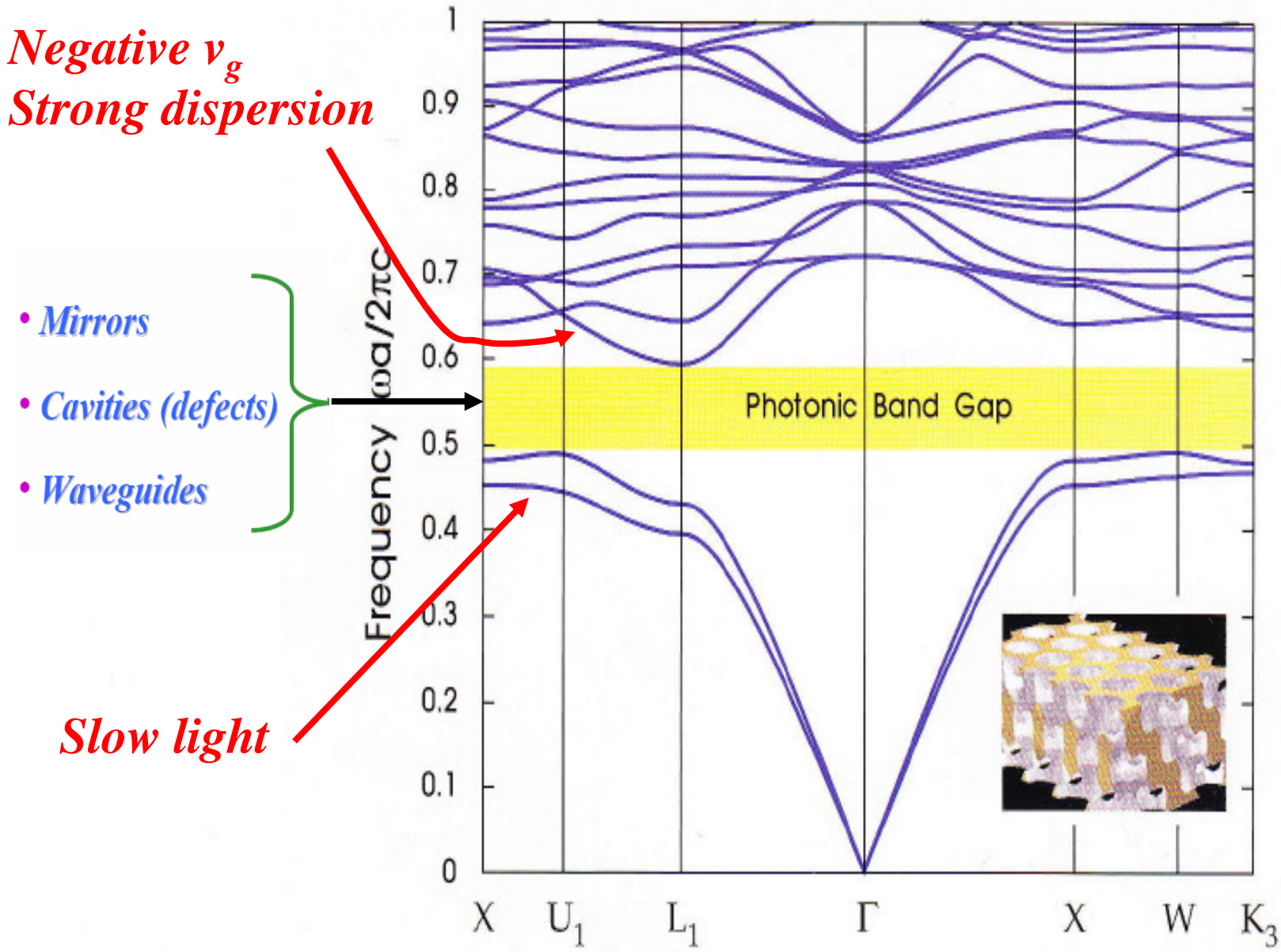
Applicazioni Fotonica 2: Emissione

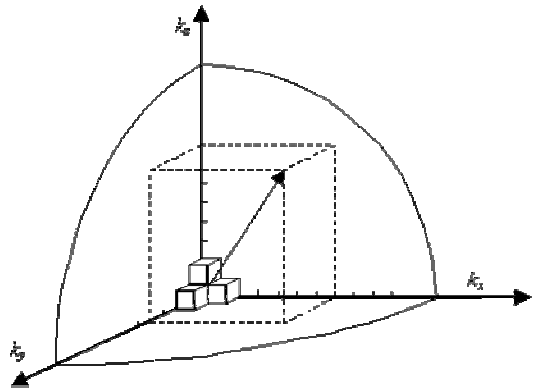

Negative v_g
Strong dispersion
• Mirrors
• Cavities (defects)
• Waveguides
Slow light
Frequency ωa/2πc
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Photonic Band Gap
X
U_1
L_1
Γ
X
W
K_3

Densità di stati fotonici nel vuoto

Volume in k-space per state $(2\pi / L)^3$

Volume in k-space occupied by states with frequency less than ω

$$V_k = 4\pi k^3 / 3 \quad where \quad k = \omega / c$$



Number of photon states in this volume

$$N(\omega) = 2\frac{4\pi k^3 / 3}{(2\pi / L)^3} = \frac{L^3}{3\pi^2}\frac{(\omega)^3}{c^3}$$

Density of states with frequency (ω,ω+dω) per unit volume

$$\rho(\omega) = \frac{1}{L^3}\frac{dN(\omega)}{d\omega} = \frac{\omega^2}{\pi^2 c^3}$$

Emissione spontanea

$$\Gamma = \frac{2\pi}{\hbar} \sum_{\vec{k},\lambda} \left| \left\langle \vec{d} \cdot \vec{E}_{\vec{k},\lambda}(\vec{r}) \right\rangle \right|^2 \delta(\hbar\omega_0 - \hbar\omega_k)$$

Regola aurea Fermi

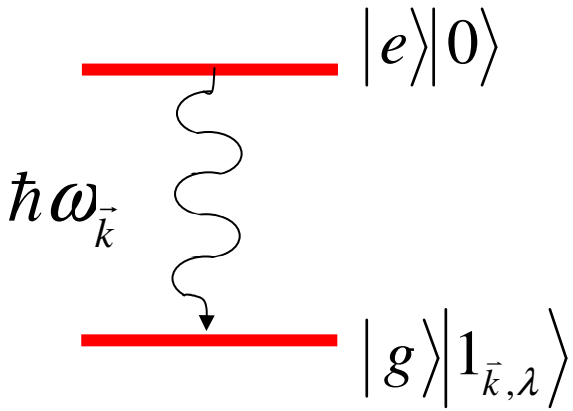
Campo quantizzato

$$\vec{E}_{\vec{k},\lambda}(\vec{r}) = i\left(\frac{\hbar\omega_k}{2\varepsilon_0 V}\right)^{1/2} \left[\hat{a}_{\vec{k},\lambda} e^{i\vec{k}\cdot\vec{r}} - \hat{a}^{+}_{\vec{k},\lambda} e^{-i\vec{k}\cdot\vec{r}}\right] \vec{e}_{\vec{k},\lambda}$$

Stati coinvolti

$$|0, e\rangle \Rightarrow |1_{\vec{k},\lambda}, g\rangle$$

$$\hbar\omega_{\vec{k}} = E_e - E_g = \hbar\omega_0$$



Emissione spontanea

Sostituendo

$$\Gamma_0 = \sum_{\vec{k},\lambda} \frac{2\pi}{\hbar} \left| \left\langle \vec{d} \cdot \vec{e}_{\vec{k},\lambda} \right\rangle \right|^2 \left(\frac{\hbar\omega_k}{2\varepsilon_0 V} \right) \delta(\hbar\omega - \hbar\omega_k) =$$

$$= \frac{\pi |d|^2 \omega}{3\varepsilon_0} \boxed{\frac{2}{V} \sum_{\vec{k}} \delta(\hbar\omega - \hbar\omega_k)} = \frac{\pi |d|^2 \omega}{3\varepsilon_0} \boxed{\rho(\hbar\omega)} =$$

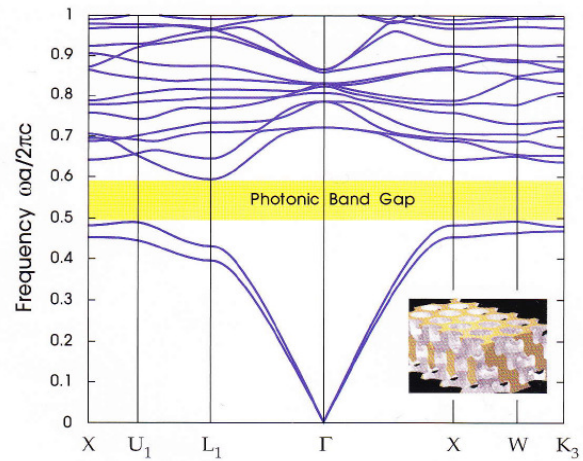
$$= \frac{\pi |d|^2 \omega}{3\varepsilon_0} \boxed{\frac{\omega^2}{\pi^2 c^3} \frac{1}{\hbar}} = \frac{4}{3} \frac{1}{4\pi\varepsilon_0} \frac{|d|^2 \omega^3}{\hbar \pi^2 c^3}$$

Densità di stati fotonici PhC

Volume in k-space per state $(2\pi / L)^3$

Volume in k-space occupied by states with frequency less than ω

$$V_k = V(\omega(\vec{k})) \quad where \quad \omega = \omega(\vec{k})$$



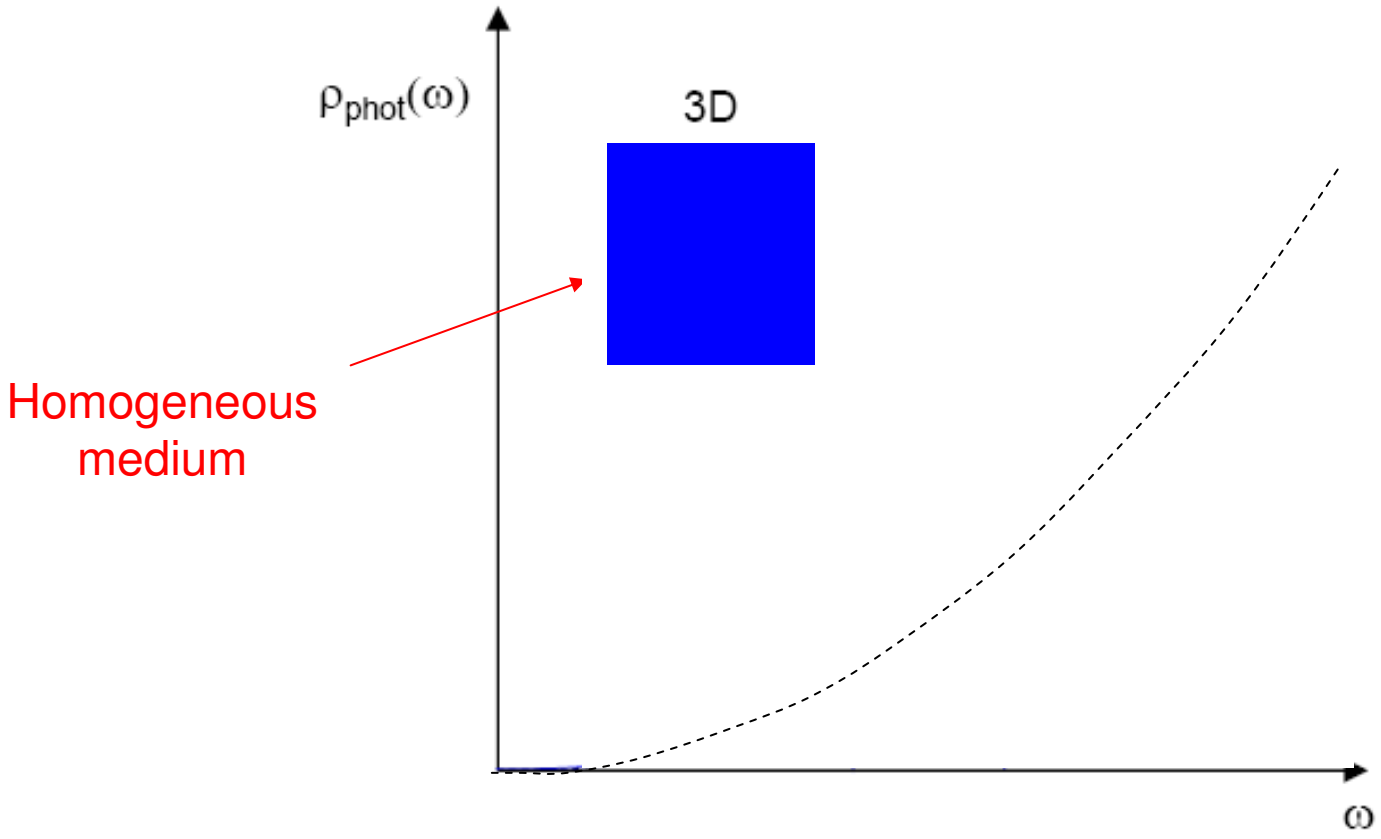
Number of photon states in this volume

$$N(\omega(\vec{k})) = 2\frac{V(\omega(\vec{k}))}{(2\pi / L)^3}$$

Density of states with frequency ($\omega, \omega + d\omega$) per unit volume

$$\rho(\omega) = \frac{1}{L^3}\frac{dN(\omega(\vec{k}))}{d\omega} = \frac{1}{L^3}\vec{\nabla}_{\vec{k}} N(\omega(\vec{k})) \cdot \frac{d\vec{k}}{d\omega} \quad \Rightarrow \quad \approx \frac{1}{v_g}$$

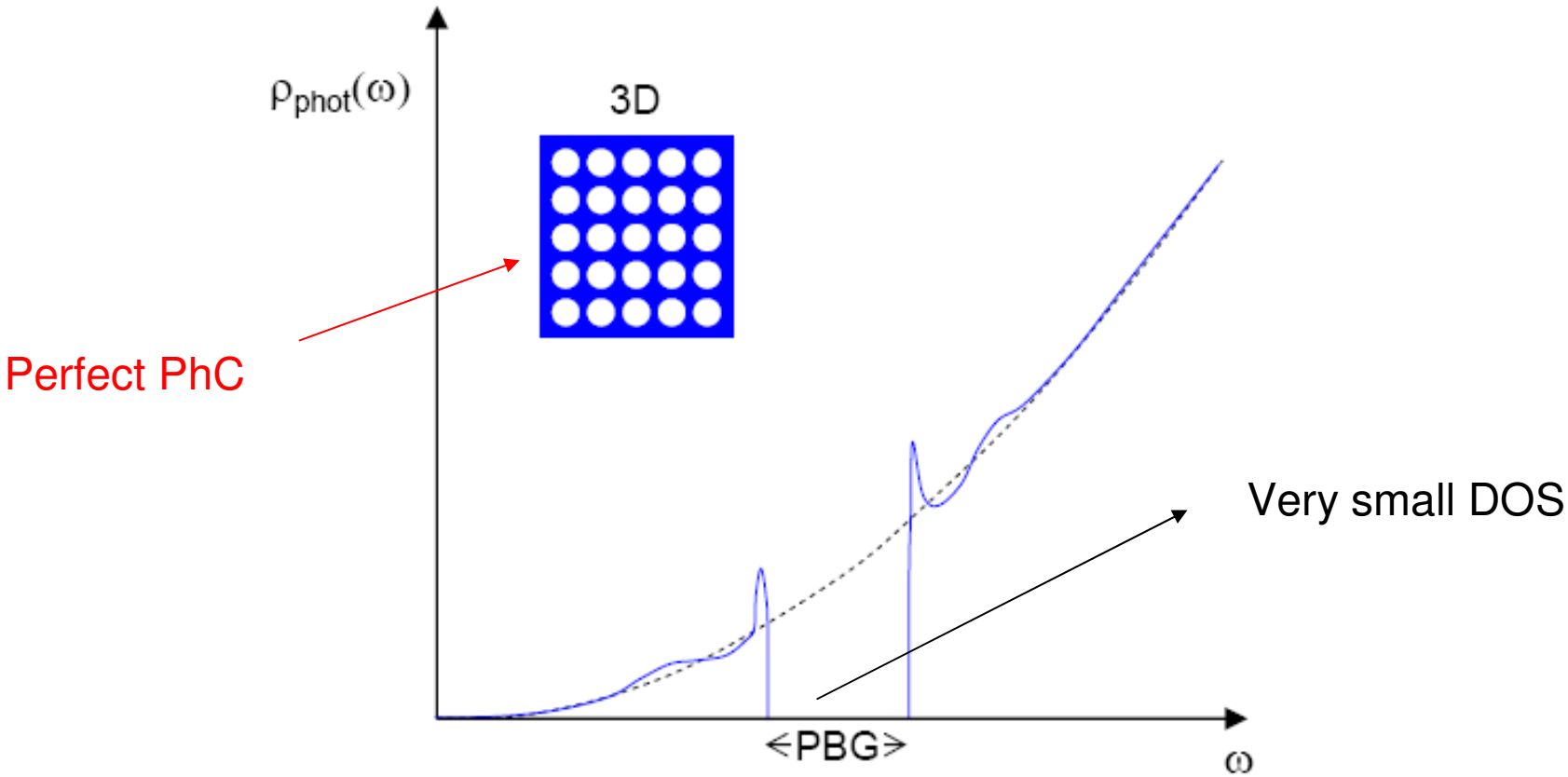
Densità di stati fotonici in mezzi isotropi



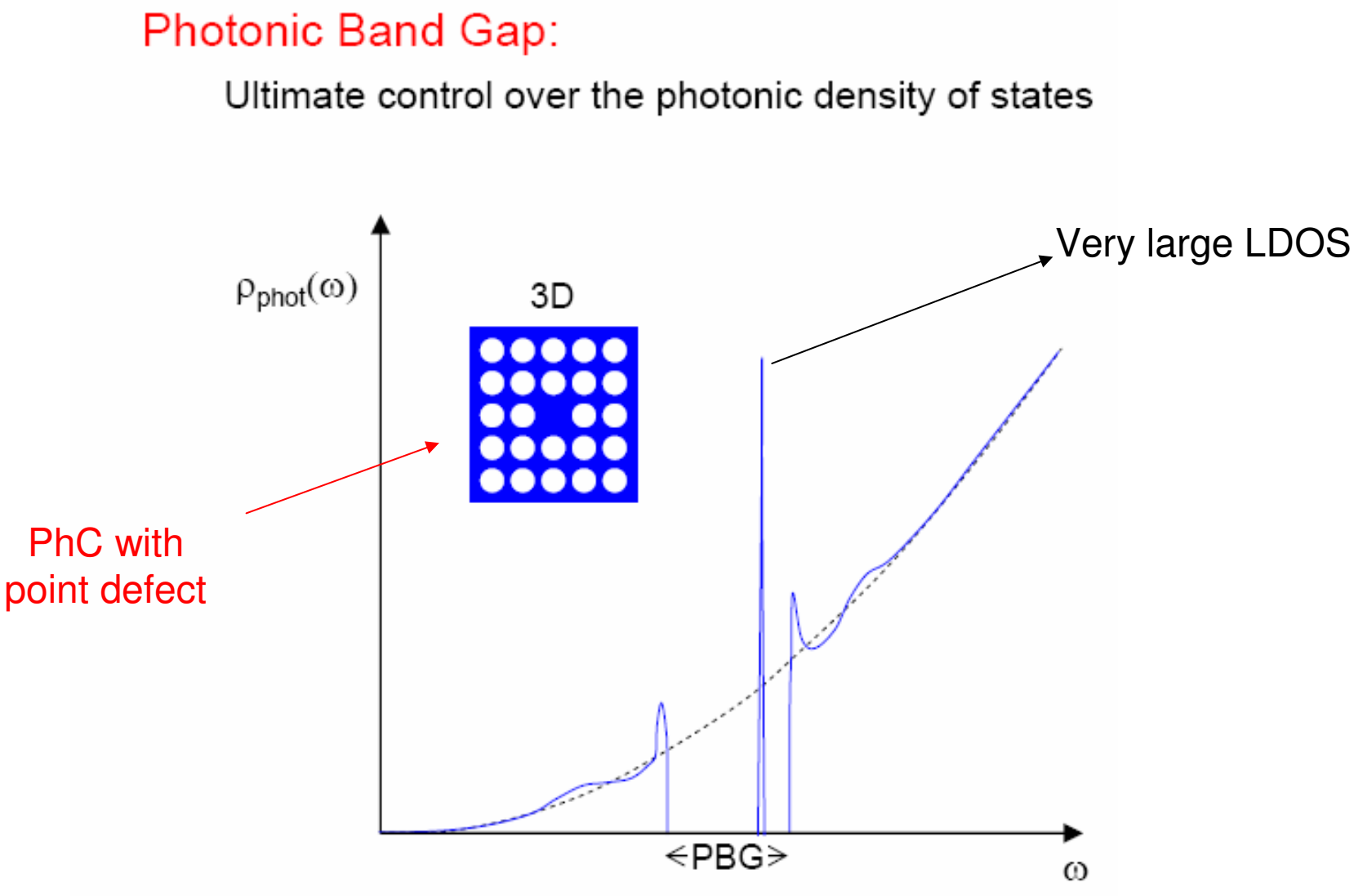
Densità di stati fotonici PhC

Photonic Band Gap:

Ultimate control over the photonic density of states



Densità di stati fotonici PhC con difetto



Purcell Effect - Control Spontaneous Emission

B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University.*—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2)\ \text{sec.}^{-1},$$

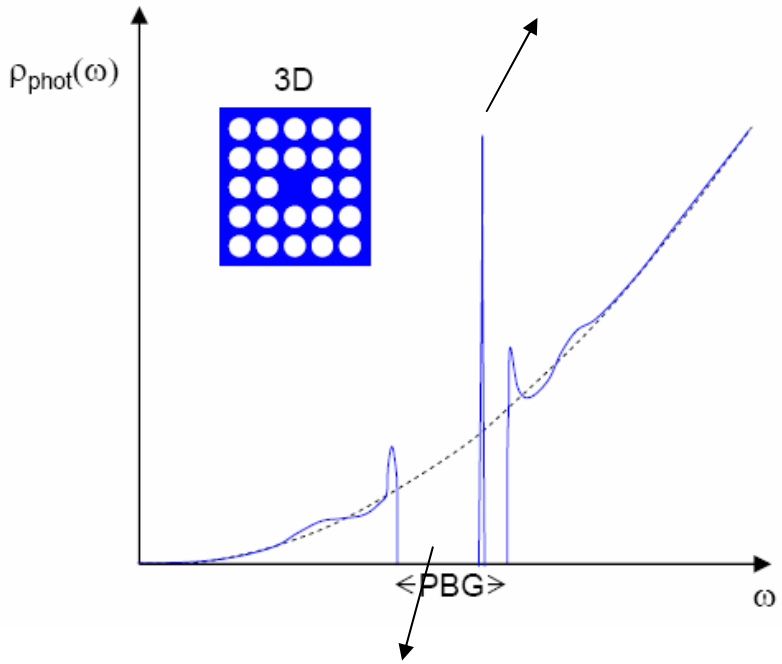
is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7$ sec.$^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V\sim a^3$, and if δ is the skin-depth at frequency ν, $f\sim\lambda^3/a^2\delta$. For a non-resonant circuit $f\sim\lambda^3/a^3$, and for $a<\delta$ it can be shown that $f\sim\lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7$ sec.$^{-1}$.

Purcell, Phys. Rev. **69**, 681 (1946).

Proceedings of APS meeting April 1946

$$A(\nu) = A_0\frac{8\pi\nu^2}{c^3} = A_0\rho_{ph}(\nu)$$

Acceleration of the emission



Suppression of the emission

Emissione spontanea in microcavità

$$\Gamma = \frac{2\pi}{\hbar}\sum_{\mu}\left|\left\langle \vec{d}\cdot\vec{E}_{\mu}(\vec{r})\right\rangle\right|^{2}\delta\left(\hbar\omega-\hbar\omega_{\mu}\right)$$

Regola aurea Fermi

Campo quantizzato in microcavità

$$\vec{E}_{\mu}(\vec{r}) = i\left(\frac{\hbar\omega_{\mu}}{2\varepsilon_0}\right)^{1/2}\left[\hat{a}_{\mu}\alpha_{\mu}(\vec{r})-\hat{a}_{\mu}^{+}\alpha_{\mu}^{*}(\vec{r})\right]\vec{e}_{\mu}$$

$$\int_V \left|\alpha_{\vec{k}}(\vec{r})\right|^2 dV = 1$$

Nel vuoto

$$\alpha_{\vec{k}}(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{V}}$$

Sostituendo

$$\Gamma = \sum_{\mu}\frac{2\pi}{\hbar}\left|\left\langle \vec{d}\cdot\vec{e}_{\mu}\alpha_{\mu}(\vec{r})\right\rangle\right|^{2}\left(\frac{\hbar\omega_{\mu}}{2\varepsilon_0}\right)\delta\left(\hbar\omega-\hbar\omega_{\mu}\right) =$$

$$= \frac{\pi\left|d\right|^{2}\omega}{\varepsilon_0}\sum_{\mu}\left|\alpha_{\mu}(\vec{r})\right|^{2}\delta\left(\hbar\omega-\hbar\omega_{\mu}\right) =$$

$$\Gamma_{\mu} = \frac{\pi\left|d\right|^{2}\omega}{\varepsilon_0}\boxed{\left|\alpha_{\mu}(\vec{r})\right|^{2}\rho_{\mu}(\hbar\omega)}$$

LDOS

Emissione spontanea in cavità

$$\Gamma_\mu = \frac{\pi |d|^2 \omega}{\varepsilon_0} \left|\alpha_\mu(\vec{r})\right|^2 \rho_\mu(\hbar\omega) =$$

$$= \frac{\pi |d|^2 \omega}{\varepsilon_0} \left|\alpha_\mu(\vec{r})\right|^2 \frac{1}{\hbar} \frac{1}{2\pi} \frac{\gamma_\mu}{(\omega - \omega_\mu)^2 + \gamma_\mu{}^2 / 4}$$

Alla risonanza

$$\Gamma_\mu = \frac{|d|^2 \omega}{\varepsilon_0} \left|\alpha_\mu(\vec{r})\right|^2 \frac{1}{\hbar} \frac{2}{\gamma_\mu} =$$

$$= \frac{2|d|^2 Q}{\hbar \varepsilon_0} \left|\alpha_\mu(\vec{r})\right|^2 = \frac{2|d|^2 Q}{\hbar \varepsilon_0 V_c}$$

Il rate dipende dalla posizione dell'emettitore rispetto alla LDOS

$$\boxed{\alpha_\mu(\vec{r}) \approx \frac{1}{\sqrt{V_C}}}$$

Effetto Purcell

In cavità

$$\Gamma_\mu = \frac{2|d|^2 Q}{\hbar \varepsilon_0 V_c}$$

Nel vuoto

$$\Gamma_0 = \frac{4}{3}\frac{1}{4\pi\varepsilon_0}\frac{|d|^2 \omega^3}{\hbar c^3} = \frac{8\pi^2}{3\varepsilon_0 \hbar}\frac{|d|^2}{\lambda^3}$$

Fattore di Purcell

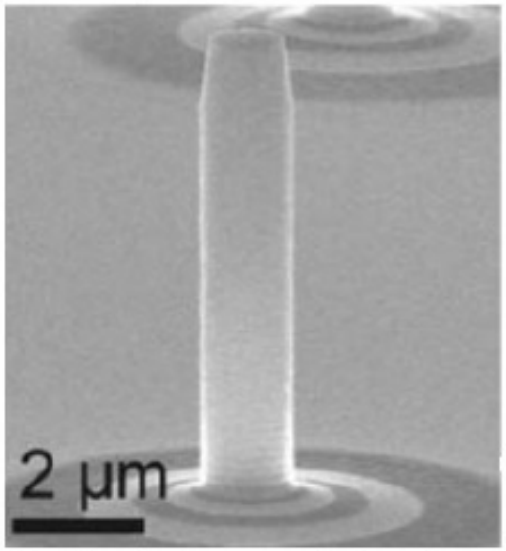
$$F_P = \frac{3}{4\pi^2}\frac{\lambda^3}{V_c}Q$$

Confronto

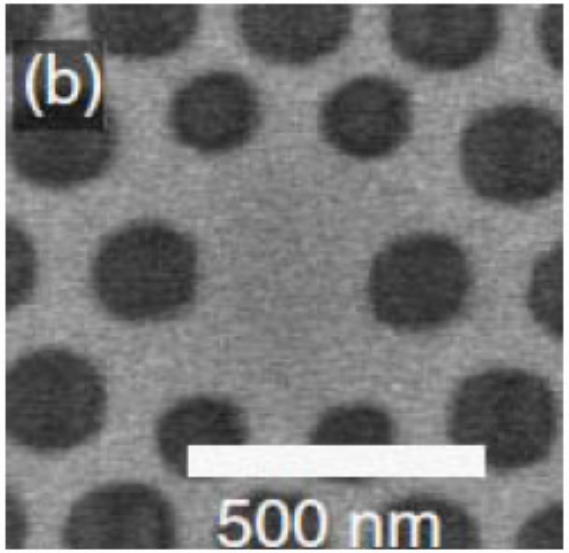
$$\Gamma_\mu = \frac{2|d|^2 Q}{\hbar \varepsilon_0 V_c} = \frac{3}{4\pi^2}\frac{\lambda^3}{V_c}Q\Gamma_0 = F_P \Gamma_0$$

Buone approssimazioni di cavità 3D

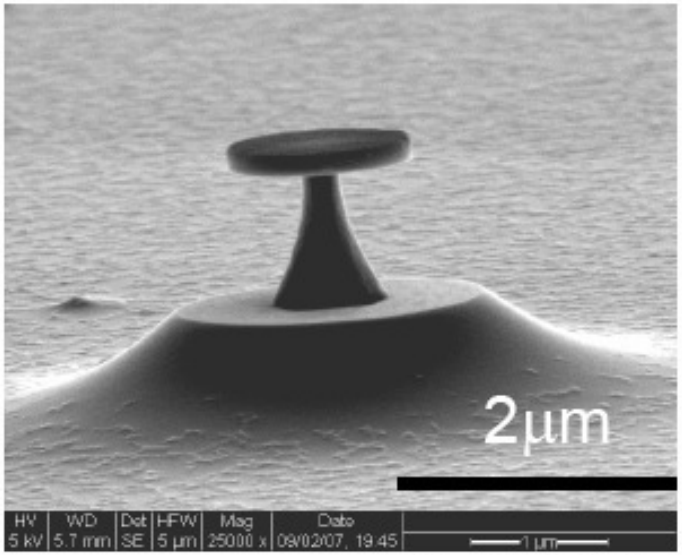
Micropillar



Photonic Crystal



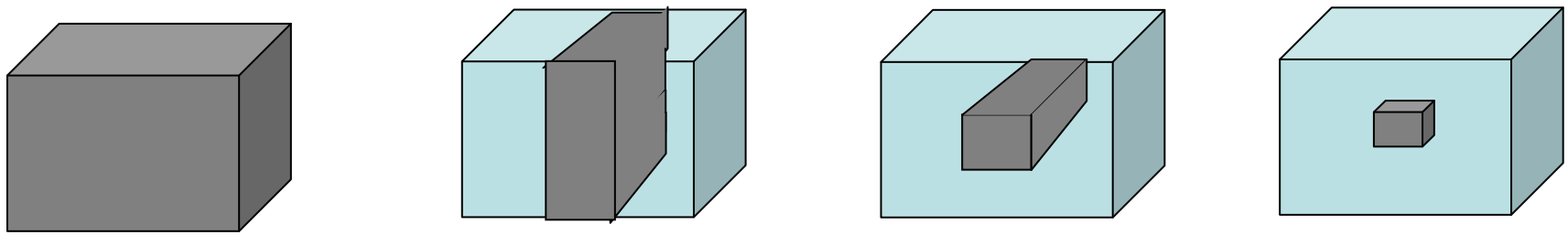
Microdisk

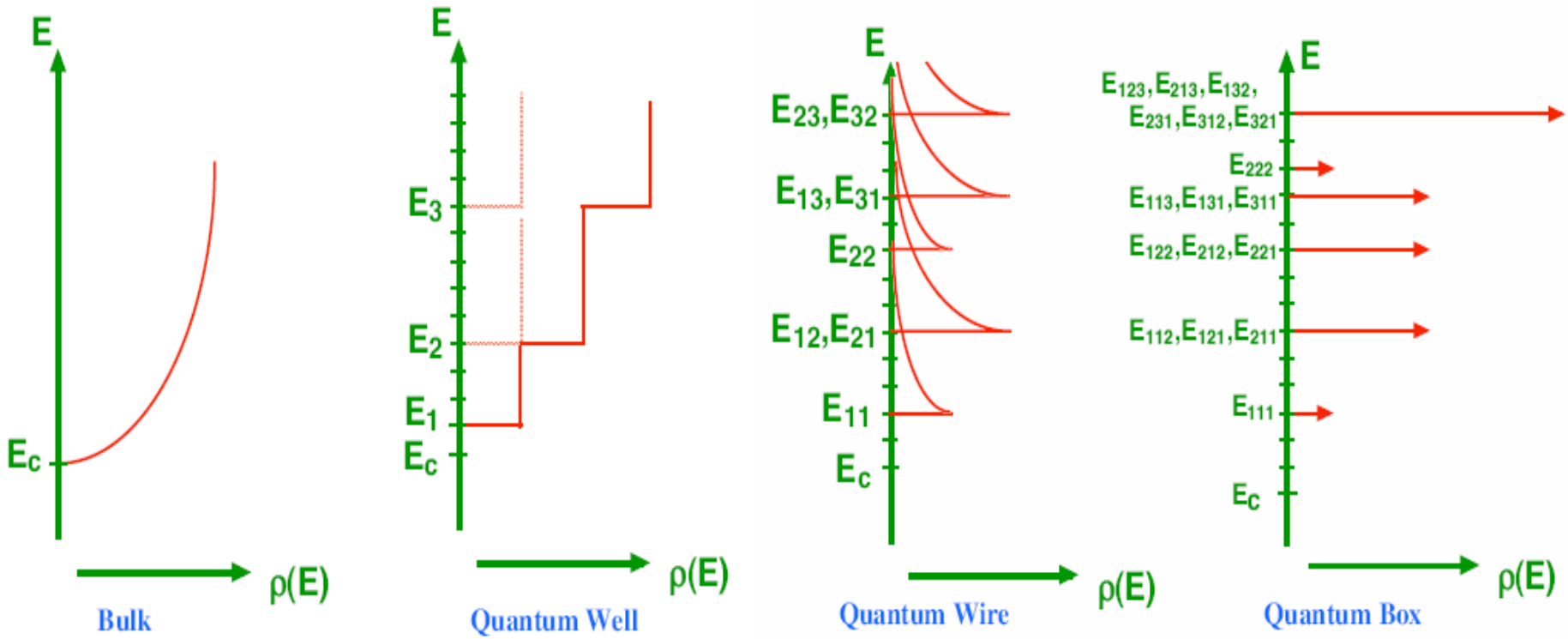


	Micropillar	Photonic Crystal	Microdisk
Q @930nm	23000	30000	20000
V	$5\ (\lambda/n)^3\sim$	$0.5\ (\lambda/n)^3\sim$	$5\ (\lambda/n)^3\sim$
Fp	61	145	125

Emittitore a semiconduttore
Quantum Dots

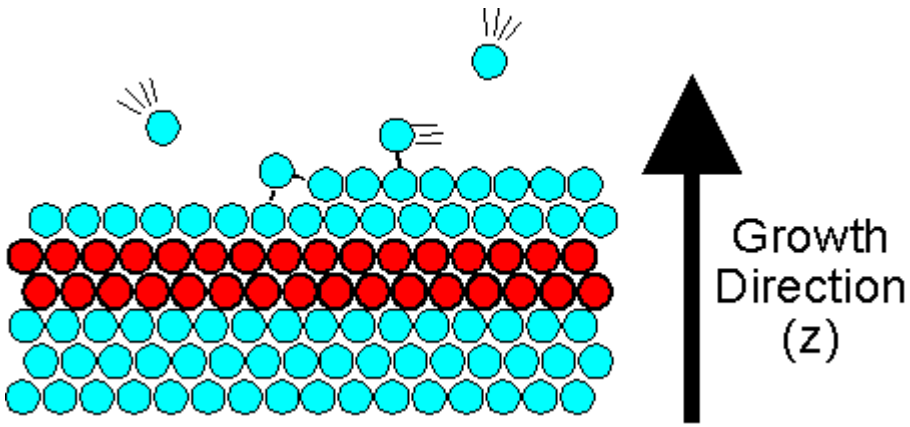
Density of states



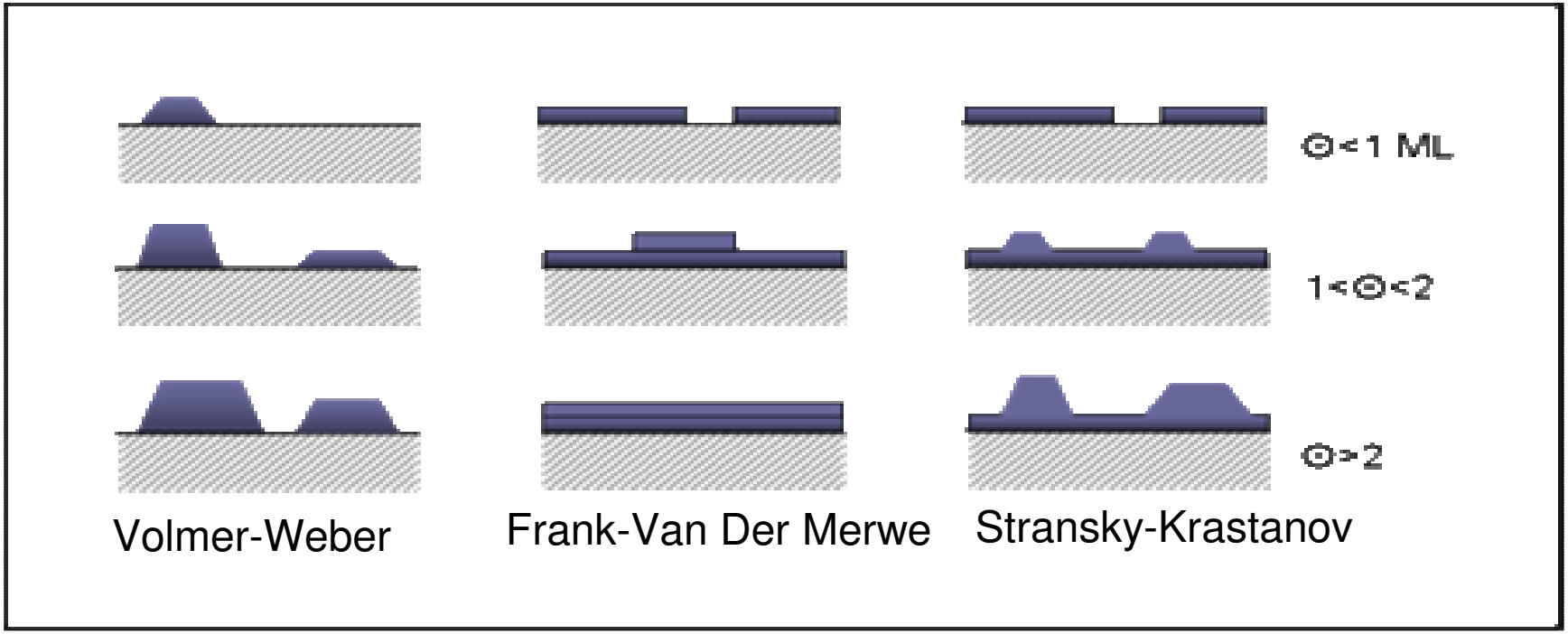


Atomic-like spectrum

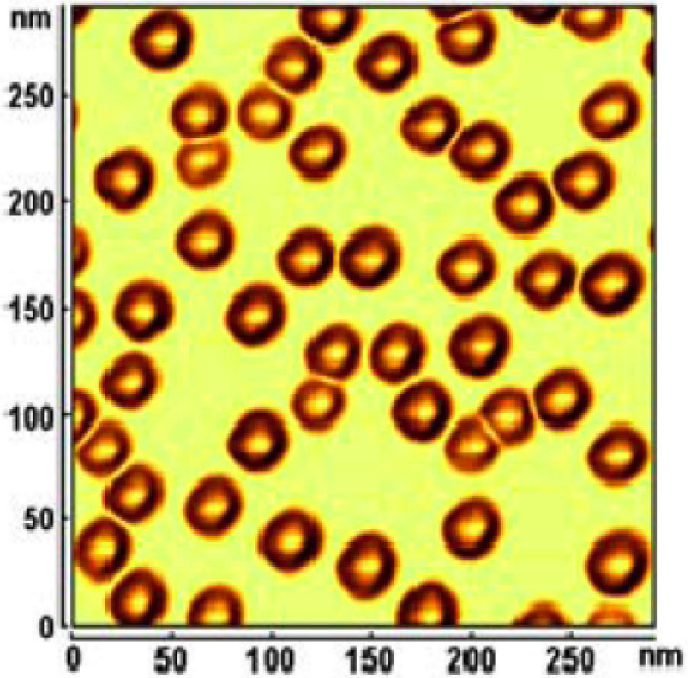
Crescita epitassiale



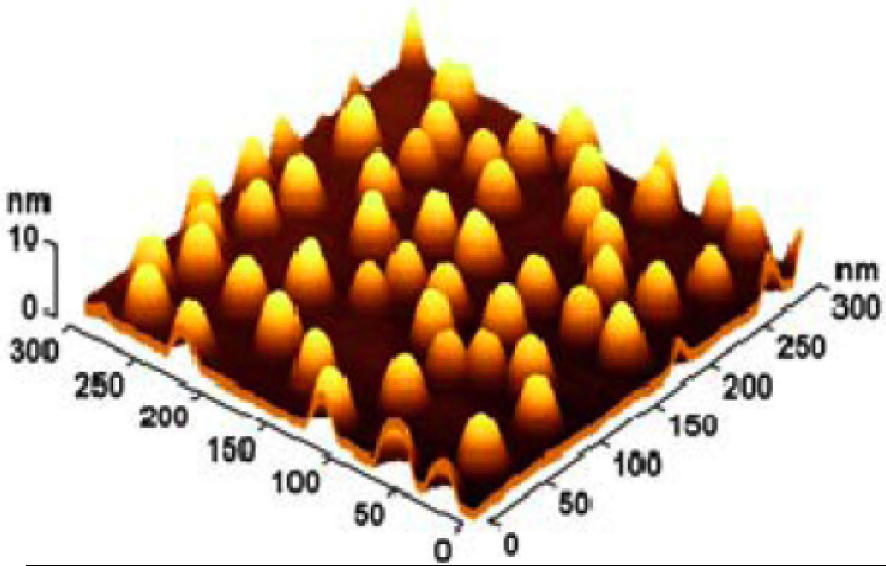
3 tipi di epitassia



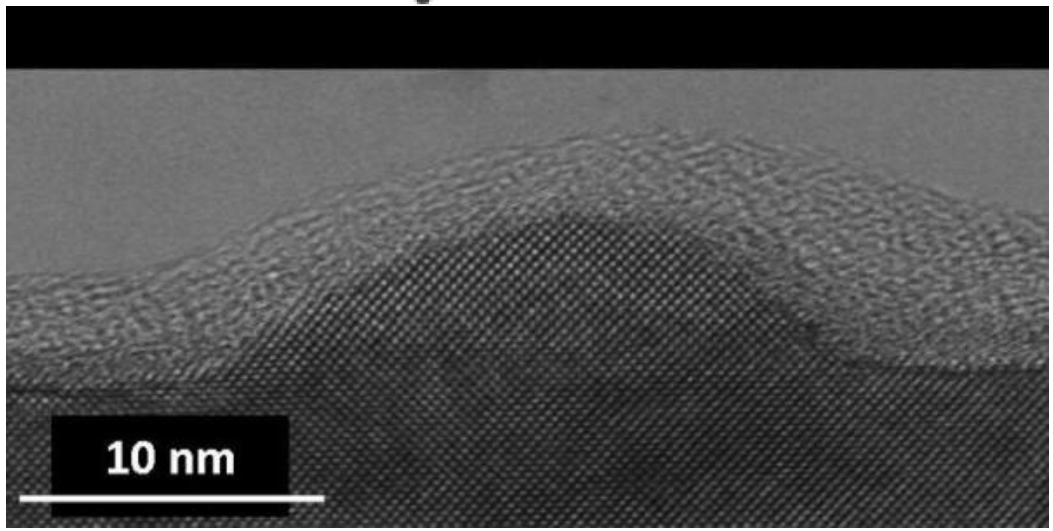
Quantum dots SK



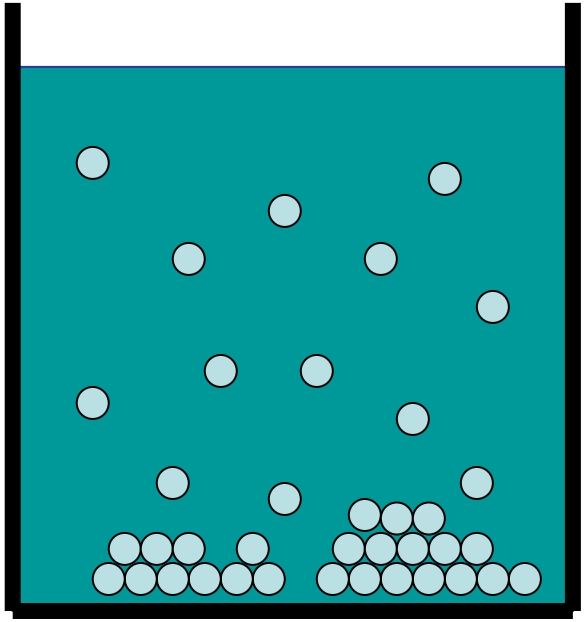
AFM

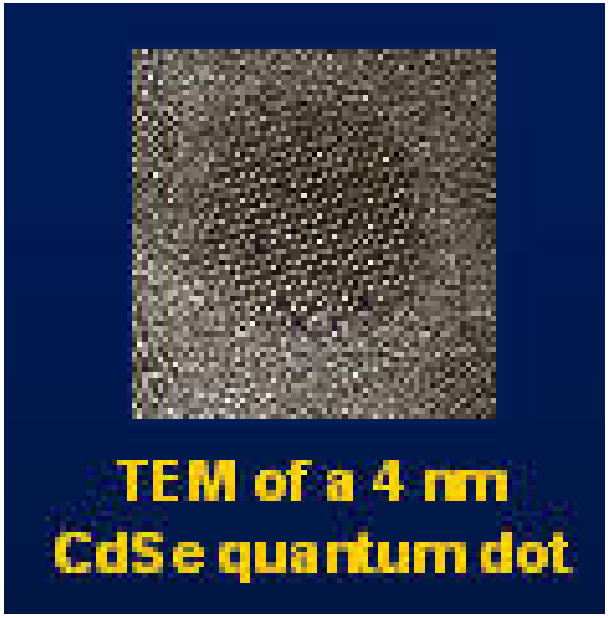


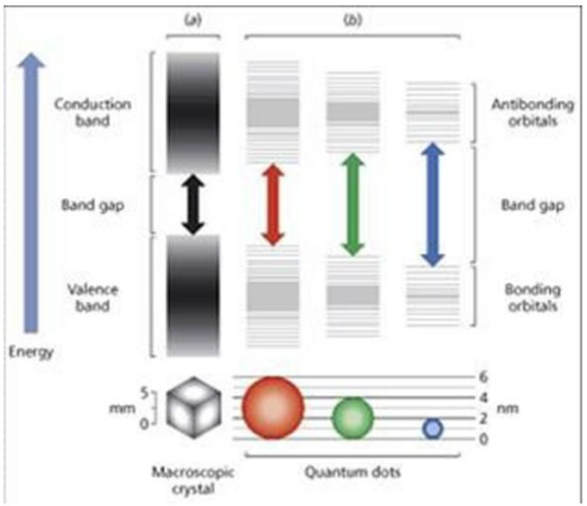
TEM



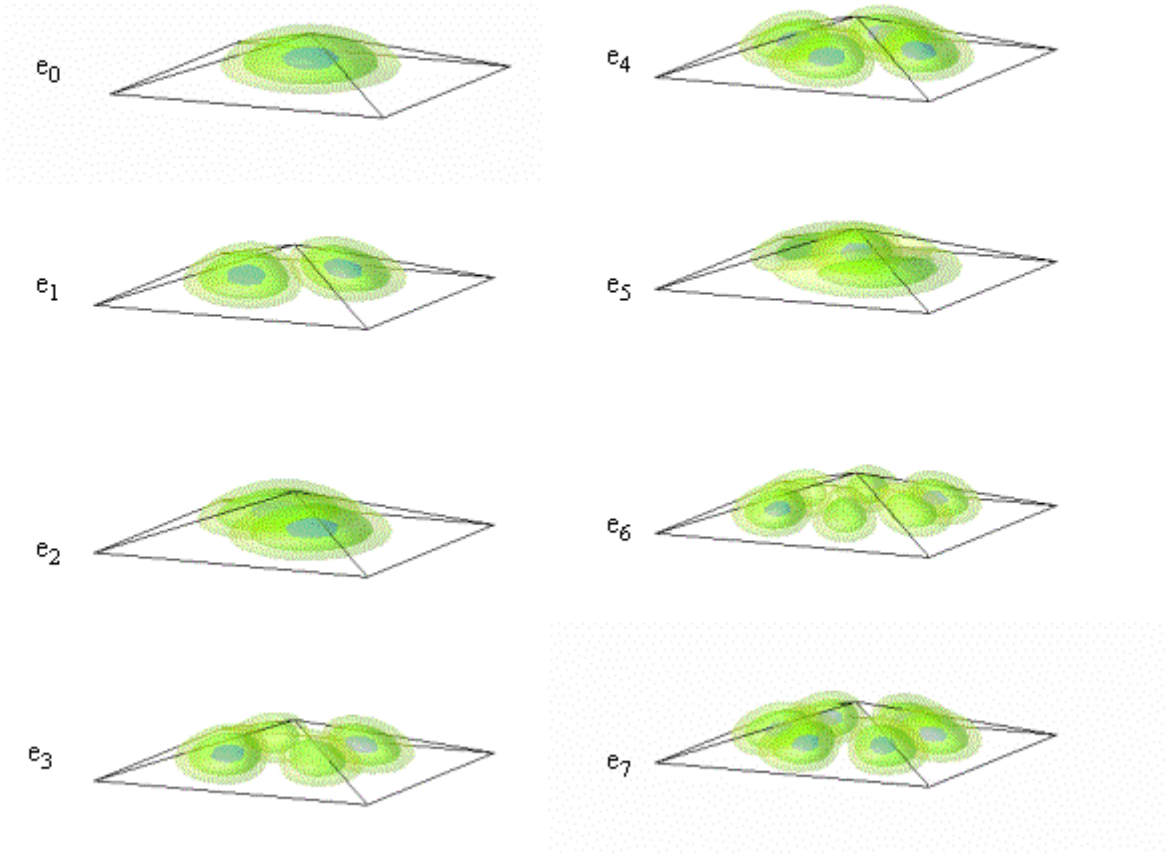
Quantum dots Colloidali

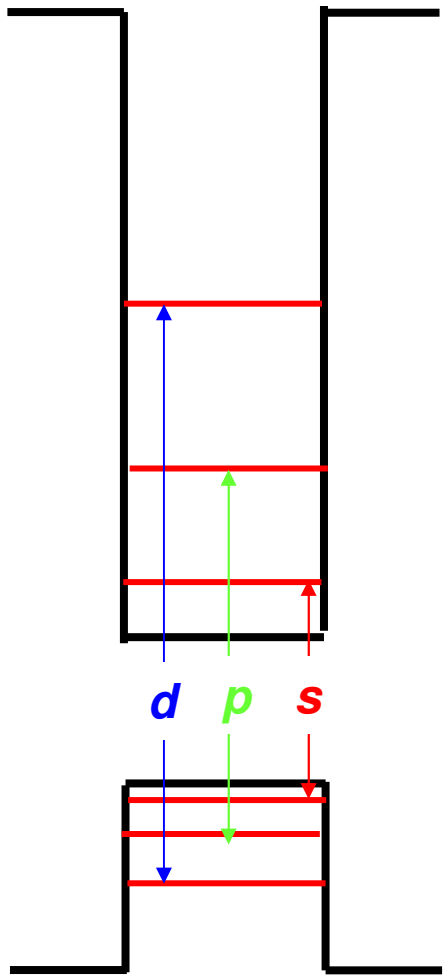






Electron states in InAs 36h/250b dot



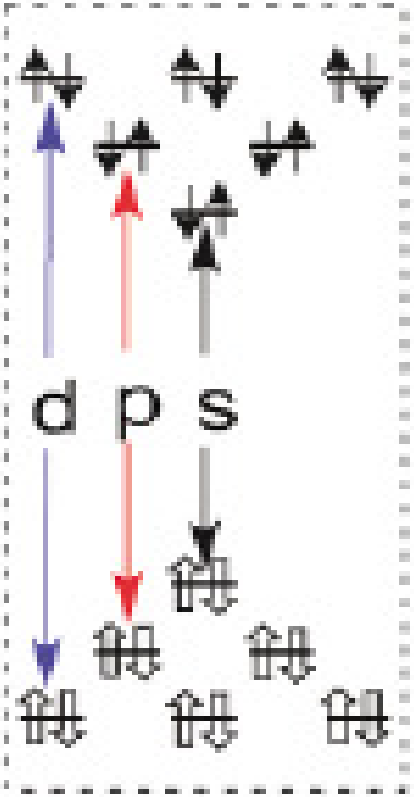


d=10 nm → **V=1000 nm^3** → **N=64000 atomi**

VOLUME 92, NUMBER 18 PHYSICAL REVIEW LETTERS week ending 7 MAY 2004

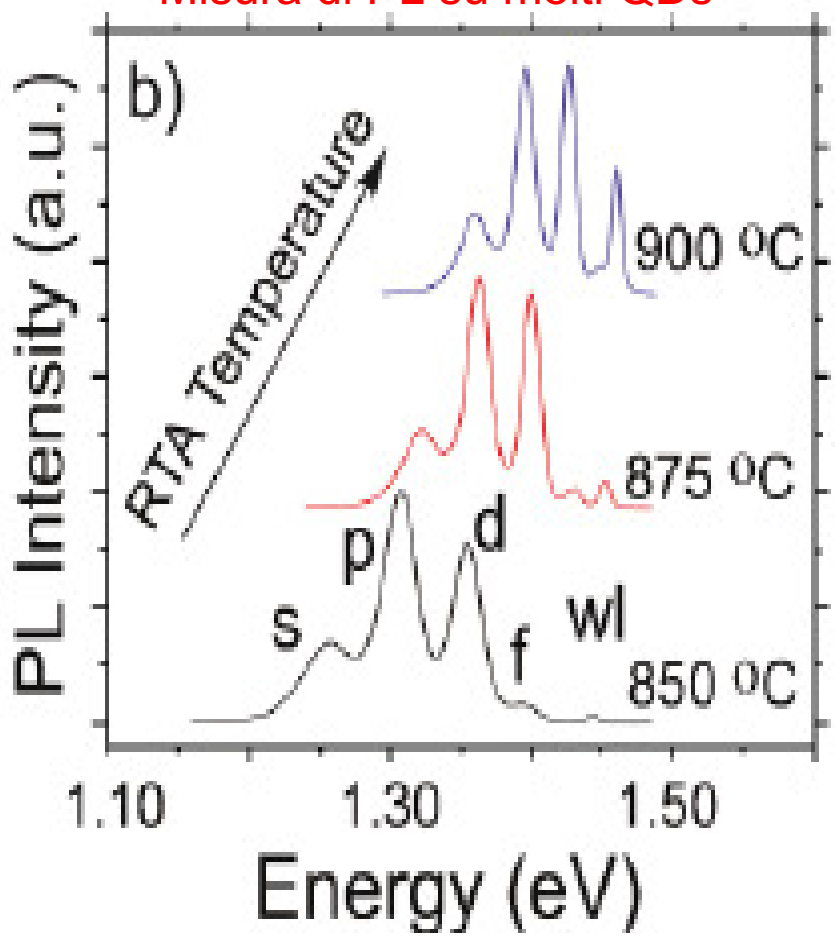
Excitonic Energy Shell Structure of Self-Assembled InGaAs/GaAs Quantum Dots

S. Raymond,[1] S. Studenikin,[1] A. Sachrajda,[1] Z. Wasilewski,[1] S. J. Cheng,[1] W. Sheng,[1] P. Hawrylak,[1] A. Babinski,[2,4] M. Potemski,[1,2] G. Ortner,[3] and M. Bayer[3]

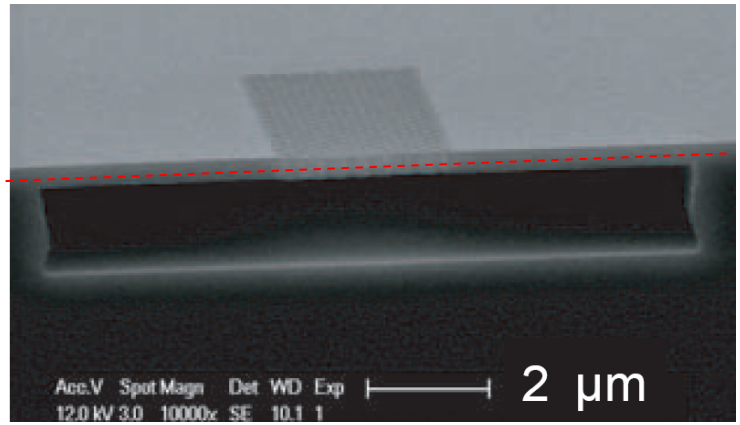


Atomic-like spectrum

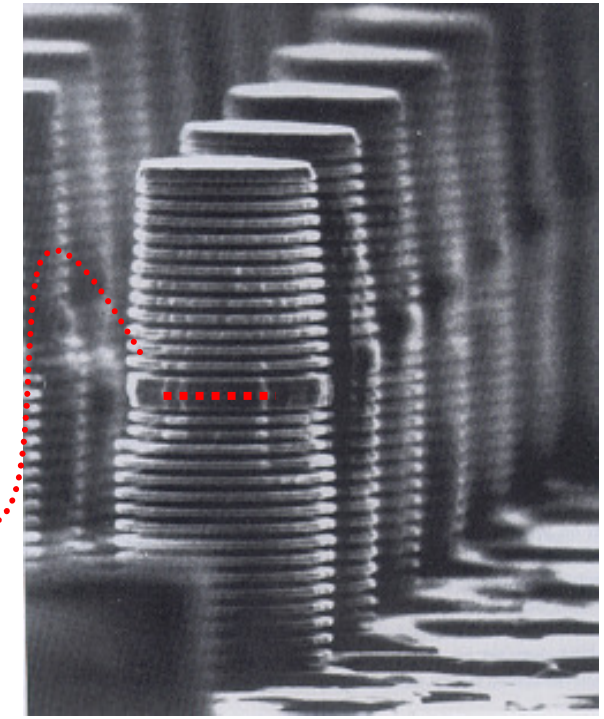
Misura di PL su molti QDs

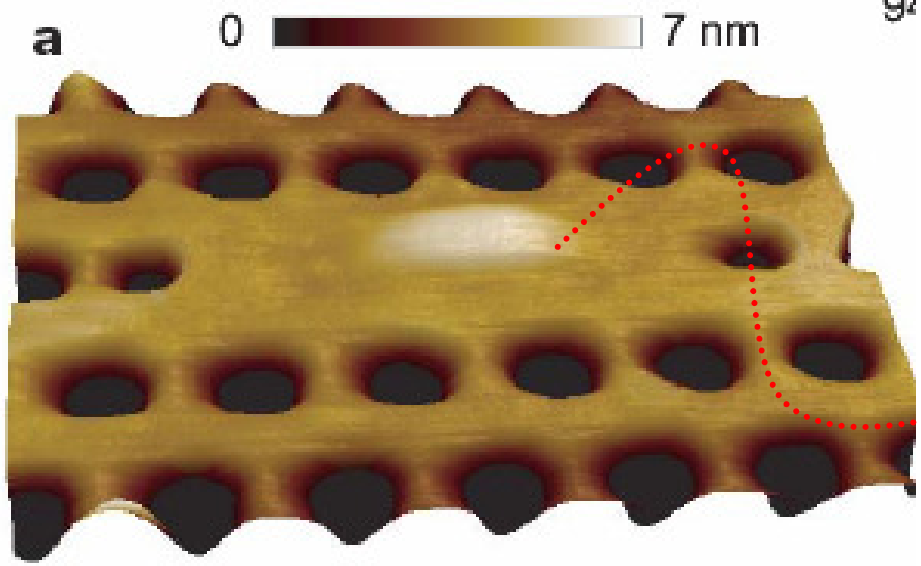


SK-QD in MC (il PhC è costruito dopo)



QD layer





QD

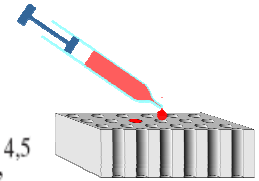
Colloidal QD in MC (il PhC è costruito prima)

RAPID COMMUNICATIONS

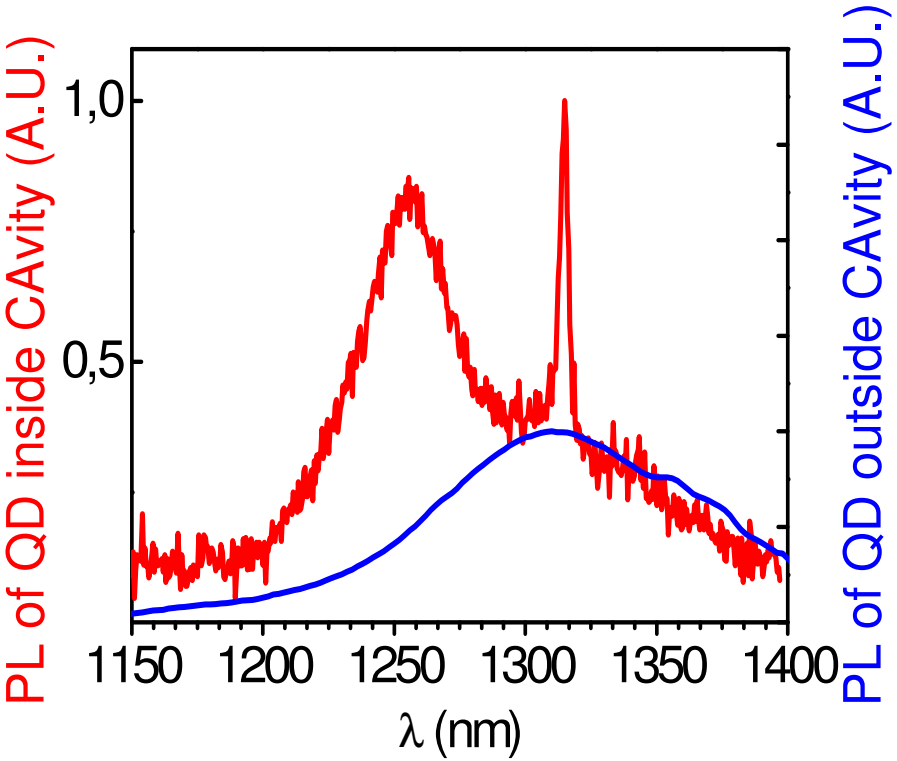
PHYSICAL REVIEW E **78**, 045603(R) (2008)

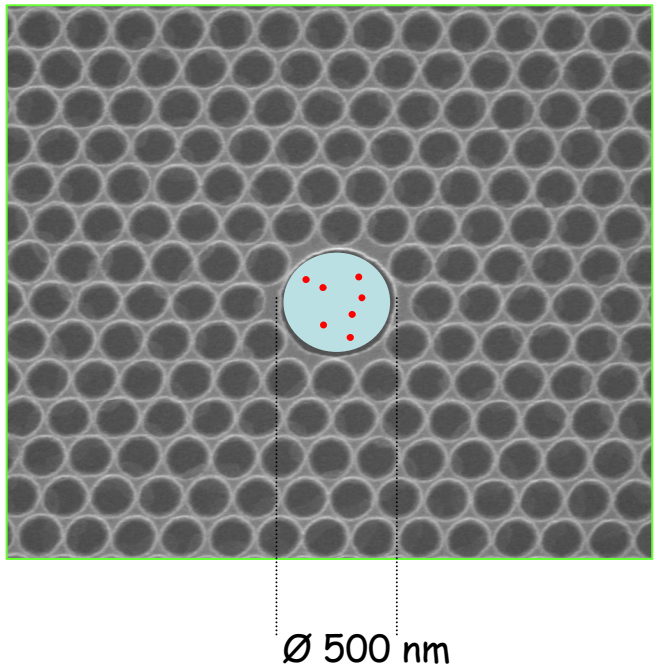
Local nanofluidic light sources in silicon photonic crystal microcavities

Silvia Vignolini,[1,*] Francesco Riboli,[1] Francesca Intonti,[2,1] Michele Belotti,[3,4] Massimo Gurioli,[1] Yong Chen,[4,5] Marcello Colocci,[1] Lucio Claudio Andreani,[3] and Diederik S. Wiersma[1]



Local Source: colloidal PbS QDs suspended in Toluene





Purcell effect in micropillars with oxidized Bragg mirrors

phys. stat. sol. (c) **5**, No. 7, 2433–2436 (2008)

L. Cavigli[1,*], L. Lunghi[2], M. Abbarchi[1], A. Vinattieri[1] B. Alloing[3], C. Zinoni[3] A. Fiore[3], A. Gerardino[2] P. Frigeri[4] L. Seravalli[4], S. Franchi[4], and M. Gurioli[1]

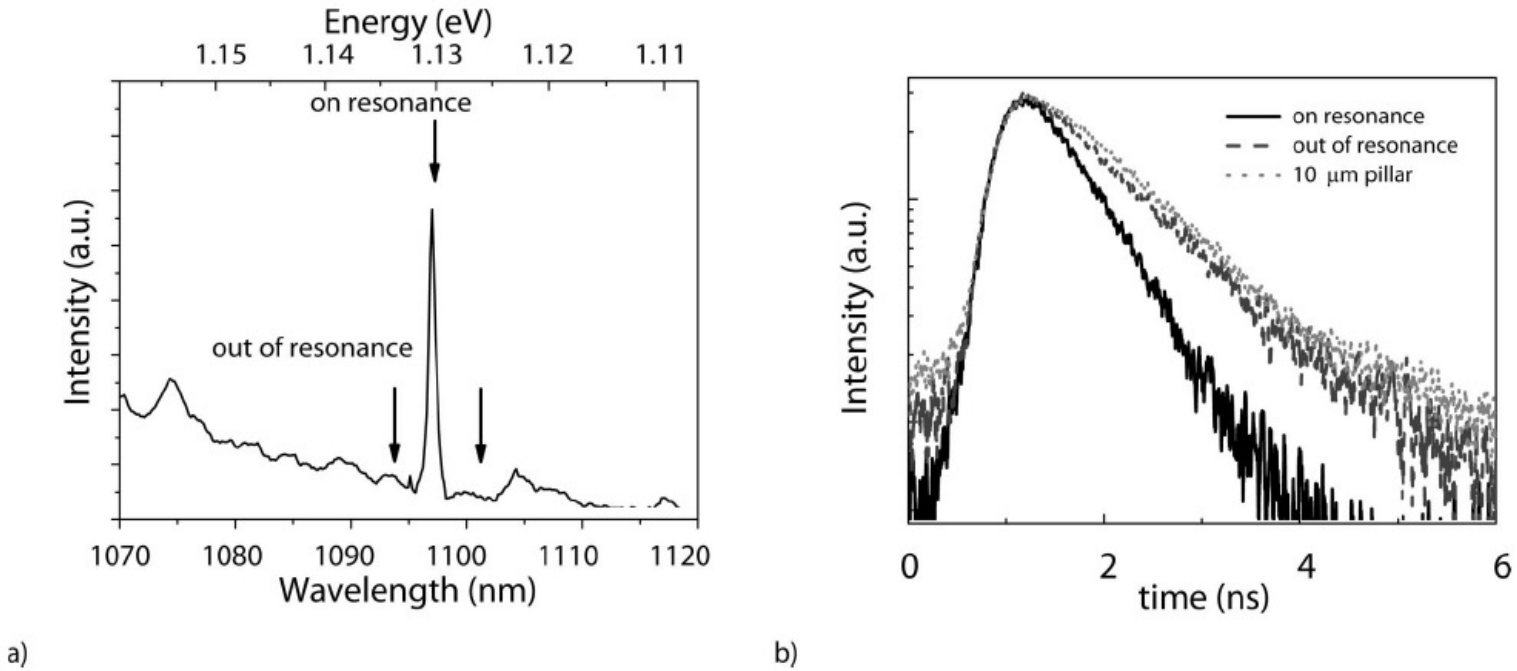
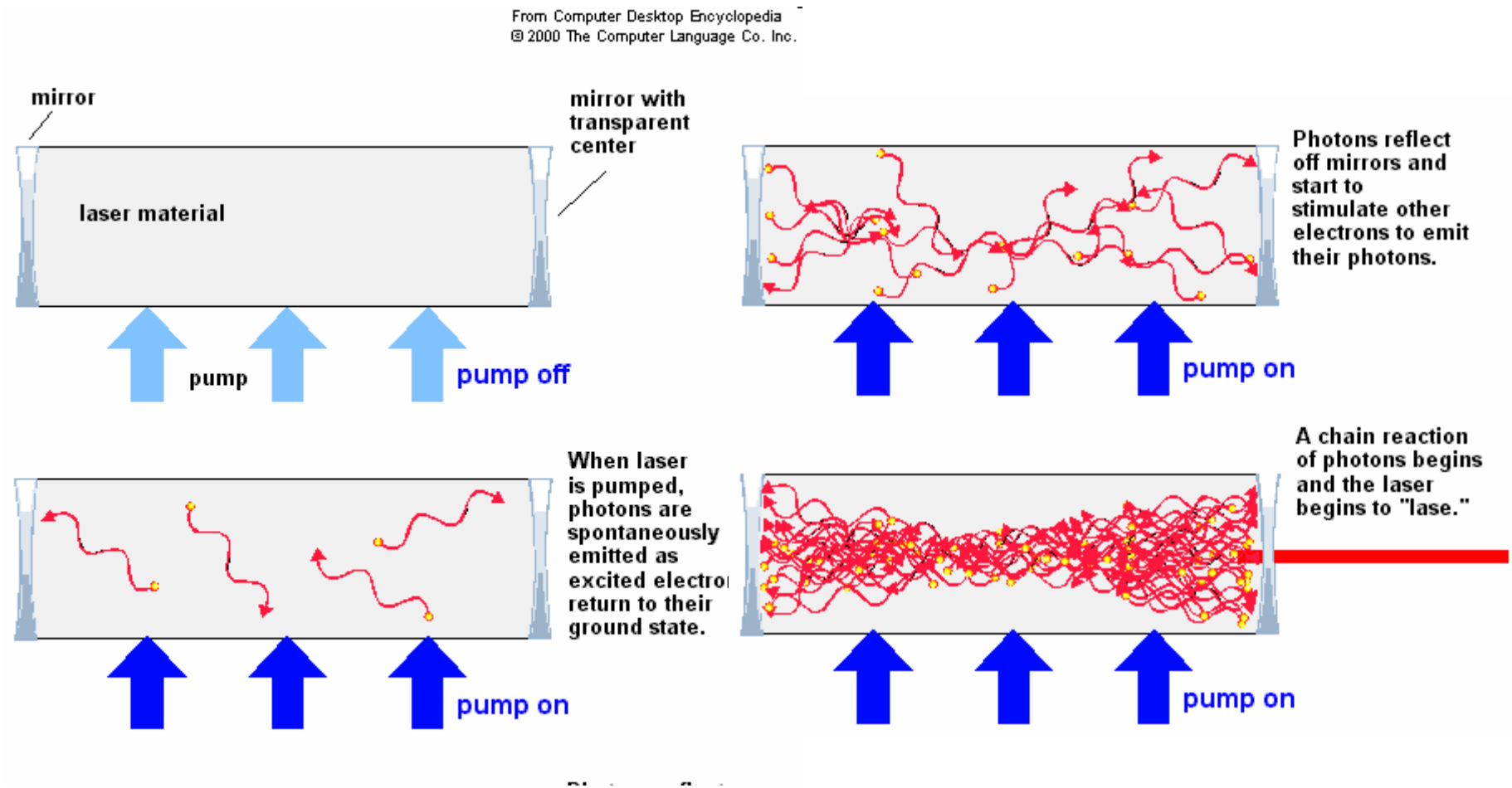


Figure 2 a) μ-PL emission spectra from single pillar with diameter $d = 2$ μm at 15 K; b) TR μ-PL emission spectra from single pillar with diameter $d = 2$ μm at 15 K (taken at two wavelengths: 1097 nm on resonance, solid line, 1099 nm out of resonance, dashed line) compared with TR μ-PL emission spectra from single pillar with diameter $d = 10$ μm at 15 K, wavelength 1097 nm (dotted line).

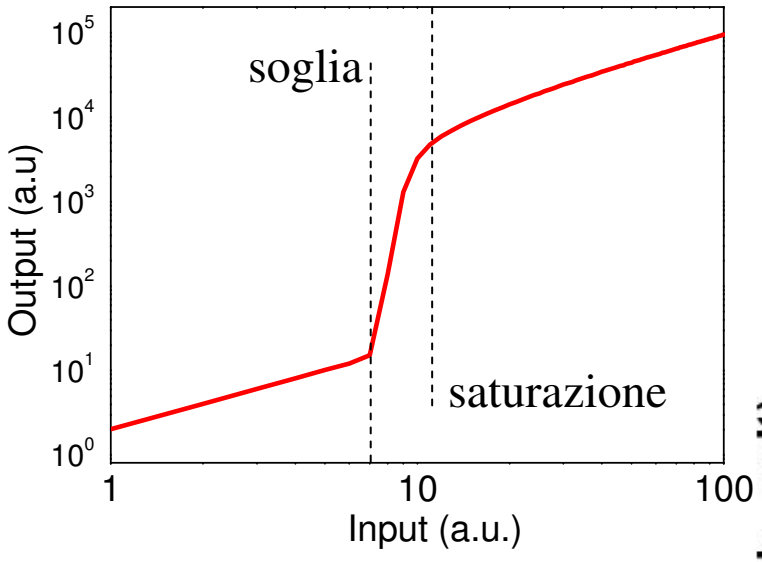
Applicazioni QDs in MC:
Optoelettronica
Laser senza soglia

Laser

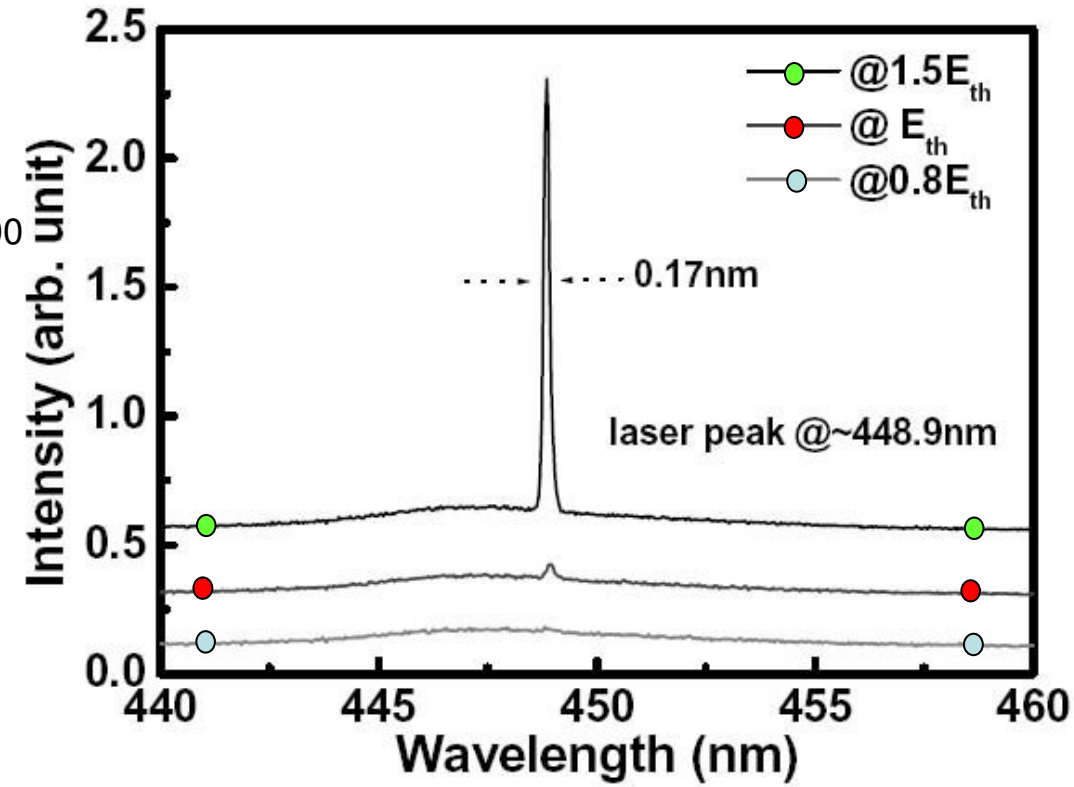


Sotto soglia domina emissione spontanea (perdite)
Sopra soglia domina emissione stimolata

Soglia dei laser



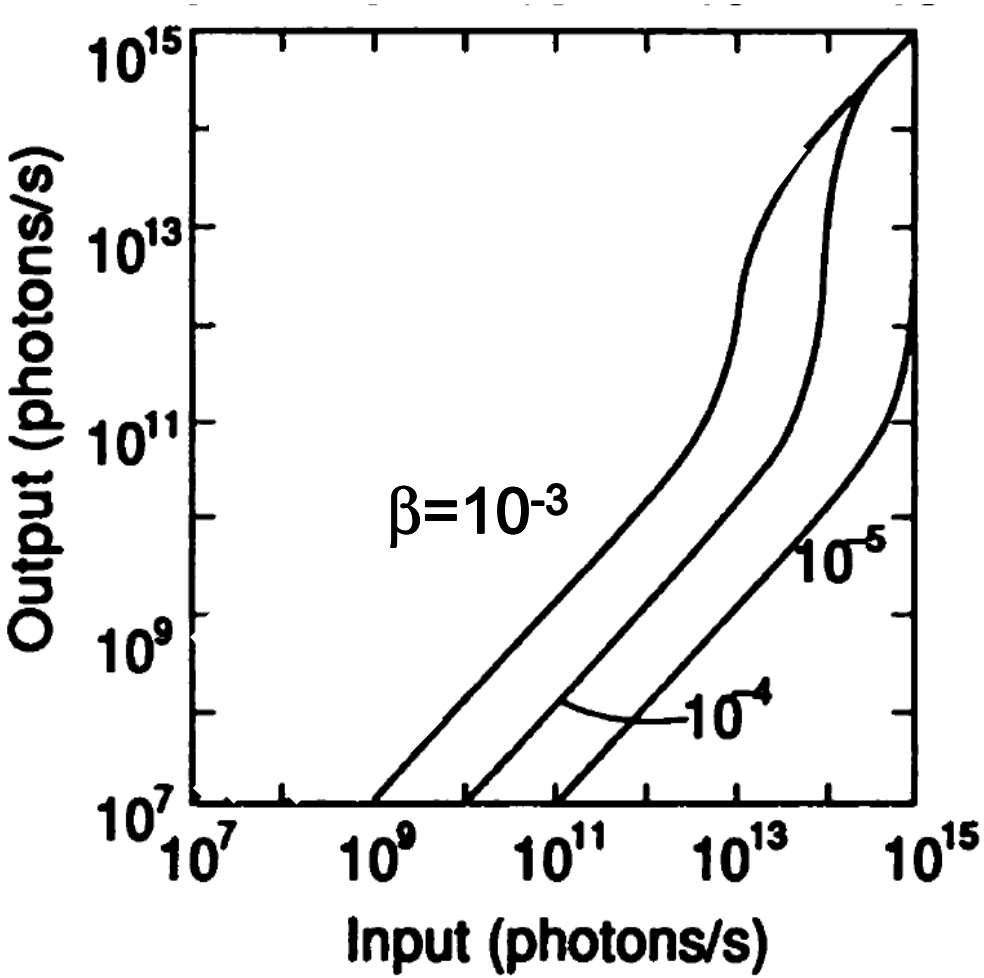
Alla soglia la probabilità di emissione stimolata eguaglia la probabilità di emissione spontanea:



Soglia dei laser

$$\beta = \frac{Numero\ di\ \ fotoni\ emessi\ nel\ modo\ laser}{Numero\ di\ \ fotoni\ emessi}$$

Al crescere di β
(qualità della cavità)
la soglia si abbassa



In microcavità l'emissione spontanea avviene nel modo di cavità (modo laser) che è l'unico presente

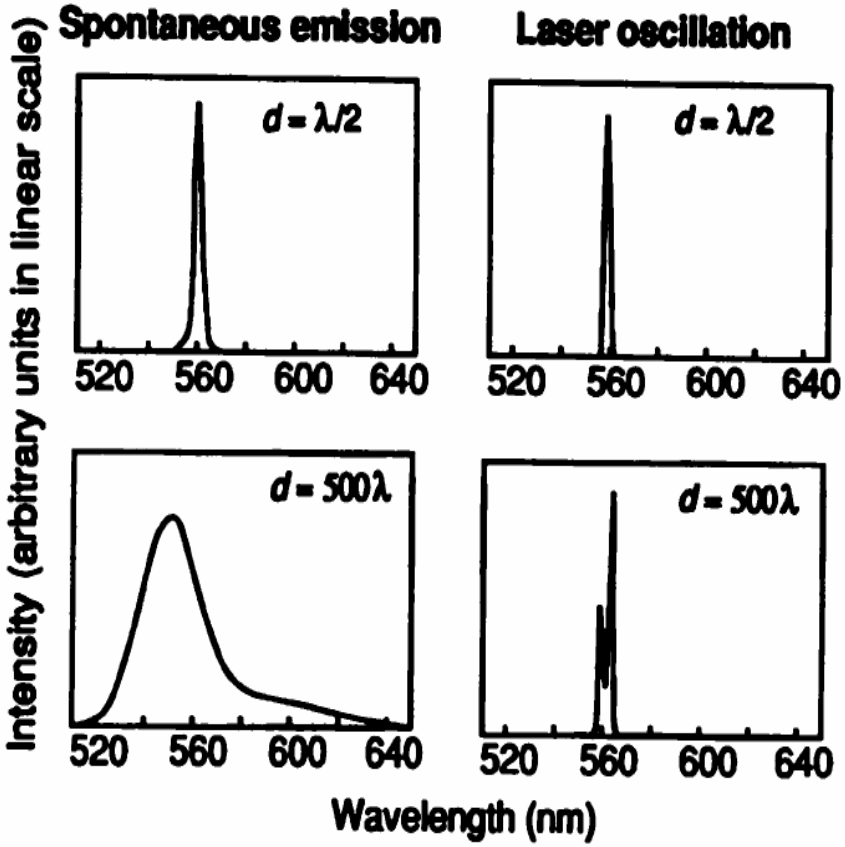


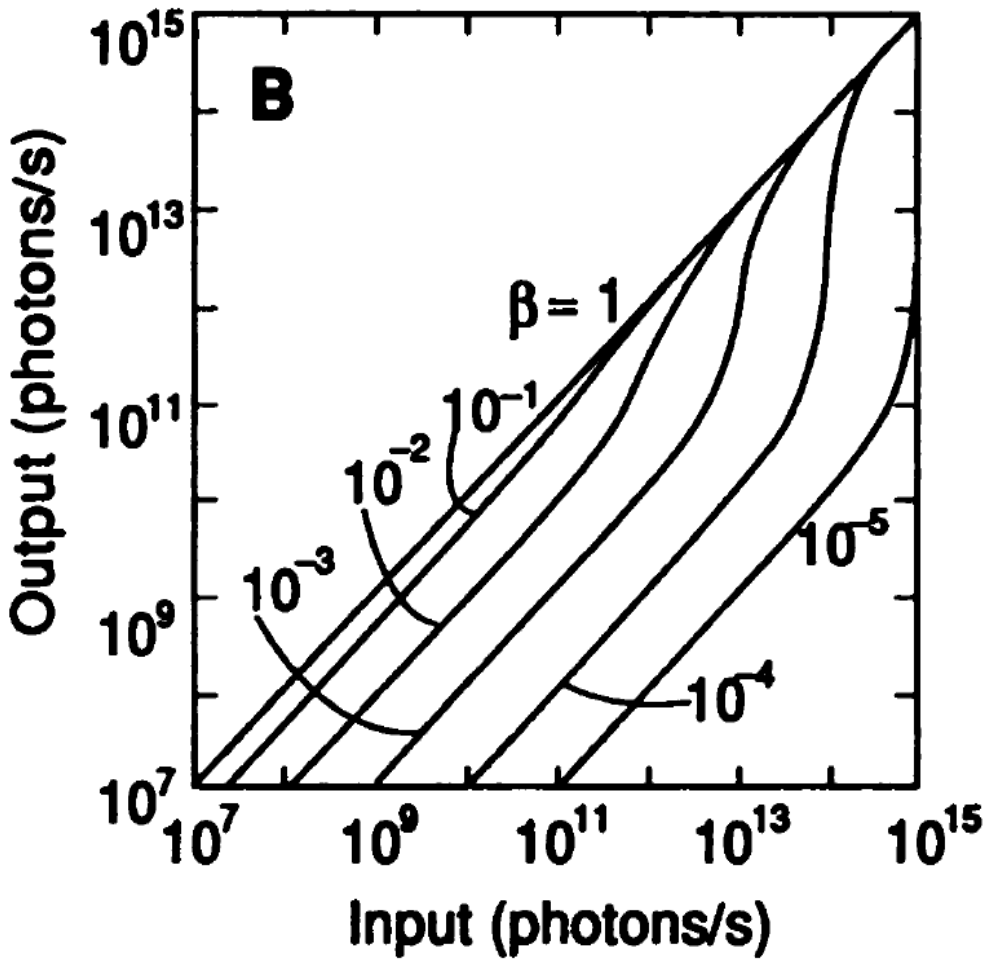
Fig. 8. Spontaneous and stimulated emission spectra of the λ/2 microcavity and the 500λ cavity.

Soglia dei laser

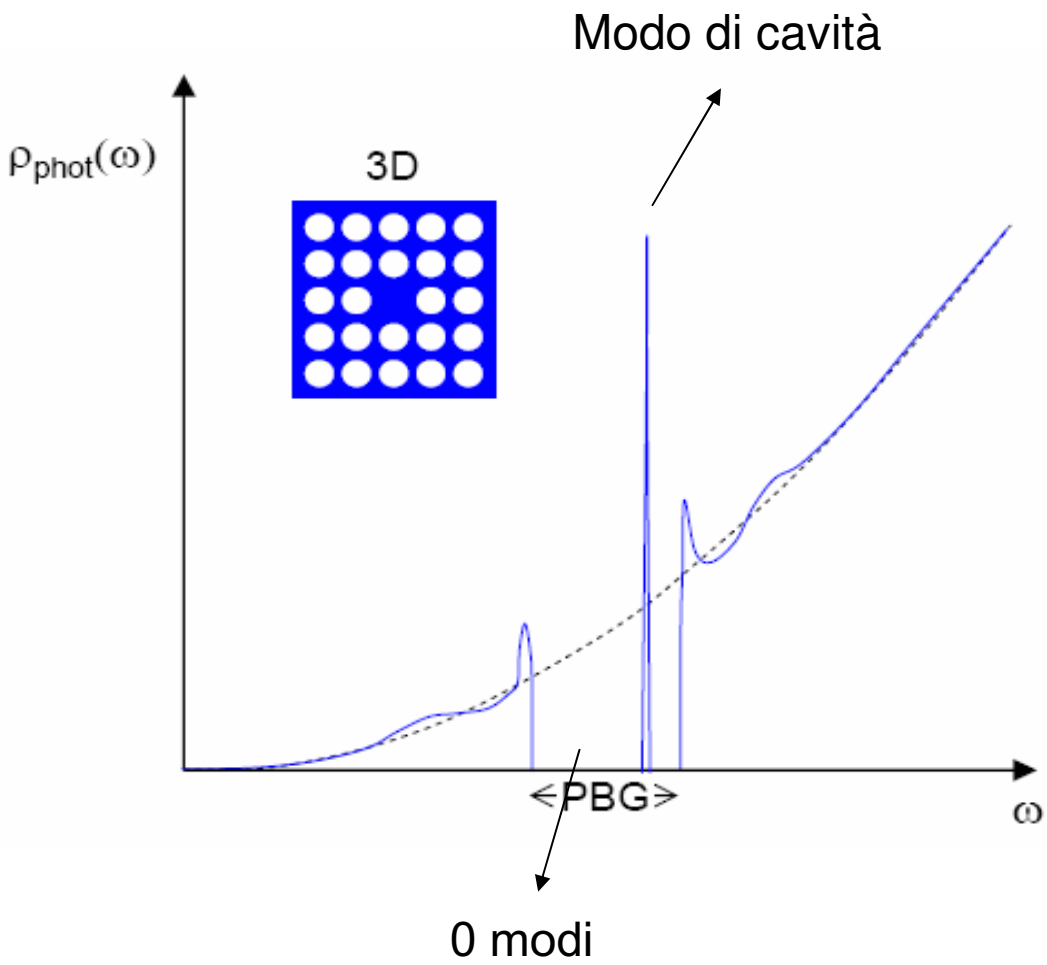
$$\beta = \frac{Numero\ di\ fotoni\ emessi\ nel\ modo\ laser}{Numero\ di\ fotoni\ emessi}$$

Al crescere di β
(qualità della cavità)
la soglia si abbassa

Per β=1
Thresholdless
Laser

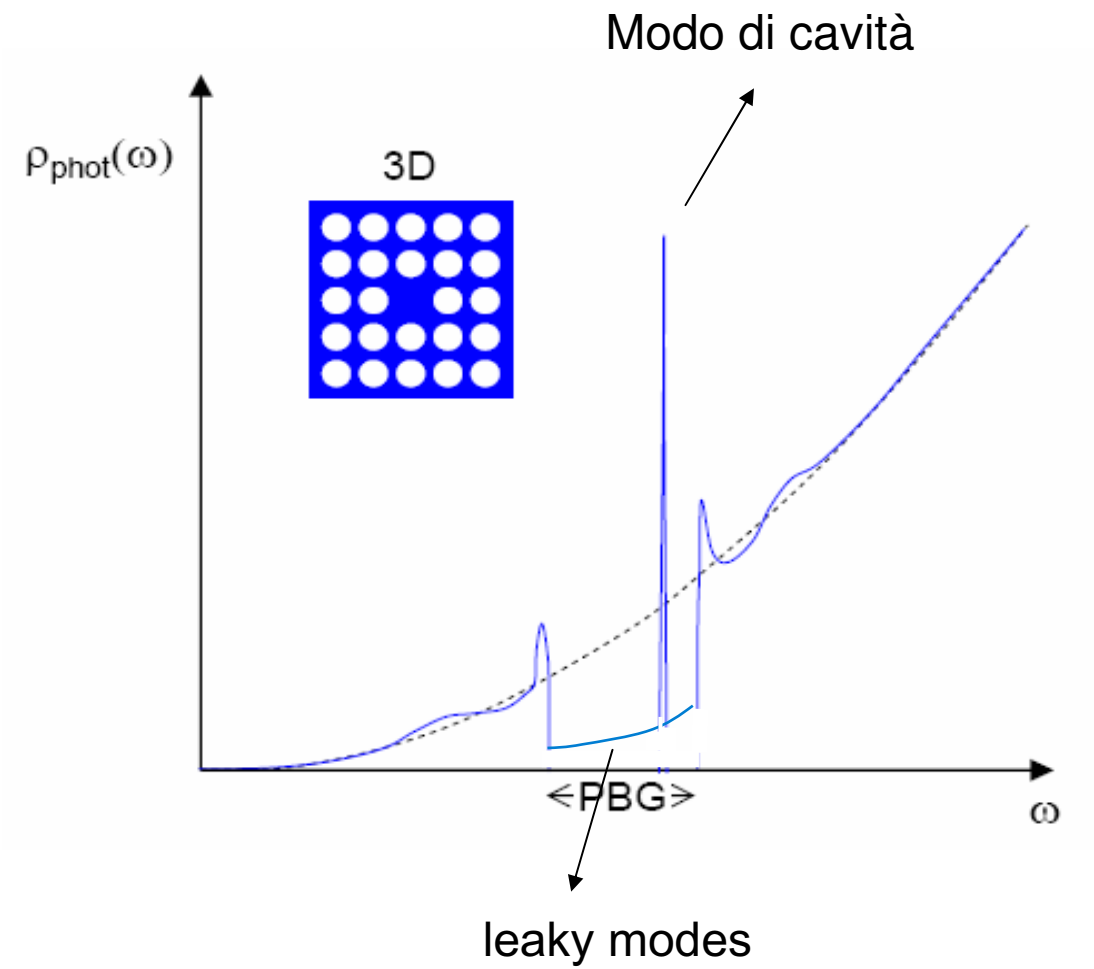


Cavità ideale



Emissione spontanea può avvenire solo nel modo di cavità **(β=1)**

Cavità reale



Emissione spontanea può avvenire anche nei leaky modes, **(β<1)**

Thresholdless Laser

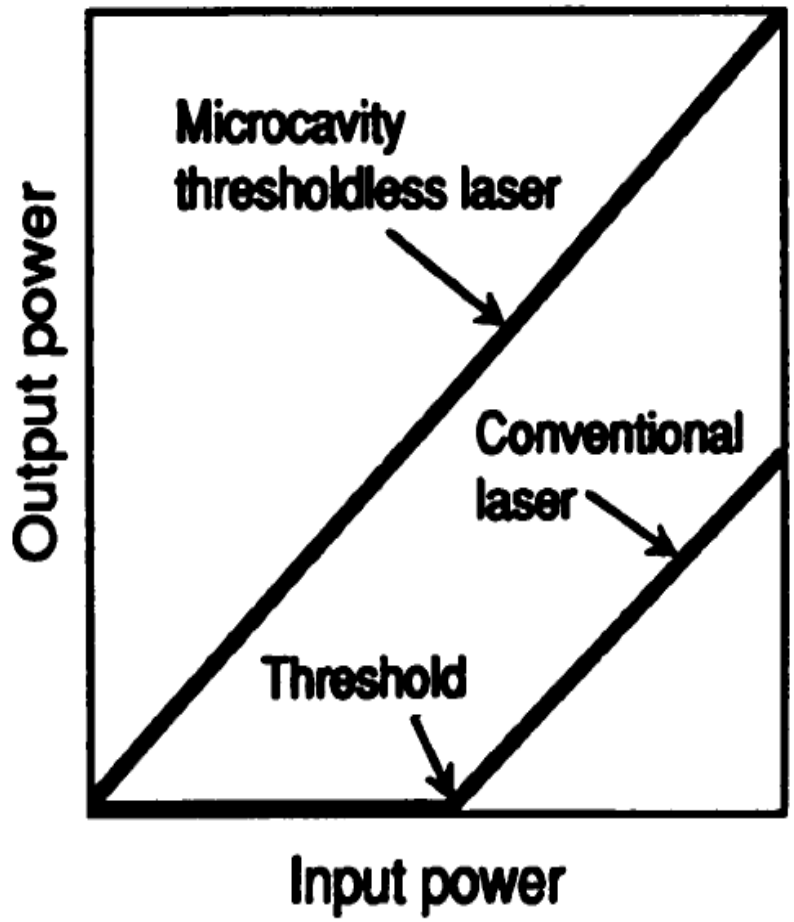
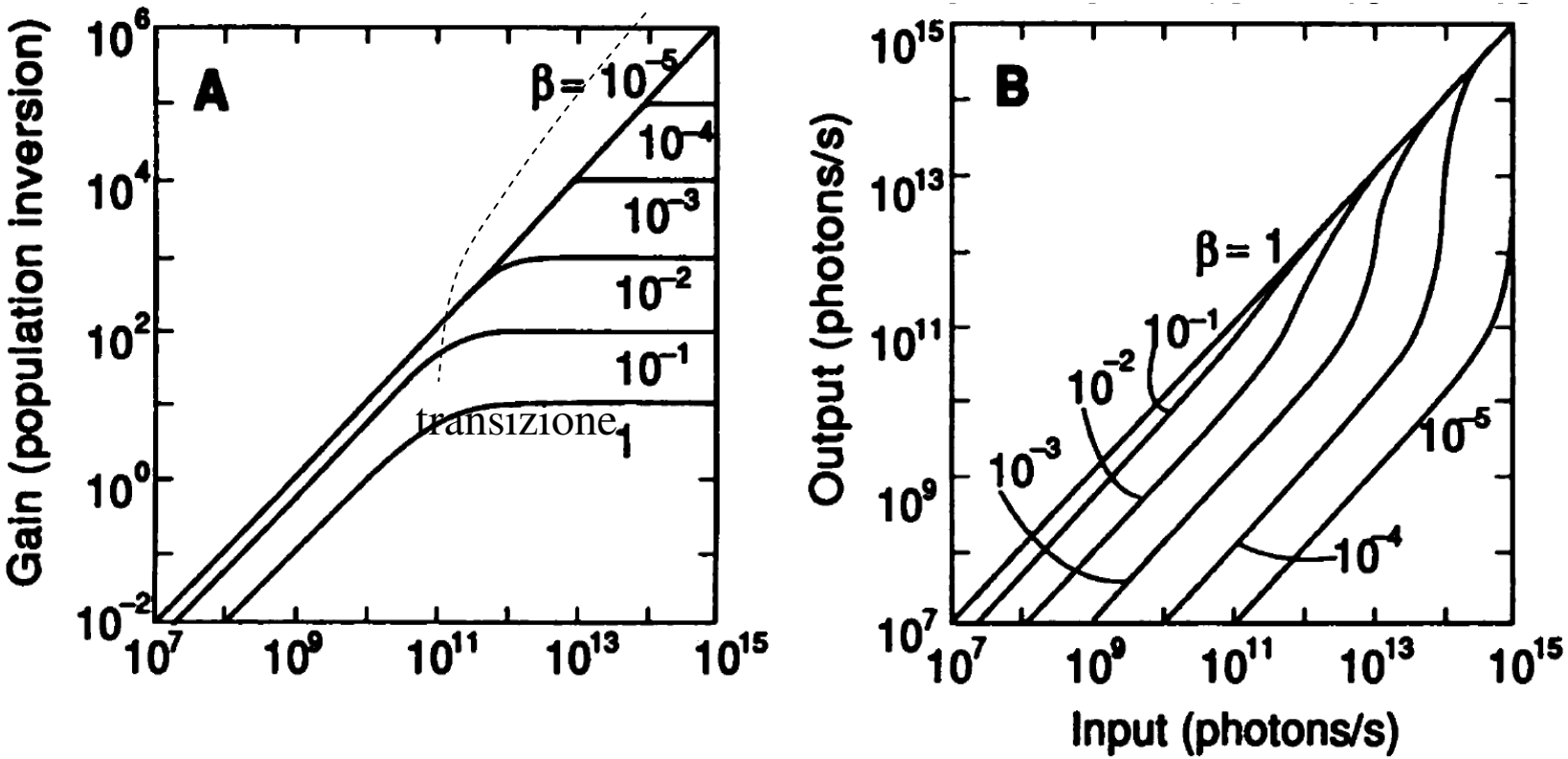


Fig. 4. Schematic of input-output characteristics of a conventional laser and a microcavity thresholdless laser.

Thresholdless Laser ma transizione da emissione spontanea a emissione stimolata



Calculated gain (A) and light output (B) versus input power for microcavities on logarithmic scales. β is the fraction of spontaneous emission coupled into the cavity mode.