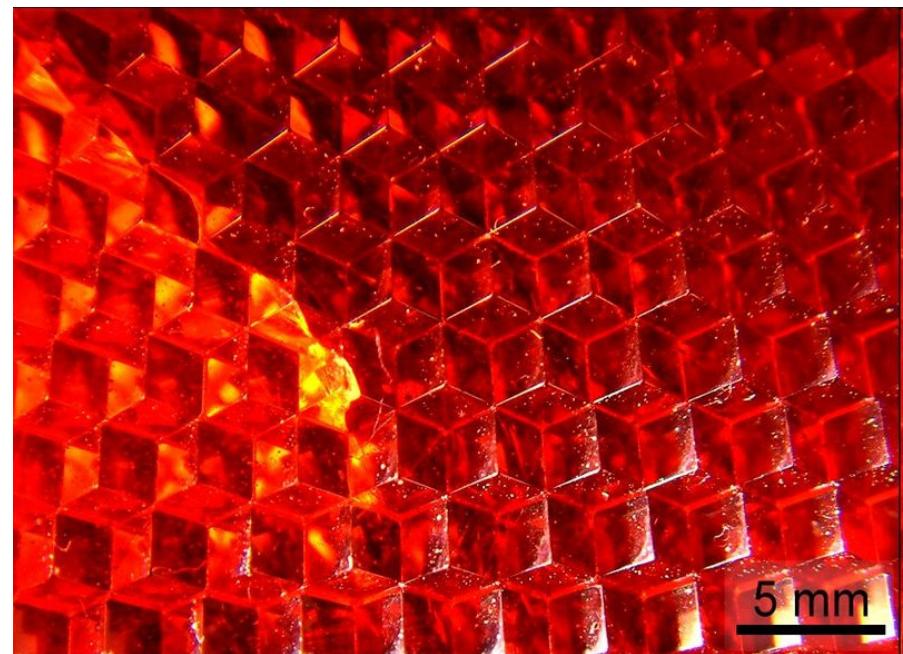
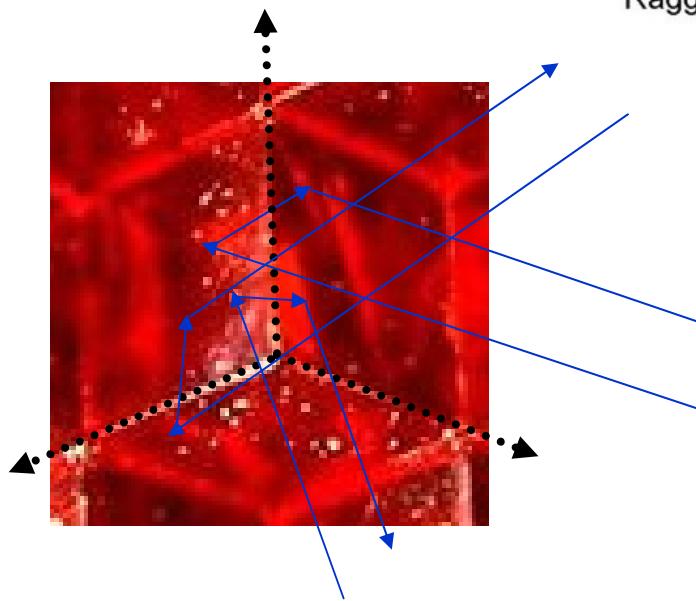
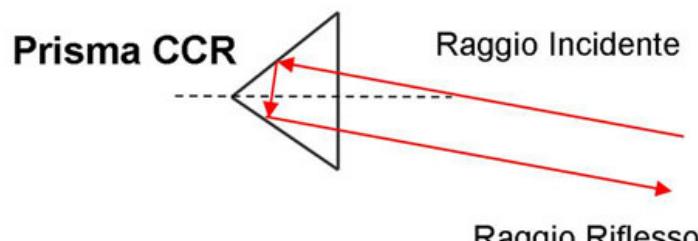
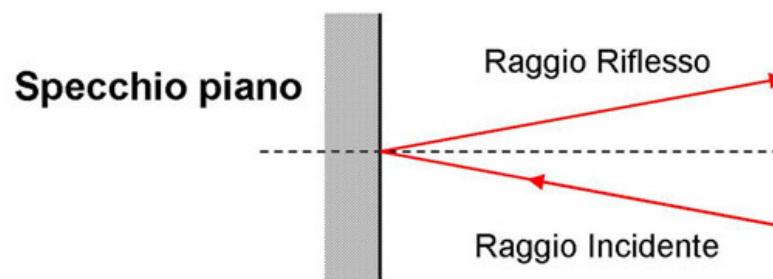
# Riflessione totale



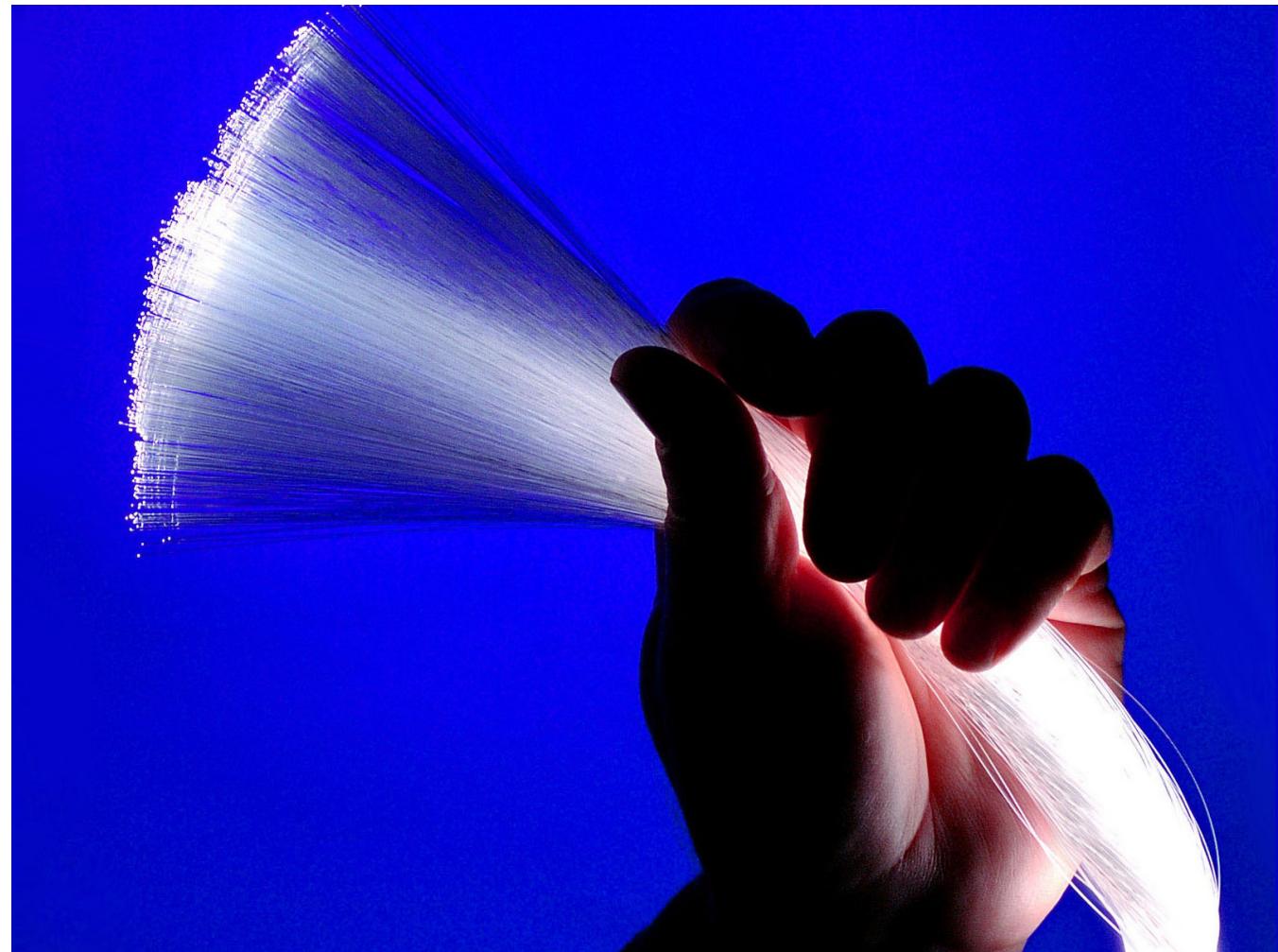
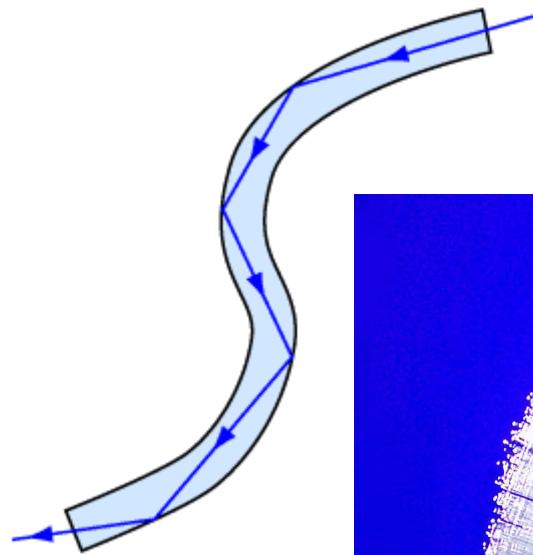
# Prismi a riflessione totale



# Retroriflettori e Catadiottri



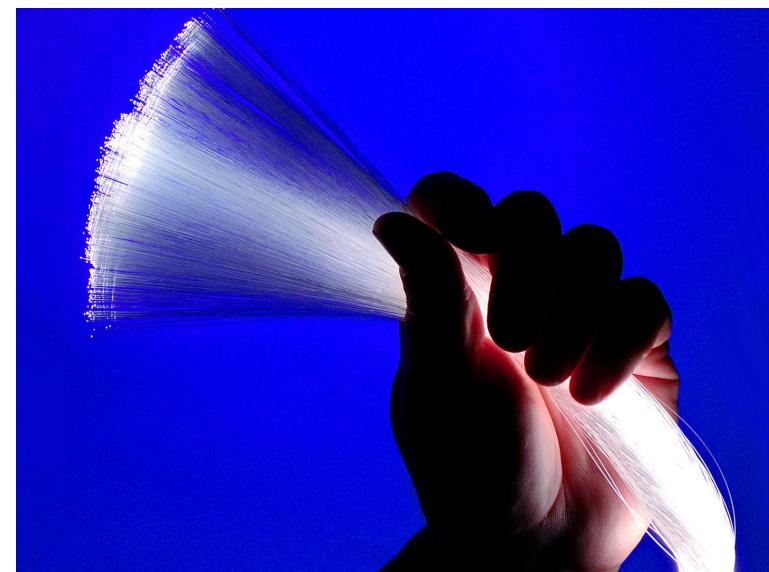
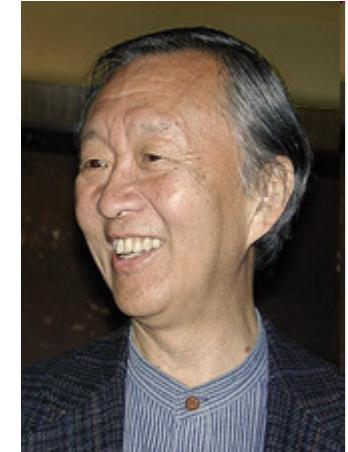
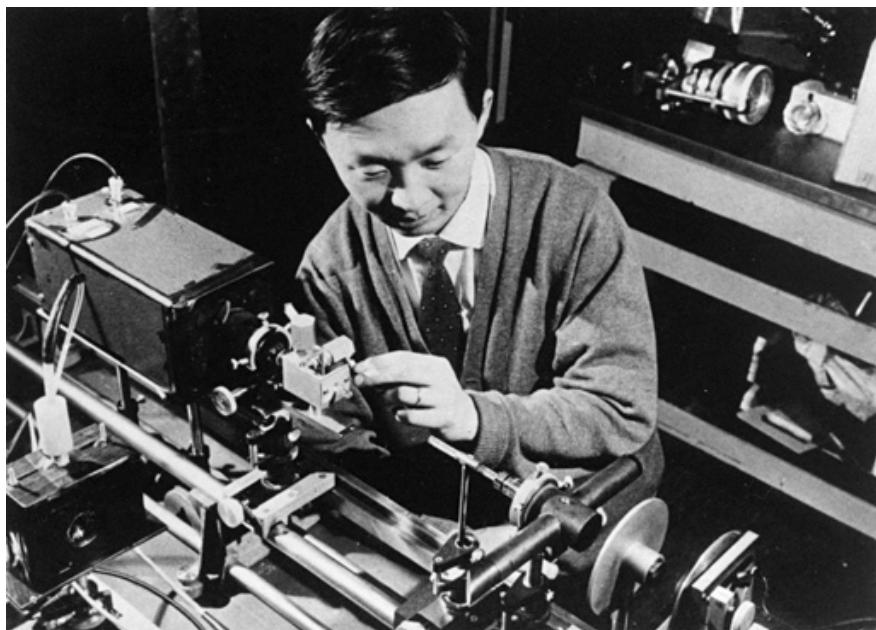
# Fibre ottiche





## The Nobel Prize in Physics 2009

Charles K. Kao



"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"

# The Guiding of Light

Total internal reflection: Johannes Kepler (before 1611)

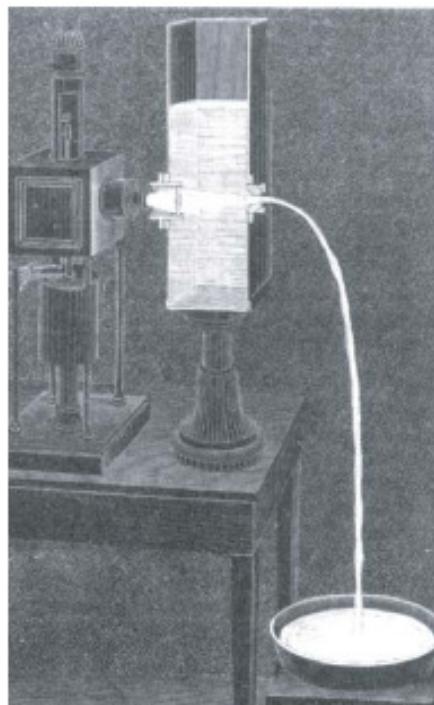
Laws of reflection and refraction: Willebrord Snell (between 1621 and 1625)

Guiding of light by total internal reflection: Daniel Colladon (1842)

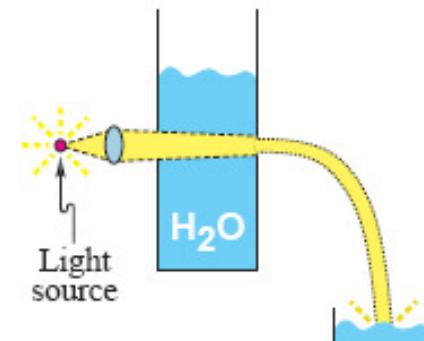
(*Comptes Rendus*, Vol. 15, p. 800, Oct. 24, 1842)



Daniel Colladon (1802 – 1893)  
"Father of light guiding"



Daniel Colladon's apparatus



"... one of the most beautiful experiments one can perform in a course on optics ..."

(after J. Hecht, Optics & Photonics News, Oct. 1999)

Fase dell'onda riflessa totalmente

$$\tan \frac{\varphi_{\perp}}{2} = \frac{-\sqrt{\sin^2 \theta_i - n^2}}{\cos \theta_i}$$

$$\tan \frac{\varphi_{//}}{2} = \frac{-\sqrt{\sin^2 \theta_i - n^2}}{n^2 \cos \theta_i}$$

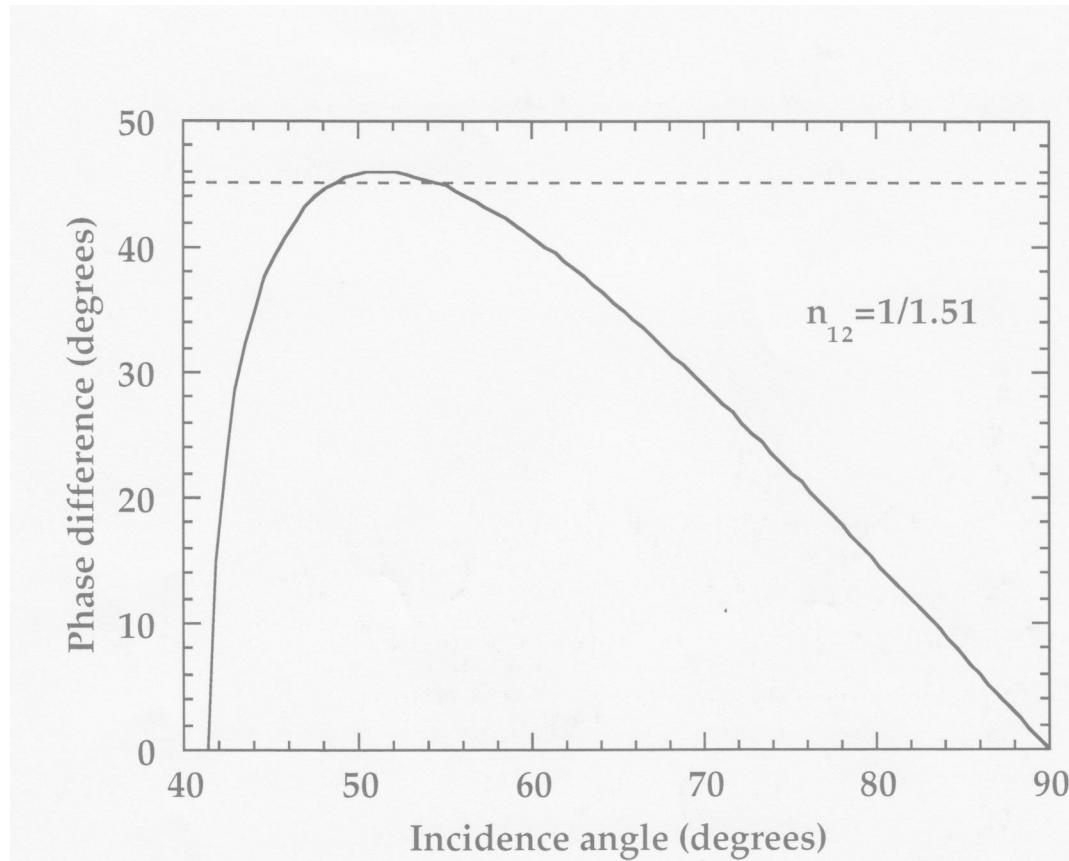
$$\tan \frac{\varphi_{\perp} - \varphi_{//}}{2} = \frac{\tan \frac{\varphi_{\perp}}{2} - \tan \frac{\varphi_{//}}{2}}{1 + \tan \frac{\varphi_{\perp}}{2} \tan \frac{\varphi_{//}}{2}} =$$

$$= \frac{\sqrt{\sin^2 \theta_i - n^2}}{\cos \theta_i} \left( \frac{1}{n^2} - 1 \right) \frac{1}{1 + \frac{\sin^2 \theta_i - n^2}{n^2 \cos^2 \theta_i}} =$$

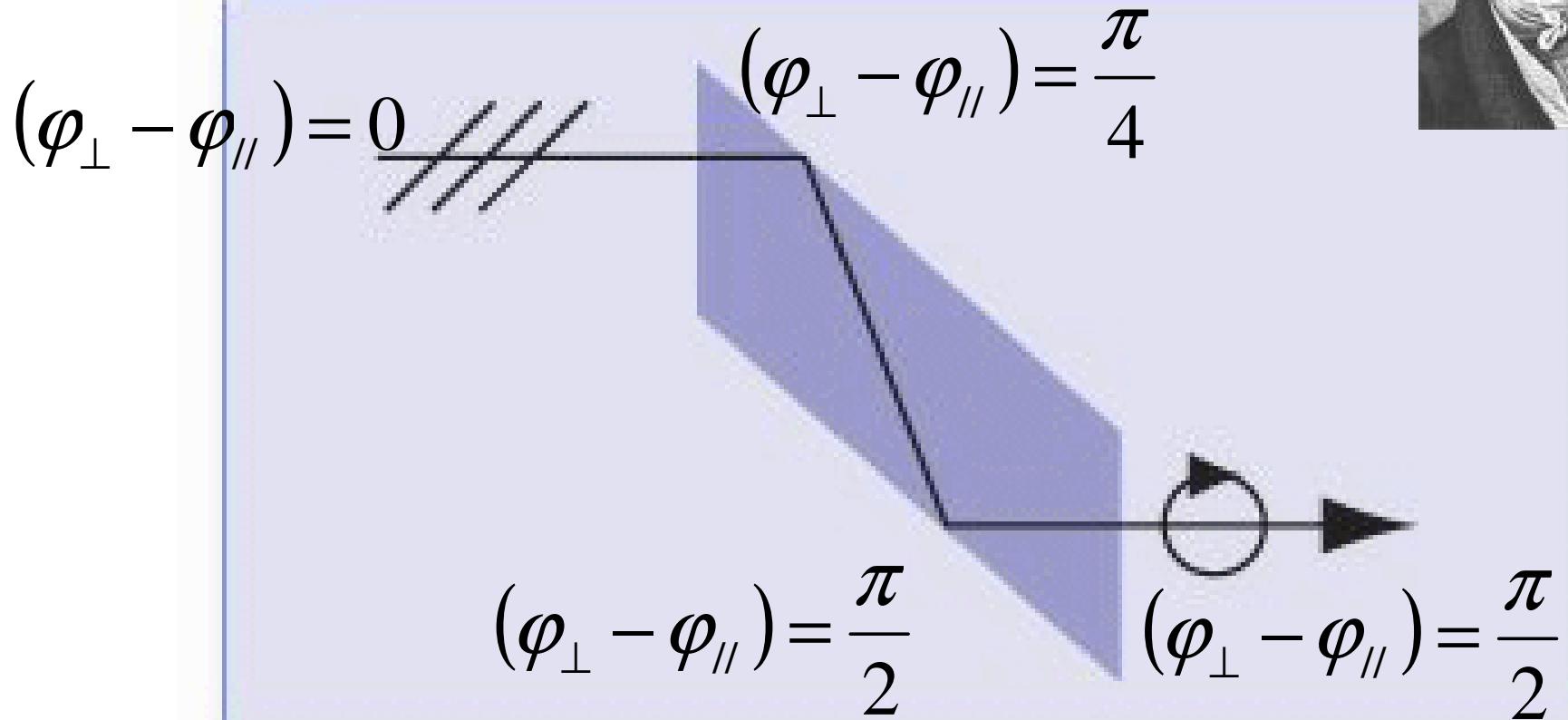
$$= \sqrt{\sin^2 \theta_i - n^2} \frac{\cos \theta_i}{\sin^2 \theta_i}$$

## Fase dell'onda riflessa totalmente

$$\varphi_{\perp} - \varphi_{\parallel} = 2 \arctan \left( \sqrt{\sin^2 \theta_i - n^2} \frac{\cos \theta_i}{\sin^2 \theta_i} \right)$$

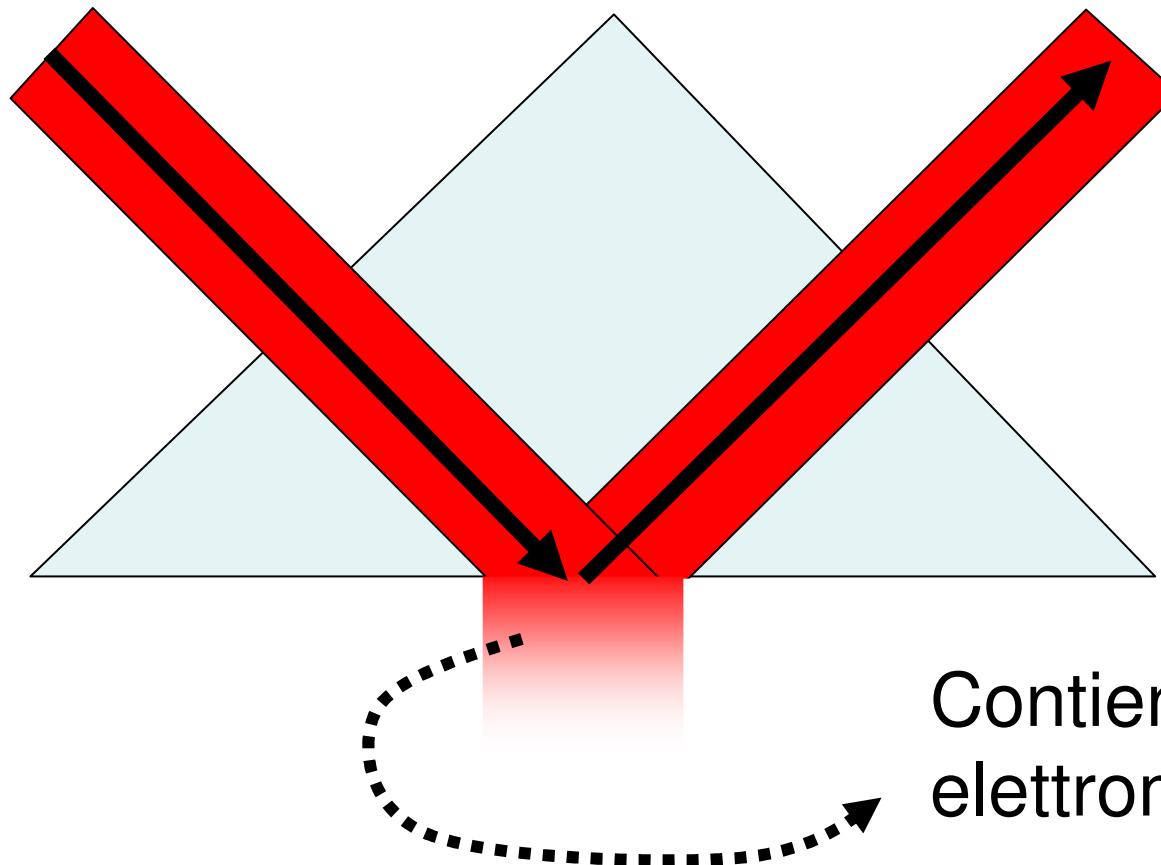


# Rombo di Fresnel

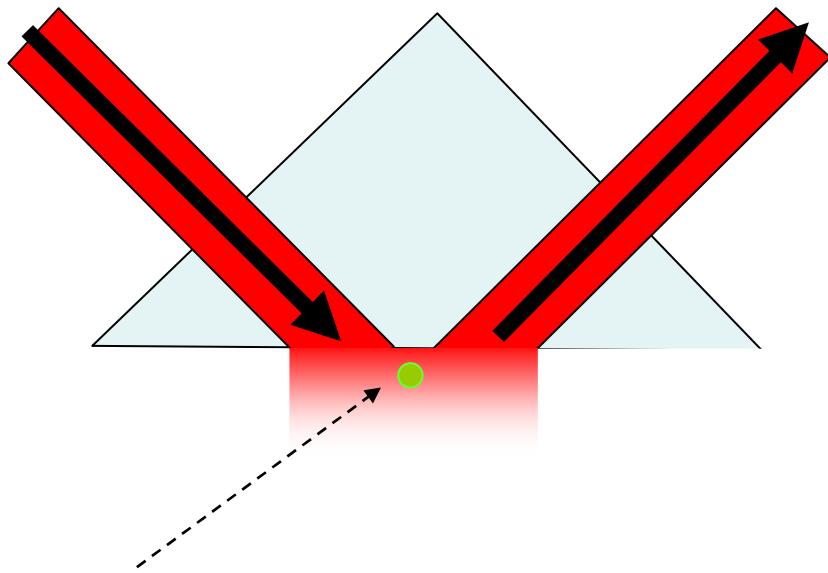


# Onda evanescente

$$|\vec{S}_r| = |\vec{S}_i|$$

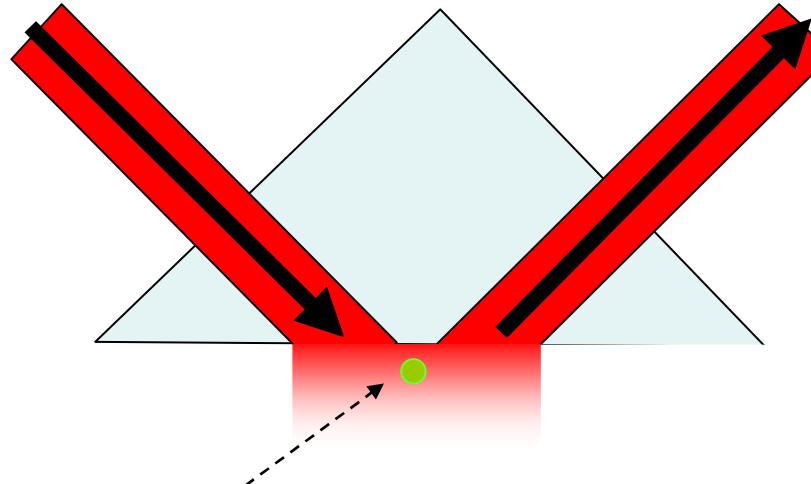


# Onda evanescente

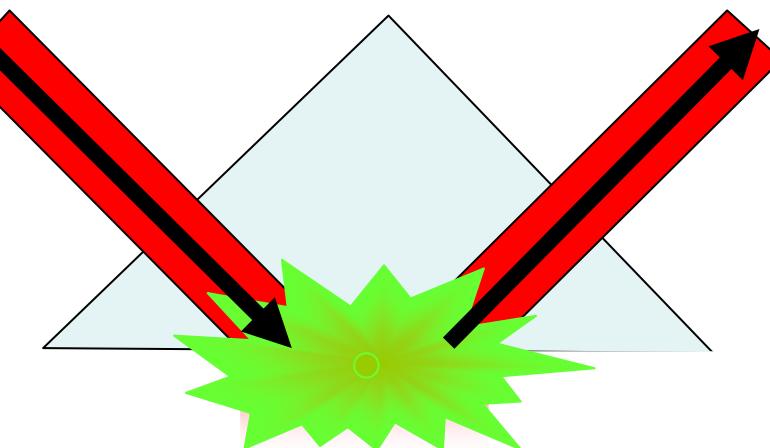


Atomo, molecola,  
centro di scattering

# Onda evanescente



Atomo, molecola,  
centro di scattering



EMISSIONE RIEMESSA

$$\Gamma \propto |\vec{d} \cdot \vec{E}|^2$$

Si puo' assorbire energia da  
un'onda evanescente

# Total Internal Reflection Fluorescence Microscopy (TIRFM)

## Total Internal Reflection Fluorescence

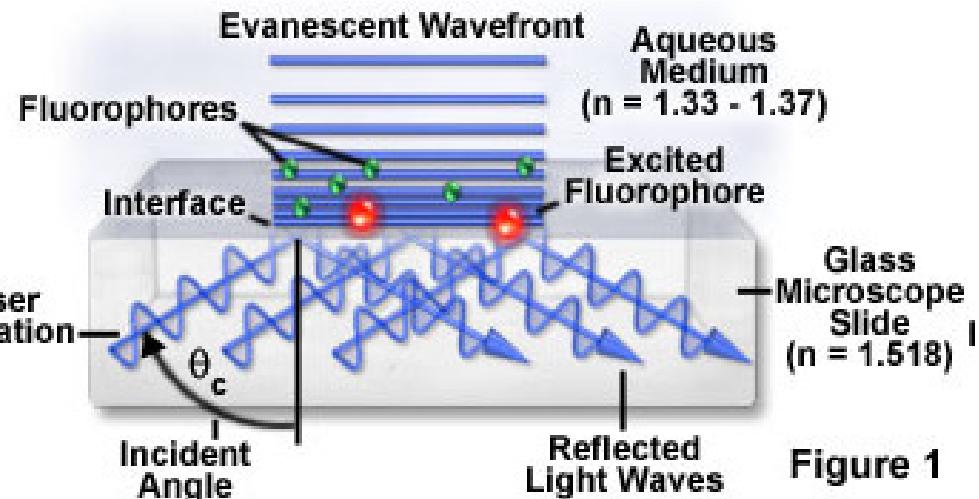


Figure 1

## TIRFM Specimen Illumination Configurations

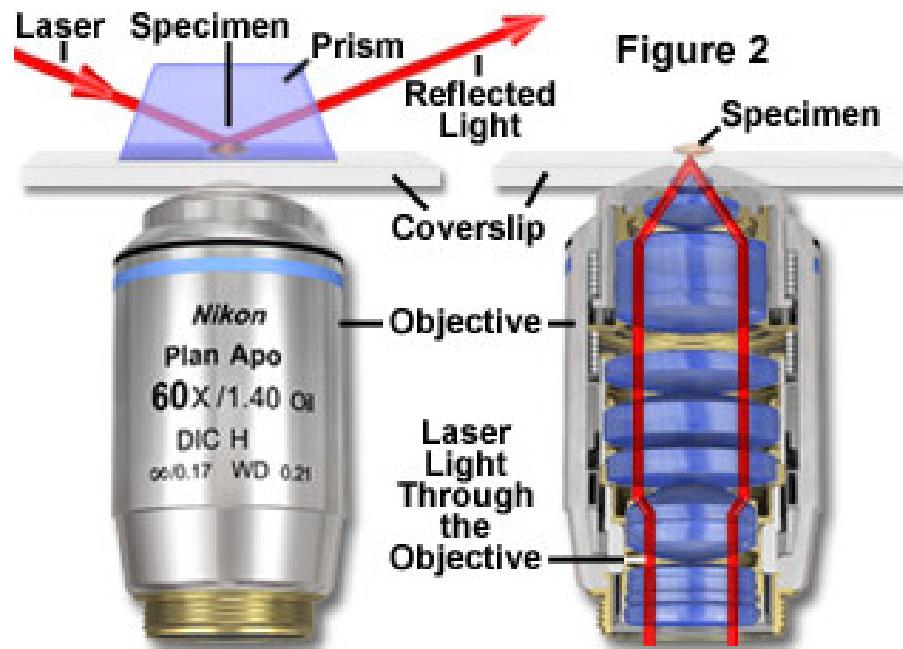
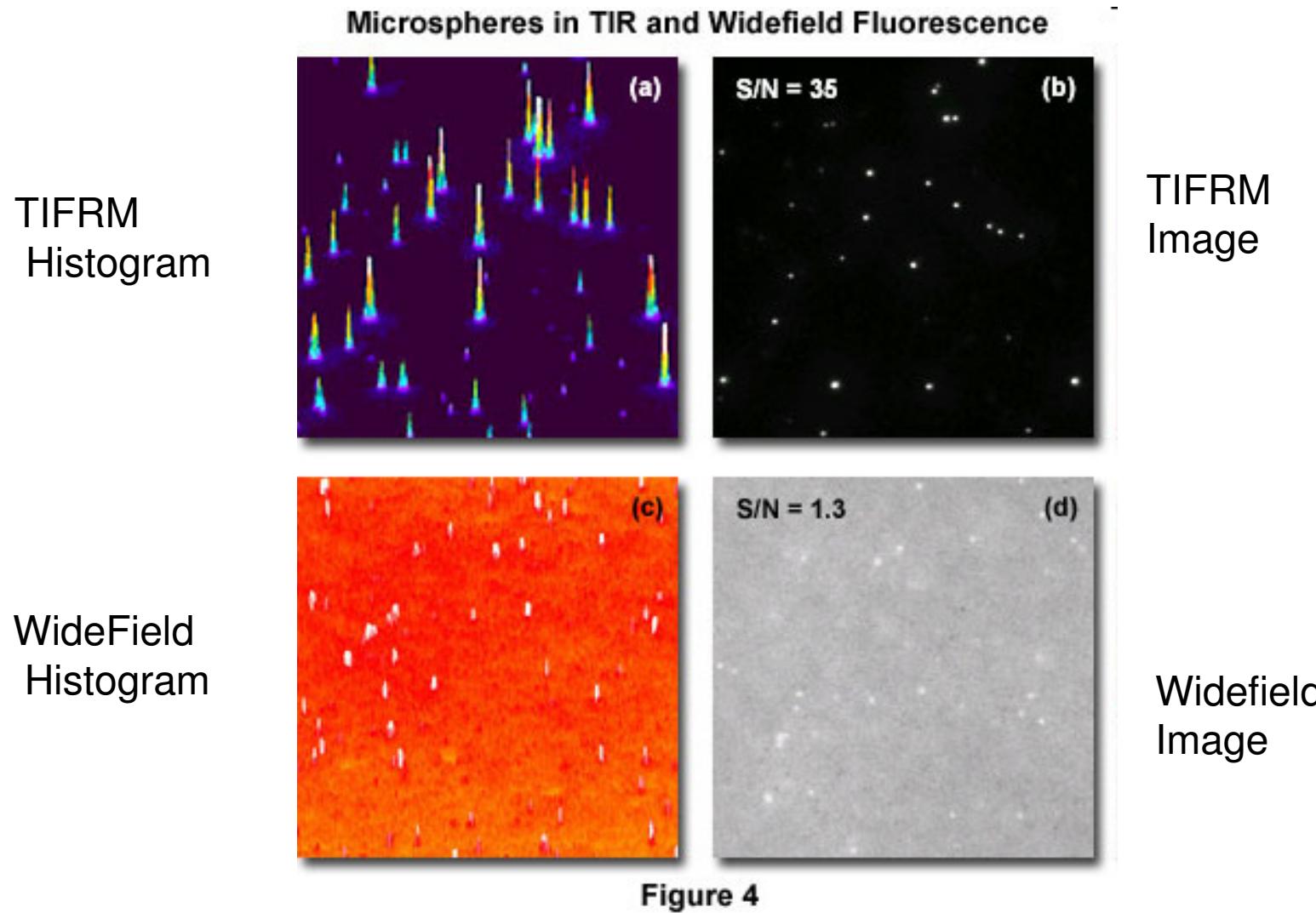
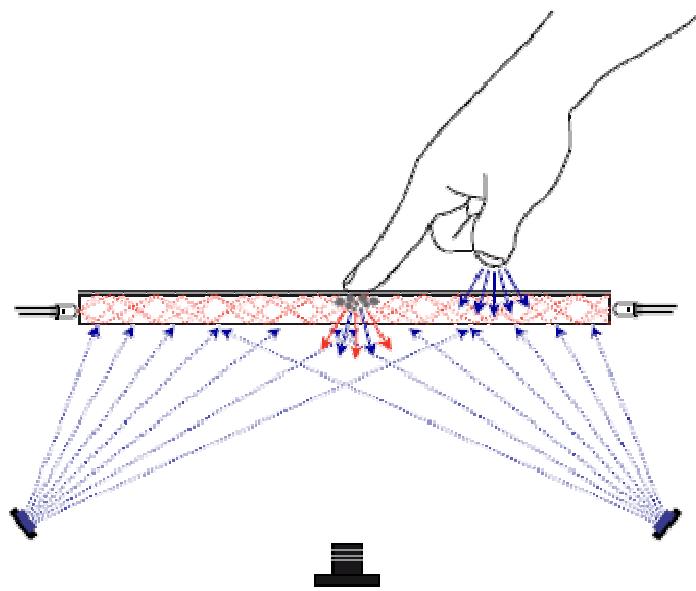


Figure 2

# TIFRM measurements on fluorescent microsphere



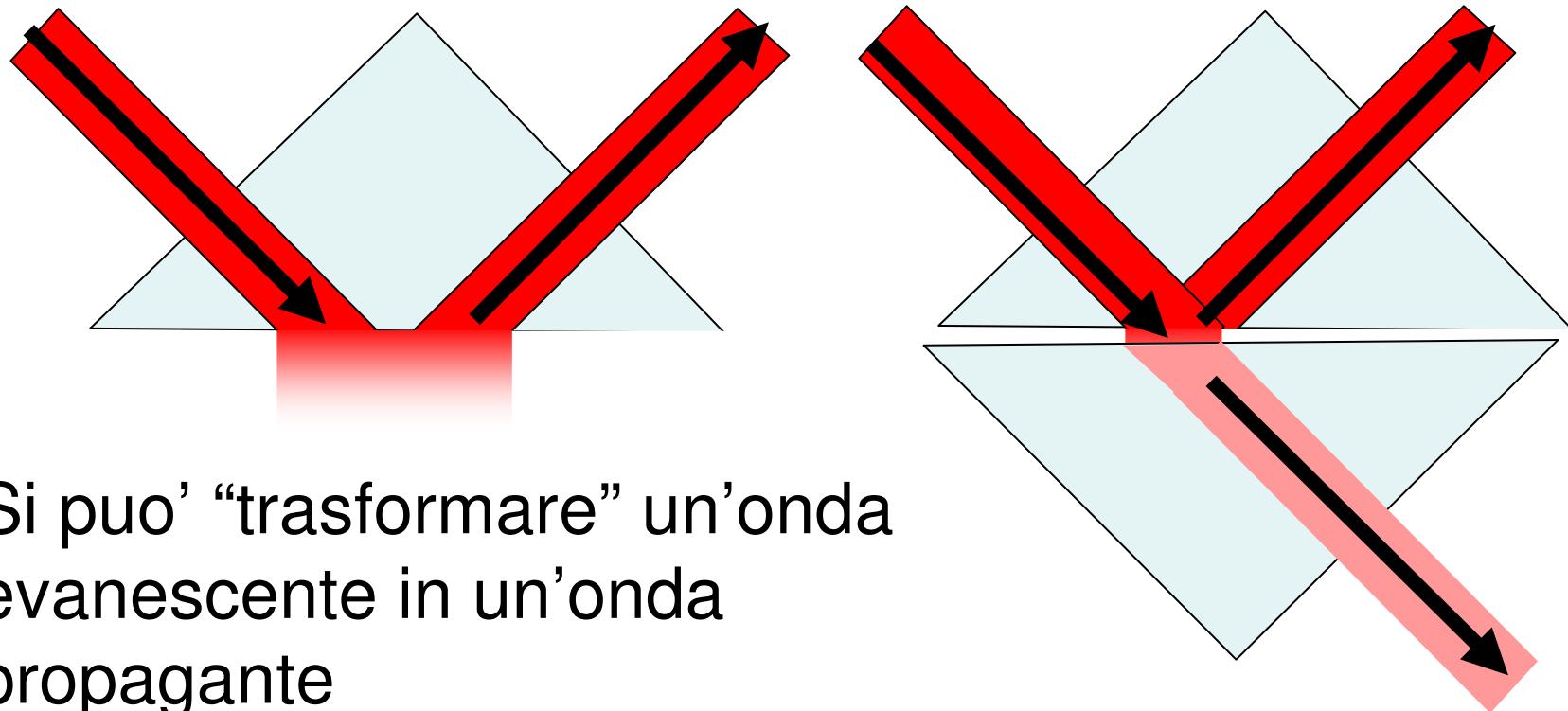
# Frustrated total internal reflection



Touching sensor

Simile all'effetto tunnel in MQ

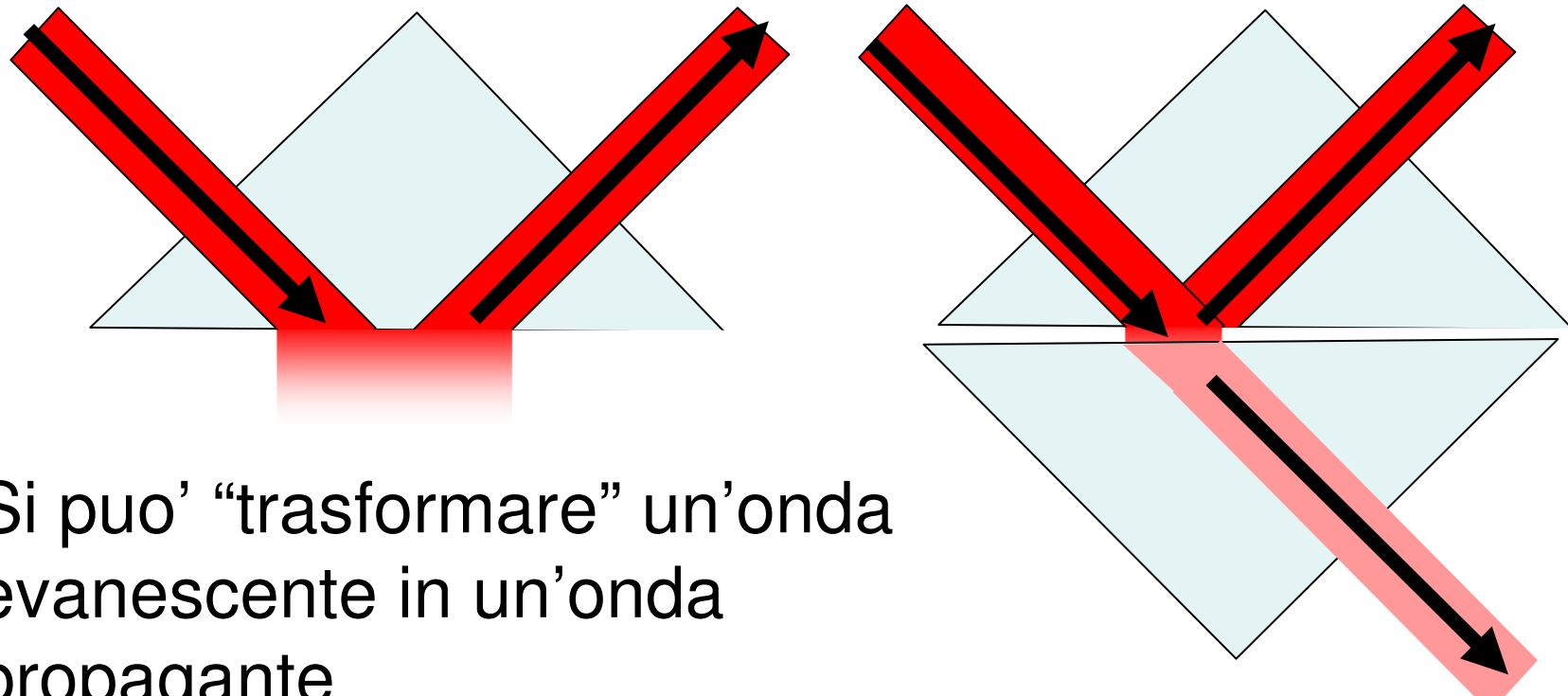
# Frustrated total internal reflection



Si puo' "trasformare" un'onda  
evanescente in un'onda  
propagante

Simile all'effetto tunnel in MQ

# Frustrated total internal reflection

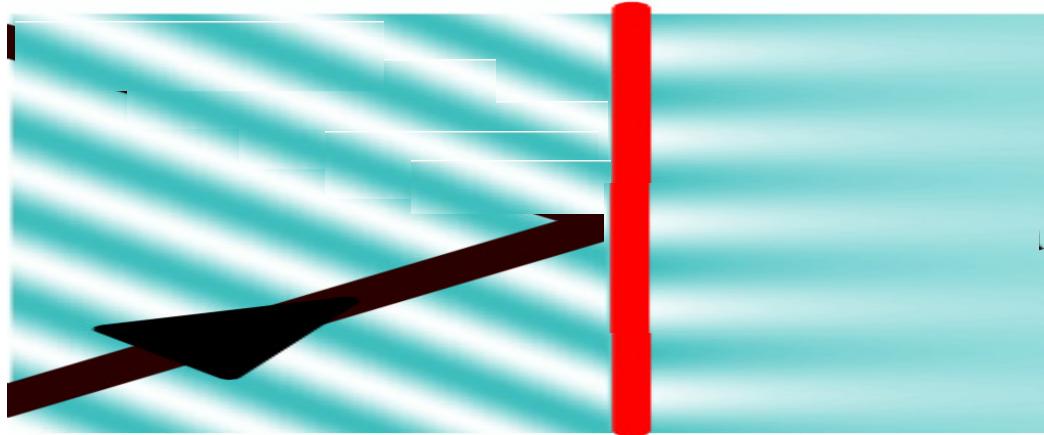


Si puo' "trasformare" un'onda evanescente in un'onda propagante

Simile all'effetto tunnel in MQ

$$? \quad |\vec{S}_r| = |\vec{S}_i| \quad ?$$

# Onda trasmessa



$$n = \frac{n_2}{n_1} < 1$$

TE                     $\cos \theta_t = ja = j\sqrt{(n \sin \theta_i)^2 - 1}$                     TM

$$\begin{cases} E_r = \frac{\cos \theta_i - jna}{\cos \theta_i + jna} E_i = r_{\perp} E_i \\ E_t = \frac{2 \cos \theta_i}{\cos \theta_i + jna} E_i = t_{\perp} E_i \end{cases} \quad \begin{cases} E_r = \frac{n \cos \theta_i - ja}{n \cos \theta_i + ja} E_i = r_{\parallel} E_i \\ E_t = \frac{2n \cos \theta_i}{n \cos \theta_i + ja} E_i = t_{\parallel} E_i \end{cases}$$

$$H_i = \frac{n_1}{\mu_1 c} E_i \quad H_r = \frac{n_1}{\mu_1 c} E_r \quad H_t = \frac{n_2}{\mu_2 c} E_t$$

# Relazioni di Fresnel

TE

$$\vec{E}_t = \frac{2 \cos \theta_i}{\cos \theta_i + jna} E_i \hat{e}_x$$

$$\vec{H}_t = \frac{1}{\omega \mu_2} \vec{k}_t \times \vec{E}_t$$

$$\vec{k}_t = \vec{k}_{t,\parallel} + j\beta \hat{e}_z$$

TM

$$\vec{H}_t = \frac{2n \cos \theta_i}{n \cos \theta_i + ja} \frac{n_2}{\mu_2 c} E_i \hat{e}_x$$

$$\vec{E}_t = -\mu_2 c^2 \vec{k}_t \times \vec{H}_t$$

$$\vec{E}_t(\vec{r}, t) = \vec{E}_t e^{j(\vec{k}_{t,\parallel} \cdot \vec{r}_{\parallel} - \omega_i t)} e^{-\beta z}$$

$$\vec{H}_t(\vec{r}, t) = \vec{H}_t e^{j(\vec{k}_{t,\parallel} \cdot \vec{r}_{\parallel} - \omega_i t)} e^{-\beta z}$$

$$\vec{S}(\vec{r}, t) = \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \quad \text{Vettore di Poynting}$$

$$\vec{S}(\vec{r}, t) = \frac{1}{4} \left( \vec{E}(\vec{r}, t) + \vec{E}^*(\vec{r}, t) \right) \times \left( \vec{H}(\vec{r}, t) + \vec{H}^*(\vec{r}, t) \right)$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{4} \left( \langle \vec{E}(\vec{r}, t) \times \vec{H}^*(\vec{r}, t) \rangle + \langle \vec{E}^*(\vec{r}, t) \times \vec{H}(\vec{r}, t) \rangle \right)$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{2} \Re e \left[ \langle \vec{E}(\vec{r}, t) \times \vec{H}^*(\vec{r}, t) \rangle \right]$$

$$\boxed{\langle \vec{S}(\vec{r}, t) \rangle = \langle \vec{E}(\vec{r}, t) \times \vec{H}^*(\vec{r}, t) \rangle}$$

# Relazioni di Fresnel

TE

$$\vec{k}_t = \vec{k}_{i,\parallel} + j\beta \hat{e}_z$$

TM

$$\vec{E}_t = \frac{2 \cos \theta_i}{\cos \theta_i + jna} E_i \hat{e}_x$$

$$\vec{H}_t = \frac{2n \cos \theta_i}{n \cos \theta_i + ja} \frac{n_2}{\mu_2 c} E_i \hat{e}_x$$

$$\vec{H}_t = \frac{1}{\omega \mu_2} \vec{k}_t \times \vec{E}_t$$

$$\vec{E}_t = -\frac{\mu_2 c^2}{\omega n^2} \vec{k}_t \times \vec{H}_t$$

$$\left\langle \vec{S}_{t,\perp}(\vec{r},t) \right\rangle = \left[ \frac{|t_{\perp}|^2 |E_i|^2}{\omega \mu_2} \vec{k}_t \right] e^{-2\beta z}$$

$$\left\langle \vec{S}_{t,\parallel}(\vec{r},t) \right\rangle = \left[ \frac{|t_{\parallel}|^2 |E_i|^2}{\omega \mu_2} \vec{k}_t \right] e^{-2\beta z}$$

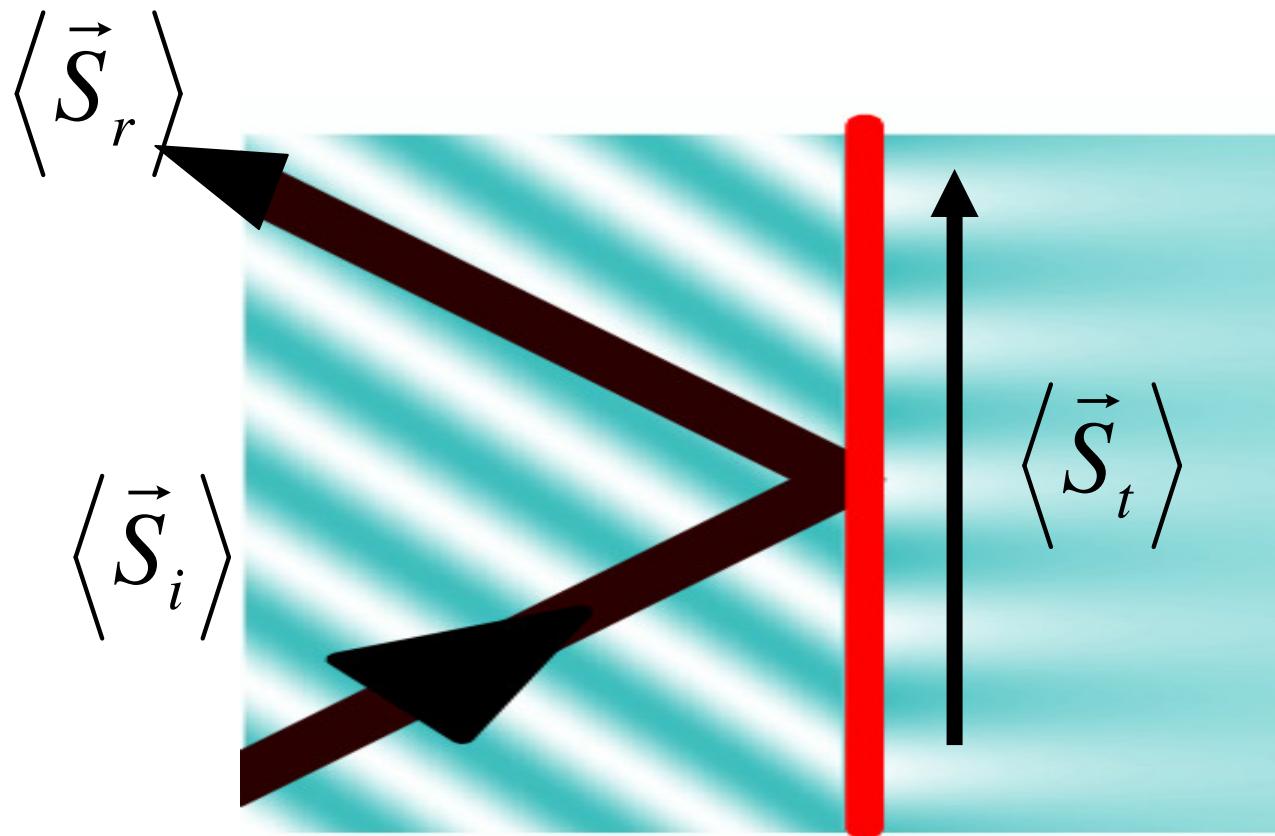
$$\boxed{\left\langle \vec{S}_{t,\ell}(\vec{r},t) \right\rangle = \left[ \frac{|t_{\ell}|^2 |E_i|^2}{\omega \mu_2} (\vec{k}_{i,\parallel} + j\beta \hat{e}_z) \right] e^{-2\beta z} \quad \ell = \perp, \parallel}$$

Onda evanescente  $\beta = \frac{\omega n_2}{c} a = \frac{2\pi n_2}{\lambda} \sqrt{n^2 \sin^2 \theta_i - 1}$

$$\text{Re}\left\langle \vec{S}_{t,\ell}(\vec{r},t) \right\rangle = \left[ \frac{|t_\ell|^2 |E_i|^2}{\omega \mu_2} \right] e^{-2\beta z} \vec{k}_{i,\parallel} \quad \ell = \perp, \parallel$$

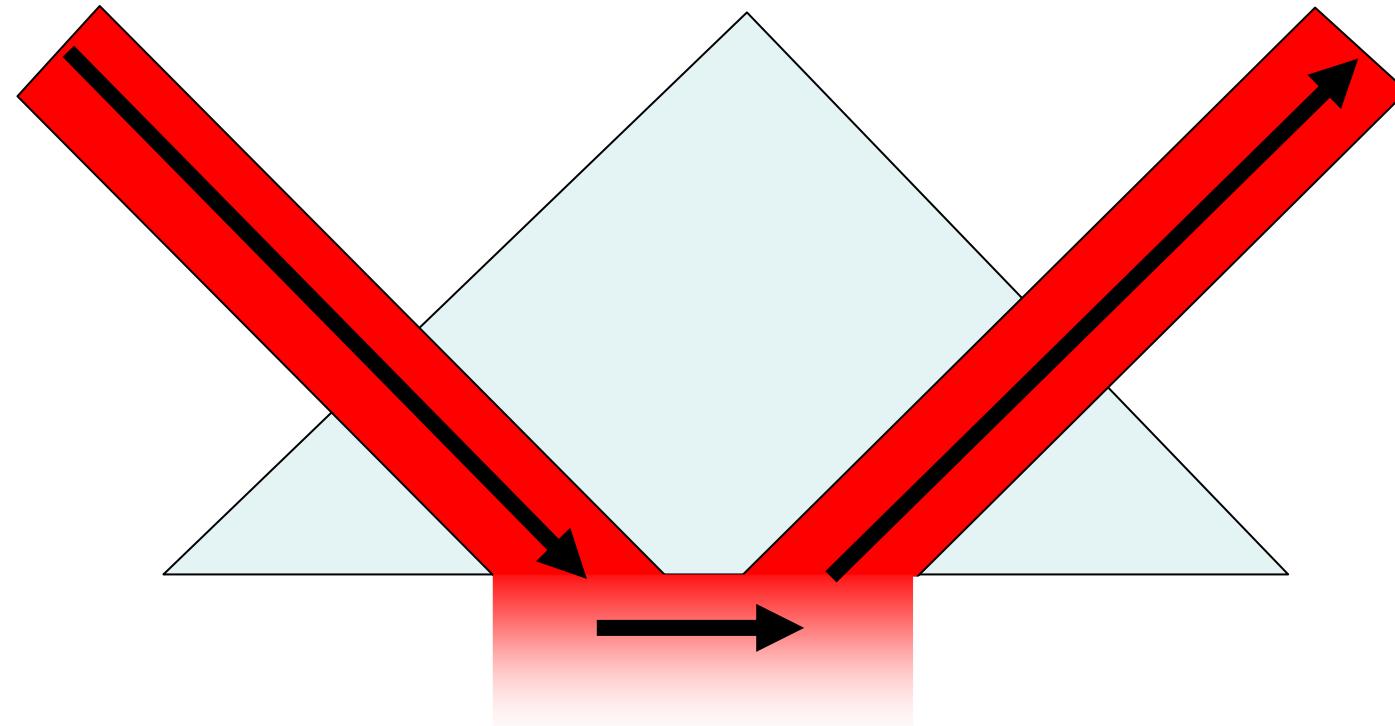


# Bilancio energia ?

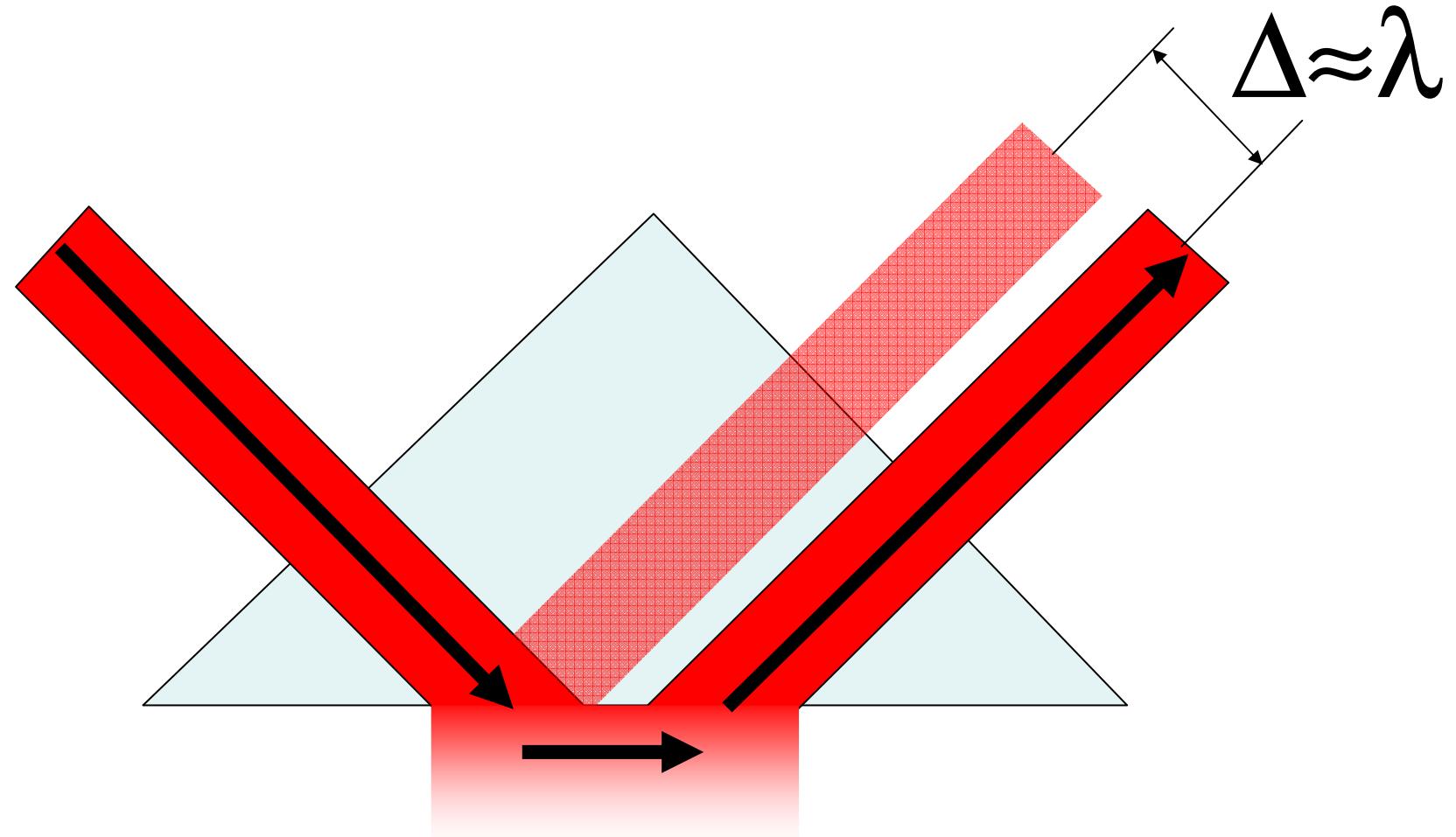


$$\langle \vec{S}_i \rangle = \langle \vec{S}_r \rangle \quad \langle \vec{S}_t \rangle = ??$$

# Goos Hanchen shift



# Goos Hanchen shift



# Goos Hanchen shift

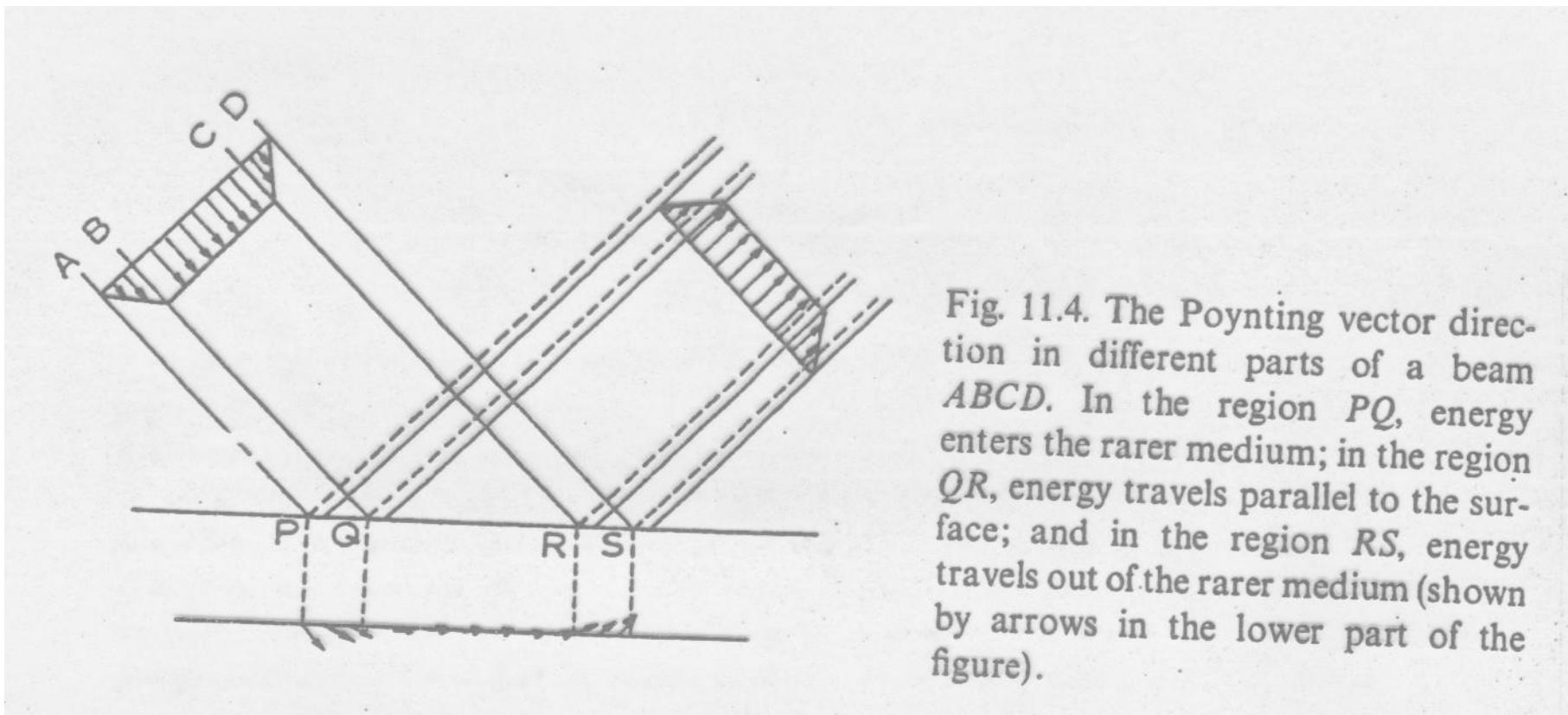
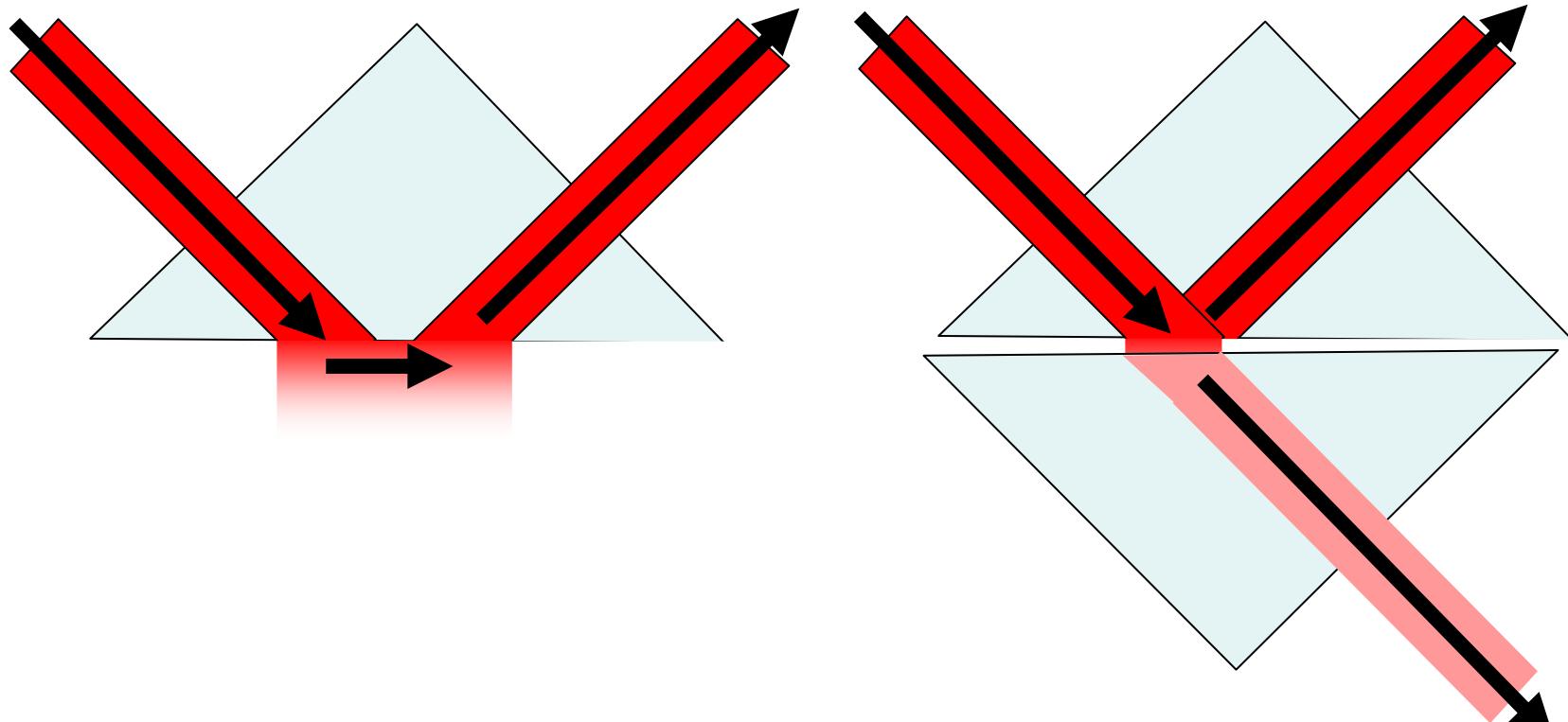


Fig. 11.4. The Poynting vector direction in different parts of a beam *ABCD*. In the region *PQ*, energy enters the rarer medium; in the region *QR*, energy travels parallel to the surface; and in the region *RS*, energy travels out of the rarer medium (shown by arrows in the lower part of the figure).

# Misura del Goos Hanchen shift



Frustrated total  
attenuated reflection

# Misura del Goos Hanchen shift

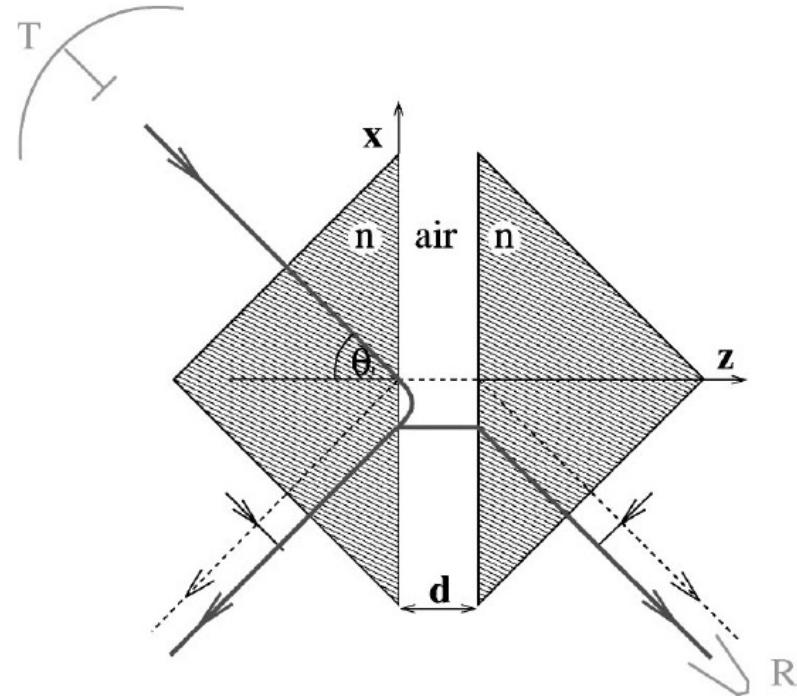
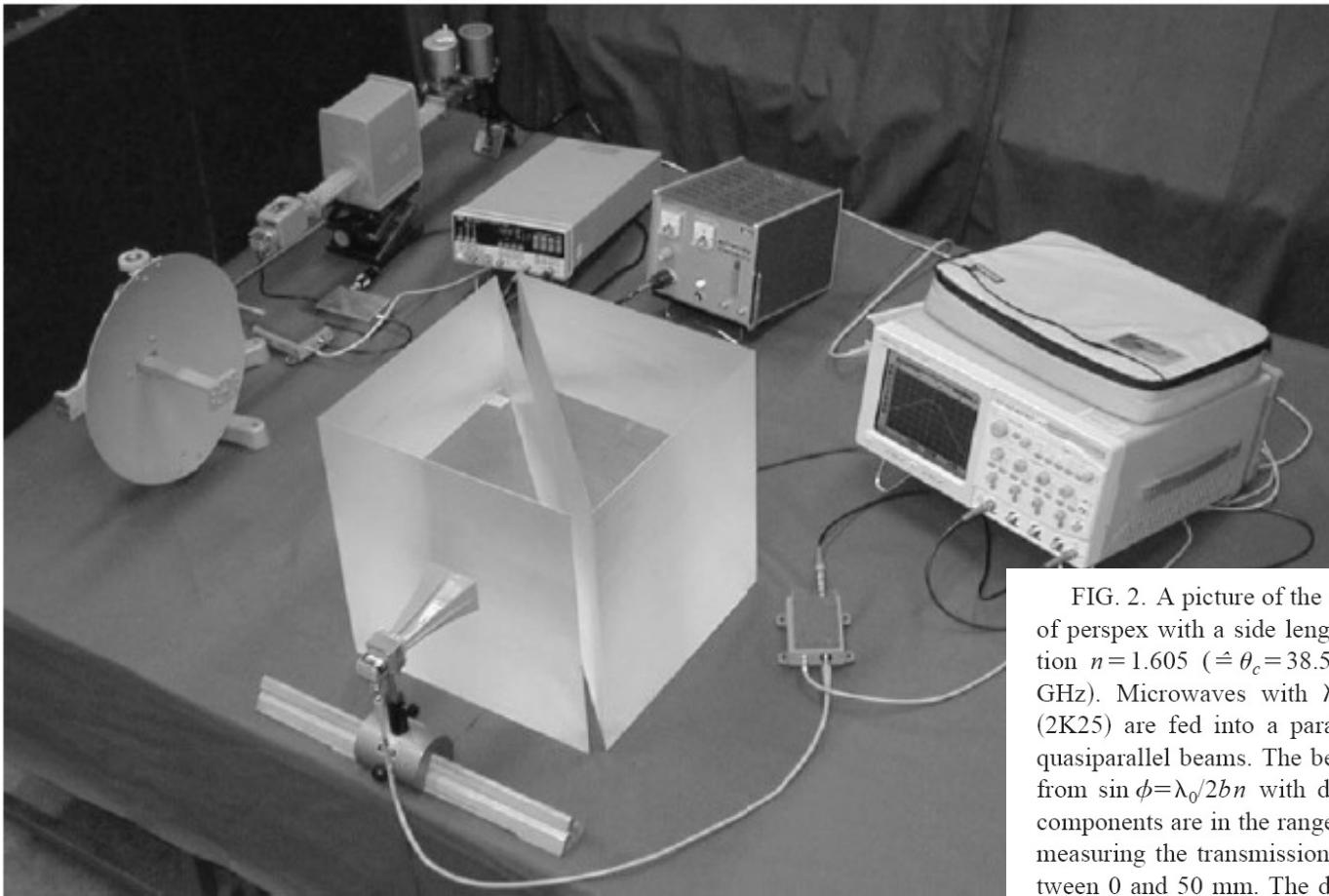


FIG. 1. Sketch of the experimental apparatus showing the parabolic transmitting antenna ( $T$ ), the prisms, the air gap of width  $d$ , the horn antenna used as receiver ( $R$ ), and the symmetrical shifts of the reflected/transmitted beams, where  $\theta_i > \theta_c = \arcsin 1/n$  is the angle of incidence.  $\theta_c$  is the critical angle of total reflection. The shift of the evanescent wave parallel to the surface in air represents the Goos-Hänchen shift  $D$ .

# Misura del Goos Hanchen shift



$\lambda=32.8 \text{ mm}$

FIG. 2. A picture of the experiment. The prisms, cut from a cube of perspex with a side length of 400 mm, have an index of refraction  $n = 1.605$  ( $\hat{\theta}_c = 38.5^\circ$ ) at the frequency in question (9.15 GHz). Microwaves with  $\lambda_0 = 32.8 \text{ mm}$ , generated in a klystron (2K25) are fed into a parabolic transmitter antenna guaranteeing quasiparallel beams. The beam spreading is less than  $2^\circ$  as follows from  $\sin \phi = \lambda_0 / 2bn$  with diameter  $b_{\text{antenna}} = 350 \text{ mm}$  and all beam components are in the range of total reflection. This was verified by measuring the transmission damping depending on the air gap between 0 and 50 mm. The damping would be 1.8 dB in the case of normal reflection compared with our measured 36 dB for the case of  $\theta_i = 45^\circ$  and a 50-mm gap. The measured value of 7.2 dB/10 mm is in agreement with the theoretical transmission Ref. [17]. The signals have been picked up by a microwave horn and fed across an amplifier to an oscilloscope (HP 54825A). A metallic reflector placed at the base of the first prism to determine the position of the reflected beam in the case of geometrical optics. The results presented here are averaged values of several runs with error bars. (For the photo we put the various components near together to present all of them in one picture.)

A. Haibel et al. Phys. Rev. E 63, 047601 (2001)

# Misura del Goos Hanchen shift

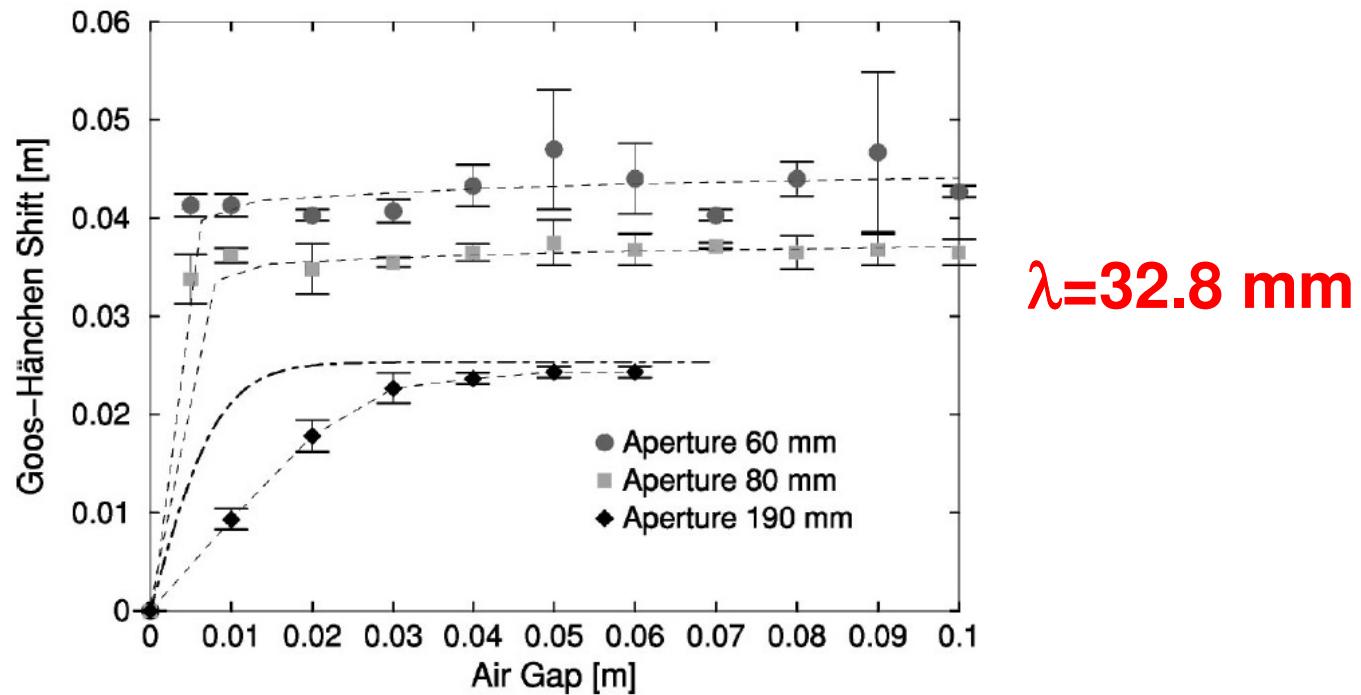


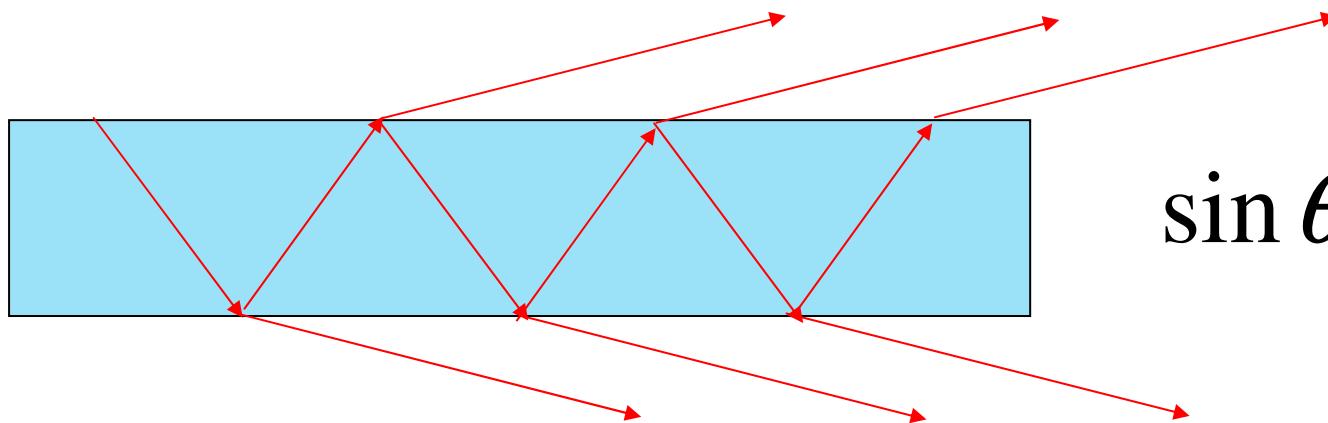
FIG. 4. The Goos-Hänchen shift vs air gap for different beam diameters in TM polarization and for  $\theta_i=45^\circ$ : the shift measured for the large beams (no aperture or an aperture of 190 mm) is roughly in agreement with theoretical prediction (dot-dashed line) [8], while decreasing beam diameters lead to increasing shifts reaching the constant asymptotic value already for very small values of the air gap. The zero point was obtained by substituting the air gap with a metallic plate.

A. Haibel et al. Phys. Rev. E 63, 047601 (2001)

# Modi guidati

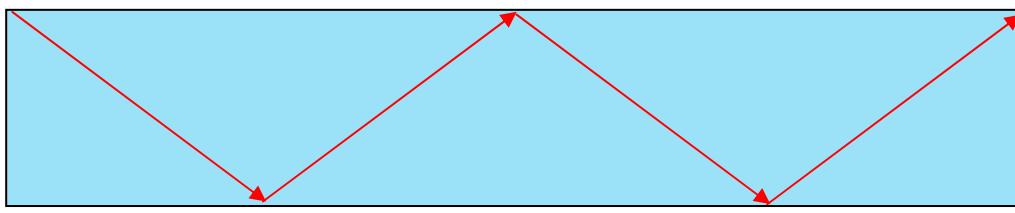
## Dielectric slab

Within the limit angle



$$\sin \theta_i < \frac{n_2}{n_1}$$

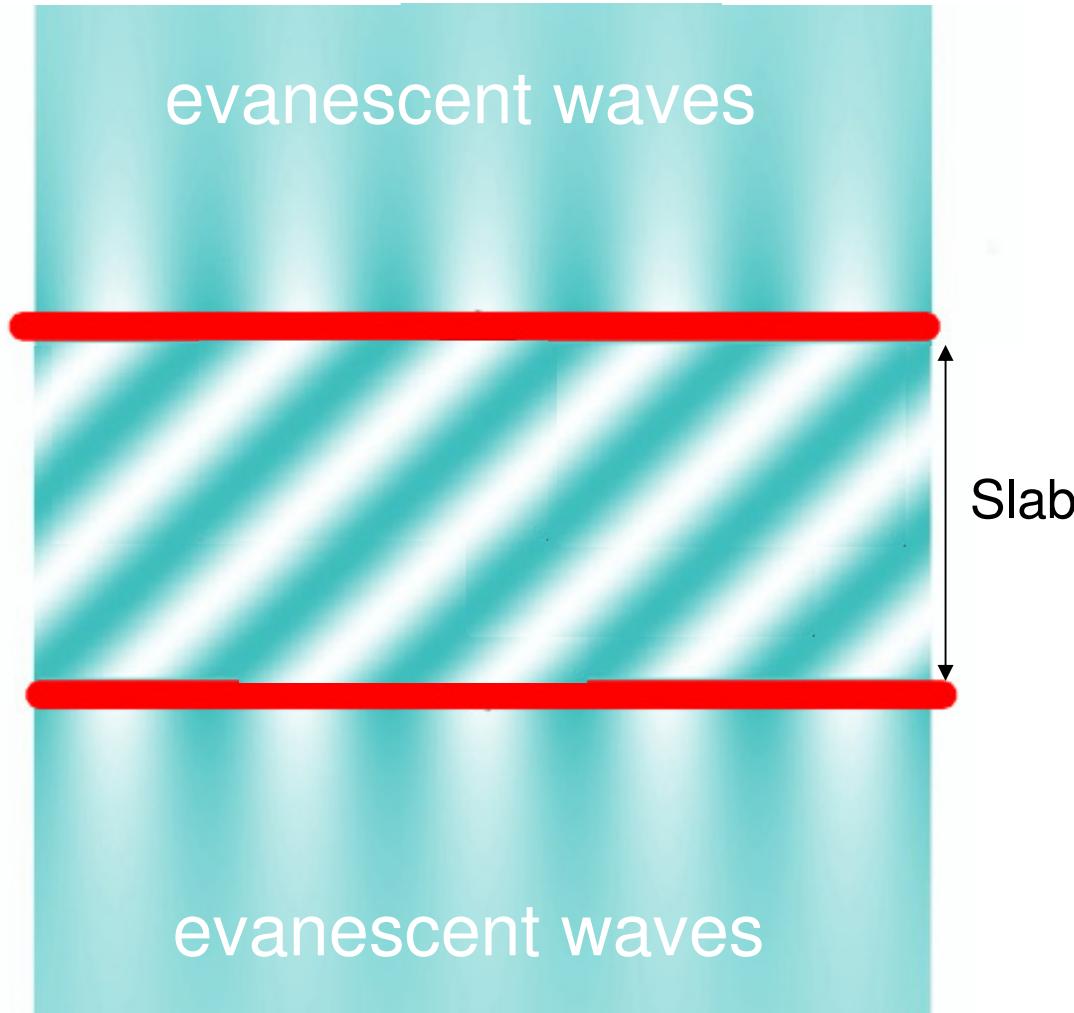
Beyond the limit angle



$$\sin \theta_i > \frac{n_2}{n_1}$$

## Dielectric slab

$$\sin \theta_i > \frac{n_2}{n_1}$$



# Beyond the limit angle

$$\vec{k} = \vec{k}_{\parallel} + \vec{k}_z \quad \left| \vec{k} \right| = \frac{\omega n}{c}$$

$$\vec{k}_{\parallel} = k_y \hat{y} \quad \frac{\omega n}{c} > |k_y| > \frac{\omega}{c}$$

In the media

$$k_z = \sqrt{n^2 \frac{\omega^2}{c^2} - k_y^2}$$

In air

$$k_z = i \sqrt{k_y^2 - \frac{\omega^2}{c^2}} = i\alpha$$

## Dielectric slab

$$\frac{\partial}{\partial z} E_x = -i\omega\mu_o\mu H_y$$

### Beyond the limit angle

$$|z| \geq a \Rightarrow E_x(\vec{r}) = A e^{ik_y y} e^{-\alpha|z|} ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} \frac{|z|}{z} A e^{ik_y y} e^{-\alpha|z|}$$

$$|z| \leq a \Rightarrow E_x(\vec{r}) = B e^{ik_y y} \cos(kz) ; \quad H_y(\vec{r}) = \frac{-ik}{\omega} B e^{ik_y y} \sin(kz)$$

**Even states**

$$|z| \geq a \Rightarrow E_x(\vec{r}) = A e^{ik_y y} e^{-\alpha|z|} ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} \frac{|z|}{z} A e^{ik_y y} e^{-\alpha|z|}$$

$$|z| \geq a \Rightarrow E_x(\vec{r}) = A \frac{|z|}{z} e^{ik_y y} e^{-\alpha|z|} ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} A e^{ik_y y} e^{-\alpha|z|}$$

$$|z| \leq a \Rightarrow E_x(\vec{r}) = B e^{ik_y y} \sin(kz) ; \quad H_y(\vec{r}) = \frac{ik}{\omega} B e^{ik_y y} \cos(kz)$$

**Odd states**

$$|z| \geq a \Rightarrow E_x(\vec{r}) = A \frac{|z|}{z} e^{ik_y y} e^{-\alpha|z|} ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} A e^{ik_y y} e^{-\alpha|z|}$$

# Beyond the limit angle

The frequency and the parallel components of the wavevector, have to be identical in the two media. We look for guided modes, that is  $|k_y| > \omega/c$ , and we define  $k = \sqrt{(n\omega/c)^2 - k_y^2}$  and  $\alpha = \sqrt{(k_y^2 - \omega/c)^2}$ . The relevant components of even  $p$  modes in the air and in the slab are given by:

$$|z| \geq a \quad \Rightarrow \quad E_x(\vec{r}) = A e^{ik_y y} e^{-\alpha|z|} \quad ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} \frac{|z|}{z} A e^{ik_y y} e^{-\alpha|z|} \quad (1.44)$$

$$|z| \leq a \quad \Rightarrow \quad E_x(\vec{r}) = B e^{ik_y y} \cos(kz) \quad ; \quad H_y(\vec{r}) = \frac{-ik}{\omega} B e^{ik_y y} \sin(kz) \quad (1.45)$$

The odd  $p$  modes in the air and in the slab are given by:

$$|z| \geq a \quad \Rightarrow \quad E_x(\vec{r}) = A \frac{|z|}{z} e^{ik_y y} e^{-\alpha|z|} \quad ; \quad H_y(\vec{r}) = \frac{-i\alpha}{\omega} A e^{ik_y y} e^{-\alpha|z|} \quad (1.46)$$

$$|z| \leq a \quad \Rightarrow \quad E_x(\vec{r}) = B e^{ik_y y} \sin(kz) \quad ; \quad H_y(\vec{r}) = \frac{ik}{\omega} B e^{ik_y y} \cos(kz) \quad (1.47)$$

and in both even and odd cases we have  $H_z(\vec{r}) = -k_y E_x / \omega$ .

# Beyond the limit angle

By imposing the conservation of both  $E_x$  and  $H_y$  at the interface  $z = a$ , we easily get:

$$\text{even modes} \Rightarrow \frac{\alpha}{k} = \tan(ka) ; A = B \cos(ka)e^{\alpha a} \quad (1.48)$$

$$\text{odd modes} \Rightarrow \frac{\alpha}{k} = -\cot(ka) ; A = B \sin(ka)e^{\alpha a} \quad (1.49)$$

where both  $k = \sqrt{(n\omega/c)^2 - k_y^2}$  and  $\alpha = \sqrt{(k_y^2 - \omega/c)^2}$  depend on  $\omega$  and  $k_y$ .

These relationships look very similar to the eigenvalue problem for a quantum well; For any fixed value of  $k_y$  only a discrete set of values of frequency (if any) can solve the eigenvalue problem. These are the guided modes and the electromagnetic field is confined in the slab, apart from evanescent tails which penetrate in the air. For  $n \rightarrow \infty$  and small values of  $k_y$  (i.e.,  $\omega < ck_y \ll n\omega$ ) the solutions are independent on  $k_y$  and we have  $ka = m\pi/2$  with  $m$  integer, similarly to the quantum well case with infinite barriers. Moreover the penetration out of the slab goes to zero, when increasing the dielectric function of the slab. There is therefore a similarity of the dielectric function for the electromagnetic modes and the potential function for electronic wavefunctions.

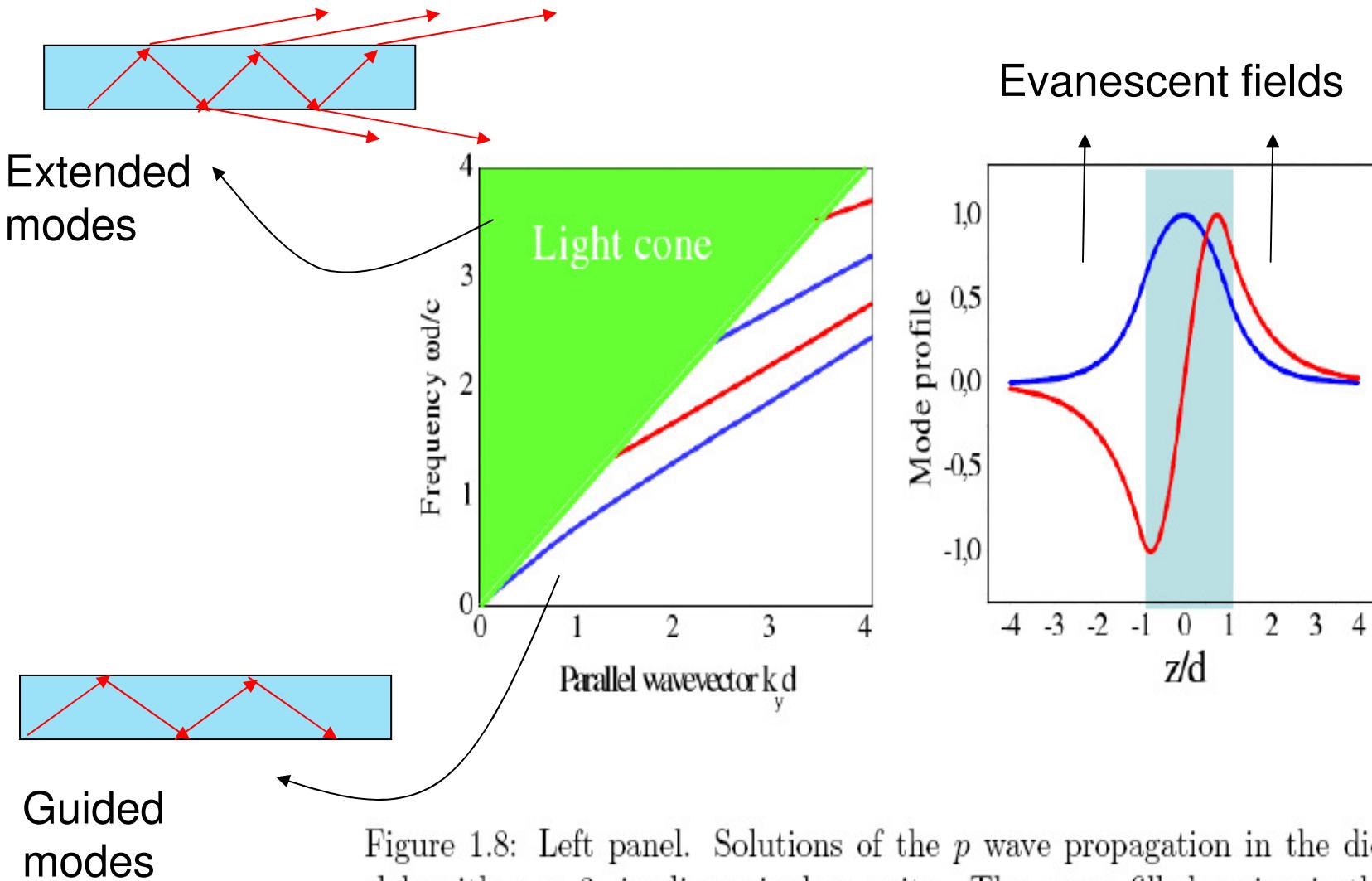
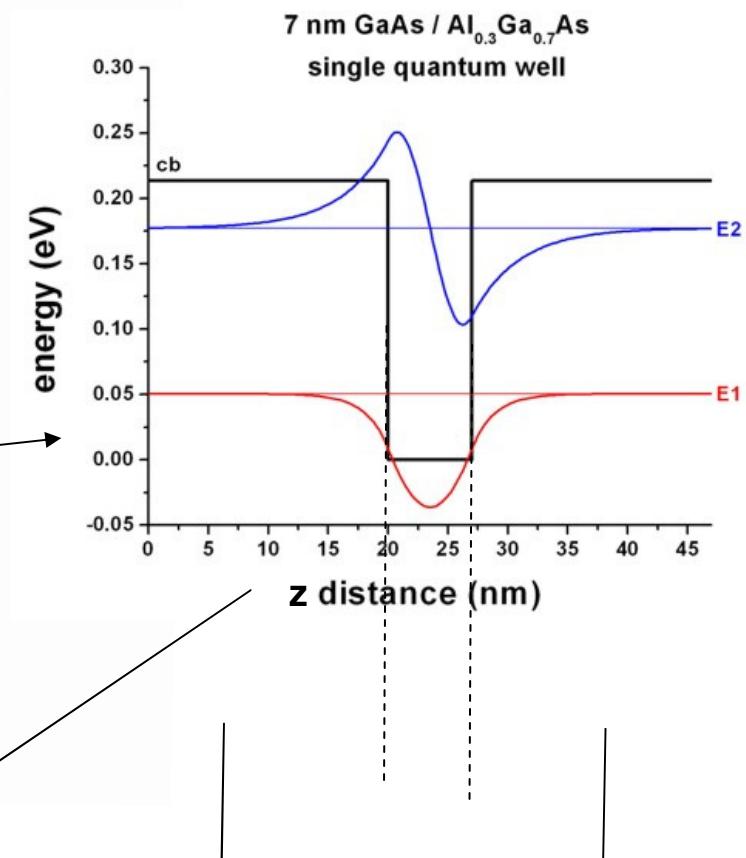
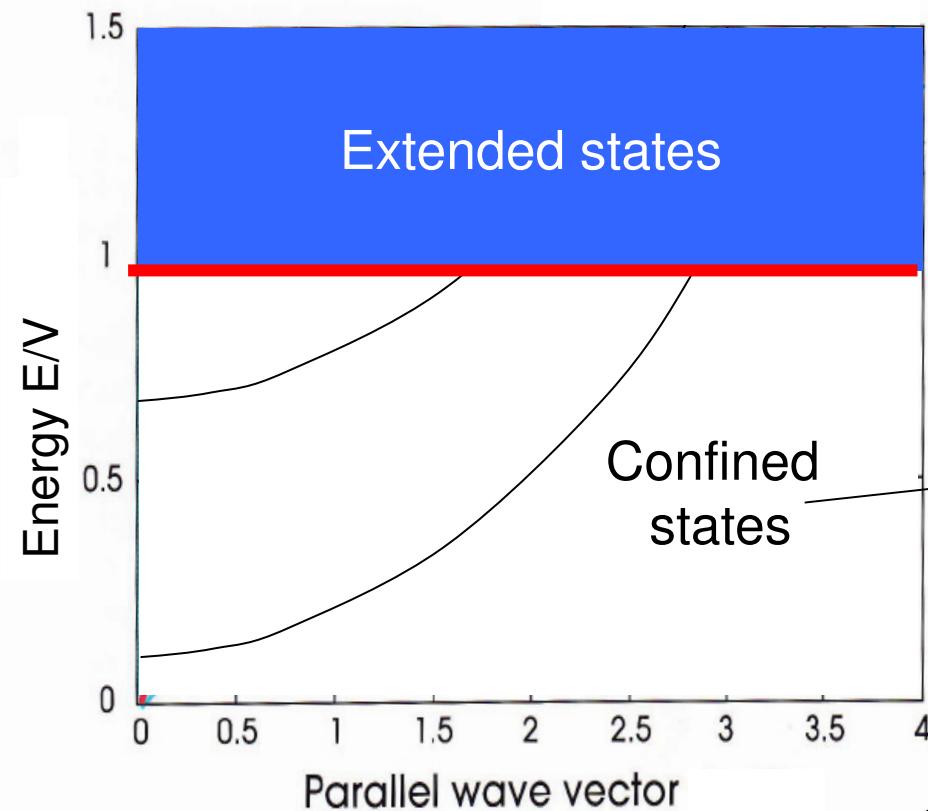


Figure 1.8: Left panel. Solutions of the  $p$  wave propagation in the dielectric slab with  $\epsilon = 3$ , in dimensionless unity. The green filled region is the light cone  $|k_y| \leq \omega/c$ , the blue and red lines are the even and odd guided modes, respectively. Right panel, Amplitude of the first even (blue line) and odd (red line) electric field  $p$  modes, for  $k_y a = 2$  along the  $z$  direction. The light blue region indicates the dielectric slab.

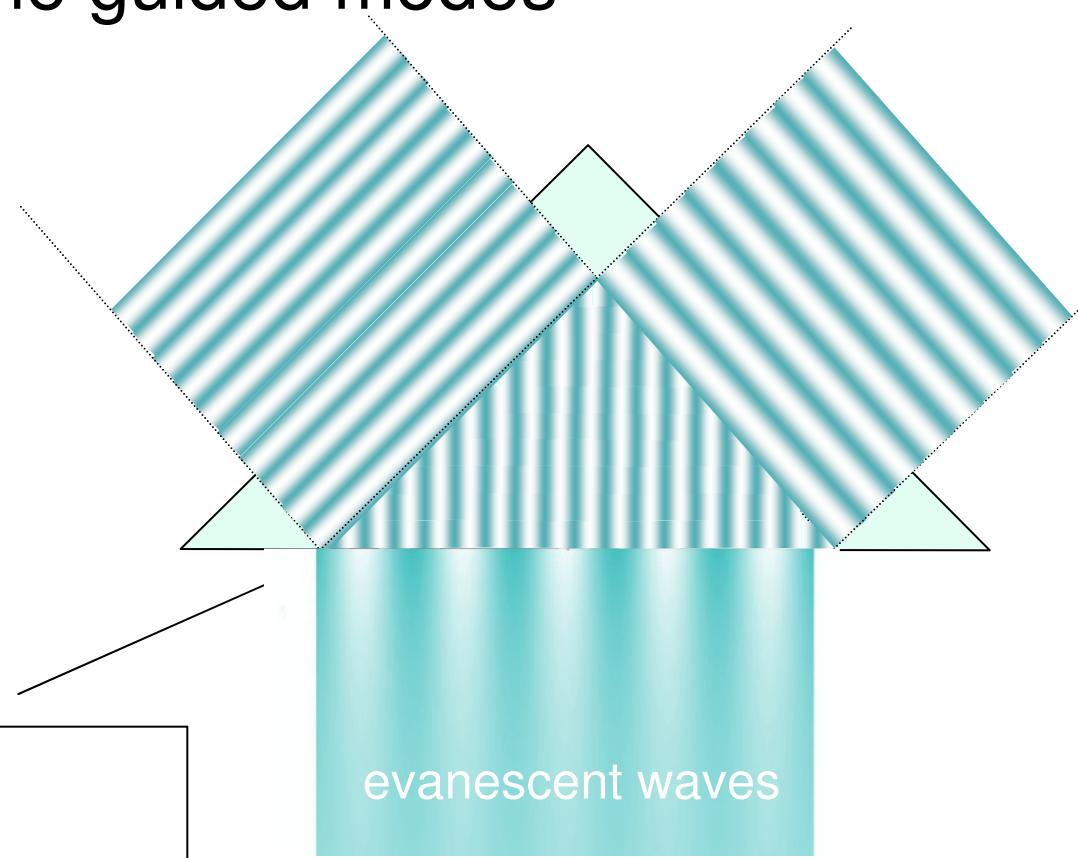
# Analogy with QWell



Confinamento moto lungo z

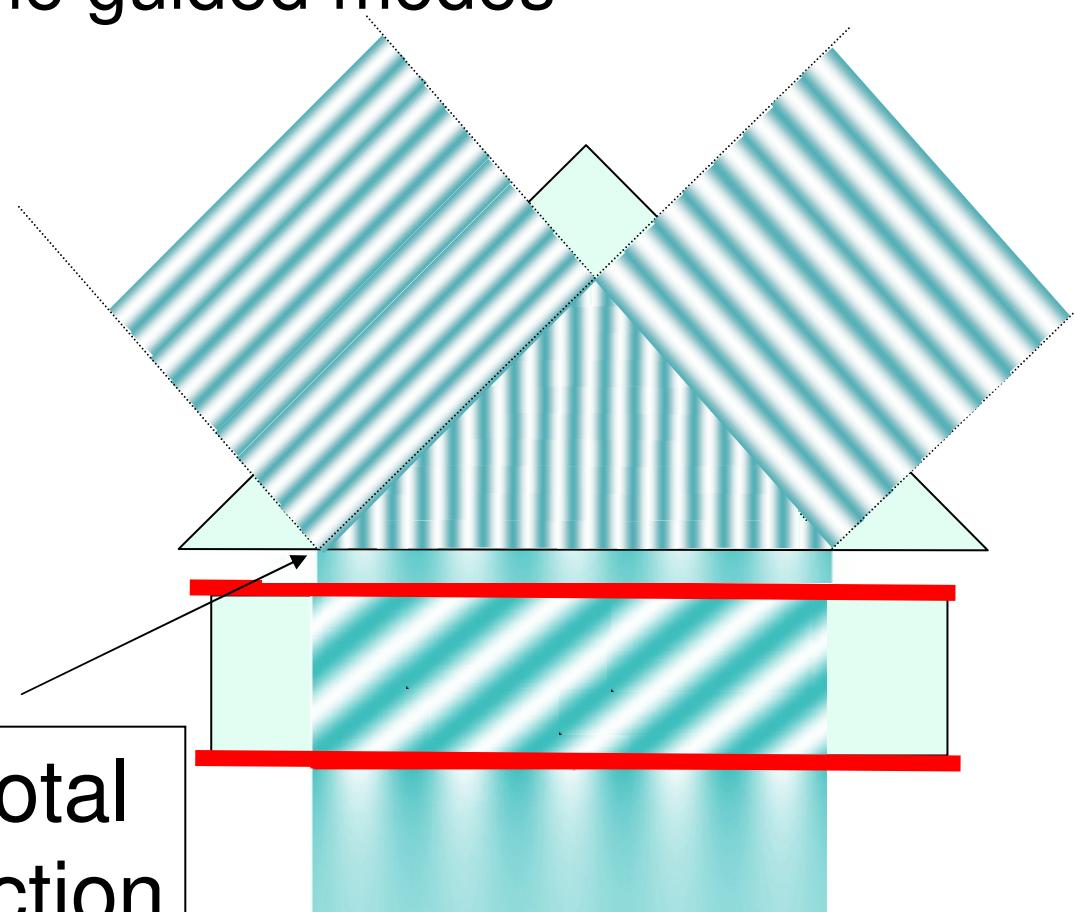
Evanescence wavefunction

# Excitation of the guided modes



Total  
internal reflection

# Excitation of the guided modes



Frustrated Total  
internal reflection