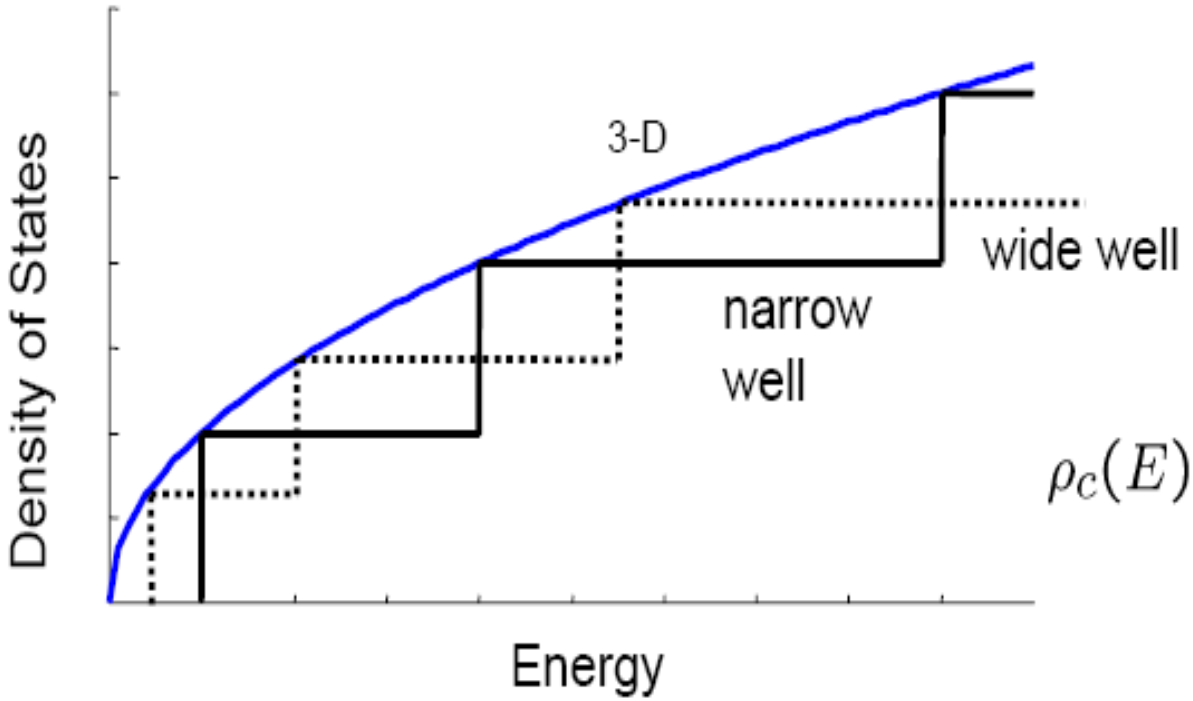
QW in MC

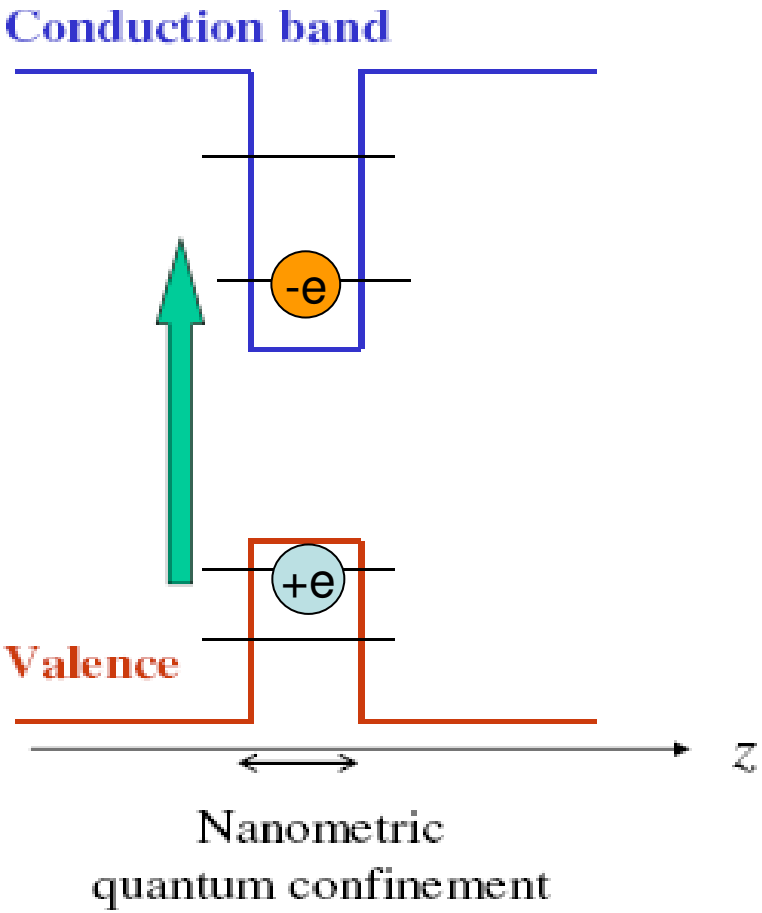
Assorbimento, Emissione: joint DOS

$$\alpha(\omega) = \frac{\pi\, q^2 c \mu_0}{m_0^2 \omega\, n} |p_{cv}|^2 \, | < F_c | F_v > |^2 \; \rho_r(\hbar\omega - E_{c1} - E_{v1})$$

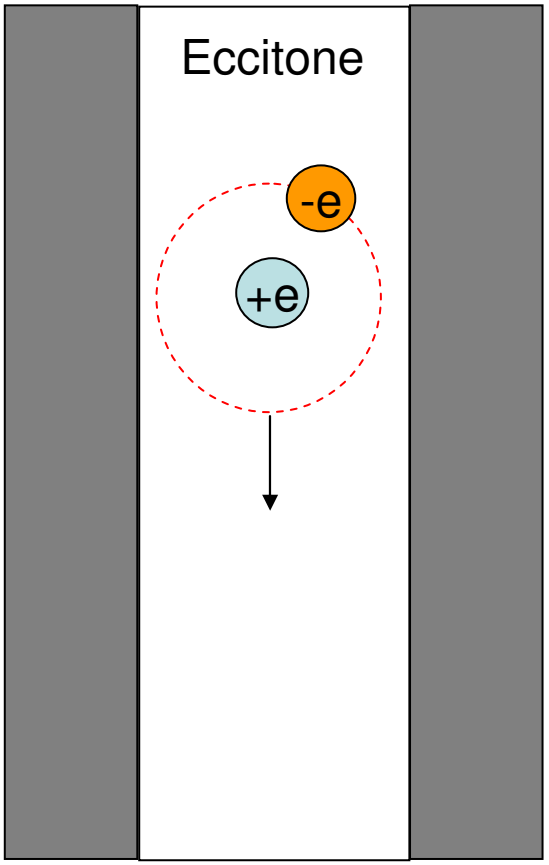


$$\rho_c(E) = \frac{m^*}{\pi \hbar^2 L_{QW}} \quad \text{(2-D)}$$

Elettrone e lacuna si attraggono e possono formare un eccitone



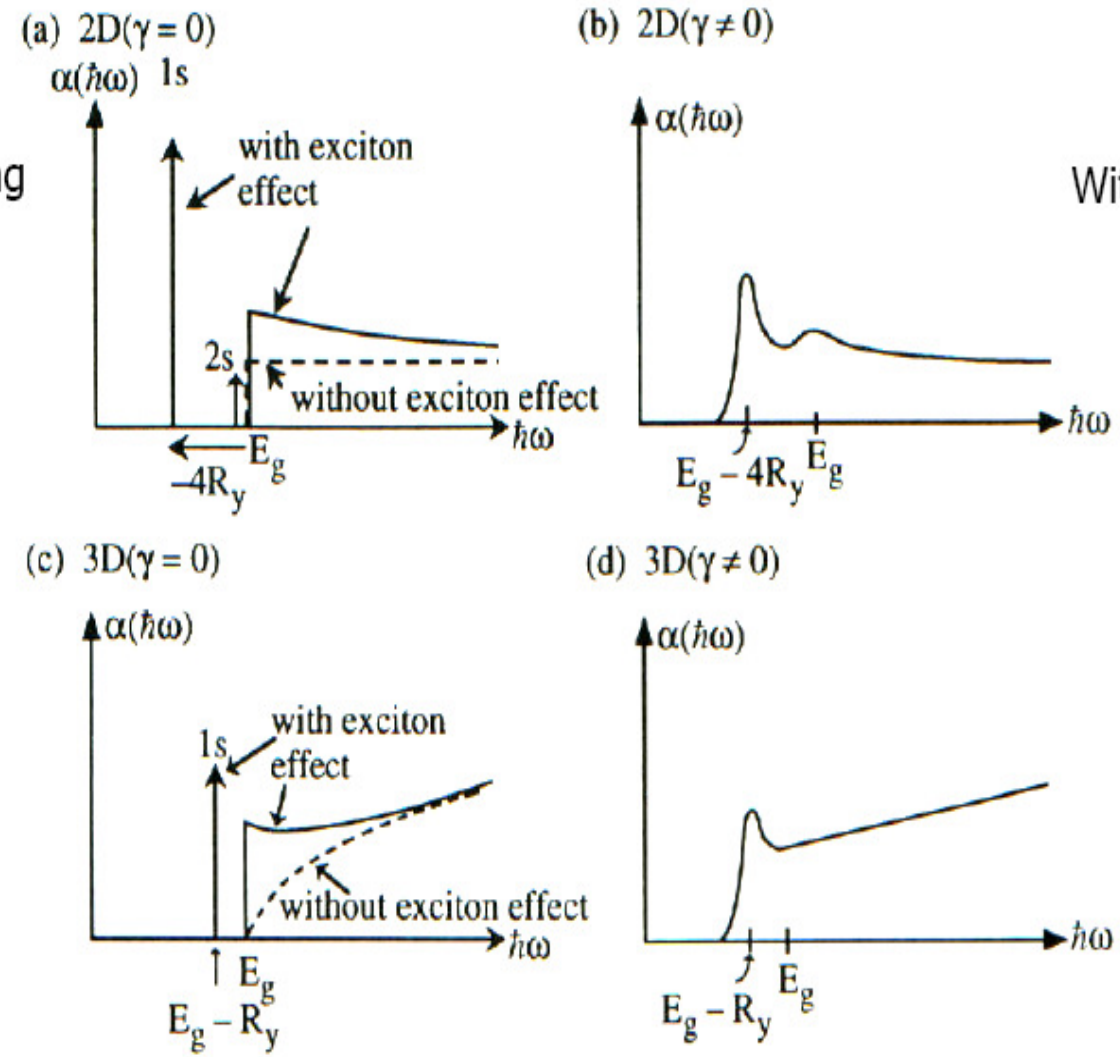
Nel piano l'eccitone
è libero di muoversi



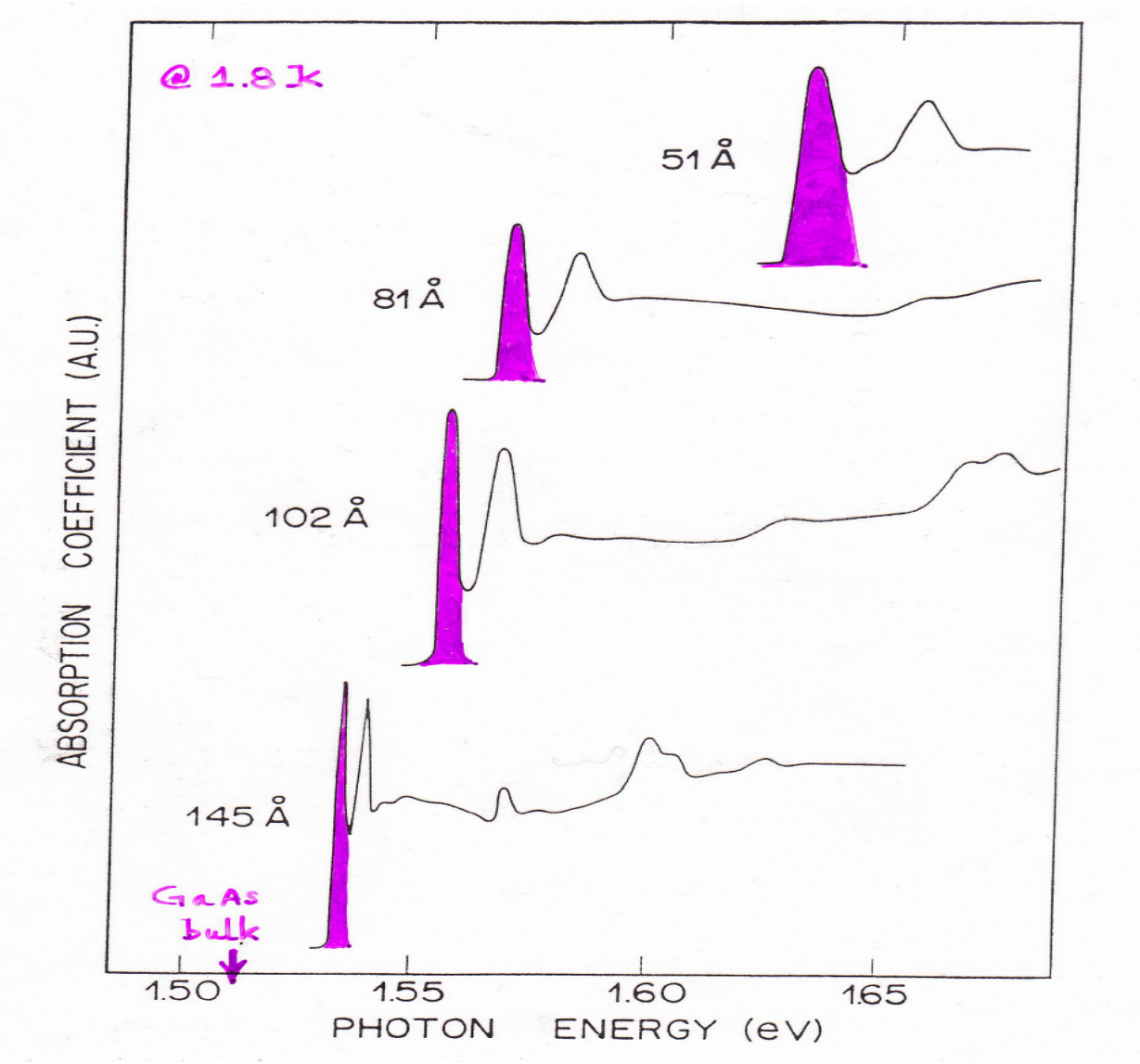
Absorption Including Coulomb Interactions

Without damping

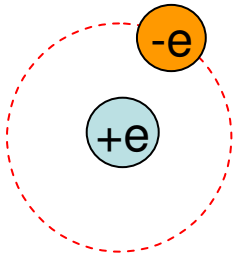
With damping



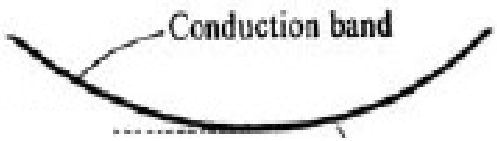
Dipendenza dallo spessore del pozzo

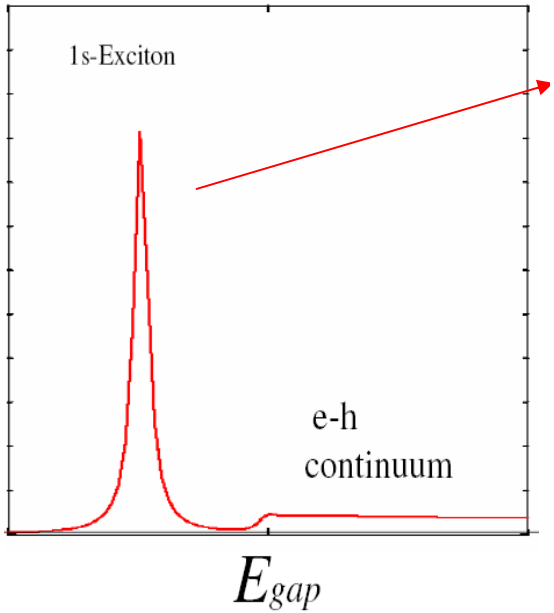


Sommario eccitone in QW

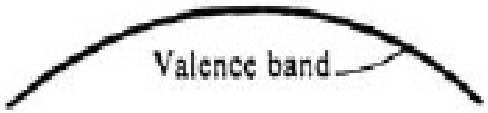


Spin intero





Picco di assorbimento ben separato dal continuo e-h



1. Stato isolato nel gap
2. Transizione tunabile
3. Statistica bosonica
4. Moto libero nel piano ($k_{//}$)

Pompaggio elettrico molto efficiente

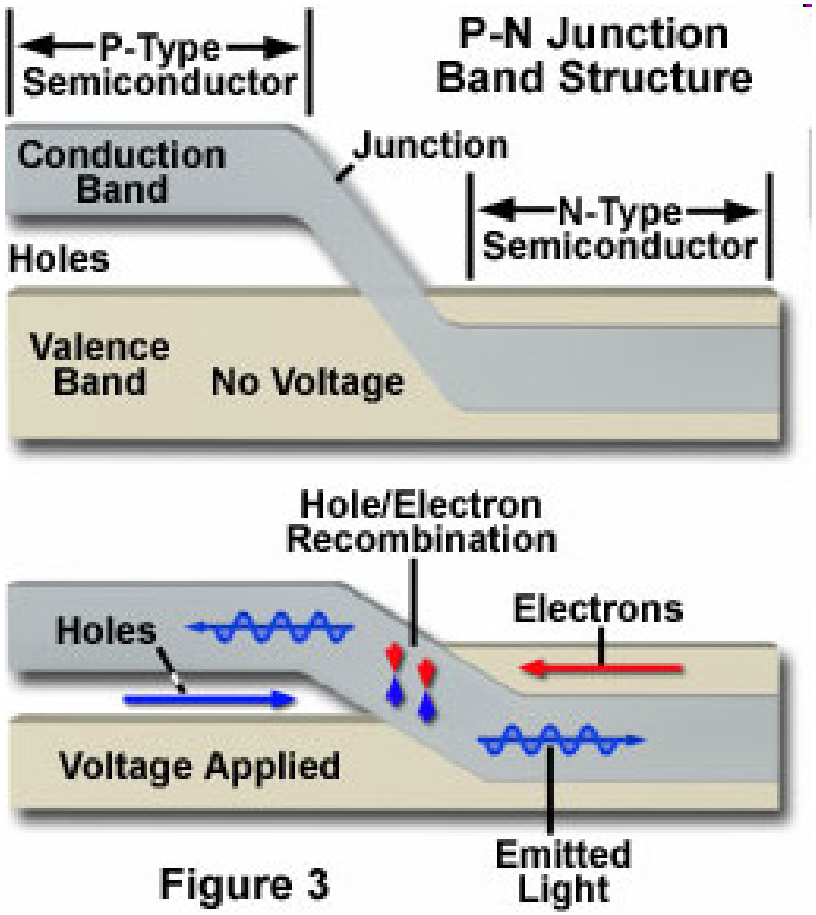
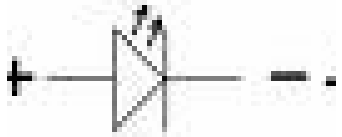
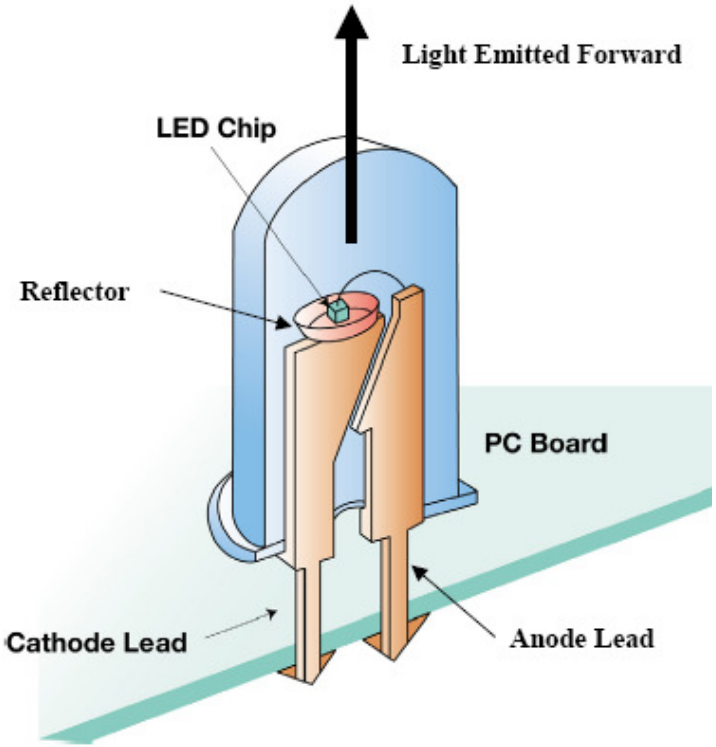


Figure 3





Sistema che ammette inversione di popolazione

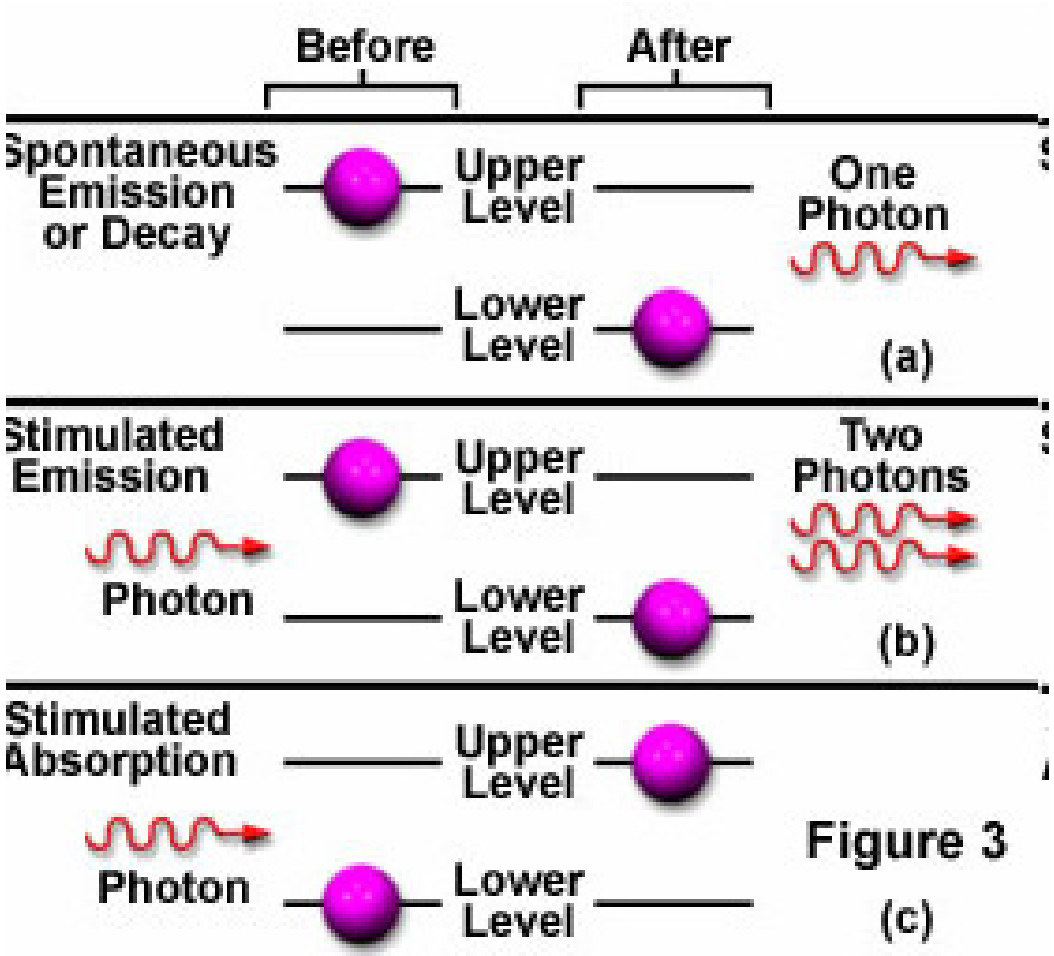
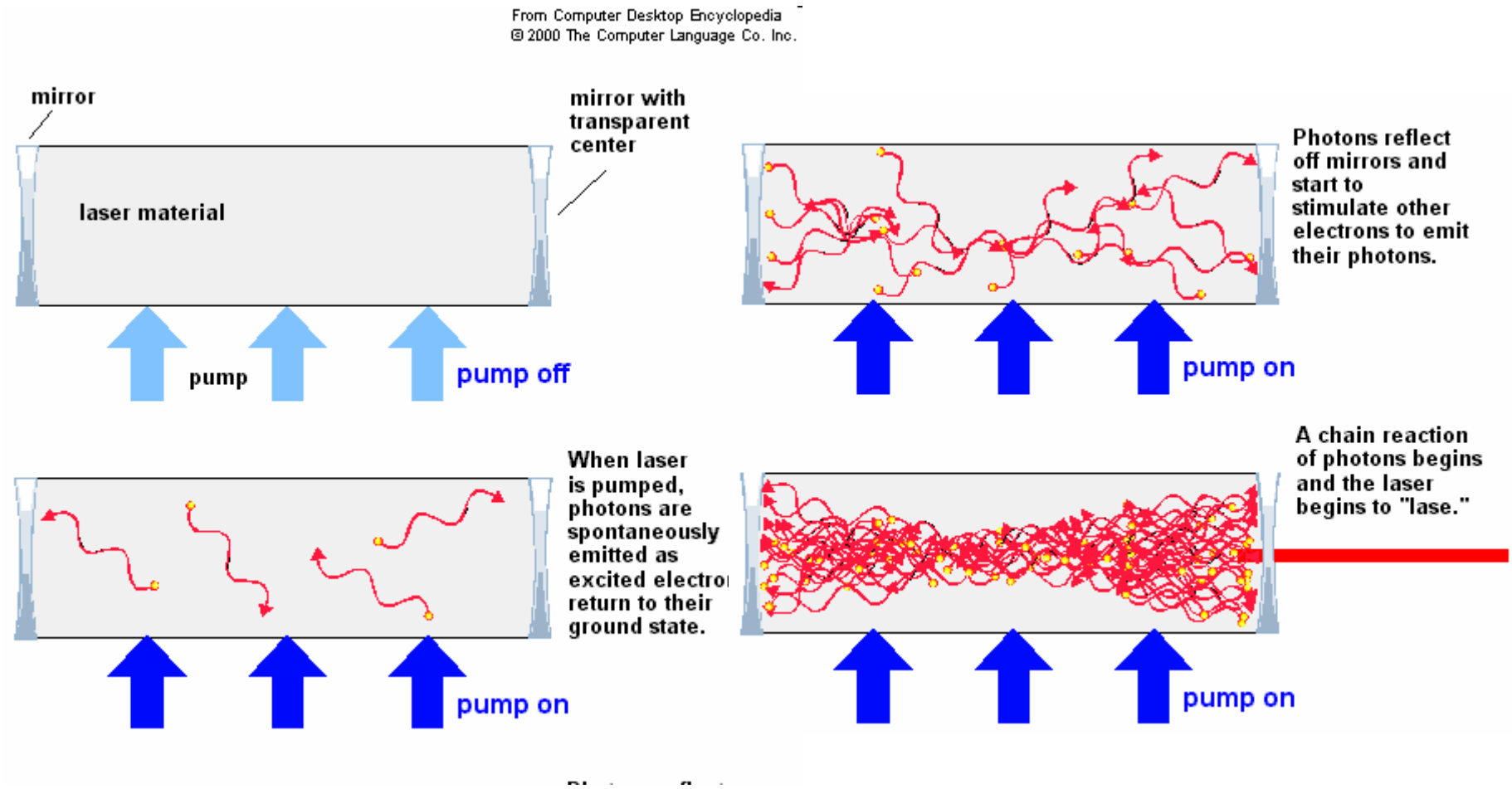


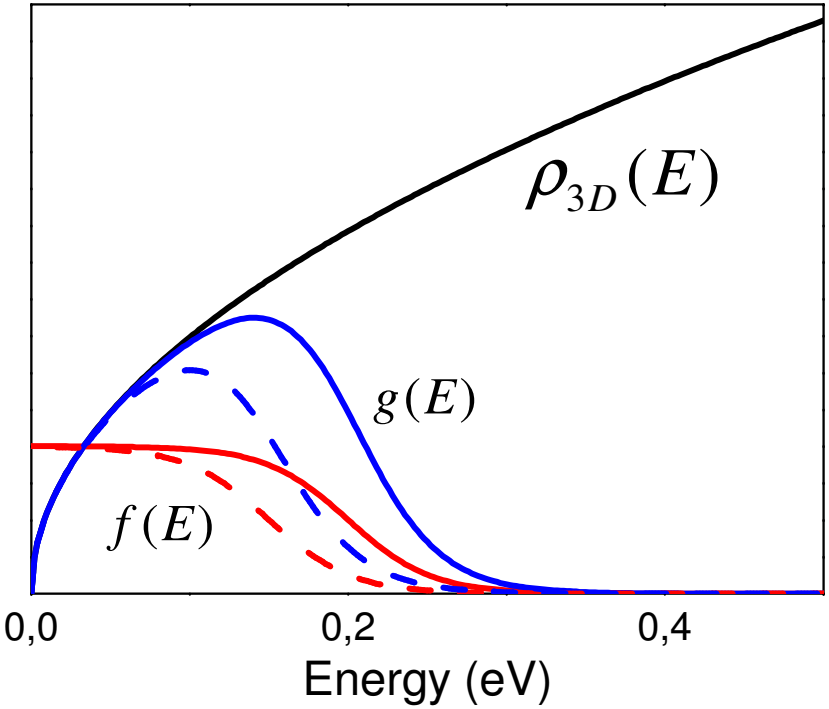
Figure 3

Azione della cavità



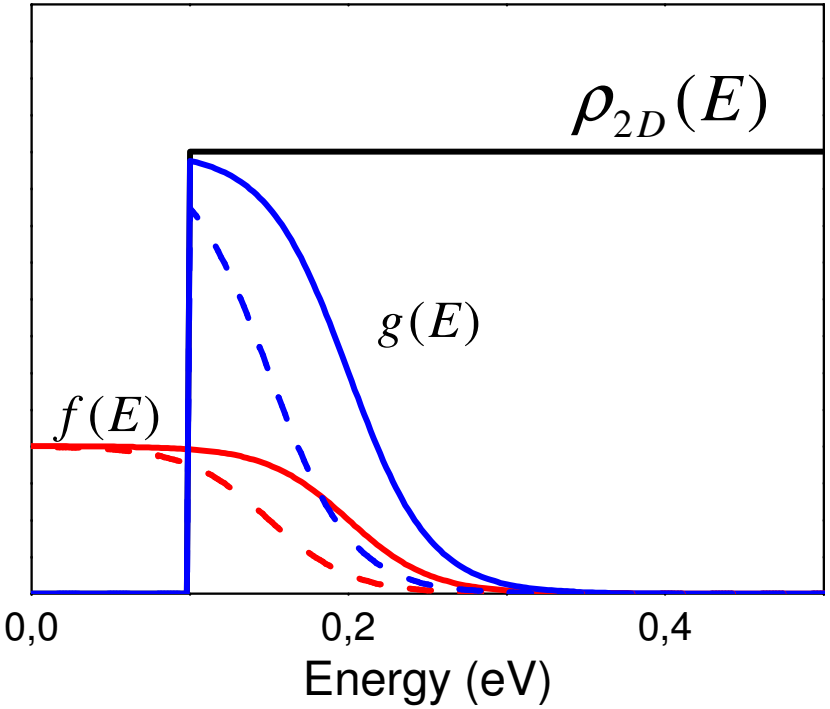
Guadagno $g(E) = A\rho(E)f(E)$

Bulk



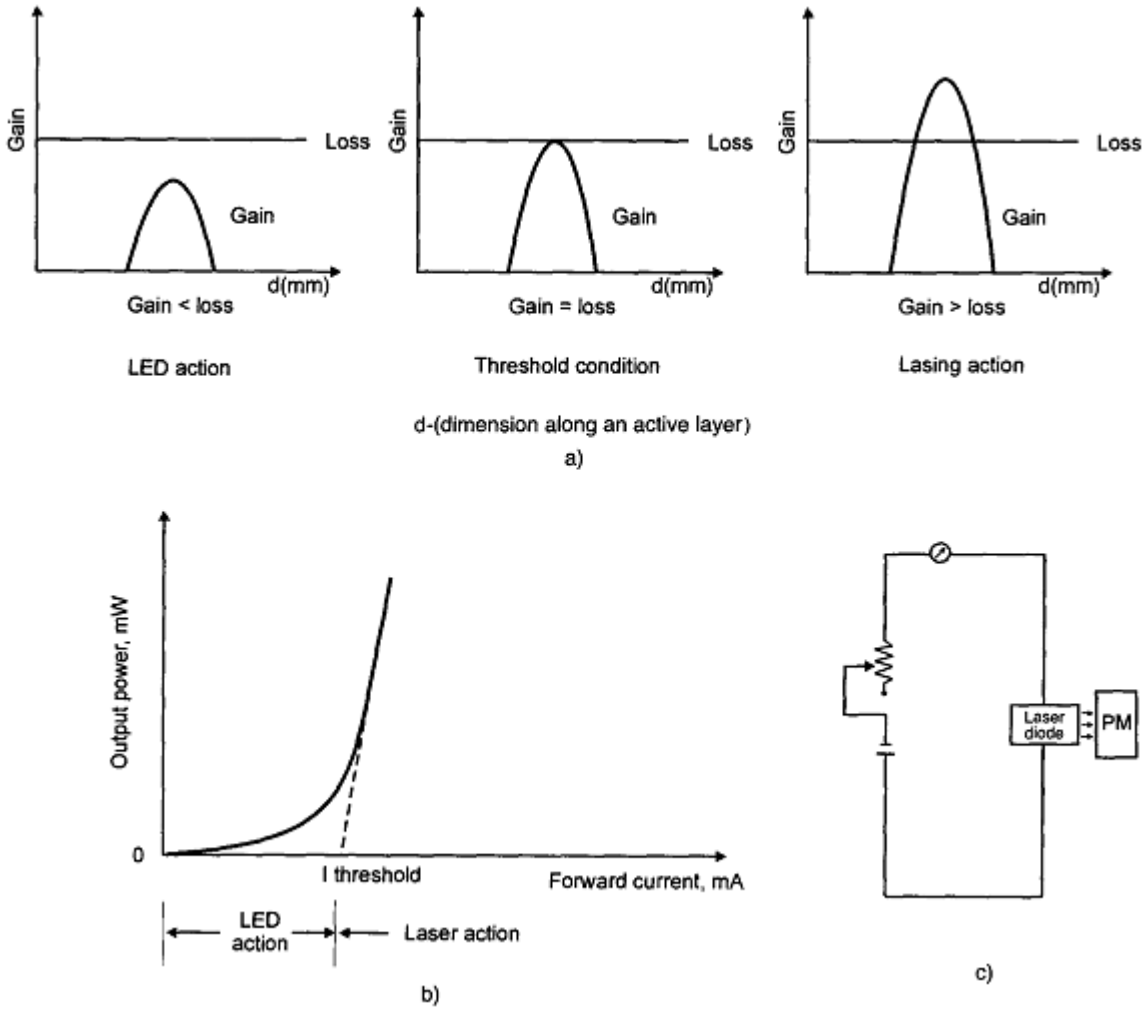
Massimo g su stato eccitato
frequenza emissione che varia

QW



Massimo g su stato fd
frequenza emissione che varia

Soglia

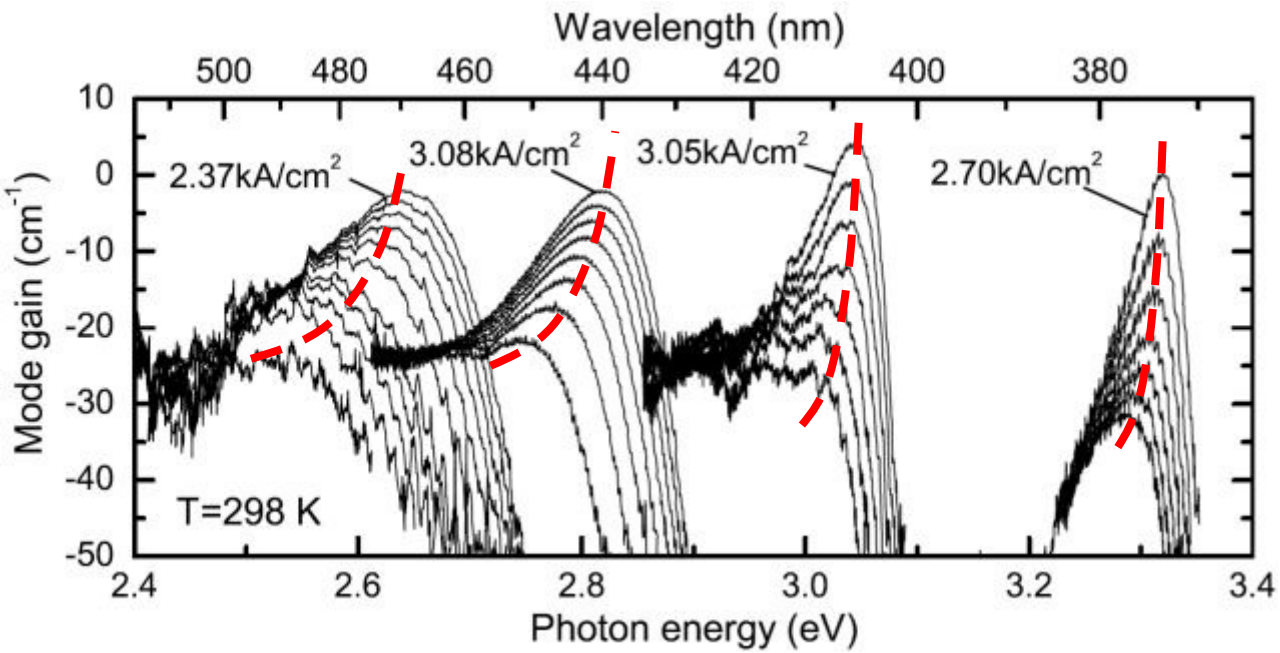


Laser a QW molto più efficiente del laser bulk

Soglia

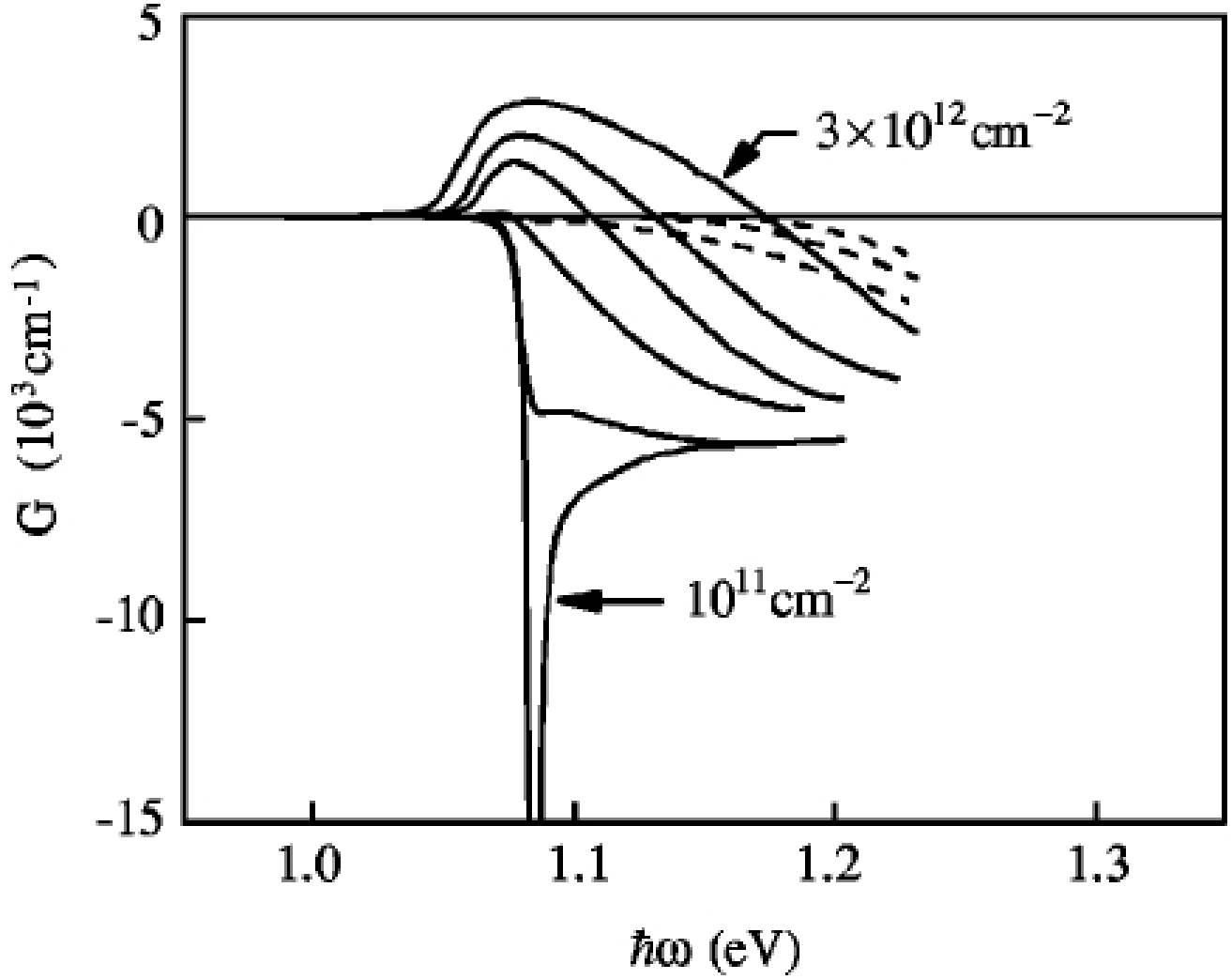
$$g(E) = A\rho(E)f(E)$$

Bulk Laser (InGaN)



Soglia

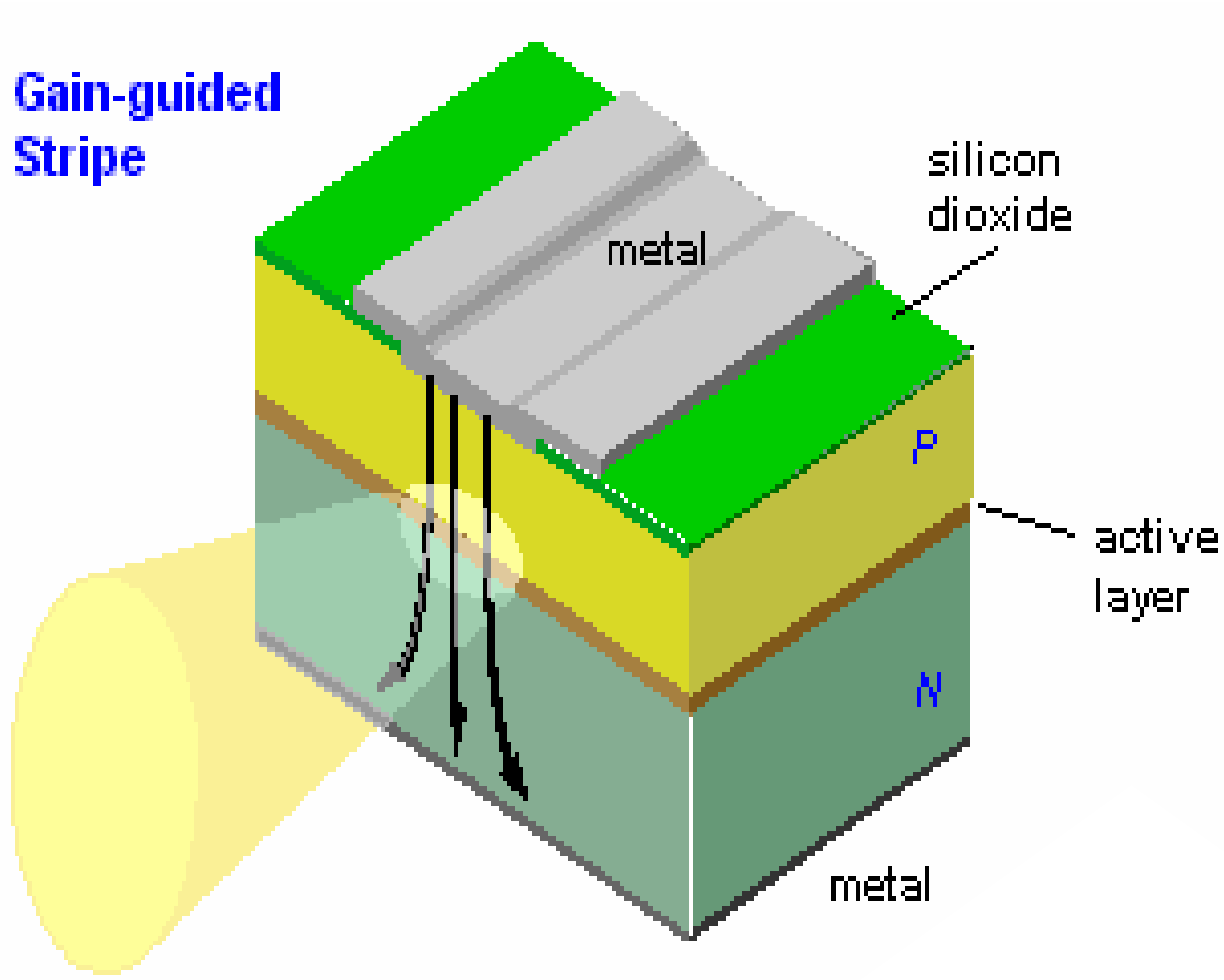
$$g(E) = A\rho(E)f(E)$$



QW
Laser

Laser a giunzione standard

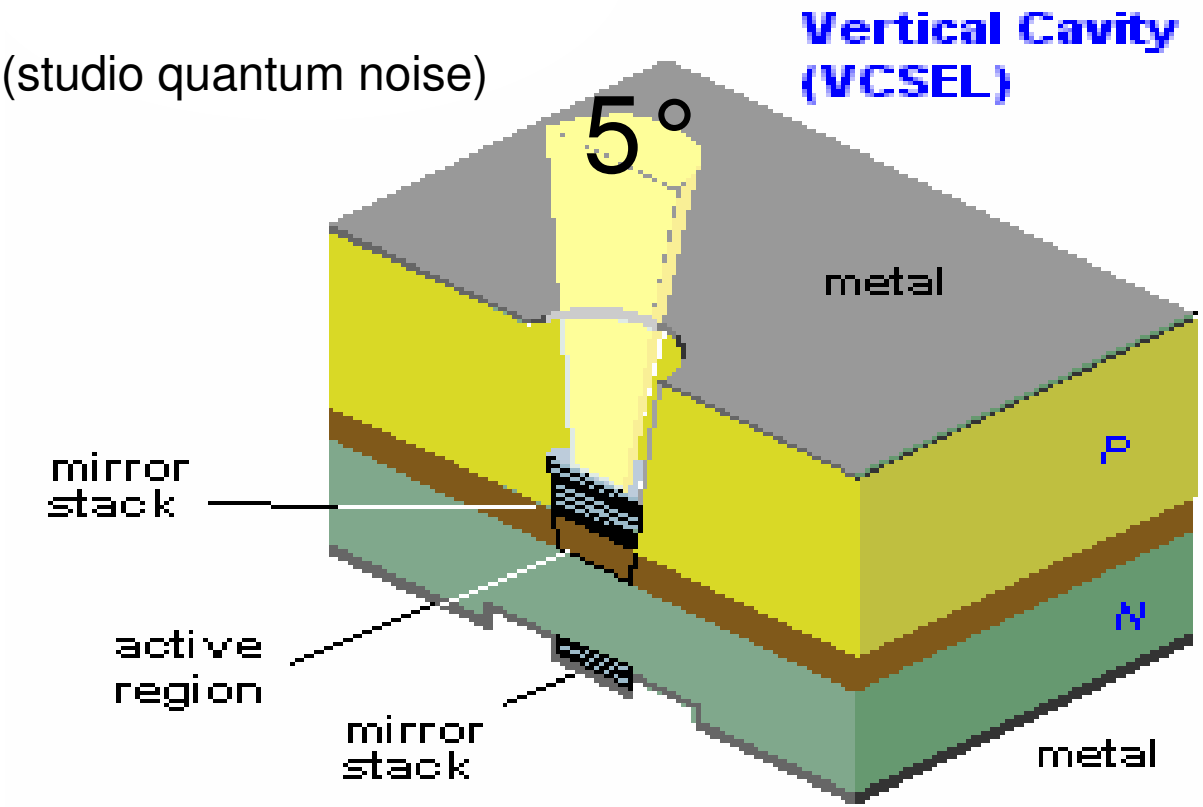
Grande divergenza
Fascio ellittico



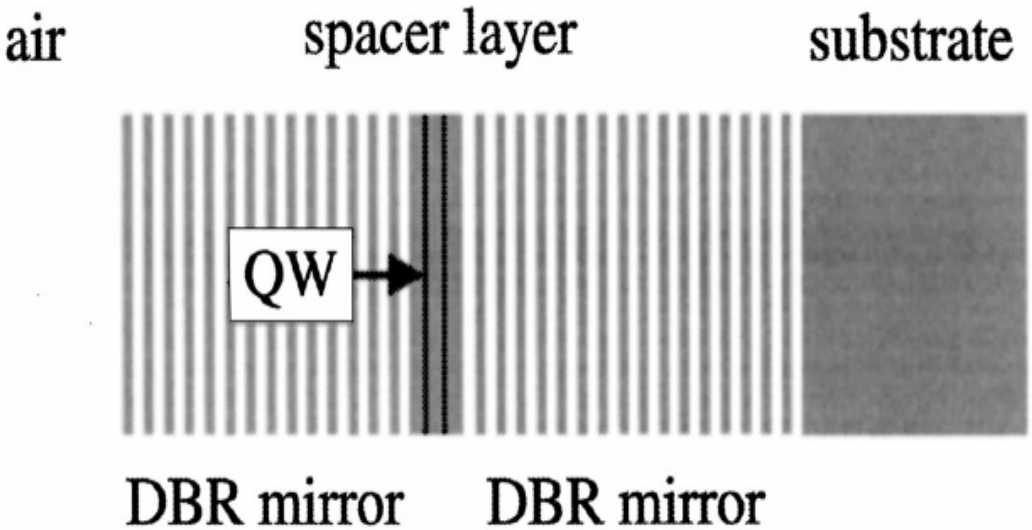
60°

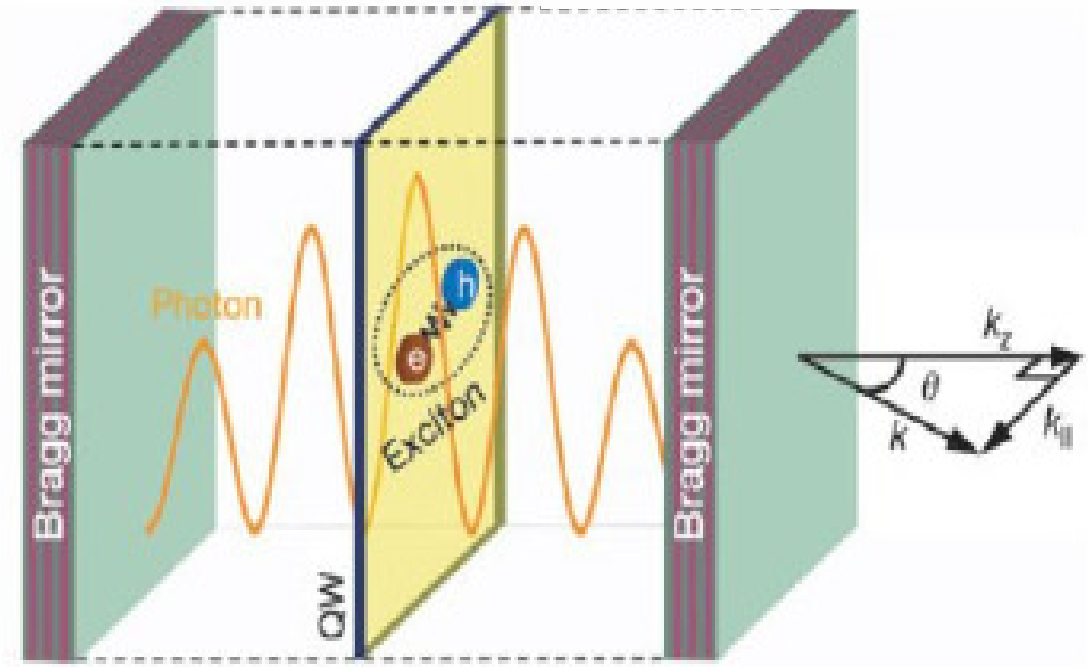
Laser a cavità verticale

Minor divergenza
Fascio circolare (miglior accoppiamento in fibra)
Soglia inferiore
Miglior stabilità
Minore rumore (studio quantum noise)
Test su wafer



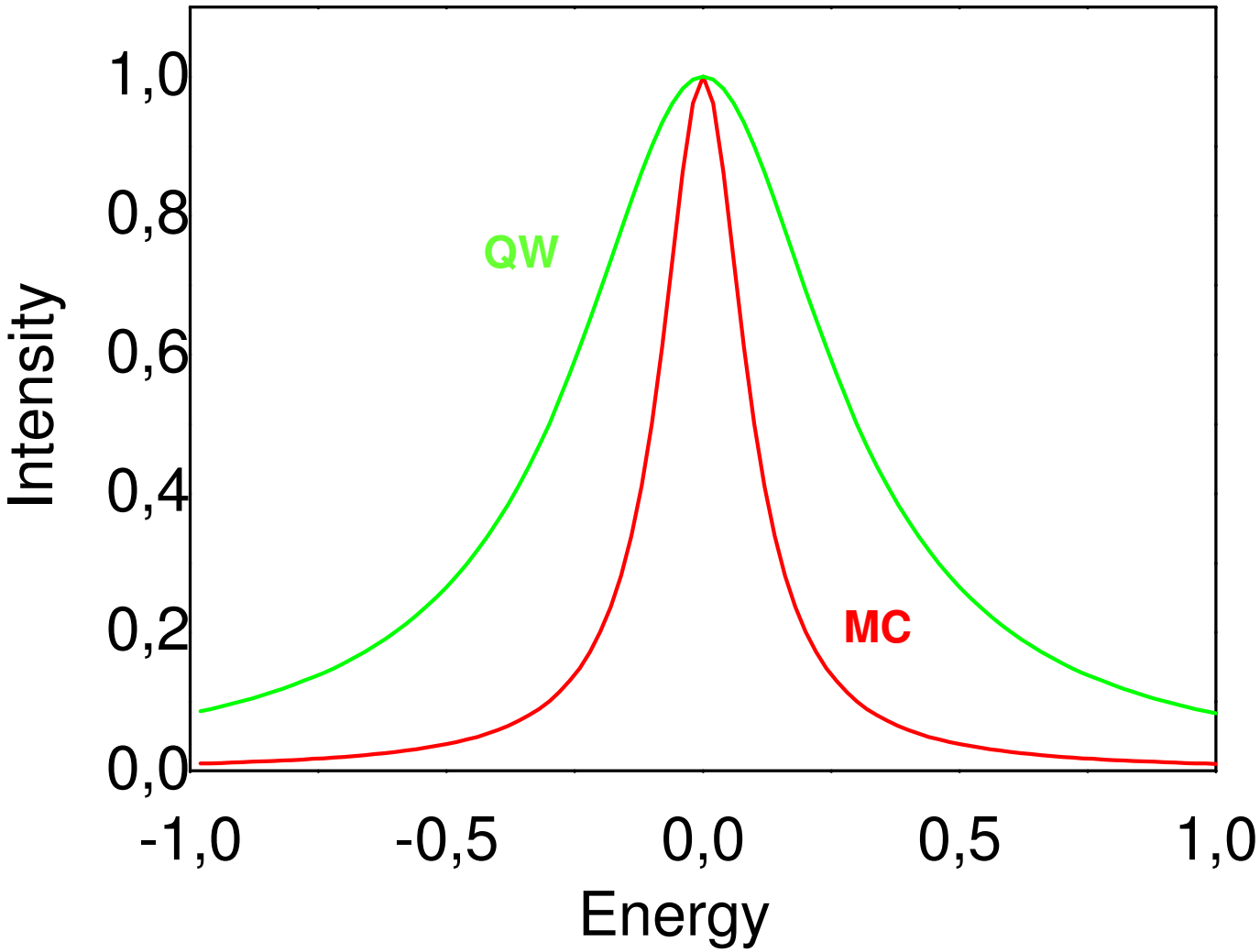
Quantum well in MC



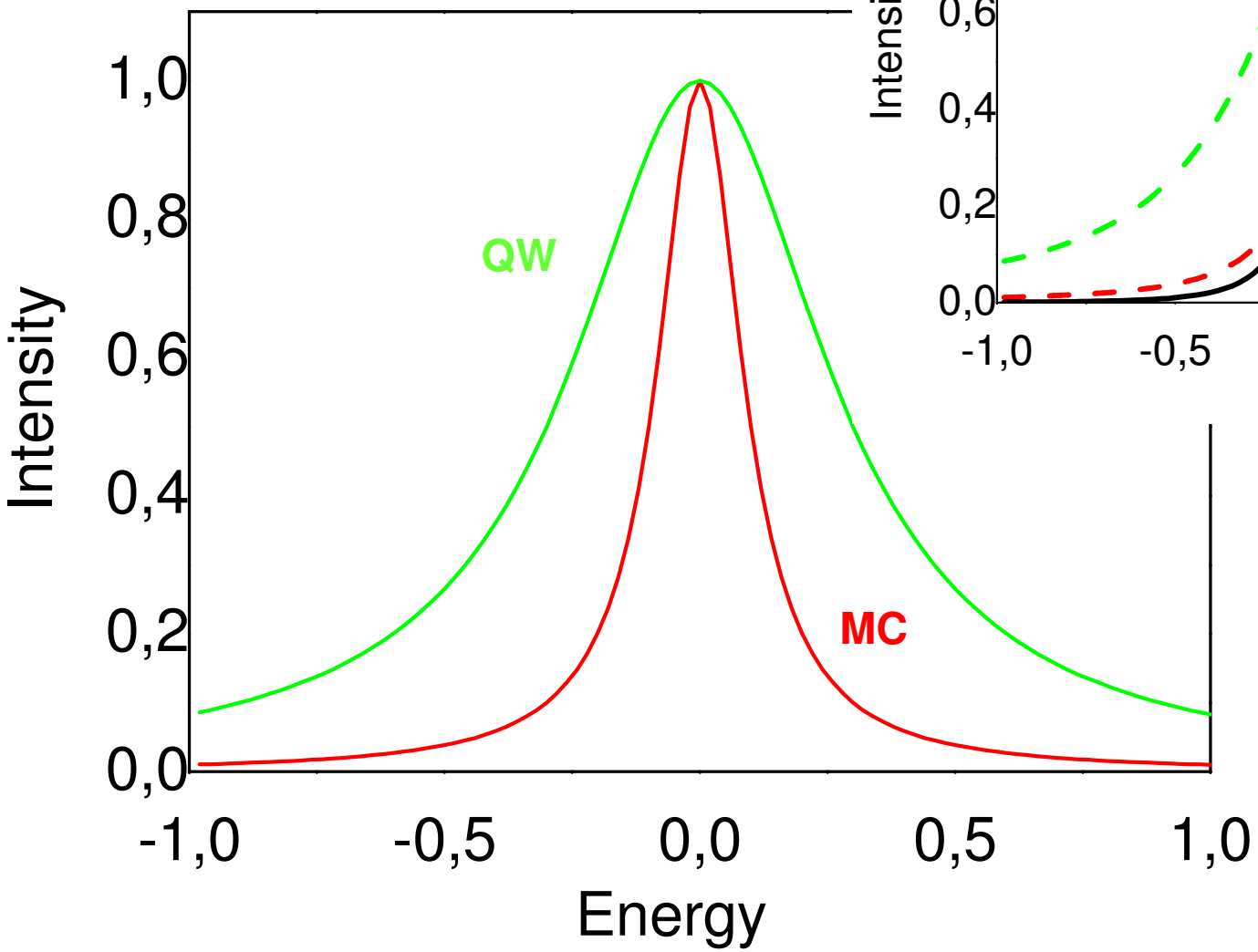


Situazione usuale RT

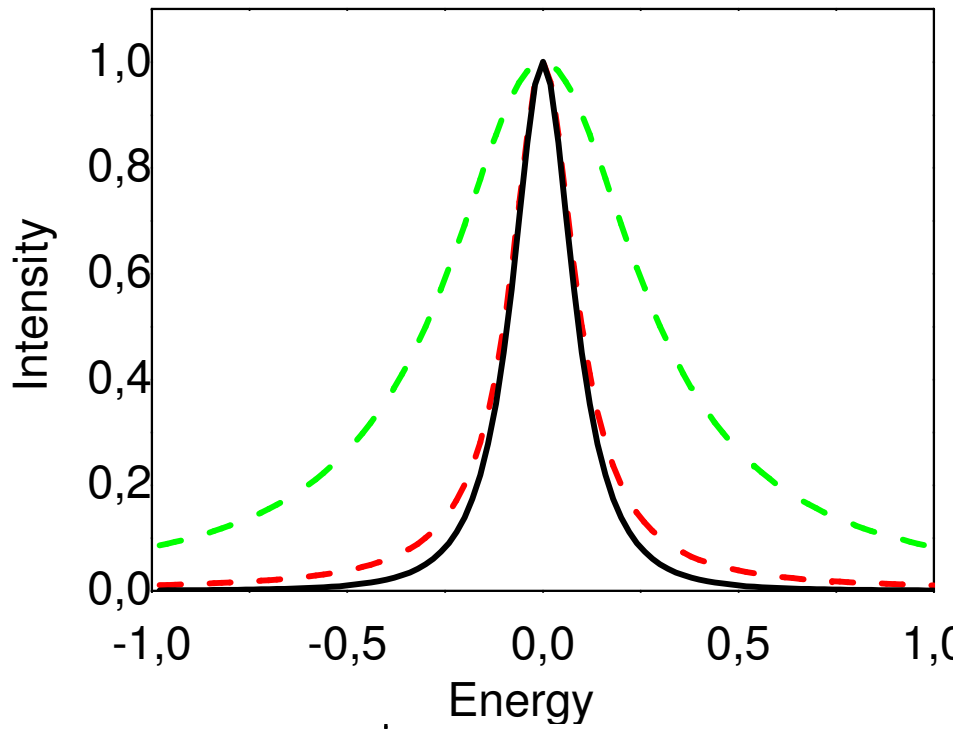
Cavità verticale con Q elevato (poche perdite, riduzione soglia)



Situazione usuale RT

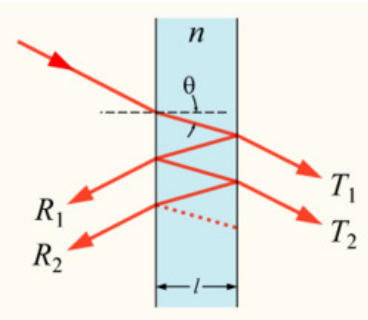


In LED effetto filtro



Emission
Narrowing

Dipendenza angolare

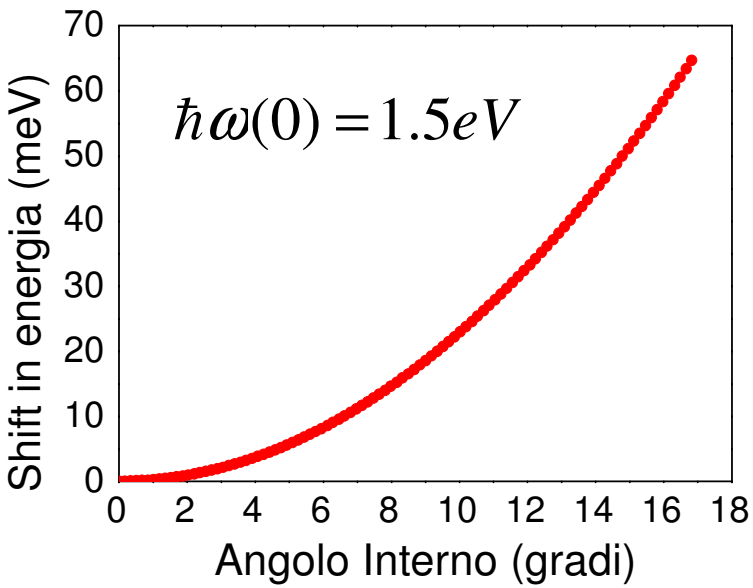


$$\hbar\omega(\theta) \approx \hbar\omega(0) + \hbar\omega(0)\frac{\theta^2}{2} =$$

$$\approx \hbar\omega(0) + \hbar\omega(0)\frac{\sin^2\theta}{2} =$$

$$= \hbar\omega(0) + \hbar\frac{ck}{n}\frac{\sin^2\theta}{2} =$$

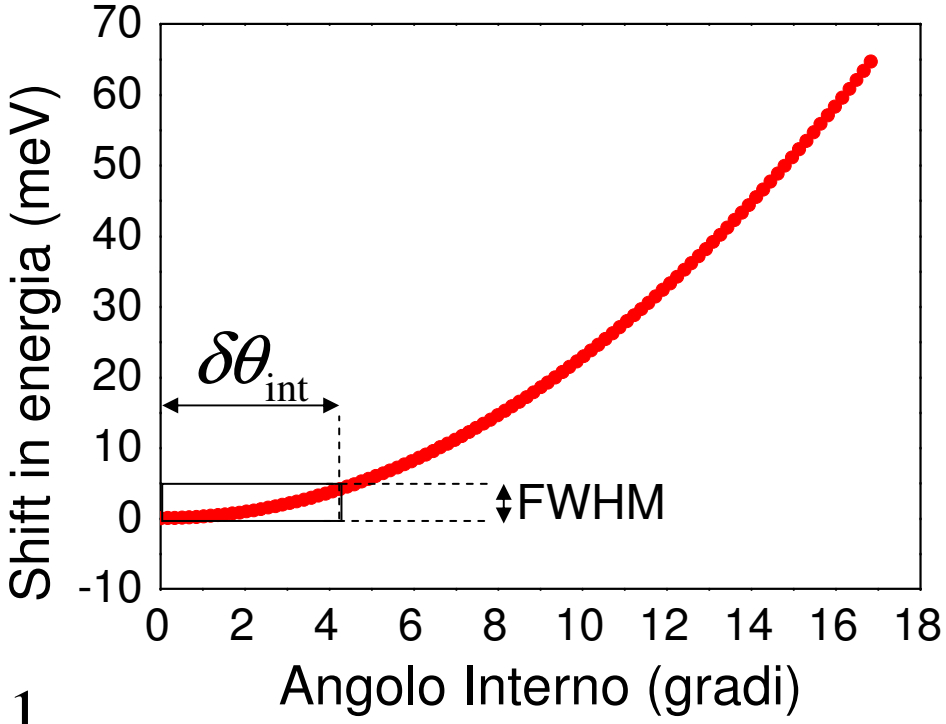
$$= \hbar\omega(0) + \frac{\hbar}{2}\frac{ck_{//}^2}{nk} \equiv \hbar\omega(0) + \frac{\hbar^2}{2}\frac{k_{//}^2}{m_{ph}}$$



$$m_{ph} = \frac{nk\hbar}{c} = \frac{n^2}{c^2}\hbar\omega \qquad \frac{m_{ph}}{m_{el}} \approx \frac{1.5\,eV}{0.5\,MeV} = 3\,10^{-6}$$

Apertura angolare cavità

Minor divergenza
Fascio circolare
(miglior accoppiamento in fibra)



$$\omega(\theta) \approx \omega(0) + \omega(0)\frac{\theta^2}{2}$$

$$\delta\theta_{\text{int}} \approx \sqrt{\frac{FWHM}{\omega(0)}} = \sqrt{\frac{1}{Q}} \approx \frac{1}{40}\,rad$$

$$\delta\theta_{ext} \approx n\sqrt{\frac{1}{Q}} \approx 0.09\ rad = 5^{\circ}$$

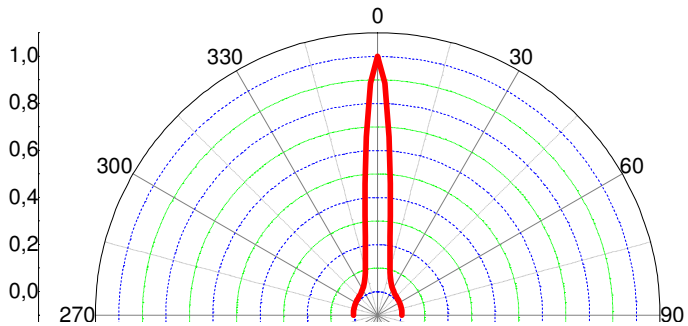
Angular patter

$$\omega(\theta) \approx \omega(0) + \omega(0)\frac{\theta^2}{2}$$

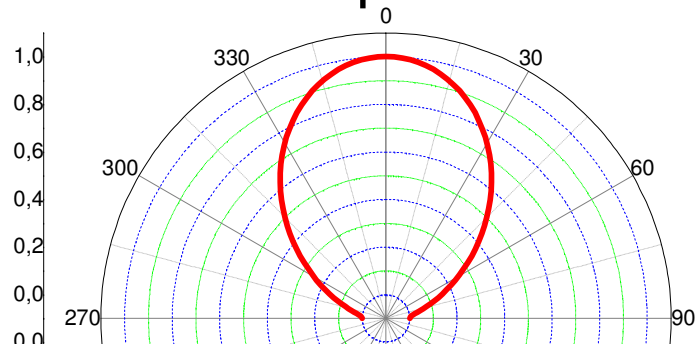
$$\delta\theta_{\text{int}} \approx \sqrt{\frac{FWHM}{\omega(0)}} = \sqrt{\frac{1}{Q}} \approx \frac{1}{40}\,rad$$

$$\delta\theta_{ext} \approx n\sqrt{\frac{1}{Q}} \approx 0.09\ rad = 5^{\circ}$$

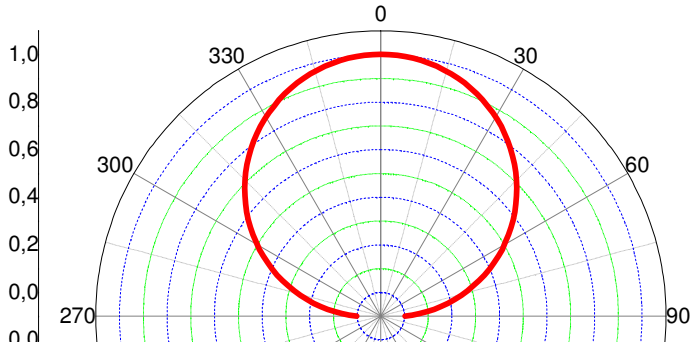
Microcavità



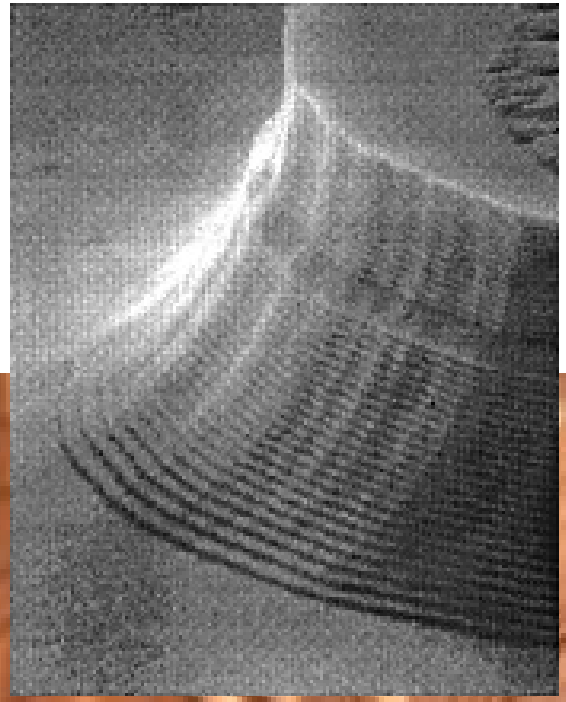
Dipolo

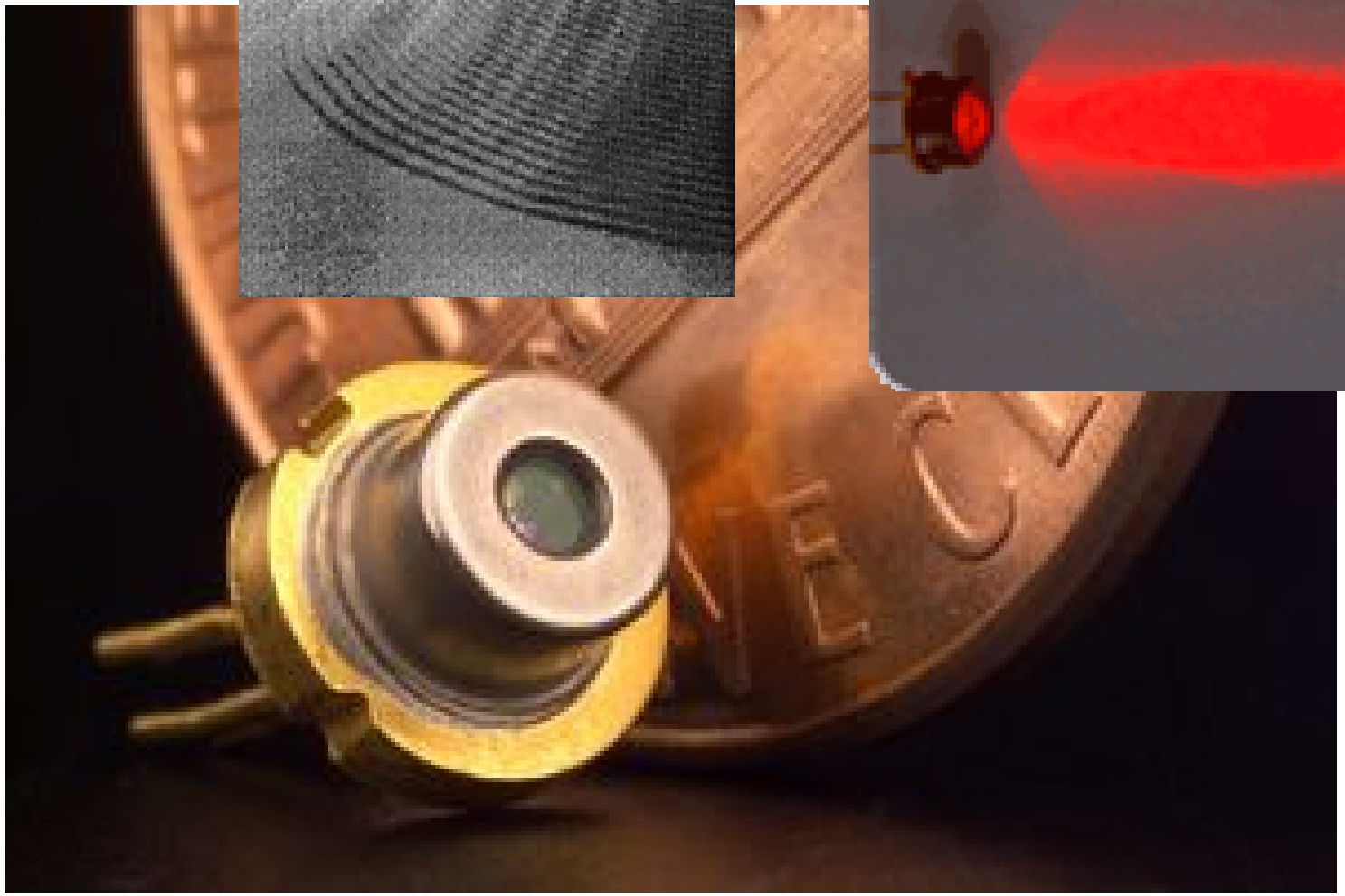


Isotropo



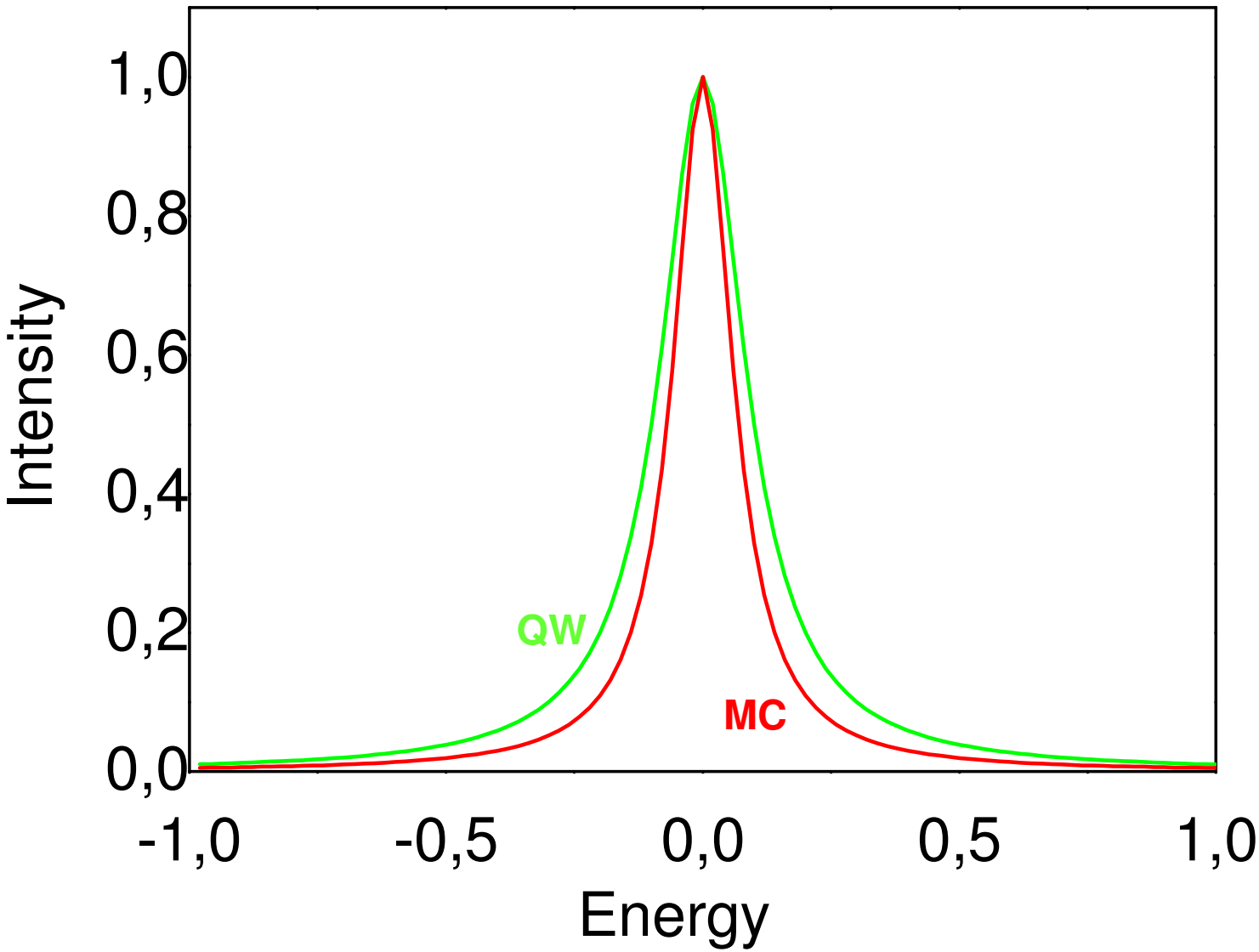
VCSEL





Strong coupling
(teoria classica)

Situazione ottimale a LT



Modello di Lorentz per l'eccitone

$$\ddot{x} = -2\gamma\dot{x} - \omega_0^2 x - \frac{eE}{m} e^{-i\omega t}$$

Oscillatore armonico forzato

$$x = x_0 e^{-i\omega t} \approx \frac{eE}{2m\omega_0} \frac{1}{(\omega - \omega_0) + i\gamma}$$

Soluzione stazionaria

$$p(\omega) = -ex = \varepsilon_0 \chi_{res}(\omega) E = \varepsilon_0 \frac{-e^2 / 2m\varepsilon_0\omega_0}{(\omega - \omega_0) + i\gamma} E$$

Dipolo elettrico indotto

$$\chi_{res}(\omega) = \frac{-e^2 / 2m\varepsilon_0\omega_0}{(\omega - \omega_0) + i\gamma} = \frac{e^2}{2m\varepsilon_0\omega_0} \frac{\left[(\omega_0 - \omega) + i\gamma\right]}{(\omega - \omega_0)^2 + \gamma^2}$$

Polarizzabilità

Modello di Lorentz per l'eccitone

$$P = \frac{N}{V} p = \varepsilon_0 \frac{N}{V} \chi_{res}(\omega) E$$

Polarizzazione macroscopica

$$D = \varepsilon_0 E + P = \varepsilon_0 \left(1 + \frac{N}{V} \chi_{res}(\omega) \right) E = \varepsilon_0 \varepsilon_r E$$

Costante dielettrica

$$\tilde{n}^2 = \varepsilon_r = \left(1 + \frac{N}{V} \chi_{res}(\omega) \right) = 1 + \frac{G\left[(\omega_0 - \omega) + i\gamma\right]}{(\omega - \omega_0)^2 + \gamma^2}$$

$$\mathrm{Re}\,\tilde{n} \approx \sqrt{1 + \frac{G(\omega_0 - \omega)}{(\omega - \omega_0)^2 + \gamma^2}} \approx 1 + \frac{1}{2} \frac{G(\omega_0 - \omega)}{(\omega - \omega_0)^2 + \gamma^2}$$

$$\mathrm{Im}\,\tilde{n} \approx \frac{1}{2} \frac{\gamma G}{(\omega - \omega_0)^2 + \gamma^2}$$

Indice di rifrazione complesso

Modello di Lorentz per l'eccitone

$$\mathrm{Re}\,\tilde{n} \approx \sqrt{1+\frac{G(\omega_0-\omega)}{(\omega-\omega_0)^2+\gamma^2}} \approx 1+\frac{1}{2}\frac{G(\omega_0-\omega)}{(\omega-\omega_0)^2+\gamma^2}$$

$$\mathrm{Im}\,\tilde{n} \approx \frac{1}{2}\frac{\gamma G}{(\omega-\omega_0)^2+\gamma^2}$$

$$E(z) = Ee^{ikz} = Ee^{i\frac{\omega}{c}\tilde{n}z} = Ee^{i\frac{\omega}{c}\mathrm{Re}(\tilde{n})z}e^{-\frac{\omega}{c}\mathrm{Im}(\tilde{n})z} = Ee^{ikz}e^{-\frac{\alpha}{2}z}$$

$$n = \mathrm{Re}(\tilde{n})$$

Indice di rifrazione

$$\alpha = 2\frac{\omega}{c}\mathrm{Im}(\tilde{n})$$

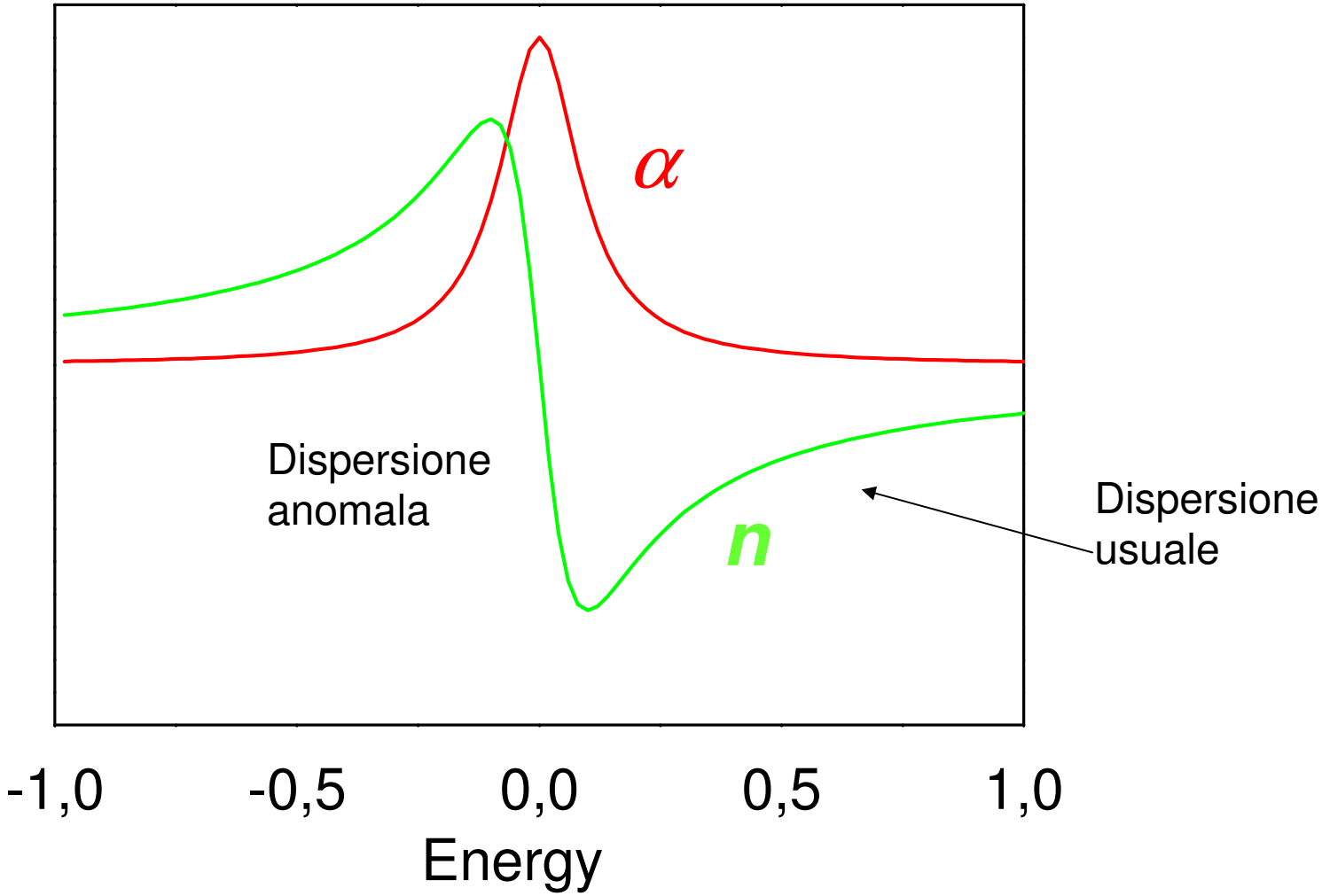
Coefficiente di assorbimento

Modello di Lorentz per l'eccitone

$$n = n_B + \frac{1}{2}\frac{G(\omega_0 - \omega)}{(\omega - \omega_0)^2 + \gamma^2}$$

$$\alpha = \frac{\omega}{c}\frac{\gamma G}{(\omega - \omega_0)^2 + \gamma^2}$$

Indice di rifrazione di background



Trasmissione FP con risonanza

$$E_t = \left[\frac{T e^{i(2\varphi_t + \Delta/2)}}{1 - R\, e^{i(2\varphi_r + \Delta)}} \right] E_i = t_C E_i$$

$$\frac{\Delta}{2} = \frac{2\pi \tilde{n}}{\lambda} \ell = \frac{\omega}{c} (n_B + n_R) \ell + i \frac{\alpha}{2} \ell$$

$$E_t = \left[\frac{T e^{i(2\varphi_t + \Delta/2)} e^{-\frac{\alpha}{2}\ell}}{1 - R\, e^{i(2\varphi_r + \Delta)} e^{-\alpha\ell}} \right] E_i = t_C E_i$$

$$T_C = |t_C|^2 = \frac{T^2 \boxed{e^{-\alpha\ell}}}{\left| 1 - R\, e^{i(2\varphi_r + \Delta)} \boxed{e^{-\alpha\ell}} \right|^2}$$

Assorbimento riduce trasmissione e allarga le risonanze

Trasmissione FP con risonanza

$$T_C = \left|t_C\right|^2 = \frac{T^2 e^{-\alpha\ell}}{\left|1 - R\, e^{i(2\varphi_r + \Delta)} e^{-\alpha\ell}\right|^2}$$

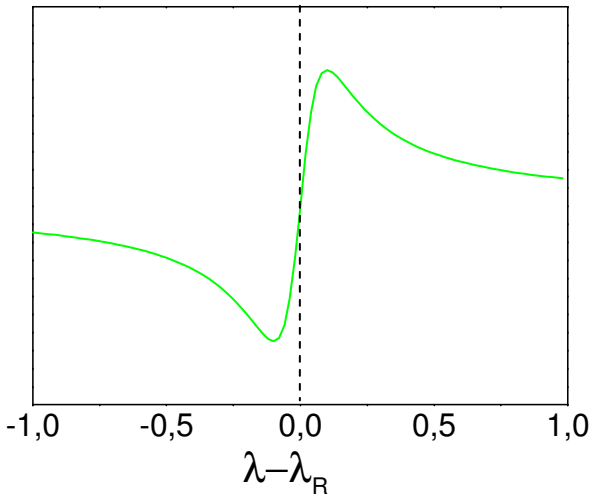
Trascurando φ_r la condizione di risonanza è

$$\frac{\Delta}{2} = \frac{2\pi n}{\lambda}\ell = \frac{\omega}{c}\left(n_B + n_R\right)\ell$$

$$Posizione\ picco \quad \left[\varphi_r + \frac{\omega_m}{c}\left(n_B + n_R\right)\ell\right] = m\pi$$

Calcolo posizione risonanze

$$\frac{\Delta}{2} = \frac{2\pi\left(n_B + n_R(\lambda)\right)}{\lambda}\ell$$



$$\textit{Posizione picco} \quad \left[\frac{2\pi}{\lambda}\left(n_B + \underset{n_B}{n_R(\lambda)}\right)\ell\right] = m\pi$$

$$\left(n_B + n_R(\lambda)\right) = \frac{\lambda}{2\ell}m$$

$$n_B = \frac{\lambda_R}{2\ell}m \qquad \lambda_R = n_B\frac{2\ell}{m}$$

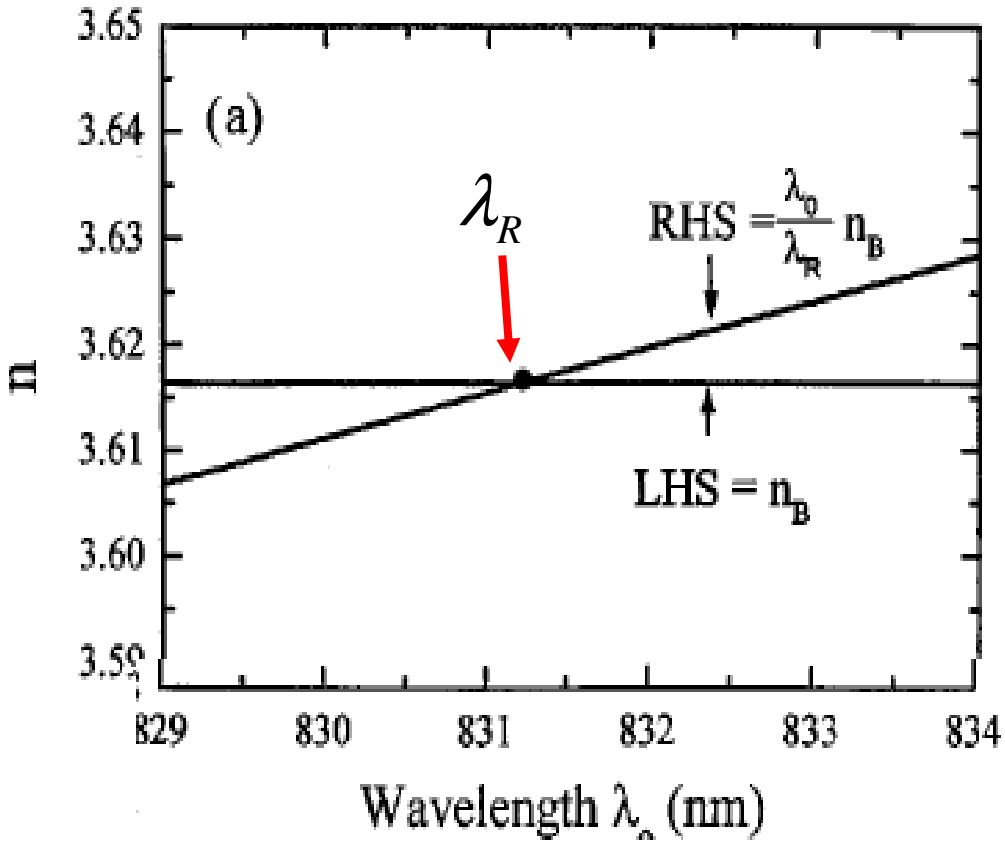
Cavità ben accordata

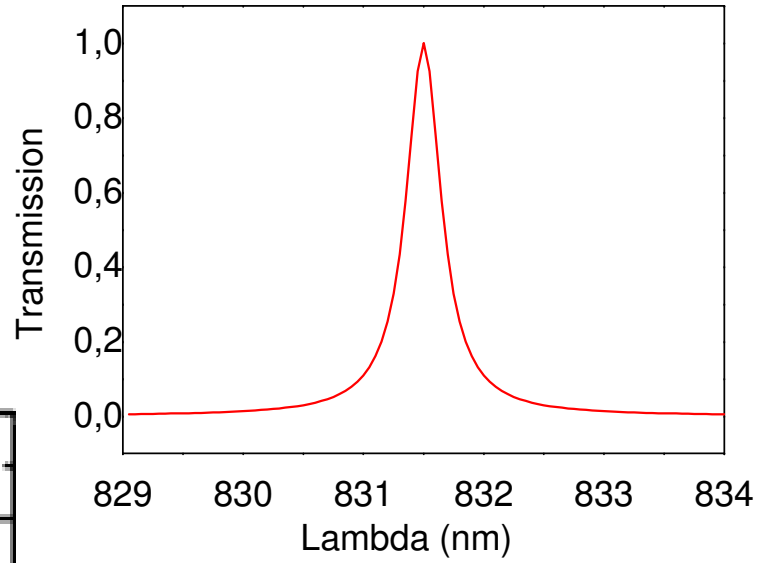
$$\left(n_B + n_R(\lambda)\right) = \frac{\lambda}{\lambda_R}n_B$$

Metodo grafico

Metodo grafico, cavità vuota

$$(n_B + 0) = \frac{\lambda}{\lambda_R} n_B$$

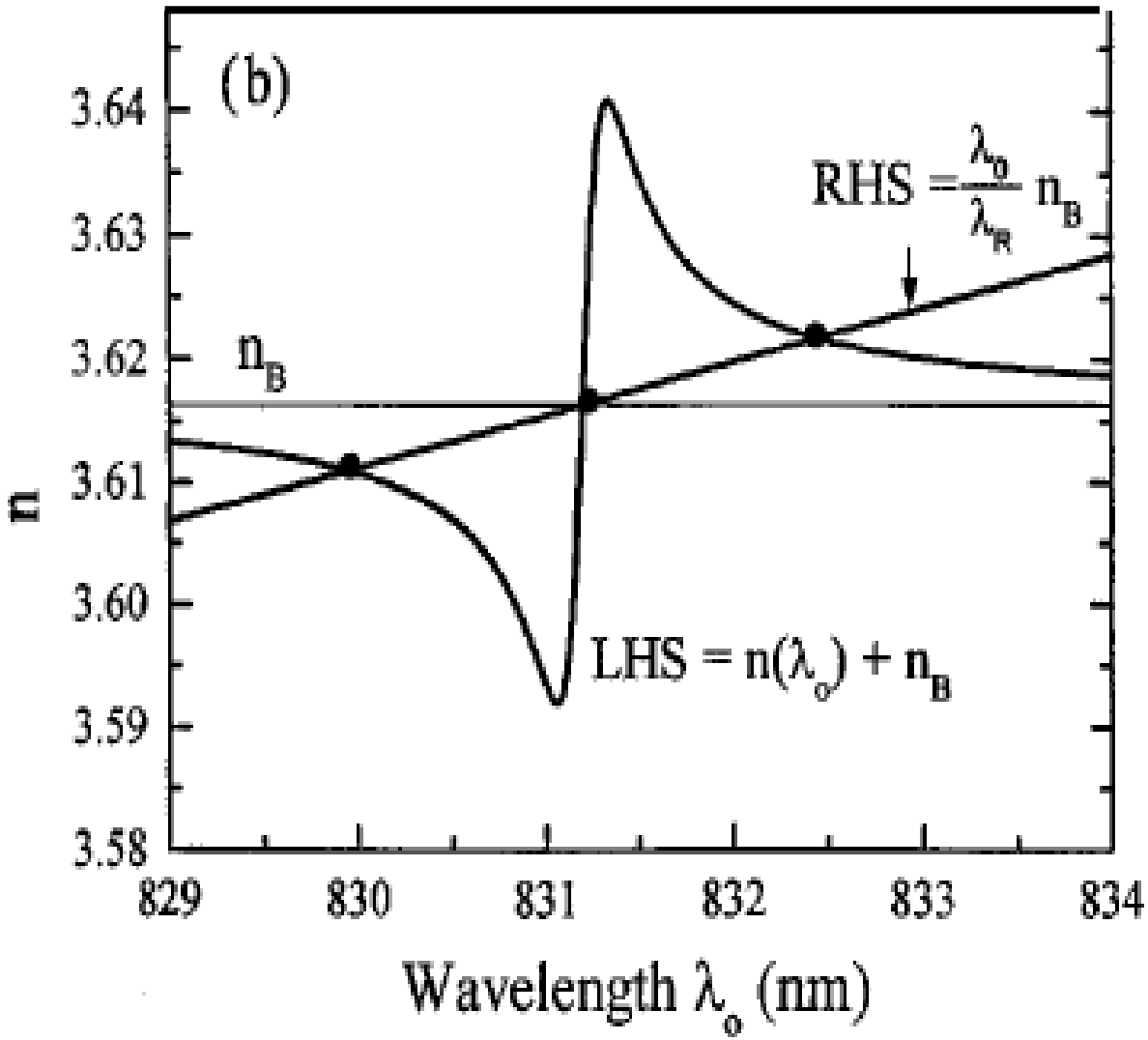




Metodo grafico, cavità con eccitone

$$\left(n_B + n_R(\lambda)\right) = \frac{\lambda}{\lambda_R} n_B$$

3 soluzioni



Spettri cavità con eccitone

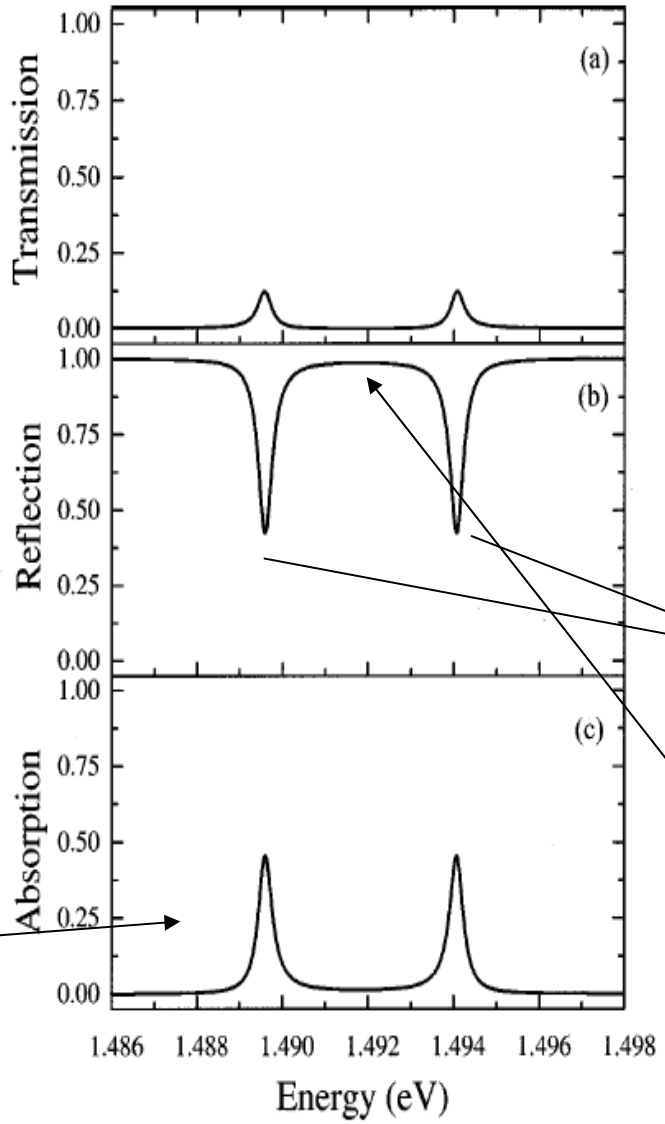


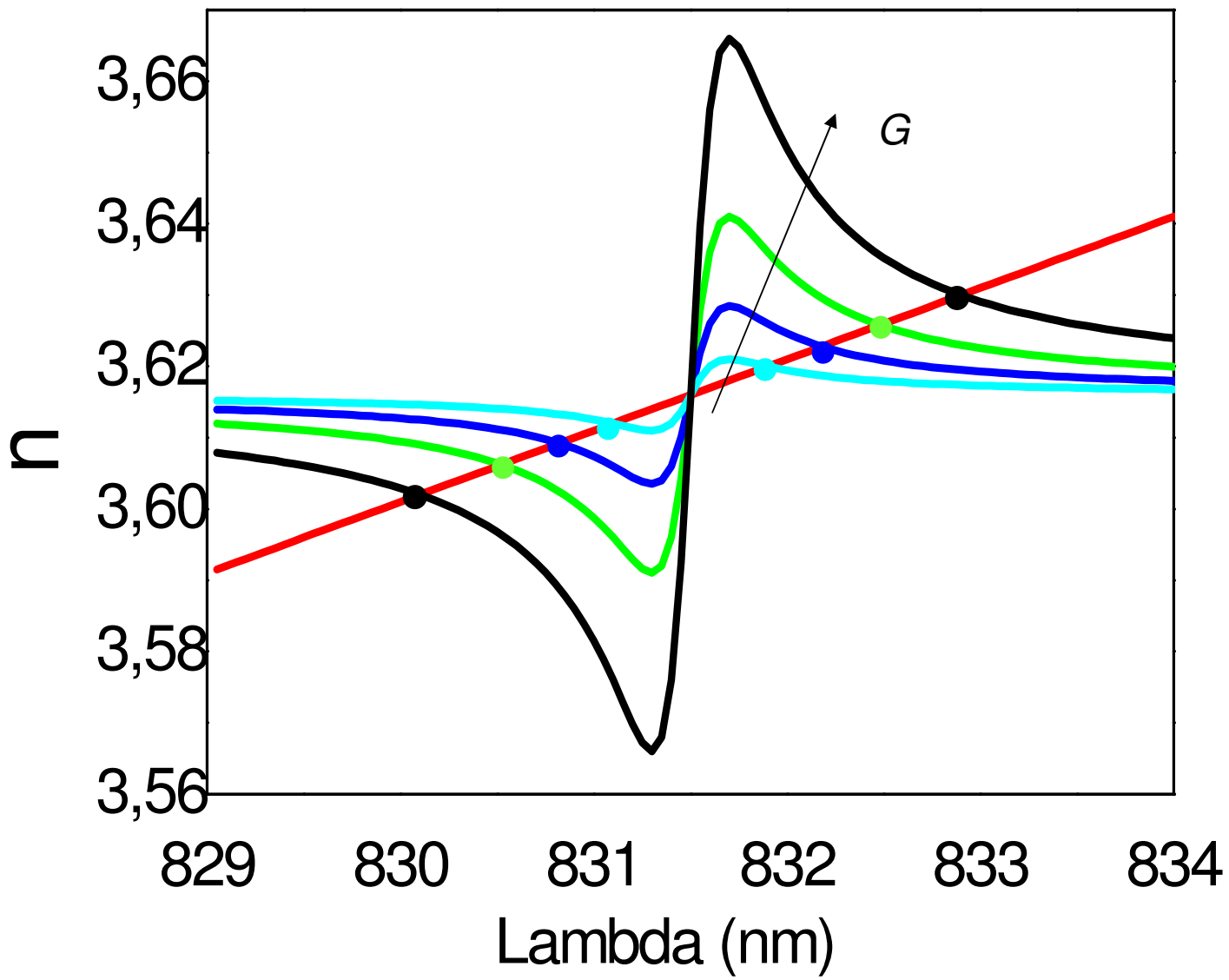
FIG. 2. Spectra of a Fabry-Pérot cavity containing Lorentz oscillators: (a) transmission; (b) reflection; (c) absorption.

2 modi normali

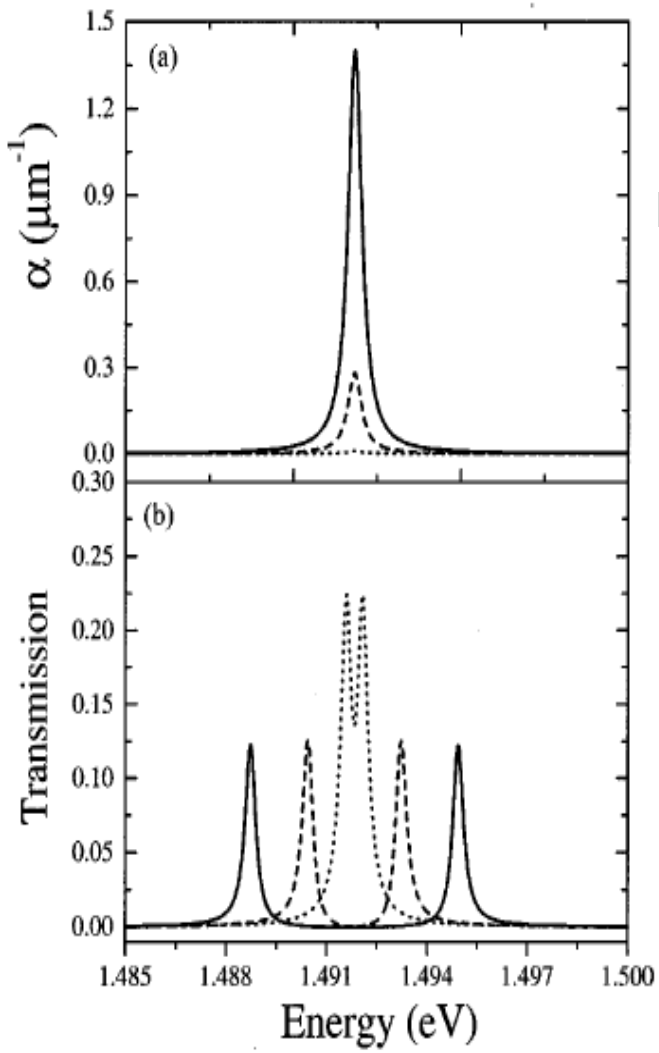
Picco centrale trova un forte assorbimento e non compare negli spettri

Resta un piccolo assorbimento sulle code della banda eccitonica

Al crescere della forza di oscillatore



Al crescere della
forza di oscillatore
lo splitting aumenta

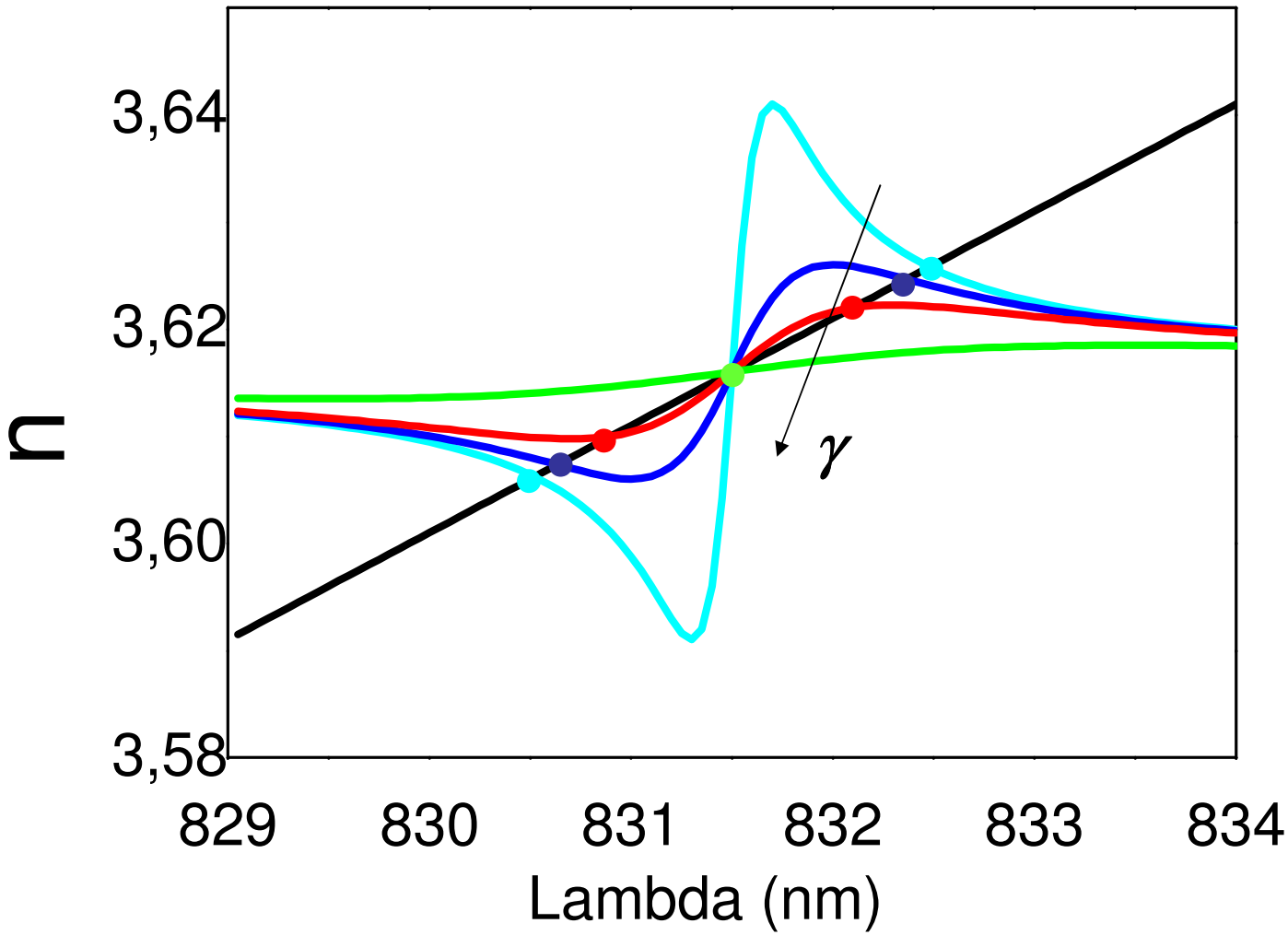


Eccitone nudo

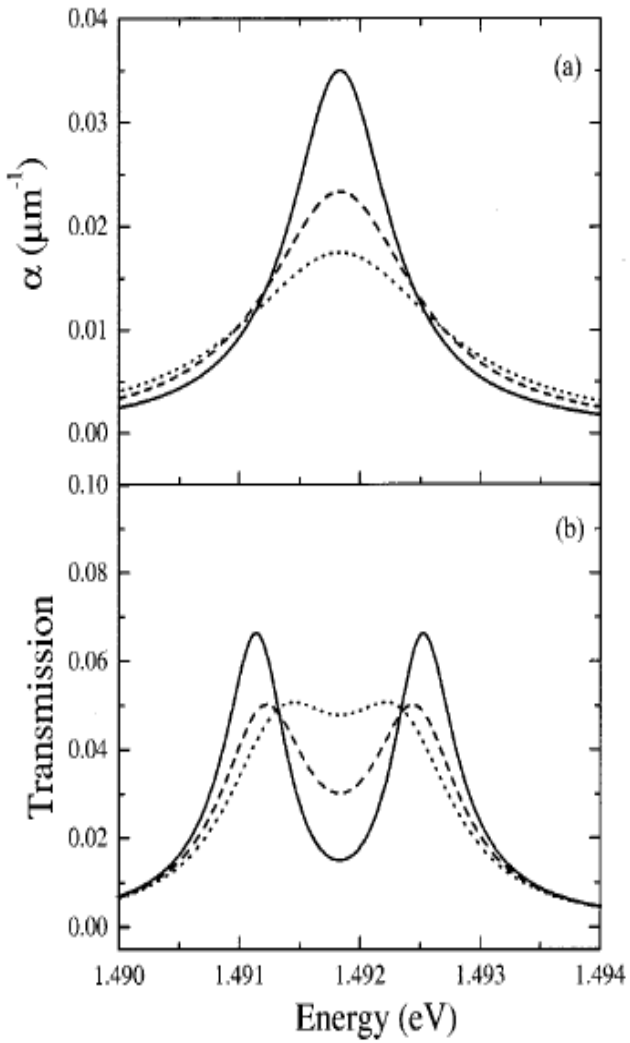
Modi normali

FIG. 10. Dependence of NMC transmission on oscillator strength: (a) Oscillator absorption coefficient for a constant HWHM broadening of $\gamma = 0.25$ meV and various oscillator strengths: solid lines, $\alpha_0 = 1.40\ \mu m^{-1}$; dashed lines, $0.28\ \mu m^{-1}$; dotted lines, $0.0084\ \mu m^{-1}$. (b) Corresponding $\Theta = 0$ NMC transmission spectra.

Al crescere dell' allargamento



Al crescere dello allargamento lo splitting diminuisce fino a sparire

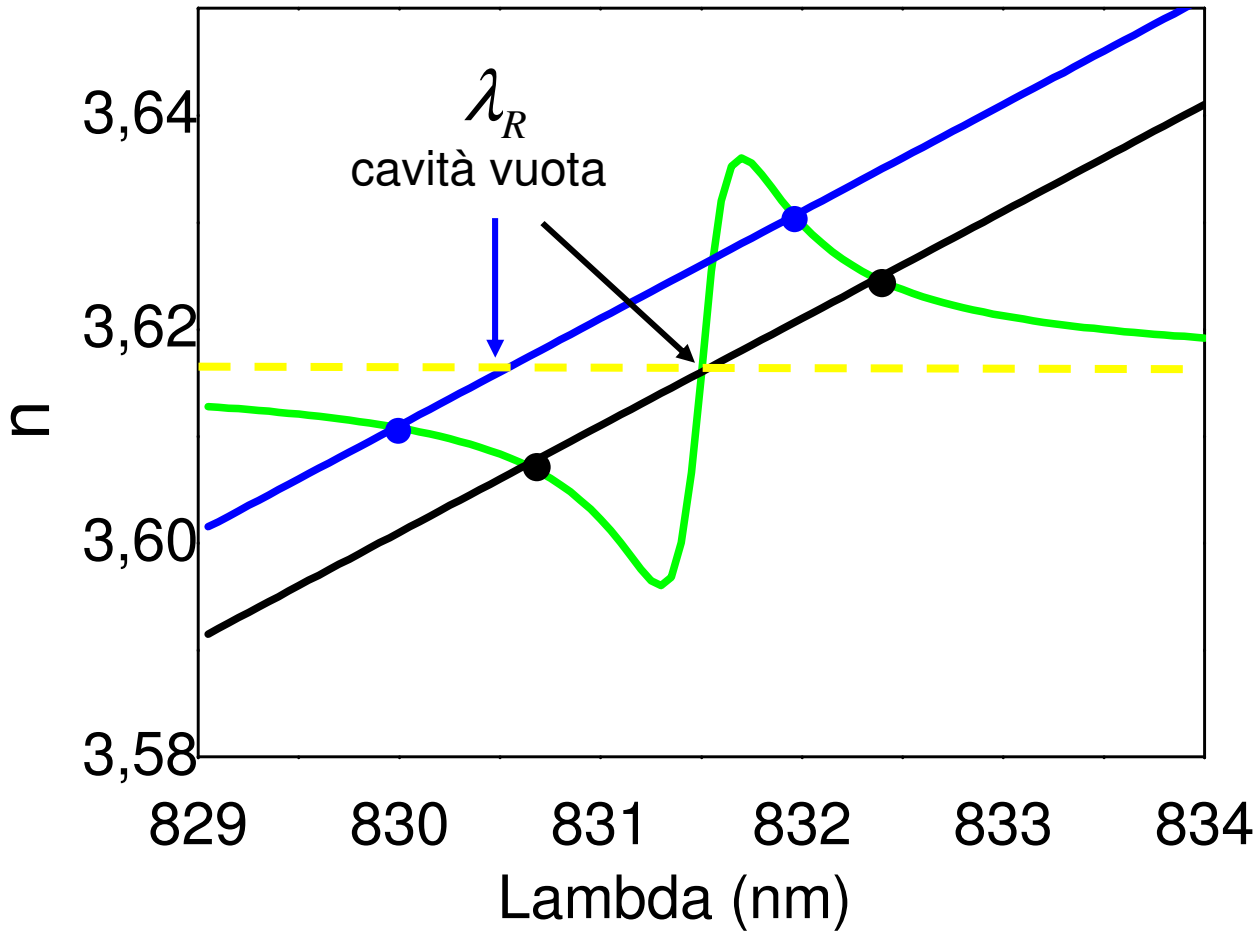


Eccitone nudo

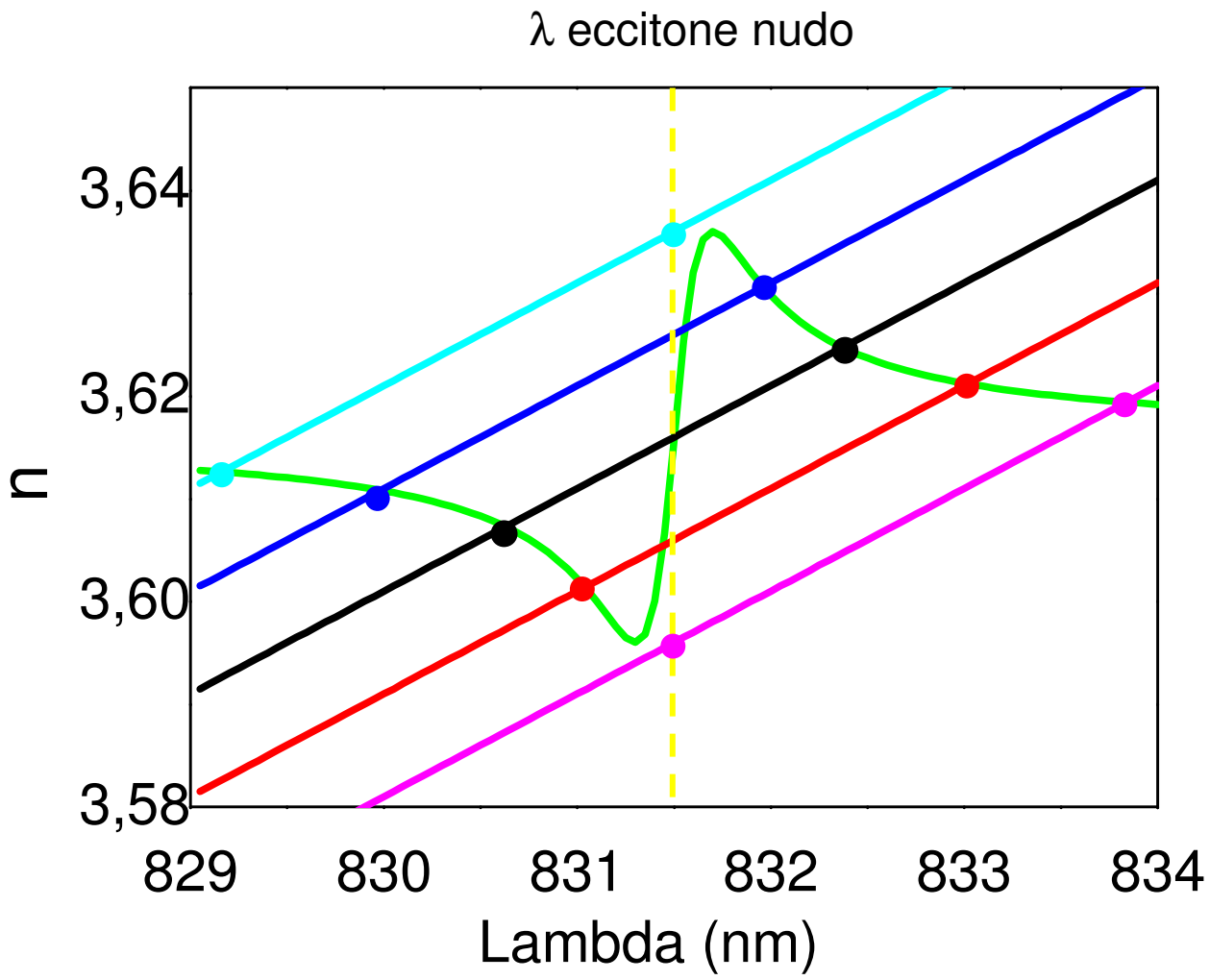
Modi normali

FIG. 9. Effect of homogeneous broadening on transmission for barely resolved normal-mode coupling: (a) Oscillator absorption coefficient for a constant oscillator strength and HWHM homogeneous broadenings: solid lines, $\gamma = 0.5$ meV; dashed lines, 0.75 meV; dotted lines, 1.0 meV. (b) Corresponding $\Theta = 0$ NMC transmission spectra at zero detuning for $\Omega \approx \gamma_{ex}$.

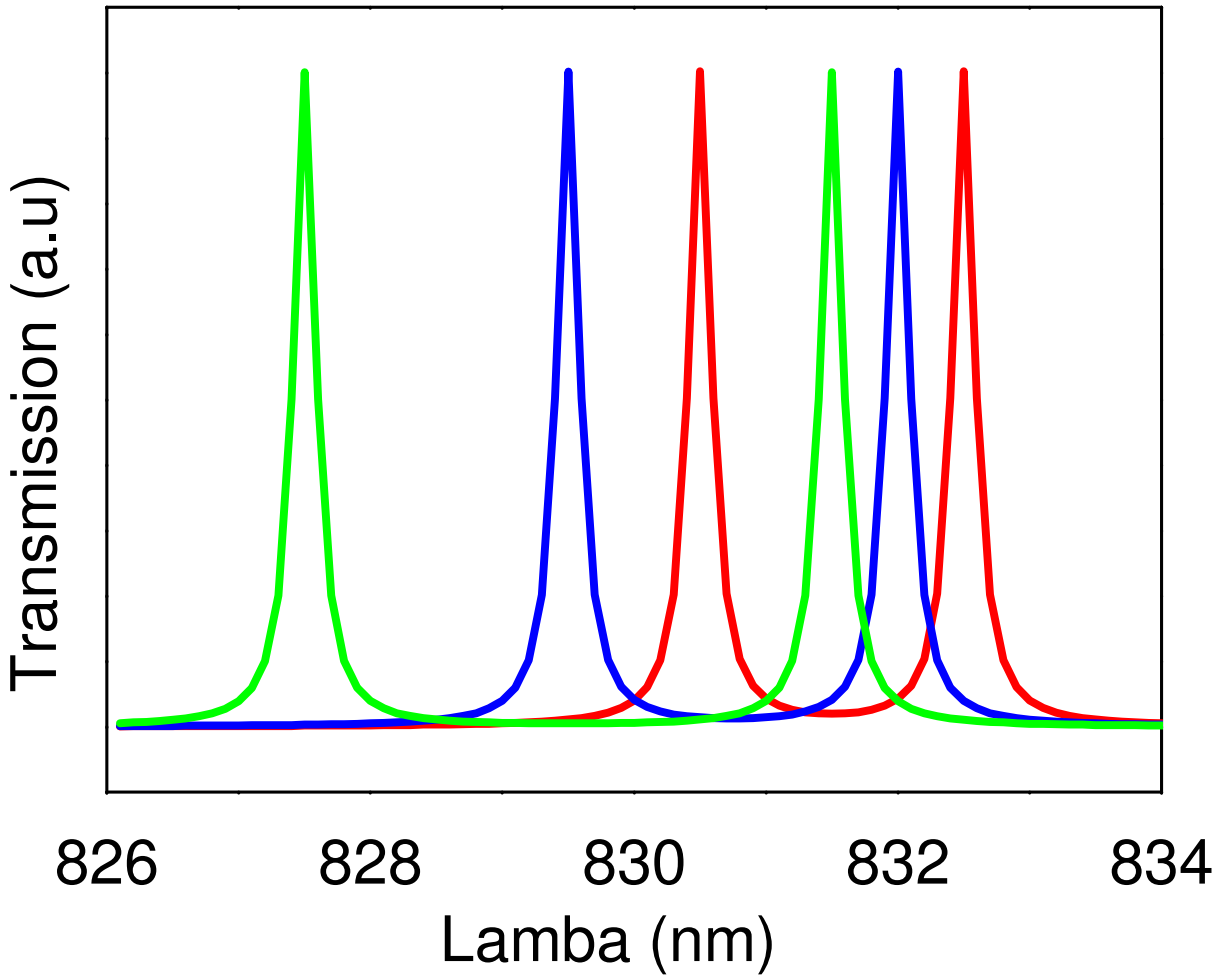
Se la cavità è fuori sintonia



Al variare del tuning



Al variare del tuning



Anticrossing $\Omega = \sqrt{(\omega - \omega_0)^2 + 2\varepsilon_0 \omega_0 G / n_B - (\gamma_X - \gamma_{Ph})^2}$

$$\Omega_0 = \sqrt{2\varepsilon_0 \omega_0 G / n_B}$$

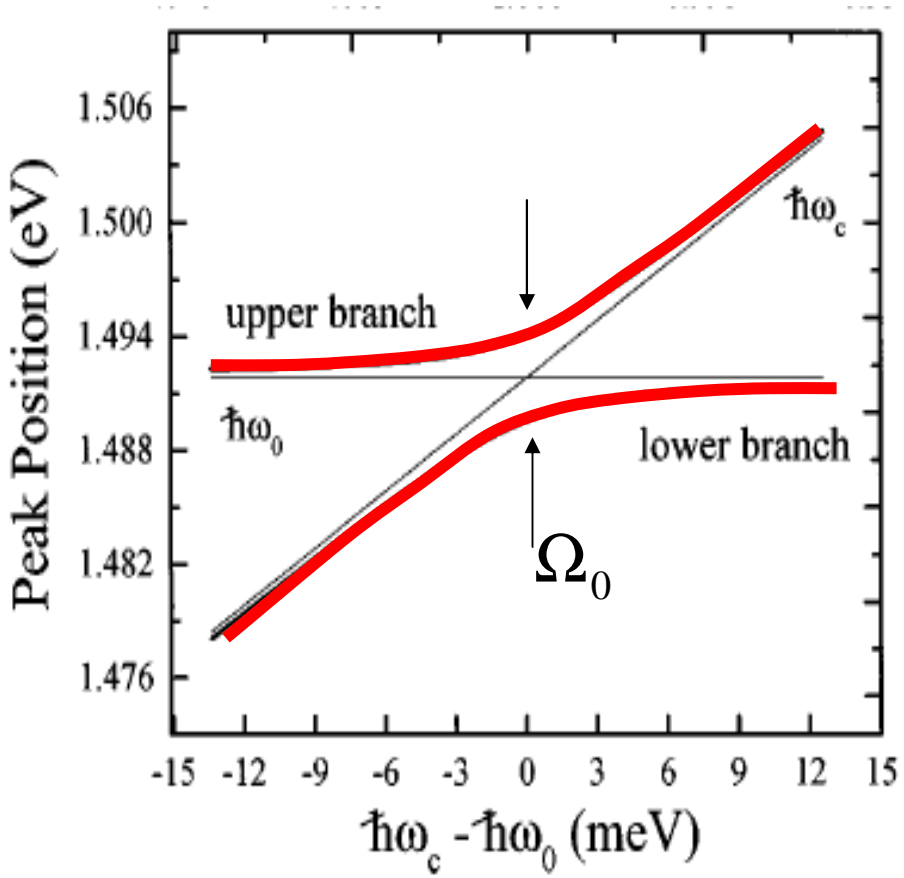


FIG. 3. NMC anticrossing curve vs oscillator-cavity detuning: heavy solid lines, transmission peak positions; thin solid lines, uncoupled oscillator and cavity positions.