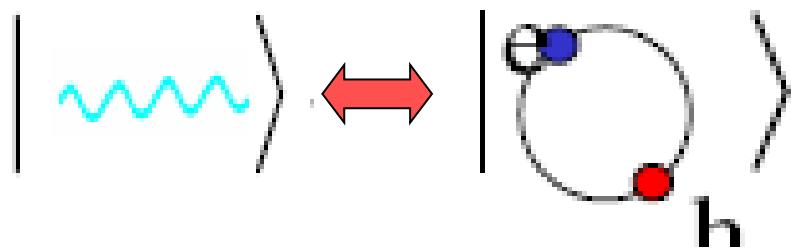


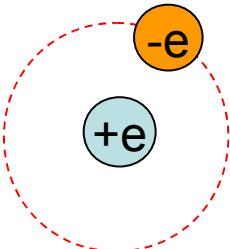
# Teoria quantistica: Polaritone

# Exciton-photon coupling

## Teoria quantistica: Polaritone



# Sommario eccitone



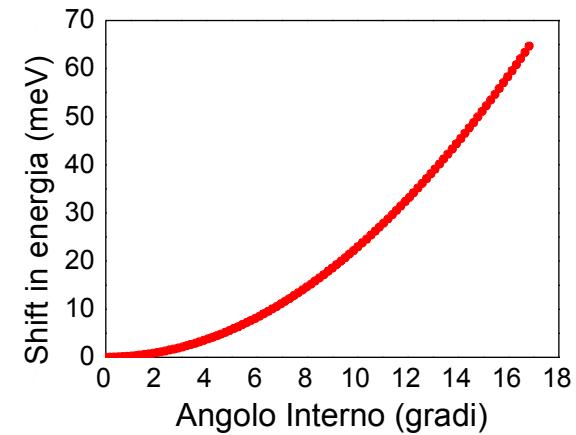
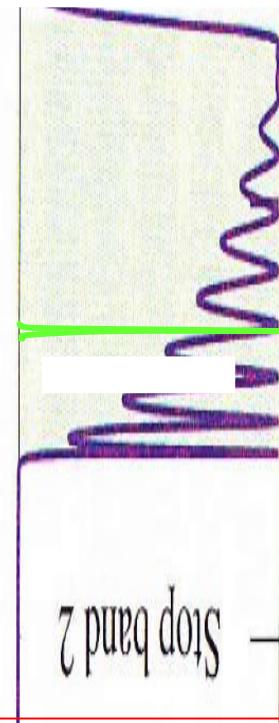
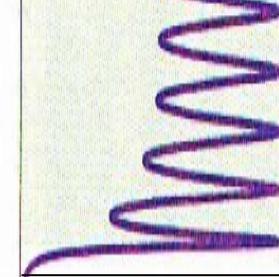
Spin intero

Picco di assorbimento  
ben separato dal  
continuo e-h

$$\vec{\hbar\omega}$$

1. Stato isolato nel gap
2. Transizione tunabile
3. Statistica bosonica
4. Moto libero nel piano ( $k_{//}$ )

# Sommario MC



1. Stato isolato nel gap
2. Transizione tunabile
3. Statistica bosonica
4. Moto libero nel piano ( $k_{\parallel}$ )

# Sommario interazione classica

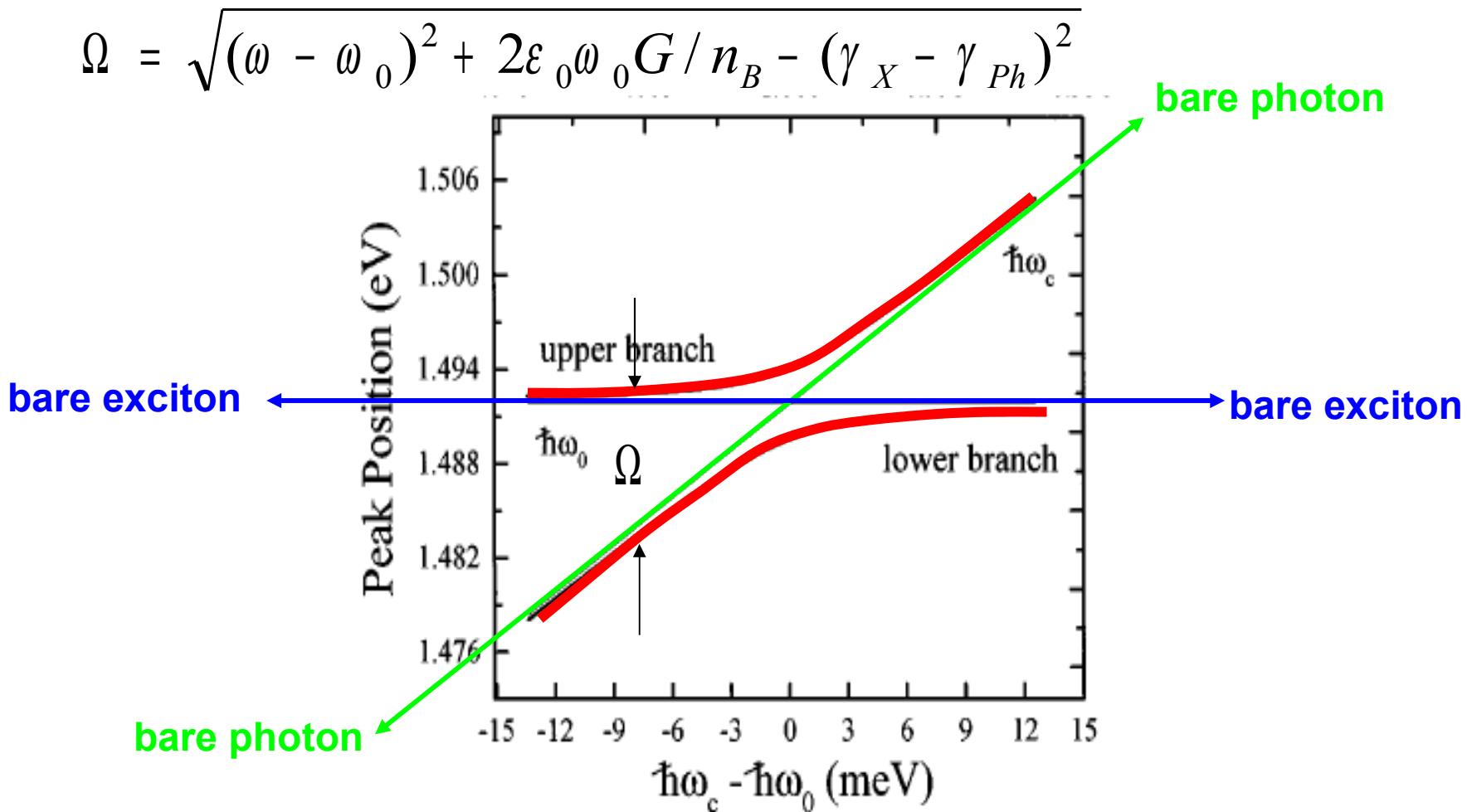


FIG. 3. NMC anticrossing curve vs oscillator-cavity detuning:  
heavy solid lines, transmission peak positions; thin solid lines,  
uncoupled oscillator and cavity positions.

## Photon state in second quantization and k space

Electromagnetic Vacuum

$$|V\rangle = |n = 0\rangle$$

$$a_{\vec{k}}^+ |V\rangle = |n = 1, \vec{k}\rangle = \left| \text{ \textcolor{cyan}{wavy line} } \right\rangle$$

$$a_{\vec{k}} |n = 1, \vec{k}\rangle = |n = 0\rangle$$

$$[a_{\vec{k}}, a_{\vec{k}'}^+] = \delta_{\vec{k}, \vec{k}'}$$

$$H_{phot} = \sum_{\vec{k}} \hbar \omega_{cav}(\vec{k}) a_{\vec{k}}^+ a_{\vec{k}}$$

# Eccitone 1s (prima quantizzazione)

$$\vec{R} = \frac{m_e \vec{r}_e + m_h \vec{r}_h}{m_e + m_h}$$

$$\vec{r} = \vec{r}_e - \vec{r}_h$$

$$\vec{r}_e = \vec{R} + \frac{m_h}{m_e + m_h} \vec{r}$$

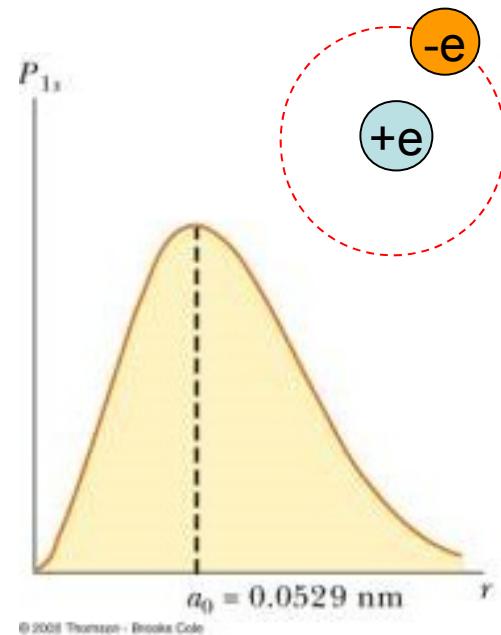
$$\vec{r}_h = \vec{R} - \frac{m_e}{m_e + m_h} \vec{r}$$

$$\vec{k} = \vec{k}_e + \vec{k}_h$$

$$\vec{k} = \frac{m_h}{m_e + m_h} \vec{k}_e - \frac{m_e}{m_e + m_h} \vec{k}_h$$

Funzione  
inviluppo  
eccitone

$$\begin{aligned} \psi_{\vec{q}}(\vec{r}_e, \vec{r}_h) &= \Phi_{1s}(\vec{r}) e^{i\vec{q}\cdot\vec{R}} = \sum_{\vec{k}_h, \vec{k}_e} \tilde{\Phi}_{1s}(\vec{k}) e^{i\vec{k}_e \cdot \vec{r}_e} e^{i\vec{k}_h \cdot \vec{r}_h} e^{i\vec{q}\cdot\vec{R}} = \\ &= \sum_{\vec{k}_h, \vec{k}_e} \tilde{\Phi}_{1s} \left( \frac{m_h}{m_e + m_h} \vec{k}_e - \frac{m_e}{m_e + m_h} \vec{k}_h \right) e^{i\vec{k}_e \cdot \vec{r}_e} e^{i\vec{k}_h \cdot \vec{r}_h} \delta_{\vec{k}_e + \vec{k}_h, \vec{q}} = \\ &= \sum_{\vec{k}_h} \tilde{\Phi}_{1s} \left( \frac{m_h}{m_e + m_h} \vec{q} - \vec{k}_h \right) e^{i(\vec{q} - \vec{k}_h) \cdot \vec{r}_e} e^{i\vec{k}_h \cdot \vec{r}_h} \end{aligned}$$



# Elettroni e lacune in seconda quantizzazione

Onda CB

$$e^{i(\vec{q} + \vec{k}) \cdot \vec{r}_e} c_{\vec{k} + \vec{q}}^+ |V\rangle = \left| \begin{array}{c} \text{e} \\ \backslash \\ \text{h} \end{array} \right\rangle \quad \{c_{\vec{k}}, c_{\vec{k}'}^+\} = \delta_{\vec{k}, \vec{k}'}$$

$$e^{i\vec{k} \cdot \vec{r}_h} v_{\vec{k}}^- |V\rangle = \left| \begin{array}{c} \text{h} \\ \circ \\ \text{e} \end{array} \right\rangle \quad \{v_{\vec{k}}, v_{\vec{k}'}^-\} = \delta_{\vec{k}, \vec{k}'}$$

Onda VB

Linguaggio elettroni

## Exciton state in second quantization and k-space

$$|V\rangle = \left| \begin{array}{c} \text{blue wavy line} \\ \text{red shaded band} \end{array} \right\rangle \quad \text{Electronic ground state}$$

$$c_{\vec{k}+\vec{q}}^+ |V\rangle = \left| \begin{array}{c} \text{blue wavy line} \\ \text{blue dot} \\ \text{red shaded band} \end{array} \right\rangle \quad \{c_{\vec{k}}, c_{\vec{k}'}^+\} = \delta_{\vec{k}, \vec{k}'}$$

$$v_{\vec{k}} |V\rangle = \left| \begin{array}{c} \text{blue wavy line} \\ \text{red shaded band} \\ \text{blue circle} \end{array} \right\rangle \quad \{v_{\vec{k}}, v_{\vec{k}'}^+\} = \delta_{\vec{k}, \vec{k}'} \quad \text{Linguaggio elettroni}$$

$$\psi_{\vec{q}}(\vec{r}_e, \vec{r}_h) = \sum_{\vec{k}_h} \tilde{\Phi}_{1s} \left( \frac{m_h}{m_e + m_h} \vec{q} - \vec{k}_h \right) e^{i(\vec{q} - \vec{k}_h) \cdot \vec{r}_e} e^{i\vec{k}_h \cdot \vec{r}_h}$$

$$\Rightarrow \sum_{\vec{k}} \tilde{\Phi}_{1s} \left( \frac{m_h}{m_e + m_h} \vec{q} + \vec{k} \right) c_{\vec{q} + \vec{k}}^+ v_{\vec{k}} |V\rangle$$

## Exciton creation operator

$$|1s, \vec{q}\rangle = b_{\vec{q}}^+ |V\rangle = \left| \begin{array}{c} e^- \\ \text{---} \\ \text{---} \\ h^+ \end{array} \right. \vec{q} \left. \begin{array}{c} \bullet \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\rangle$$

$$b_{\vec{q}}^+ = \sum_{\vec{k}} \tilde{\phi}_{1s}(\vec{k} + \frac{m_h}{M} \vec{q}) c_{\vec{k} + \vec{q}}^+ v_{\vec{k}}$$

$$[b_{\vec{q}}, b_{\vec{q}'}^+] = \delta_{\vec{q}, \vec{q}'} + \hat{D}_{\vec{q}, \vec{q}'}$$

Operator describing deviations from elementary boson behavior

$$H_{exc} = \sum_{\vec{k}} \hbar \omega_{exc}(\vec{k}) b_{\vec{k}}^+ b_{\vec{k}}$$



$$H = H_0 + H_{\text{int}}$$

$$H_0 = \sum_q \hbar \omega_{cav}(k) a_{\vec{k}}^+ a_{\vec{k}} + \sum_q \hbar \omega_{exc}(k) b_{\vec{k}}^+ b_{\vec{k}}$$

$$H_{\text{int}} = \sum_{\vec{k}} \hbar \Omega_0 a_{\vec{k}}^+ b_{\vec{k}} + \sum_{\vec{k}} \hbar \Omega_0 b_{\vec{k}}^+ a_{\vec{k}}$$

↗                      ↘

Photon emission              Photon absorption

- In-plane wavevector is conserved
  - Different in-plane wavevectors are decoupled
-

## Cavity polariton states and operators

$$p_{1,\vec{k}}^+ = -C_k a_{\vec{k}}^+ + X_k b_{\vec{k}}^+$$
$$p_{2,\vec{k}}^+ = X_k a_{\vec{k}}^+ + C_k b_{\vec{k}}^+$$

$$M = \begin{pmatrix} \omega_{cav}(k) & \Omega_0 \\ \Omega_0 & \omega_{exc}(k) \end{pmatrix}$$



$$H_0 + H_{int} = \sum_{\vec{k}} \hbar \omega_{LP}(k) p_{1,\vec{k}}^+ p_{1,\vec{k}}^- + \sum_{\vec{k}} \hbar \omega_{UP}(k) p_{2,\vec{k}}^+ p_{2,\vec{k}}^-$$

$$|LP\rangle = -C_k | \text{wavy line} \rangle + X_k | \text{electron hole} \rangle$$

$$|UP\rangle = X_k | \text{wavy line} \rangle + C_k | \text{electron hole} \rangle$$

$|C_k|^2$  = Photonic fraction of the LP mode

$|X_k|^2$  = Excitonic fraction of the LP mode

## *Half-photon, half-exciton*

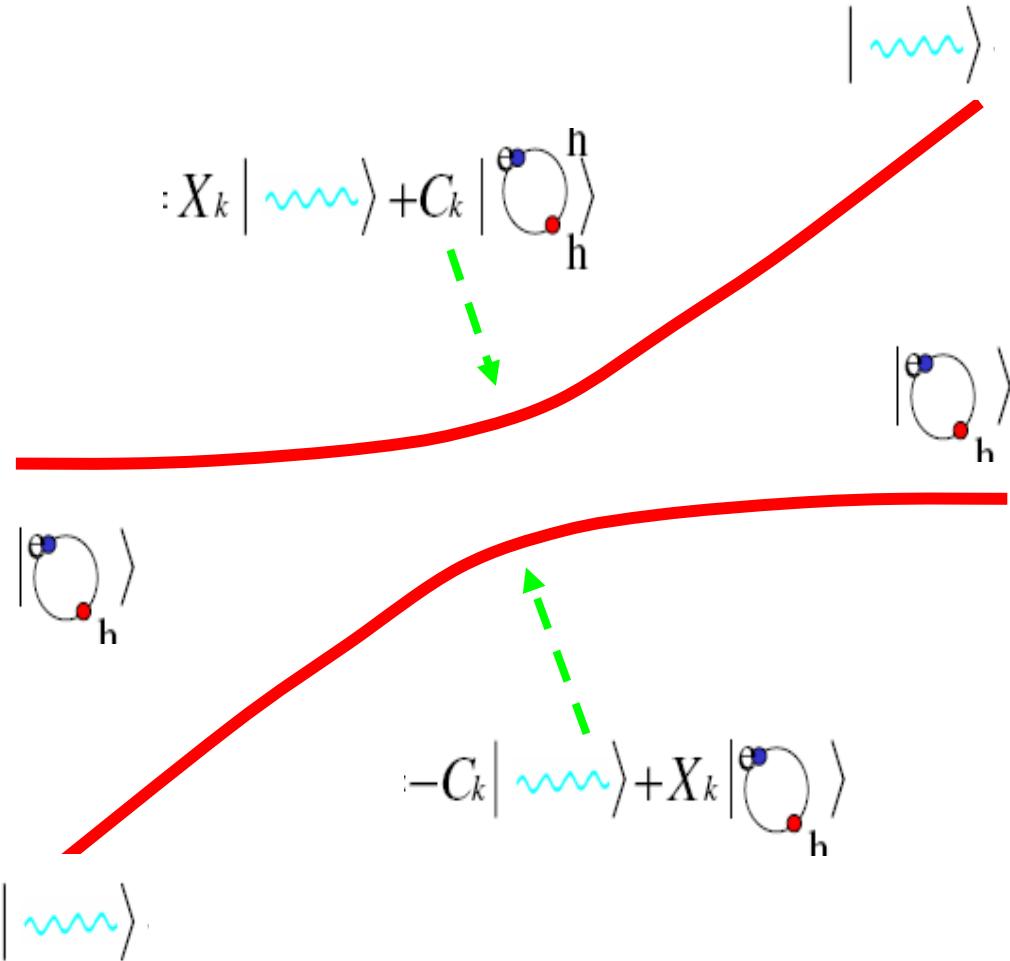
$$|LP\rangle = -C_k \left| \text{wavy} \right\rangle + X_k \left| \begin{array}{c} e \\ \circlearrowleft \\ h \end{array} \right\rangle$$

$$|UP\rangle = X_k \left| \text{wavy} \right\rangle + C_k \left| \begin{array}{c} e \\ \circlearrowright \\ h \end{array} \right\rangle$$

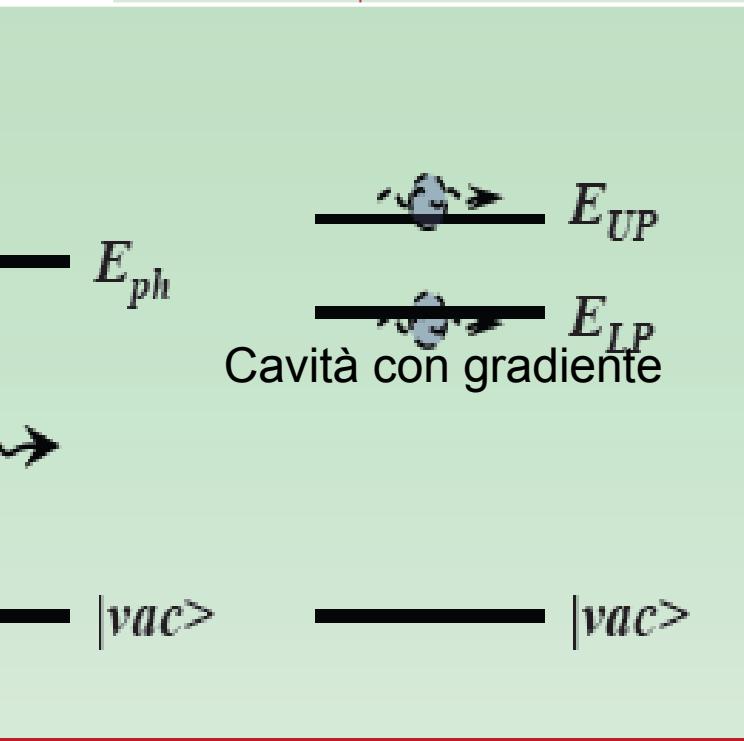
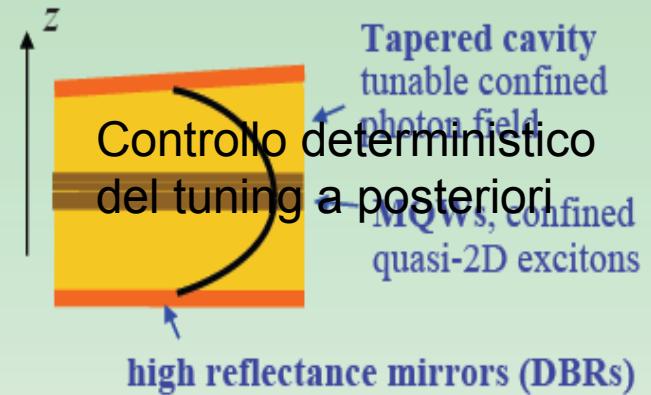
$|C_k|^2$  = Photonic fraction of the LP mode

$|X_k|^2$  = Excitonic fraction of the LP mode

# Anticrossing $k_{\parallel}=0$



# Accordo in frequenza

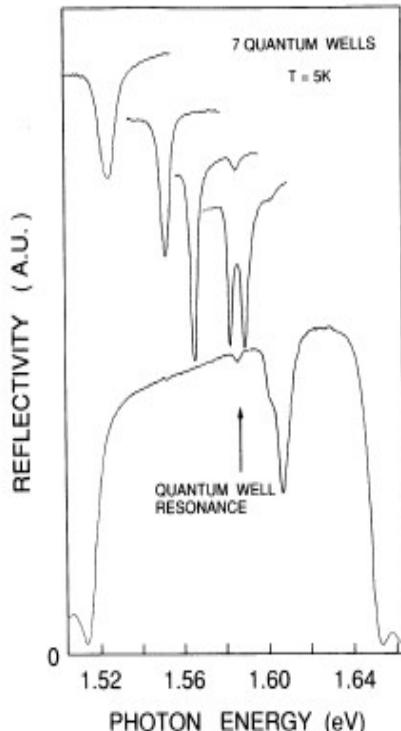


## Observation of the Coupled Exciton-Photon Mode Splitting in a Semiconductor Quantum Microcavity

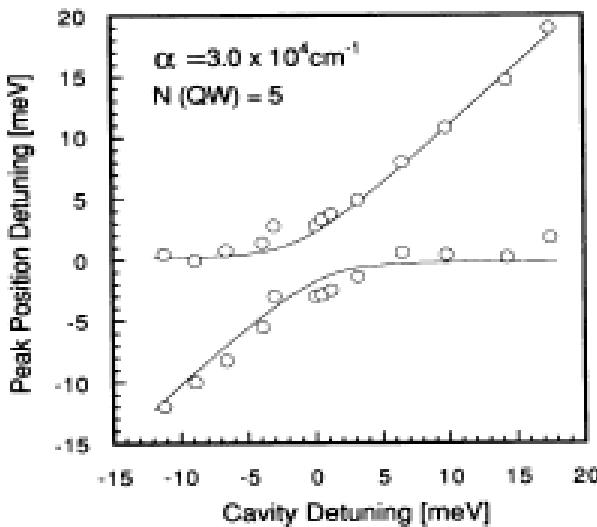
C. Weisbuch,<sup>(a)</sup> M. Nishioka,<sup>(b)</sup> A. Ishikawa, and Y. Arakawa

*Research Center for Advanced Science and Technology, University of Tokyo, 4-6-1 Meguro-ku, Tokyo 153, Japan*

(Received 12 May 1992)



GaAs



**FIG. 3.** Reflectivity peak positions as a function of cavity detuning for a five-quantum-well sample at  $T = 5$  K. The theoretical fit is obtained through a standard multiple-interference analysis of the DBR-Fabry-Pérot-quantum-well structure.

# Effetti quantistici BEC polaritoni

Anticrossing  $k_{\parallel}=0$

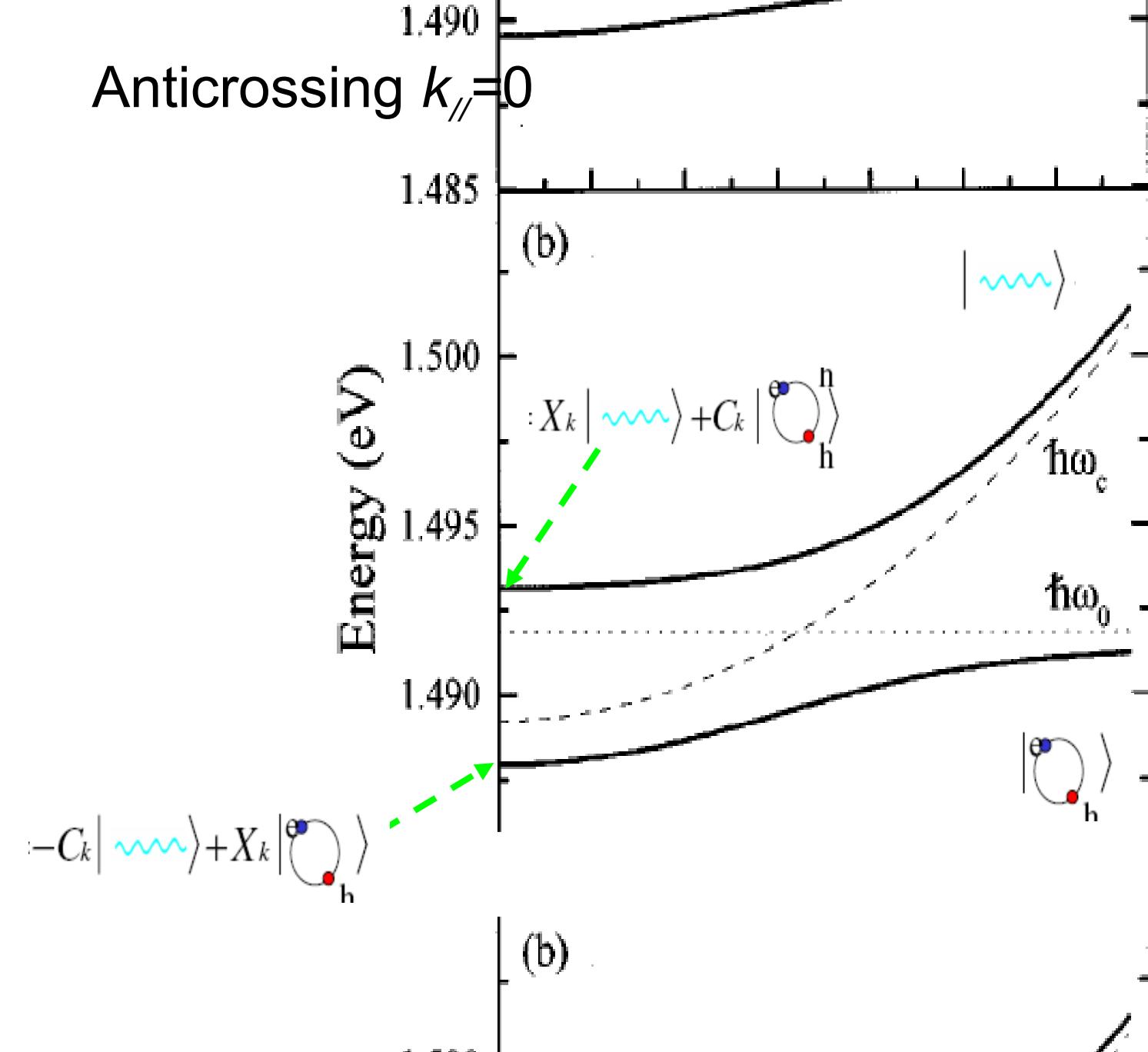
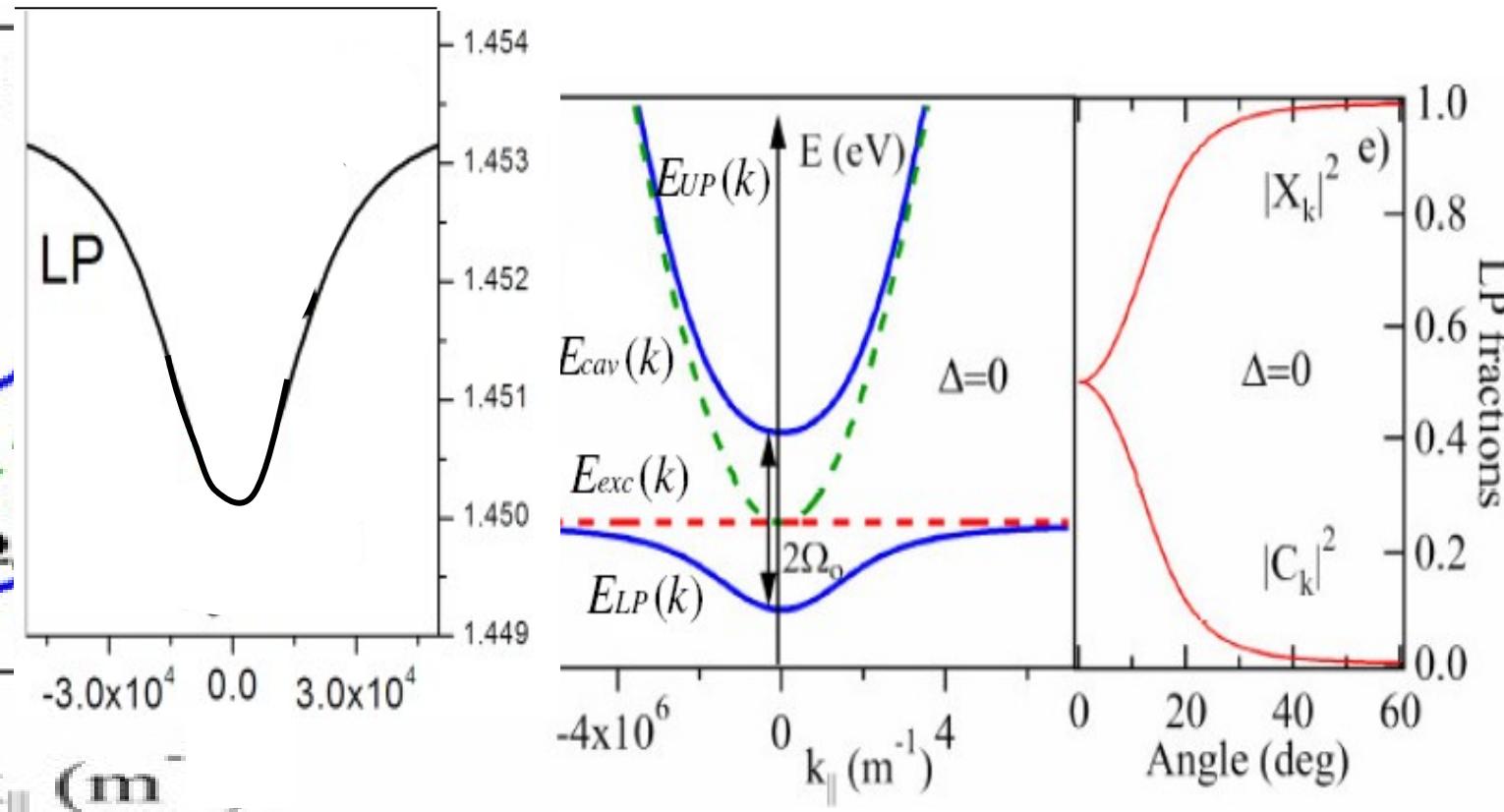


FIG. 5. Cavity polariton dispersion curves (heavy solid lines).

# Lower polariton states

## In-plane dispersion of the polariton modes



itons is very small  
dispersion is strongly non-parabolic

## Bose-Einstein condensation (BEC) of an ideal Bose gas<sup>1</sup>

- The Bose-Einstein distribution function:

$$f_B(\vec{k}, T, \mu) = \frac{1}{\exp\left[\frac{E(\vec{k}) - E(0) - \mu}{k_B T}\right] - 1}, \quad \mu < 0$$

- In a d-dimensional system with a parabolic dispersion around k=0:

$$T_c(n) = 4\pi \frac{\hbar^2}{2m} n^{2/d} \zeta(d/2)^{-2/d}$$

- In a 3D (d=3) system with a parabolic dispersion around k=0:

$$T_c = \frac{2\pi \hbar^2}{1.897 \cancel{mk_b}} n^{2/3}$$

<sup>1</sup> S.N. Bose, Z. Phys. **26**, 178 (1924), A. Einstein, Sitzber. Kgl. Preuss. Akad. Wiss (1924).

# Where ‘Quantum’ Grows ‘Macroscopic’

-- Parameter comparison of atomic and semiconductor BEC systems

| systems                               | atomic gas                                       | exciton  | polariton   |
|---------------------------------------|--|--|---|
|                                       | realized in<br>1995                              | proposed in<br>1962                                    | proposed in<br>1968   |
| effective mass<br>$m^*/m_e$           | $10^3$   | $10^{-1}$  | $10^{-5}$   |
| Bohr radius<br>$a_B$ (A)              | $10^{-3}$  | $10^2$   | $10^2$  |
| particle spacing<br>$n_c^{-1/d}$ (nm) | $10^2$   | $10^3$   | $10^3$  |
| critical temperature $T_c$            | $1\text{nK} \sim 1\text{mK}$                     | $1\text{mK} \sim 1\text{K}$                            | $1\text{K} \sim 300\text{K}$  |
| thermalization time<br>lifetime       | $\frac{1\text{ms}}{1\text{s}}$<br>$\sim 10^{-3}$ | $\frac{10\text{ps}}{1\text{ns}}$<br>$= 10^{-2} \sim 1$ | ( $1 \sim 10 \text{ ps}$ )<br>( $1 \sim 10 \text{ ps}$ )<br>$= 10^{-1} \sim 10^1$ |

← extremely light  $m^*$   
 (due to mixing with photons)

← high  $T_c$   
 $T_c \propto m^{-1}$

← dynamic,  
 how to reach equilibrium within lifetime

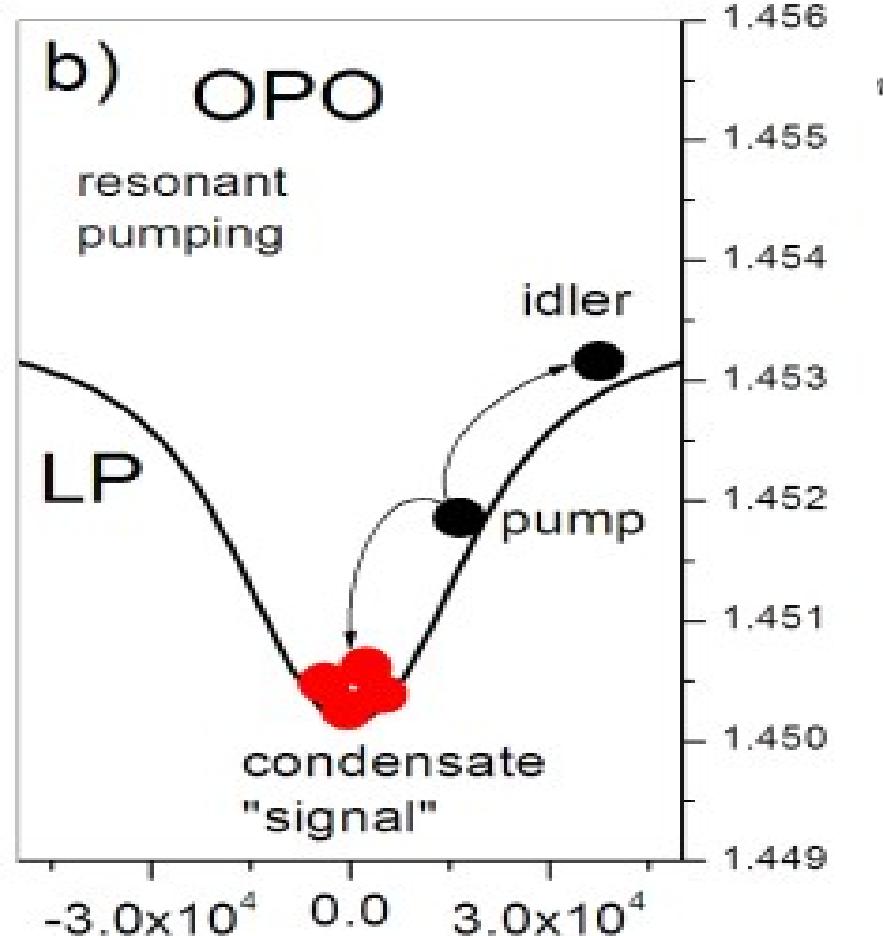
## Scattering risonante

<sup>1</sup>*Department of Physics & Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*

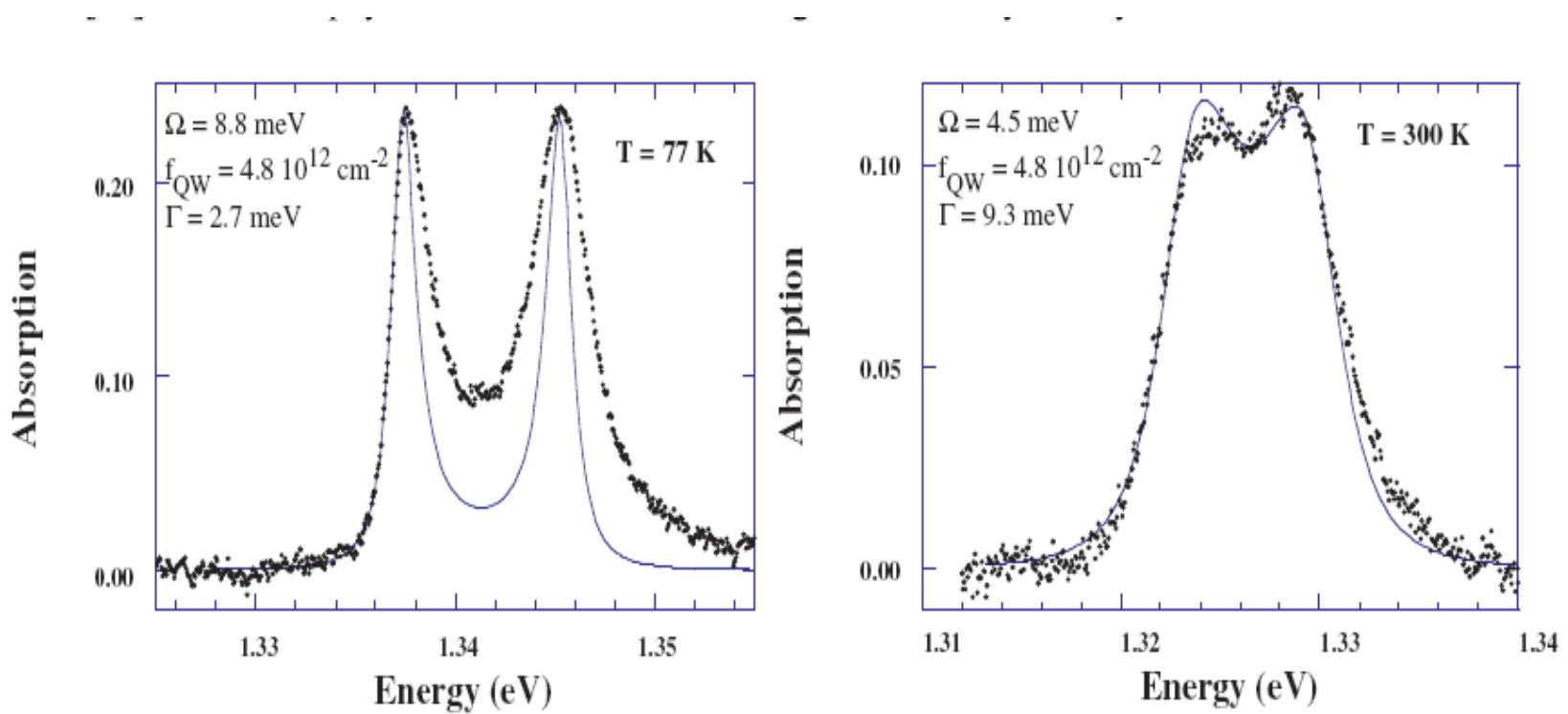
<sup>2</sup>*Department of Physics*

<sup>3</sup>*Toshiba Research*

<sup>4</sup>*Department of Electronic and Electri*

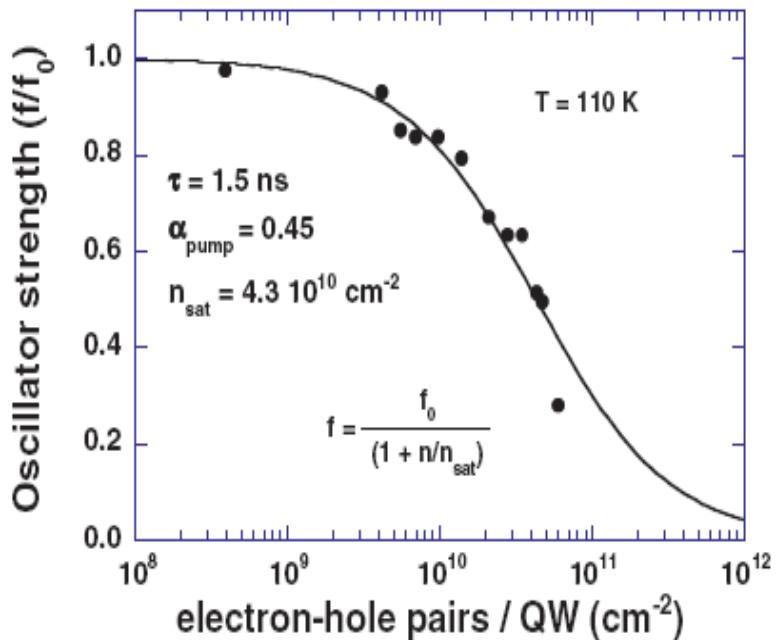


# Fononi distruggono strong coupling

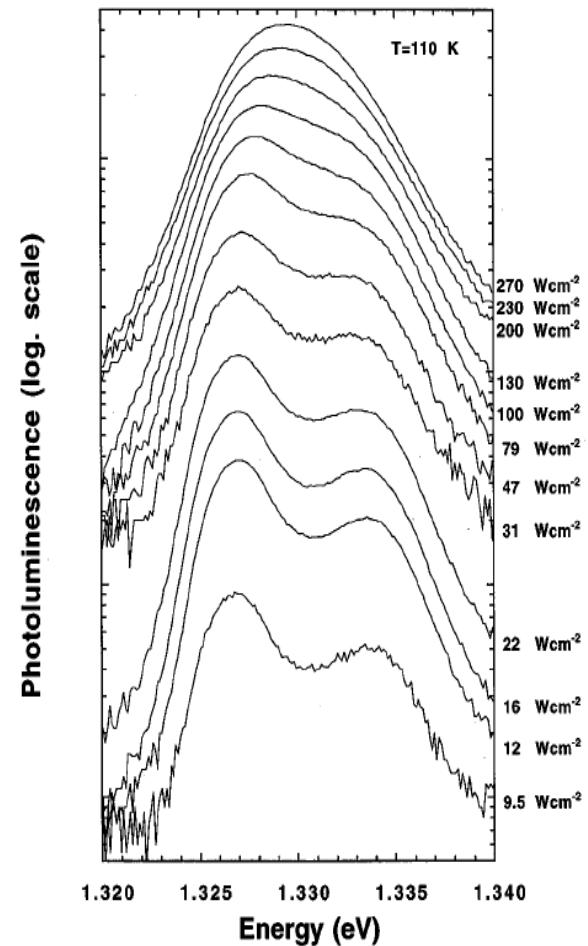


**Fig. 3** (online colour at: [www.pss-b.com](http://www.pss-b.com)) 77 K and 300 K absorption spectrum. The solid line is a linear dispersion model fit. The oscillator strength per quantum well is  $4.8 \times 10^{12} \text{ cm}^{-2}$ . Reprinted from [17].

# Exciton scattering distrugge strong coupling



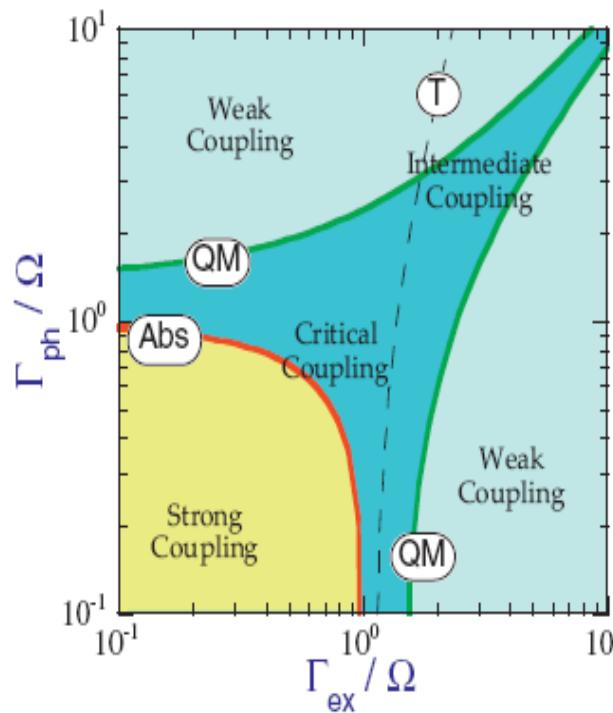
**Fig. 8** Exciton oscillator strength as a function of electron–hole pairs density. The continuous line is a fit with a usual screening function  $f(N_{e-h}) = f_0/(1 + N_{e-h}/N_{\text{sat}})$ . Reprinted from [30].



**FIG. 23.** Series of photoluminescence spectra ( $110 \text{ K}$ ) as a function of pump power showing bleaching of the strong-coupling regime. The incident power density is shown on the right axis. From Houdré *et al.*, 1995.

# Esistenza polaritone

## Coupling regimes

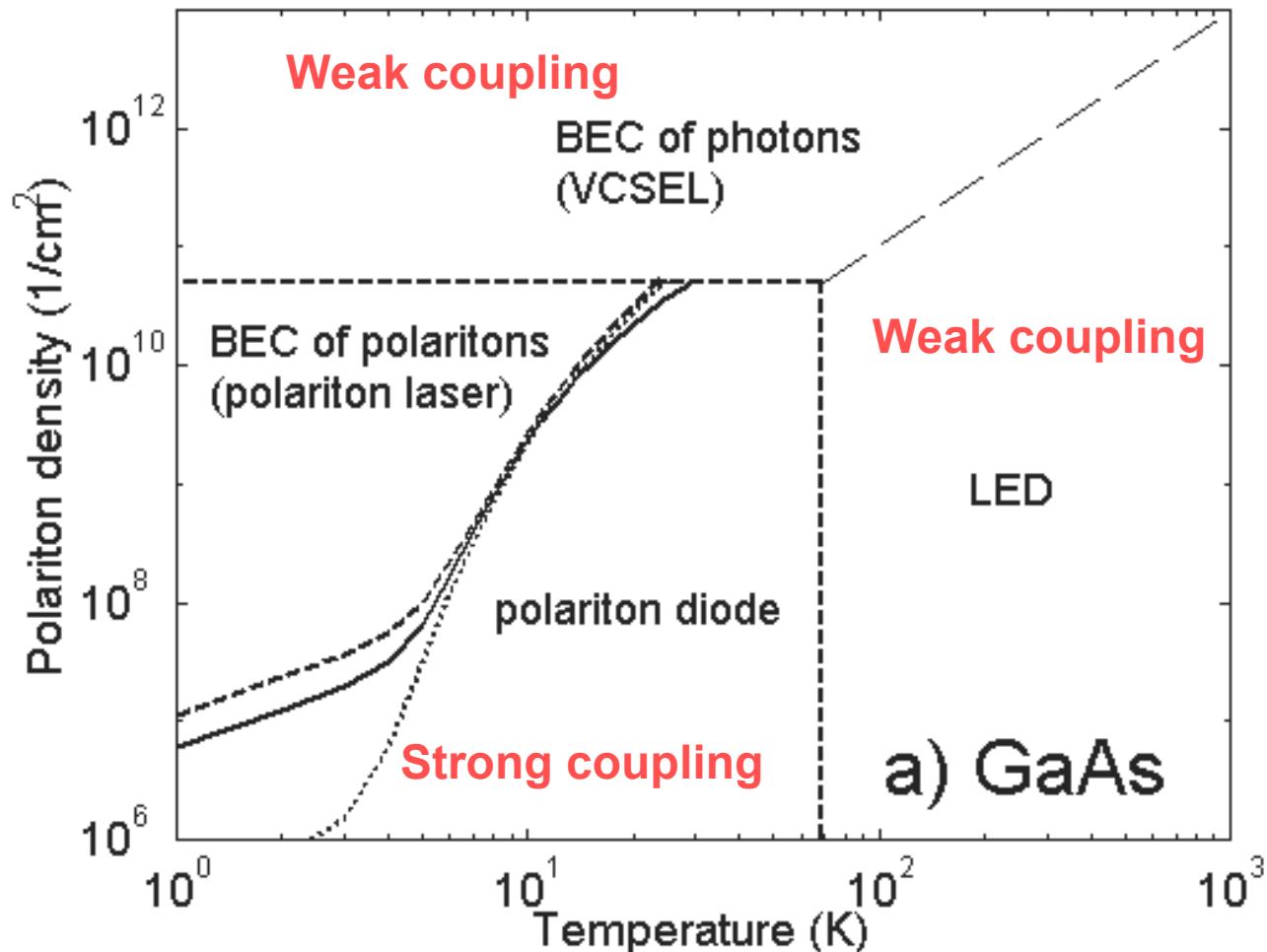


|   |
|---|
| Weak Coupling:<br>(perturbative)  |
| $\Omega^2 < \left( \frac{\Gamma_{\text{ph}} - \Gamma_{\text{ex}}}{2} \right)^2$   |
| Intermediate Coupling:<br>(non-perturbative, no ringing)  |
| $\left( \frac{\Gamma_{\text{ph}} - \Gamma_{\text{ex}}}{2} \right)^2 < \Omega^2 < \frac{\Gamma_{\text{ph}}^2 + \Gamma_{\text{ex}}^2}{2}$ |
| Strong Coupling:<br>(non-perturbative, ringing)   |
| $\Omega^2 > \frac{\Gamma_{\text{ph}}^2 + \Gamma_{\text{ex}}^2}{2}$  |

*Broadening  
distrugge  
Strong  
coupling*

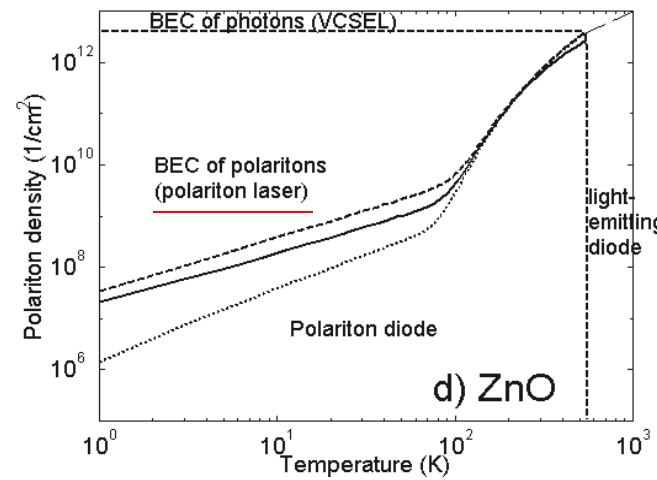
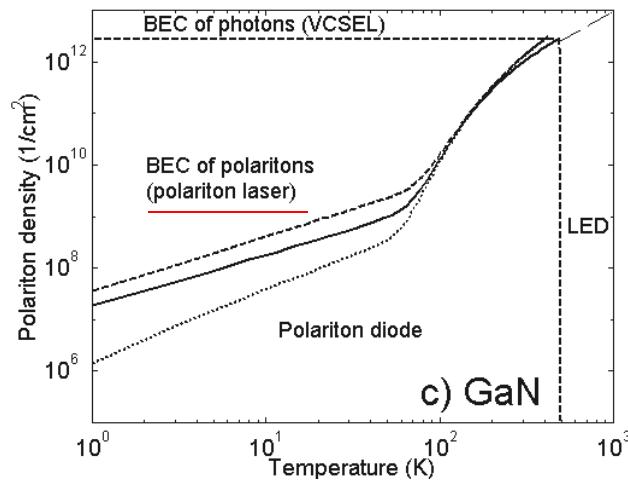
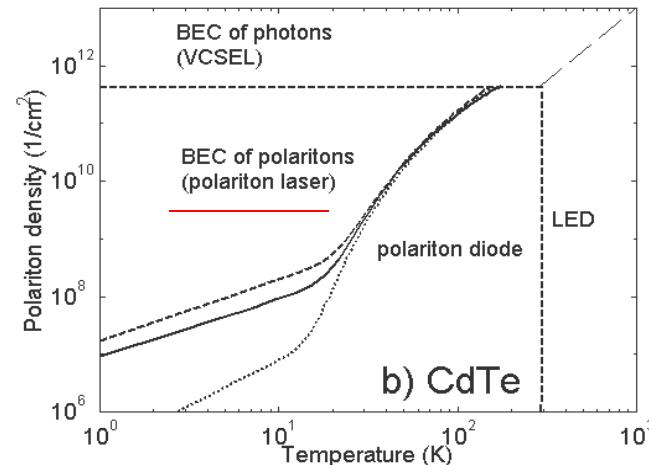
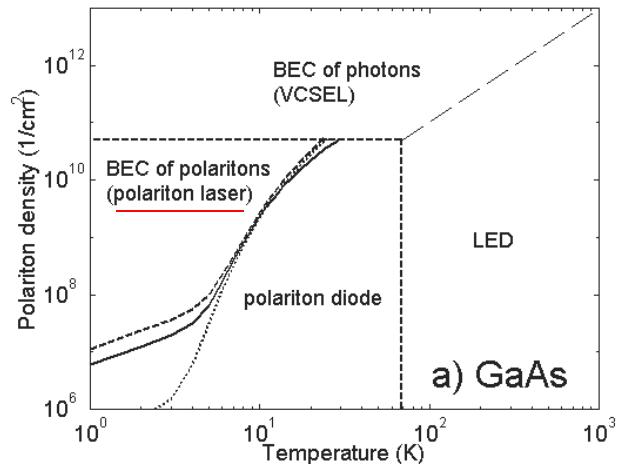
**Fig. 13** (online colour at: [www.pss-b.com](http://www.pss-b.com)) Map of the weak, strong and intermediate coupling as a function of the linewidths of both oscillators. “Abs” denotes the condition to observe a splitting in absorption, “QM” denotes the limit of the perturbative regime. The dashed line denotes the condition to observe a meaningless splitting in the transmission spectrum alone.

# Phase diagram of exciton-polaritons



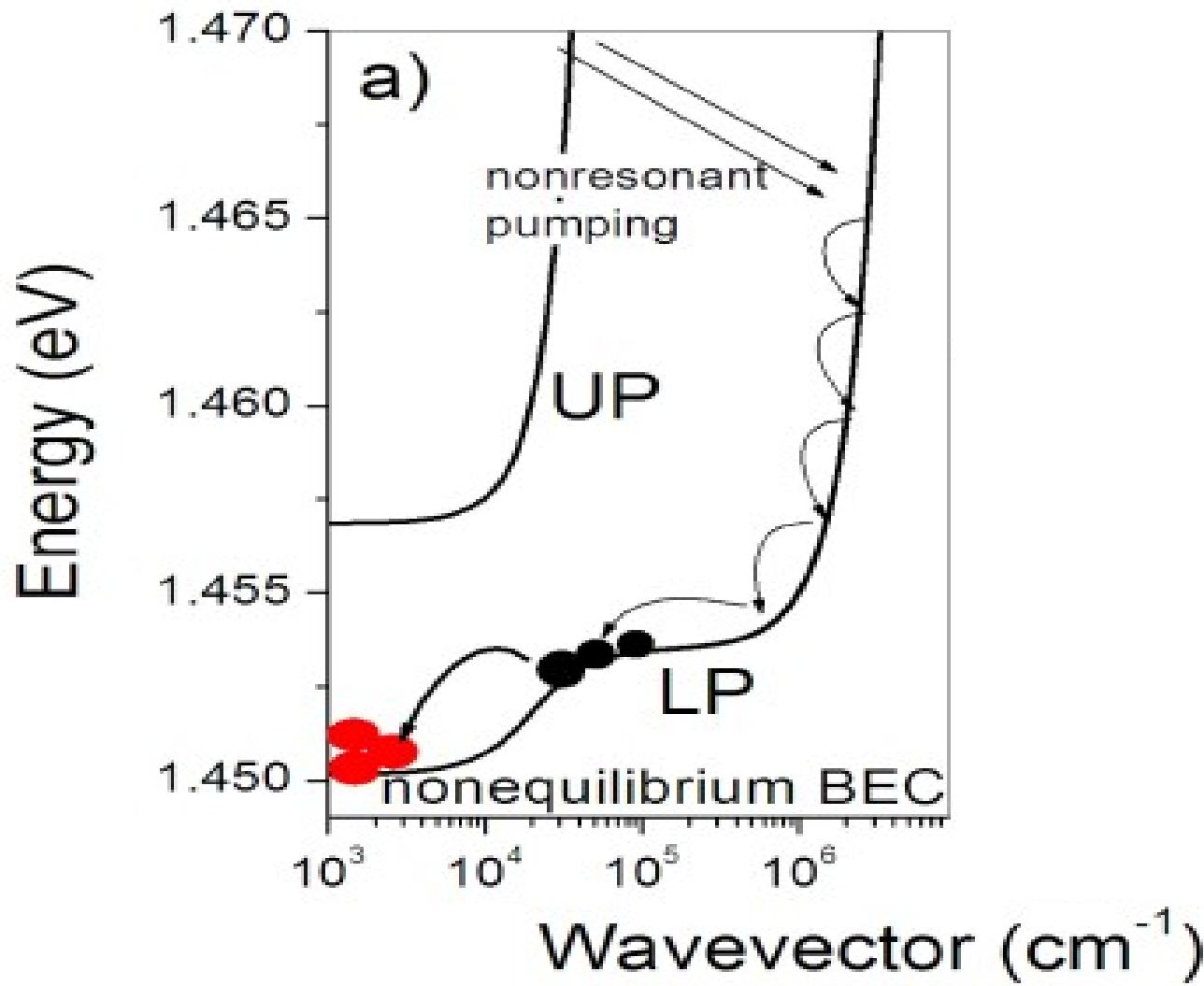
Solid lines show the critical concentration  $N_c$  versus temperature of the polariton KT phase transition. Dotted and dashed lines show the critical concentration  $N_c$  for quasi condensation in  $100 \mu\text{m}$  and  $1 \text{ meter}$  lateral size systems, respectively.

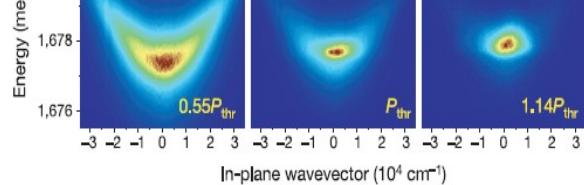
# Phase diagrams of exciton-polaritons in different materials



Solid lines show the critical concentration  $N_c$  versus temperature of the polariton KT phase transition. Dotted and dashed lines show the critical concentration  $N_c$  for quasi condensation in  $100 \mu\text{m}$  and 1 meter lateral size systems, respectively.

# Trappola in $k$ space per polariton





## ARTICLES

## Bose-Einstein condensation of exciton polaritons

J. Kasprzak<sup>1</sup>, M. Richard<sup>2</sup>, S. Kundermann<sup>2</sup>, A. Baas<sup>2</sup>, P. Jeambrun<sup>2</sup>, J. M. J. Keeling<sup>3</sup>, F. M. Marchetti<sup>4</sup>, M. H. Szymańska<sup>5</sup>, R. André<sup>1</sup>, J. L. Staehli<sup>2</sup>, V. Savona<sup>2</sup>, P. B. Littlewood<sup>4</sup>, B. Deveaud<sup>2</sup> & Le Si Dang<sup>1</sup>

CdTe T=5K

Figure 2 | Far-field emission measured at 5 K for three excitation

# Polariton emission and reflectivity in GaN microcavities as a function of angle and temperature

I. R. Sellers,\* F Semond, M. Leroux, and J. Massies  
*CRHEA-CNRS, Rue Bernard Gregory, Valbonne 065*

M. Zamfirescu, F. Stokker-Cheregi, M. Gurioli, A. Vinatt  
*LENS, Dipartimento di Fisica, Università di Firenze, 50019 S*

F. Stokker-Cheregi et al. / *Superlattices and Microstructures* 41 (2007) 376–383

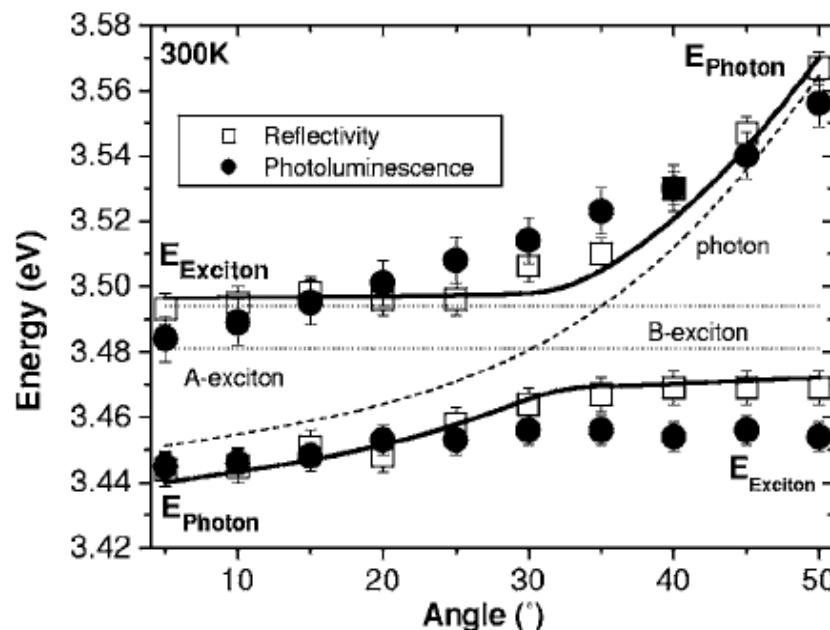
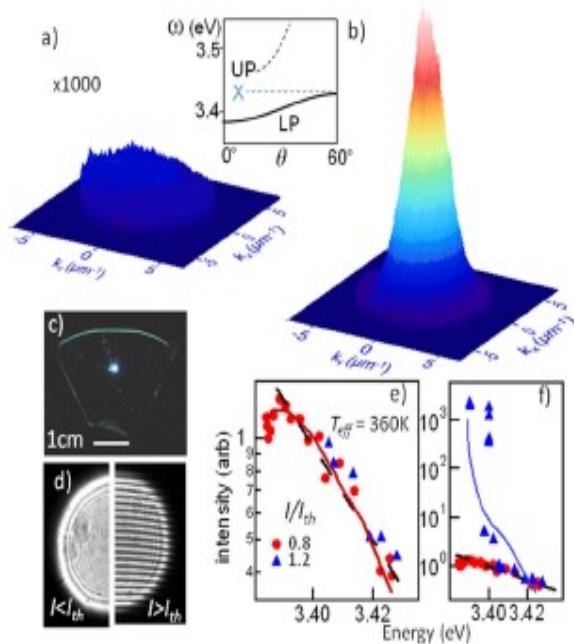


FIG. 4. Variation with the angle of incidence of the polariton mode energies at 300 K for both the reflectivity (open squares) and photoluminescence (closed circles). Also shown as a solid line is the numerical simulation using  $(3 \times 3)$  matrix analysis. The uncoupled excitonic (dotted lines) and photonic modes (dashed line) as a function of angle are also included in both cases.

Fig. 1. Simultaneous RT reflectivity (green line) and PL (red line) spectra along the wedge of the microcavity. The left vertical scale (linear) is for the reflectivity and the right vertical scale (logarithmic) is for the PL. (For interpretation of the references to colour in this figure legend, the reader is referred to the article.)

GaN  
**Polaritons**  
**at T=300K**



## BEC in GaN @ 300K

FIG. 1 (color online). Formation of polariton lasing at  $T = 300$  K in a GaN microcavity at in-plane wave vectors up to  $k_{\max} = 7 \mu\text{m}^{-1}$  for (a) just below (scaled up by  $\times 1000$ ) and (b) just above threshold ( $I_{\text{th}} \sim 1$  mW). Inset shows dispersion. (c) Image of pumped sample above threshold. (d) Interference of far-field emission cone through a slightly misaligned Michelson interferometer above and below threshold. (e),(f) Polariton emission intensity just below and above threshold as a function of energy, together with Boltzmann fit (dashed line) giving an effective temperature of 360 K, and result from kinetic simulation (solid lines).

# Polariton laser

# Laser history...

1917 Einstein derived the Plank formula, spontaneous + stimulated emission

1950 W. Lamb: idea of light amplification

1950 A. Kastler, optical pumping

1953 Weber, Twones, Basov, Prokhorov, maser

1959 T.H.Maiman, laser on rubis

1960s gaz lasers

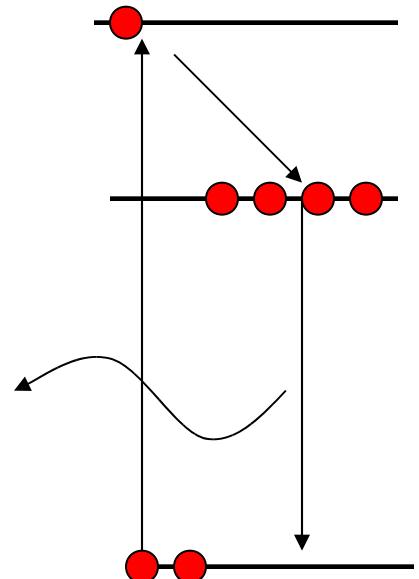
1969 first semiconductor lasers (pn-junction)

1972 Zh. Alferov, laser on heterostructures

1990s lasers on semiconductor nanostructures, VCSELs

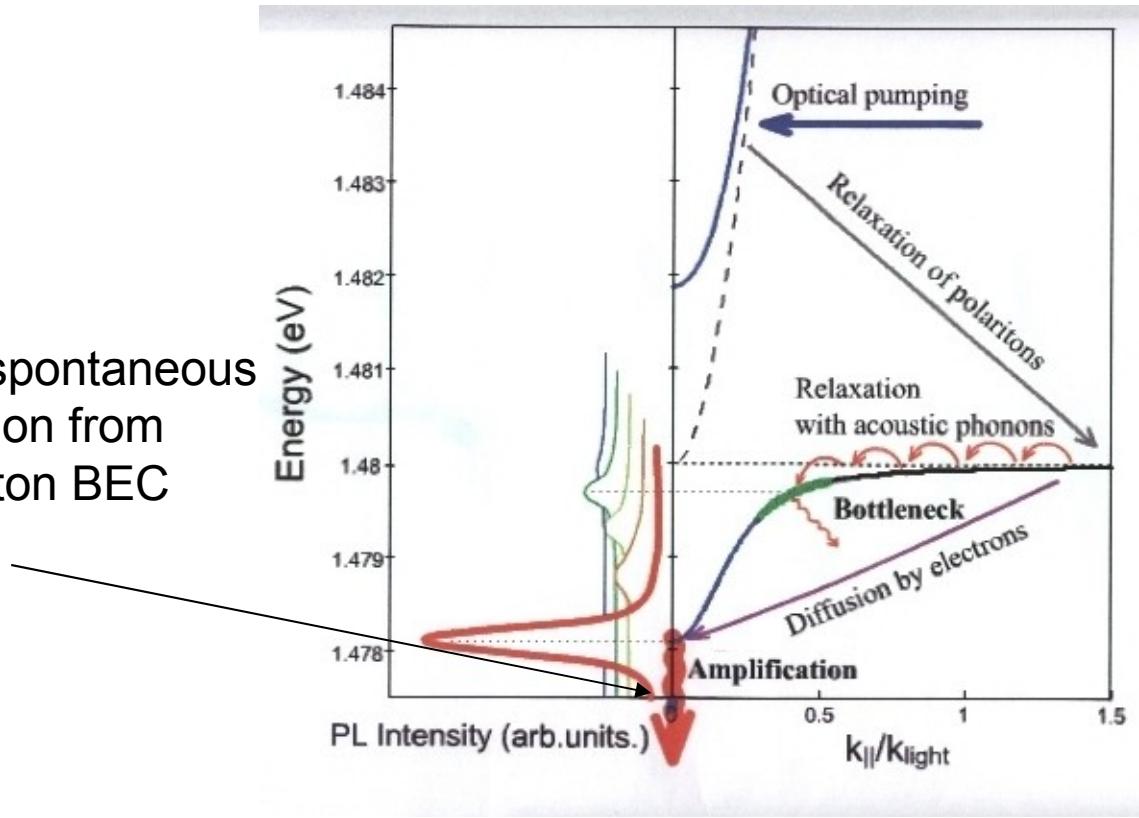
1996, Imamoglou, idea of polariton lasing

2007, RT polariton laser



To make a polariton laser one should have a microcavity in the strong-coupling regime

Coherent spontaneous emission from polariton BEC



Optically or electronically excited exciton-polaritons relax towards the ground state and Bose-condense there. Their relaxation is stimulated by final state population. The condensate emits spontaneously a coherent light

## “Normal” semiconductor laser:

- The threshold to lasing is given by the inversion of population condition.
- The absorption must be balanced by stimulated emission.
- Photon Bose condensation.
- Stimulated emission of light

## “Polariton” laser:

- The threshold condition: population of the  $k=0$  state larger than 1.
- The emission occurs at the energy lower than the absorption edge.
- Bose condensation of a half matter-half light particle.
- Spontaneous emission of light  
**Escape of polaritons from cavity**