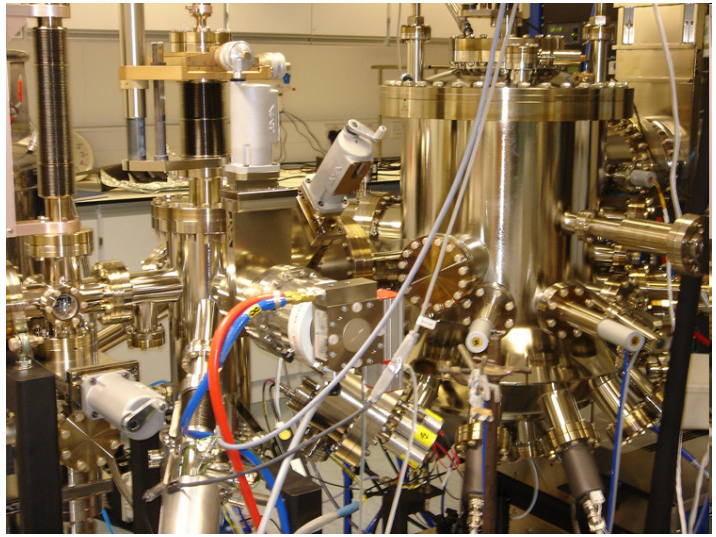
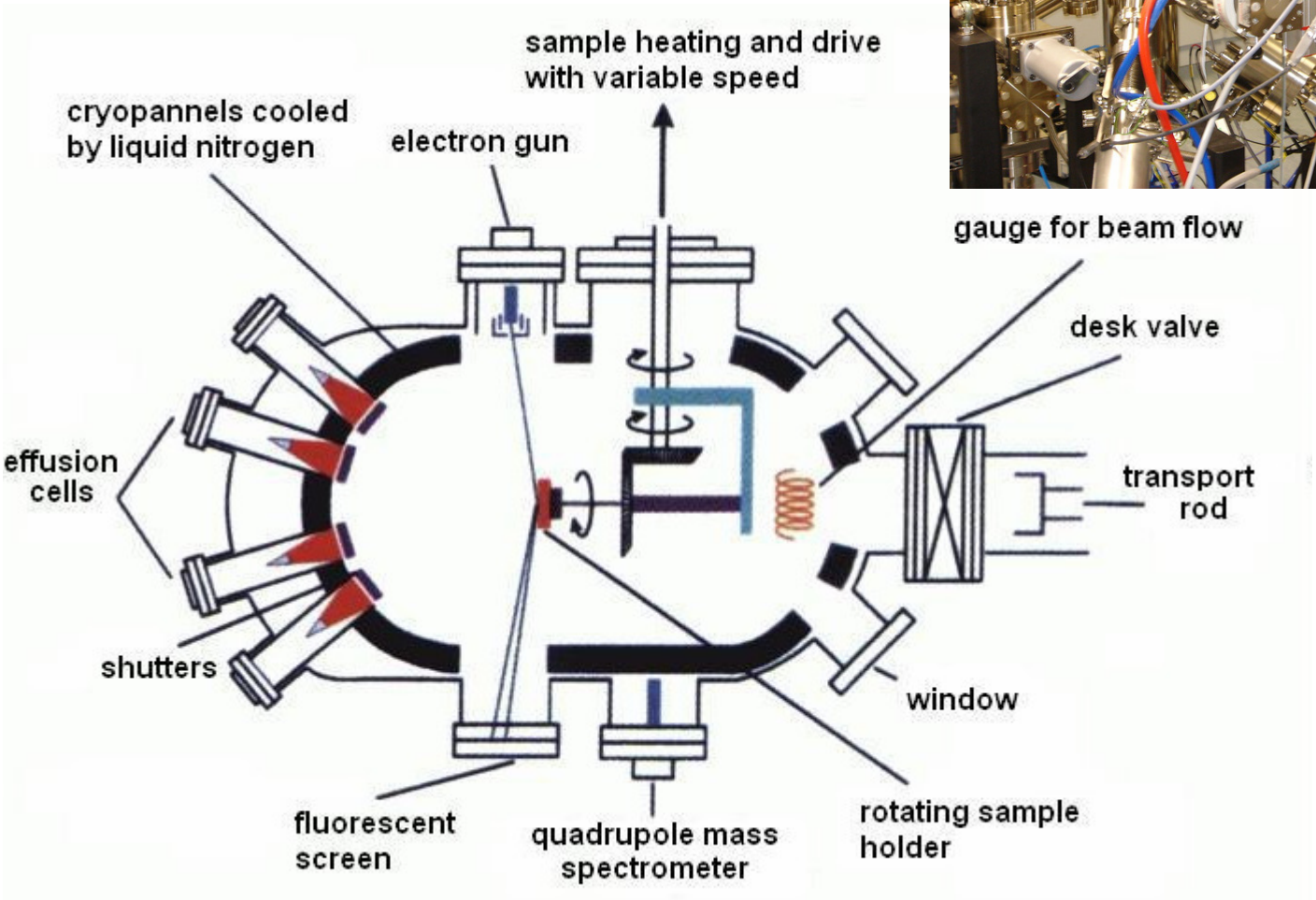
Emettitori a semiconduttore
QW

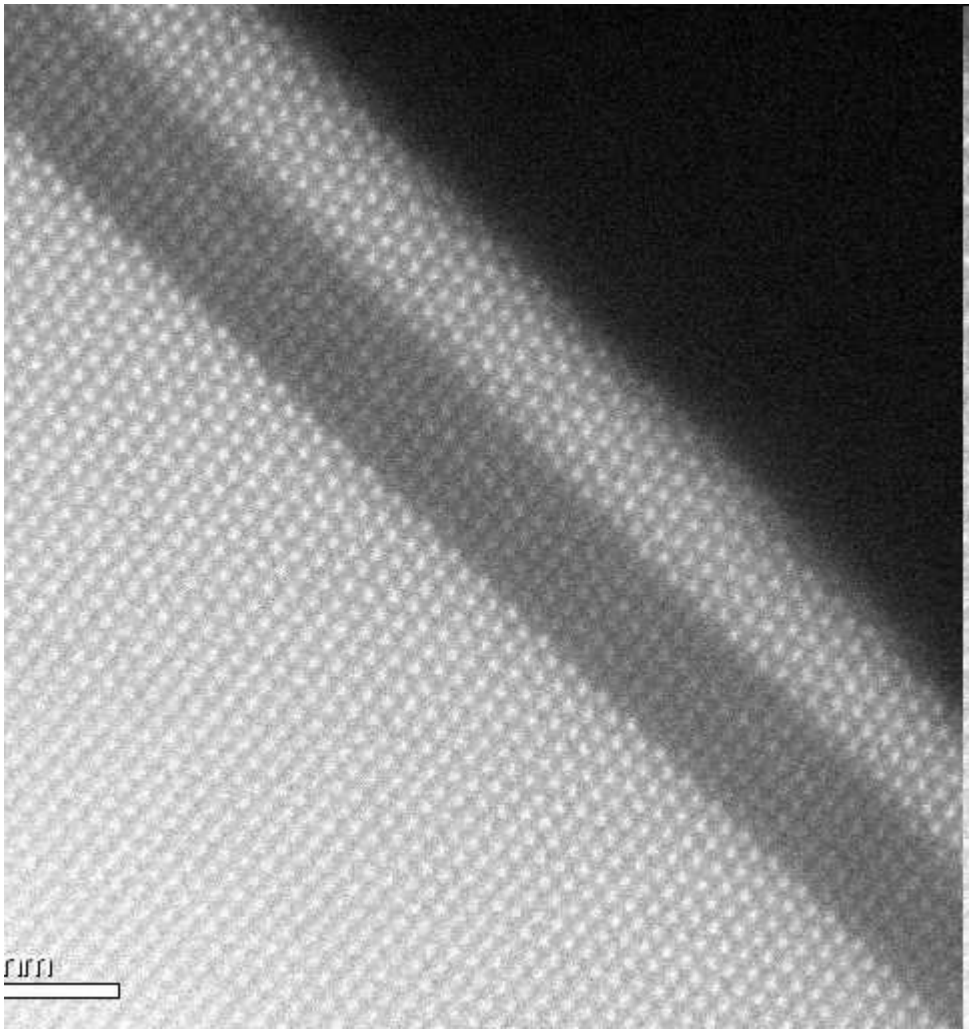
Deposizione controllata

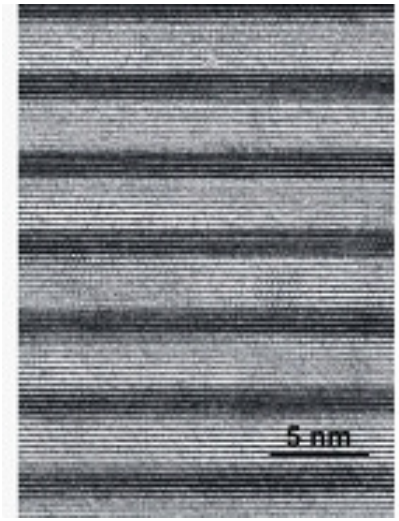
MBE

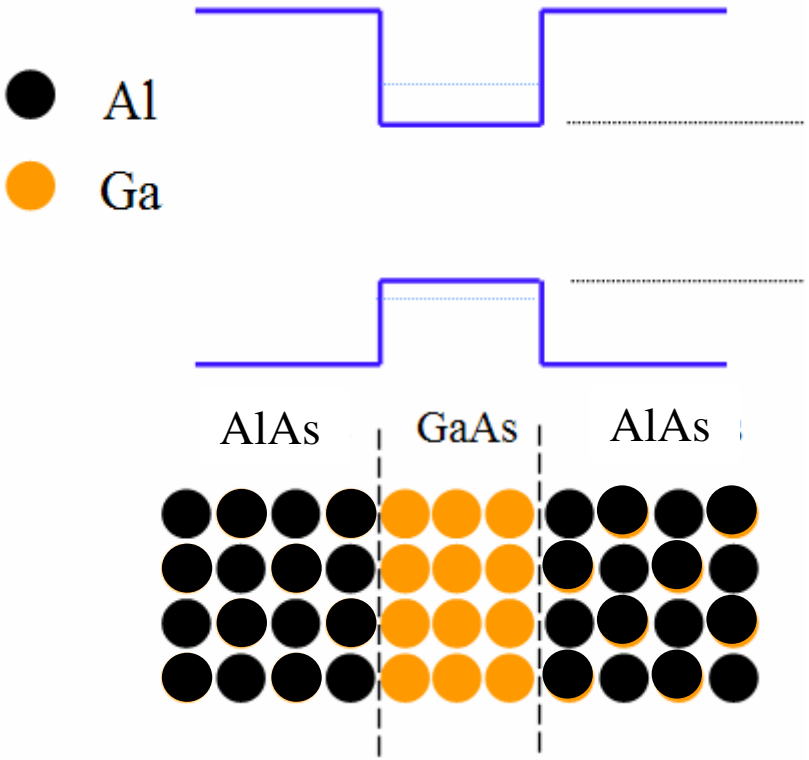




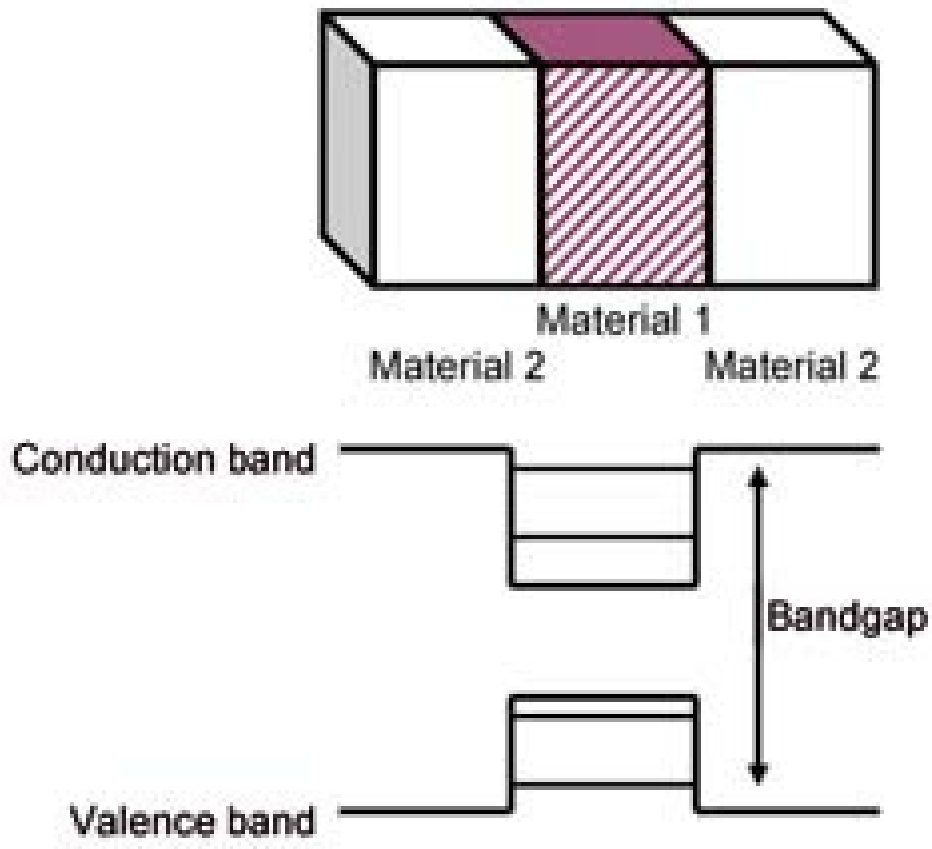
Heterointerfaces



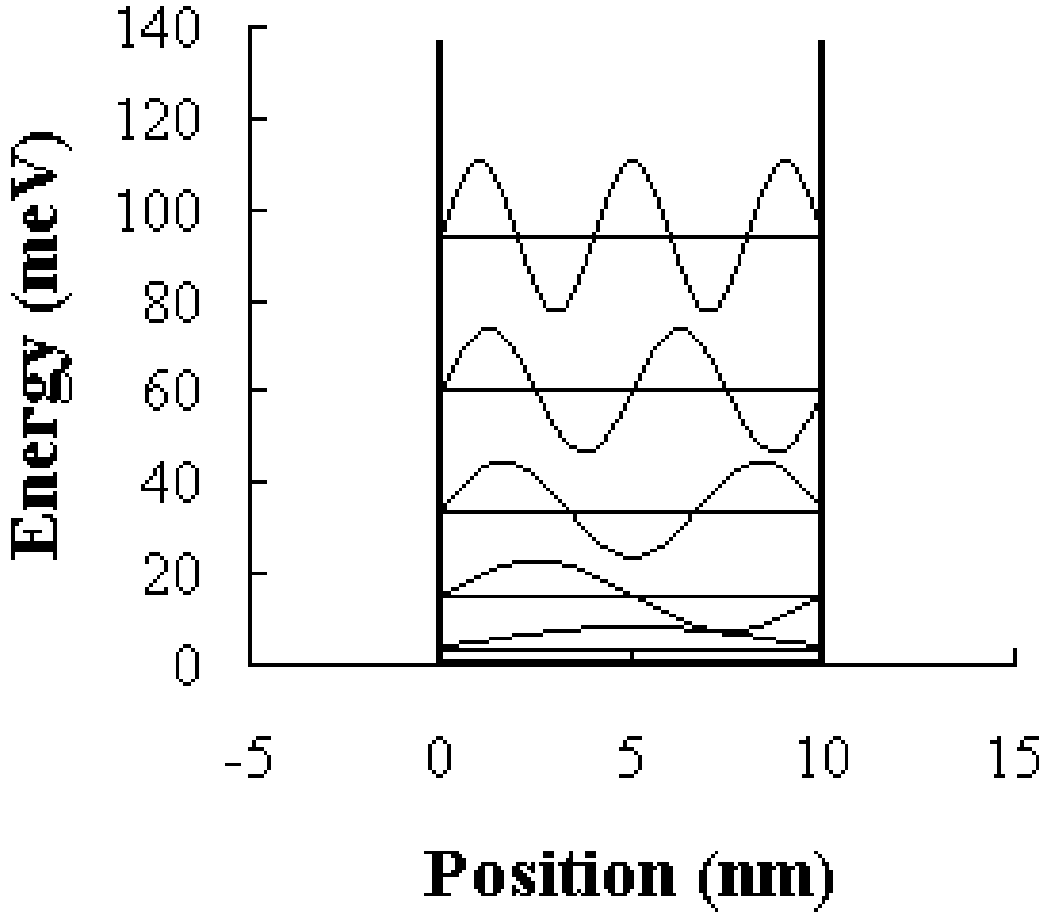


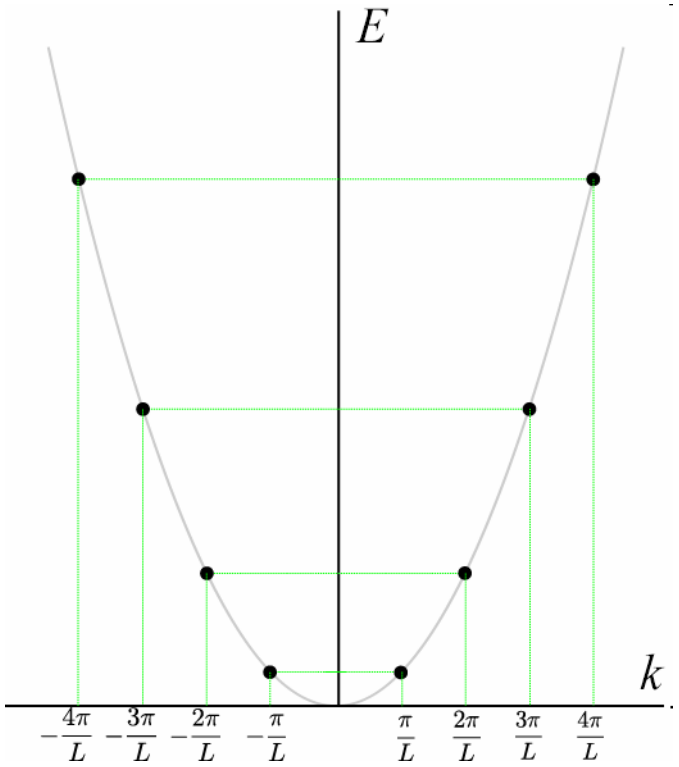


Quantum wells: $d<\lambda_{DeBroglie}$



Stati Quantici

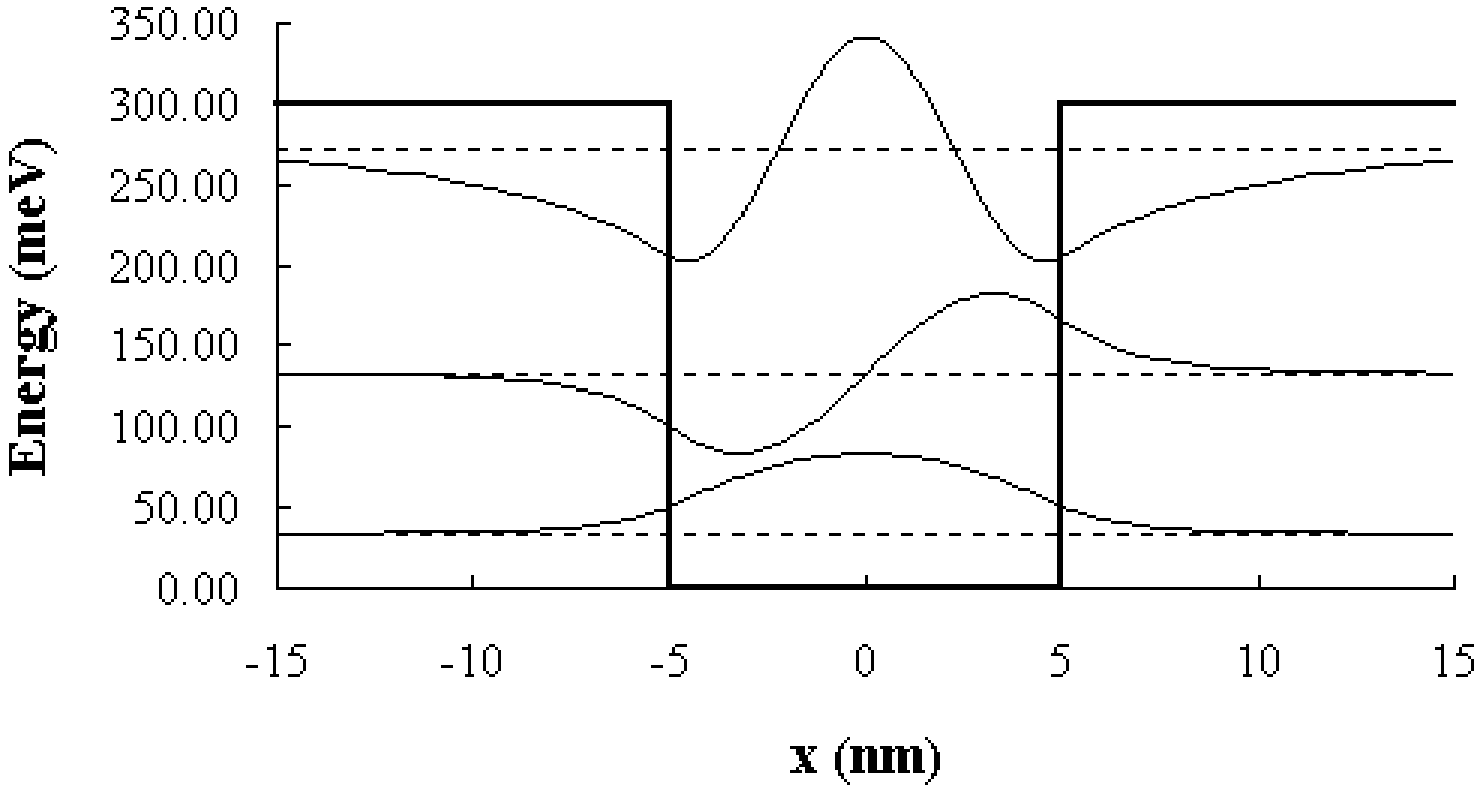




$$E_n = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 n^2$$

Pozzo con barriere finite

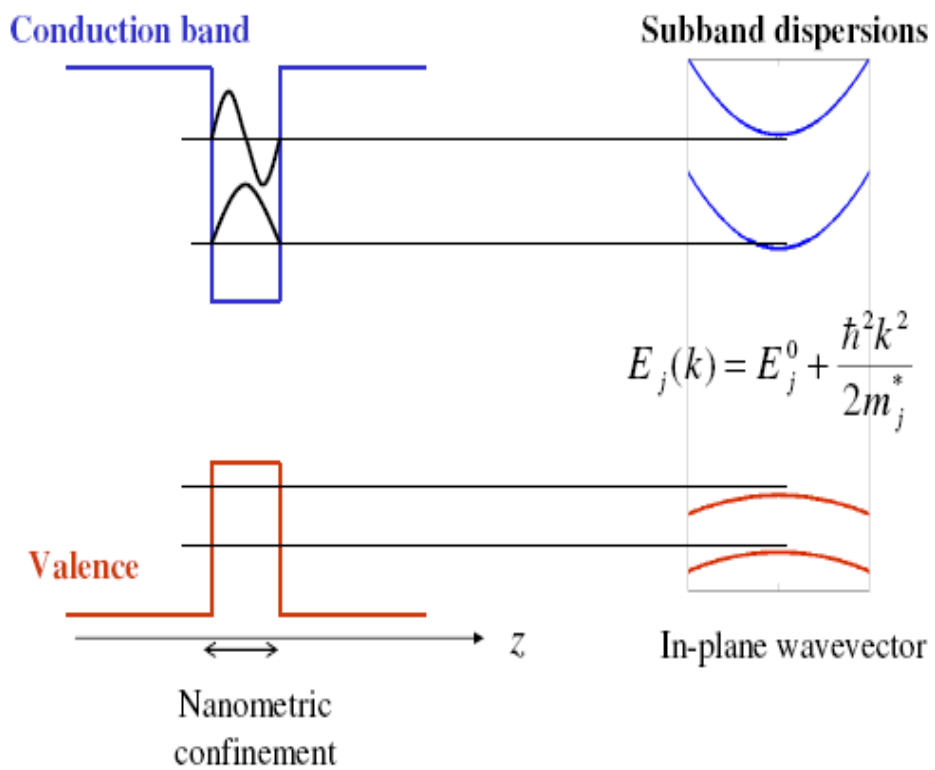
$$E_n < \frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 n^2$$



Sottobande

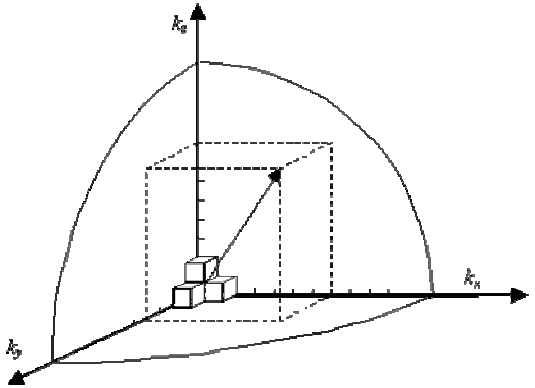
$$\psi(\vec{r}) = F(z)e^{i\vec{k}_{//}\cdot\vec{r}}$$

$$E_n(\vec{k}_{//}) = E_n + \frac{\hbar^2}{2m^*}\left(k_{//}\right)^2$$



Quantum Wells, Wires, Boxes: Density of States

1. Bulk material



Volume in k-space per state: $(2\pi/L)^3$

Volume in k-space occupied by states with energy less than E:

$$V_k = 4\pi k^3/3 \quad where \;\; k = \sqrt{\frac{2m}{\hbar^2}}\sqrt{E - E_c}$$

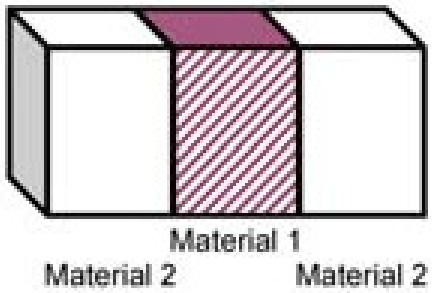
Number of electron states in this volume:

$$N(E) = 2\cdot\frac{4\pi k^3/3}{(2\pi/L)^3} = \frac{L^3}{3\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2}(E - E_c)^{3/2}$$

Density of states with energies between E and E+dE per unit volume:

$$\rho(E)\;\left[eV^{-1}\cdot m^{-3}\right] = \frac{1}{L^3}\frac{dN(E)}{dE} = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2}(E - E_c)^{1/2}$$

2. Quantum well



Area in 2-d k-space (i.e., k_y,k_z) per state: $(2\pi/L)^2$

Area in k-space occupied by staes with energy less than E_n:

$$A_k = \pi k^2 \quad where \;\; k = \sqrt{\frac{2m}{\hbar^2}}\sqrt{E - E_n - E_c}, \;\; and \;\; E_n = \frac{\pi^2\hbar^2 n^2}{2m^* d_x^2}$$

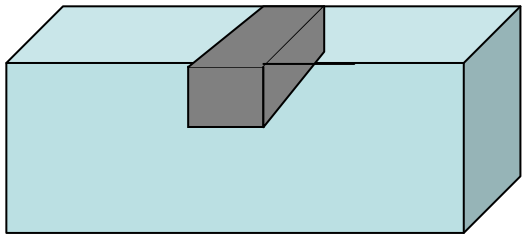
Number of electrons in this area in band n:

$$N_n(E) = 2\cdot\frac{\pi k^2}{(2\pi/L)^2} = \frac{L^2}{\pi^2}\frac{m^*}{\hbar^2}(E - E_n - E_c)$$

Density of states in well band n between E and E+dE per unit area:

$$\rho_n(E)\;\left[eV^{-1}\cdot m^{-2}\right] = \frac{1}{L^2}\frac{dN_n(E)}{dE} = \frac{m^*}{\pi^2\hbar^2}$$

$$\psi(\vec{r}) = F(y,z)e^{ikx} \qquad E_{n,m}(k) = E_n + E_m + \frac{\hbar^2}{2m^*}k^2$$



3. Quantum wire

Distance in k-space per state: $2\pi / L$

Distance in k-space occupied by states with energy less than $E_{n,m}$:

$$L_k = k \quad where \;\; k = \sqrt{\frac{2m}{\hbar^2}}\sqrt{E - E_{n,m} - E_c}, \;\; and \;\; E_{n,m} = \frac{\pi^2\hbar^2}{2m^*}\left(\frac{n^2}{d_x^2} + \frac{m^2}{d_y^2}\right)$$

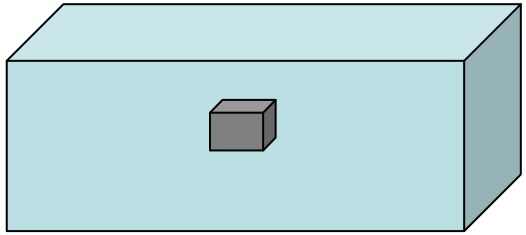
Number of electron states in this length in band n,m:

$$N_{n,m}(E) = 2 \cdot \frac{k}{2\pi / L} = \frac{L}{\pi}\sqrt{\frac{2m^*}{\hbar^2}}\sqrt{E - E_{n,m} - E_c}$$

Density of states in wire band n,m between E and E+dE per unit wire length:

$$\rho_{n,m}(E)\left[eV^{-1} \cdot m^{-1}\right] = \frac{1}{L}\frac{dN_{n,m}(E)}{dE} = \sqrt{\frac{m^*}{2\pi^2\hbar^2}}\frac{1}{\sqrt{E - E_c - E_{n,m}}}$$

$$\psi(\vec{r}) = F(x, y, z) \qquad E_{n,m,p} = E_n + E_m + E_p$$



4. Quantum box

In this situation the density of states is simply the number of states per box, which is 2, at each possible discrete energy level, $E_{n,m\,p}$, times the degeneracy of that energy level, i.e, the number of combinations of n, m, and p that result in the same value of $E_{n,m\,p}$.

$$E_{n,m,p} = \frac{\pi^2\hbar^2}{2m^*}\left(\frac{n^2}{d_x^2} + \frac{m^2}{d_y^2} + \frac{p^2}{d_z^2}\right)$$

Summary:

Bulk

$$\rho(E)\ \left[eV^{-1}\cdot m^{-3}\right] = \frac{1}{L^3}\frac{dN(E)}{dE} = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2}(E - E_c)^{1/2}$$

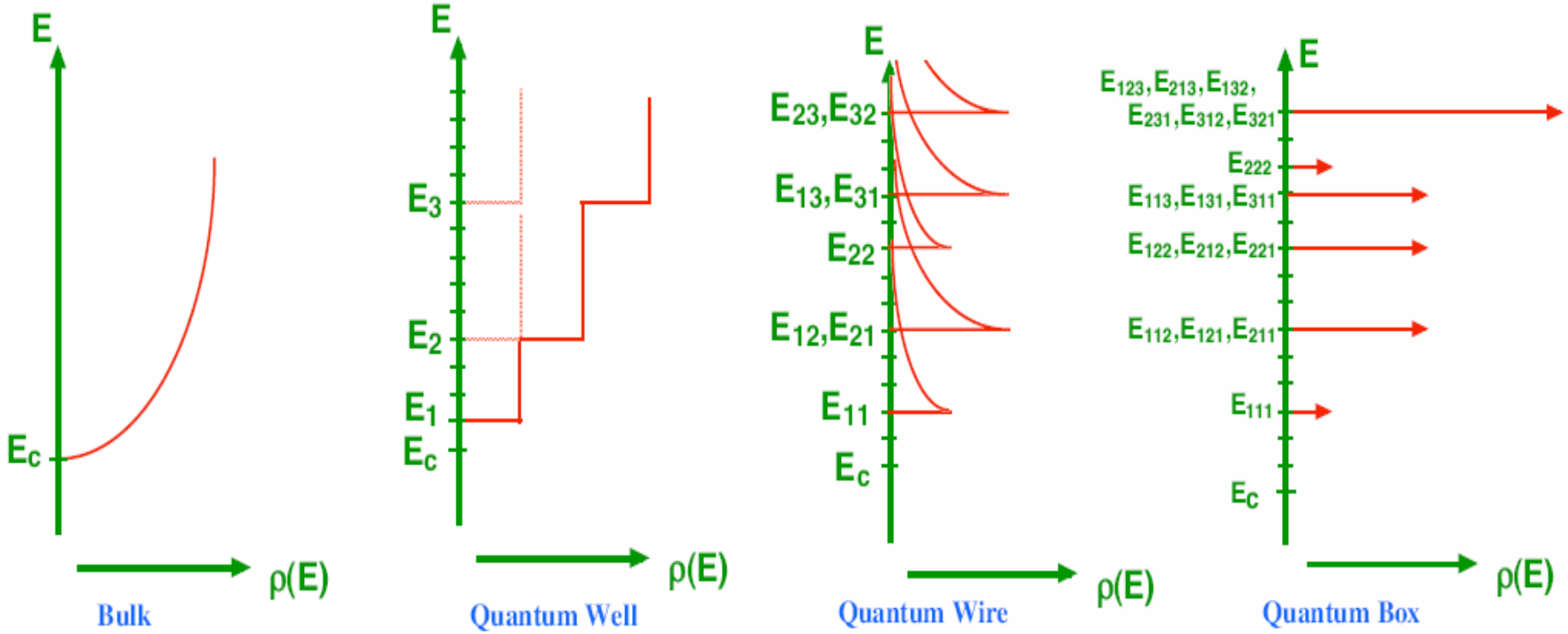
Well

$$\rho_n(E)\ \left[eV^{-1}\cdot m^{-3}\right] = \frac{1}{L^2}\frac{dN_n(E)}{dE} = \frac{m^*}{\pi^2\hbar^2}$$

Wire

$$\rho_{n,m}(E)\ \left[eV^{-1}\cdot m^{-1}\right] = \frac{1}{L}\frac{dN_{n,m}(E)}{dE} = \sqrt{\frac{m^*}{2\pi^2\hbar^2}}\frac{1}{\sqrt{E - E_c - E_{n,m}}}$$

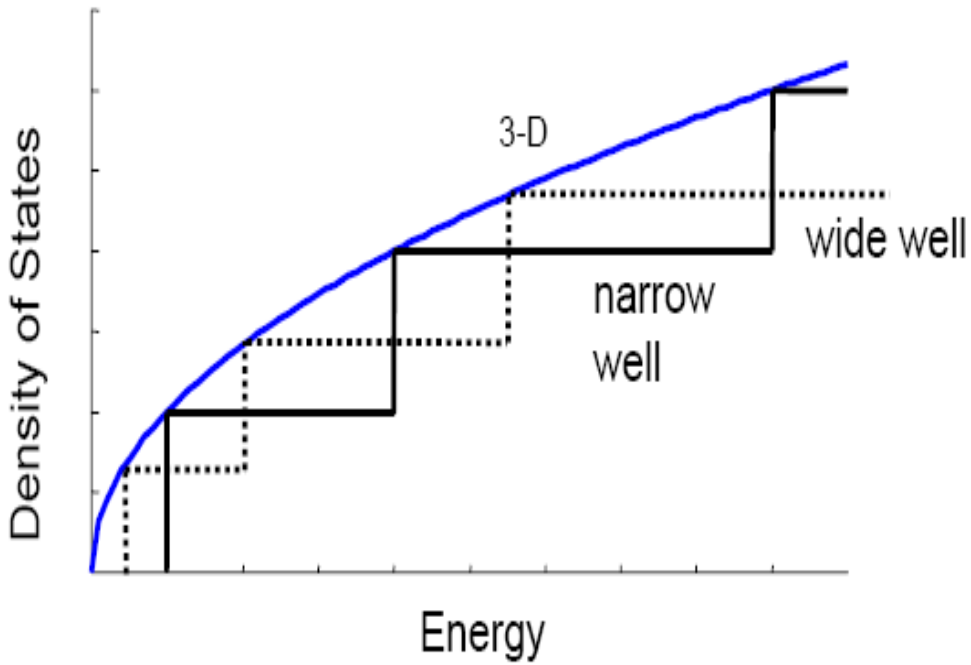
Density of states



Confronto DOS Bulk vs QW

$$\rho_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{(E-E_c)}$$

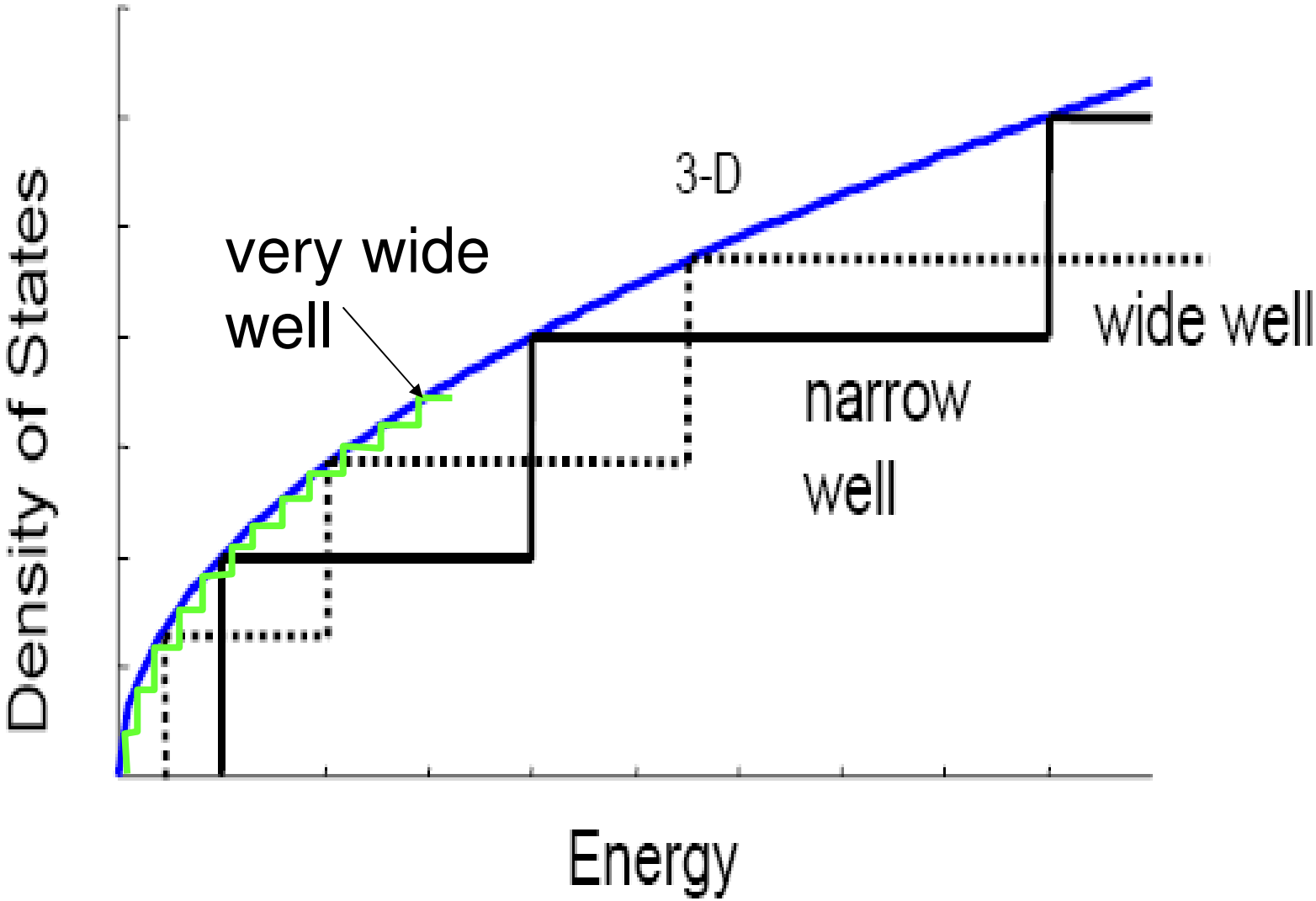
$$\rho_{2D}(E) = \frac{m^*}{\hbar^2\pi^2}\frac{1}{L}\sum_{i=1}^{N}\Theta(E-E_c-E_n)$$



$$\rho_{3D}(E_n) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{\frac{1}{2m^*}}\frac{\hbar\pi n}{L} = \frac{m^*}{\hbar^2\pi^2}\frac{n}{L} = \rho_{2D}(E_n)$$

Stessa DOS sul bordo delle sottobande

QW largo+allargamento=bulk



QW largo+allargamento=bulk

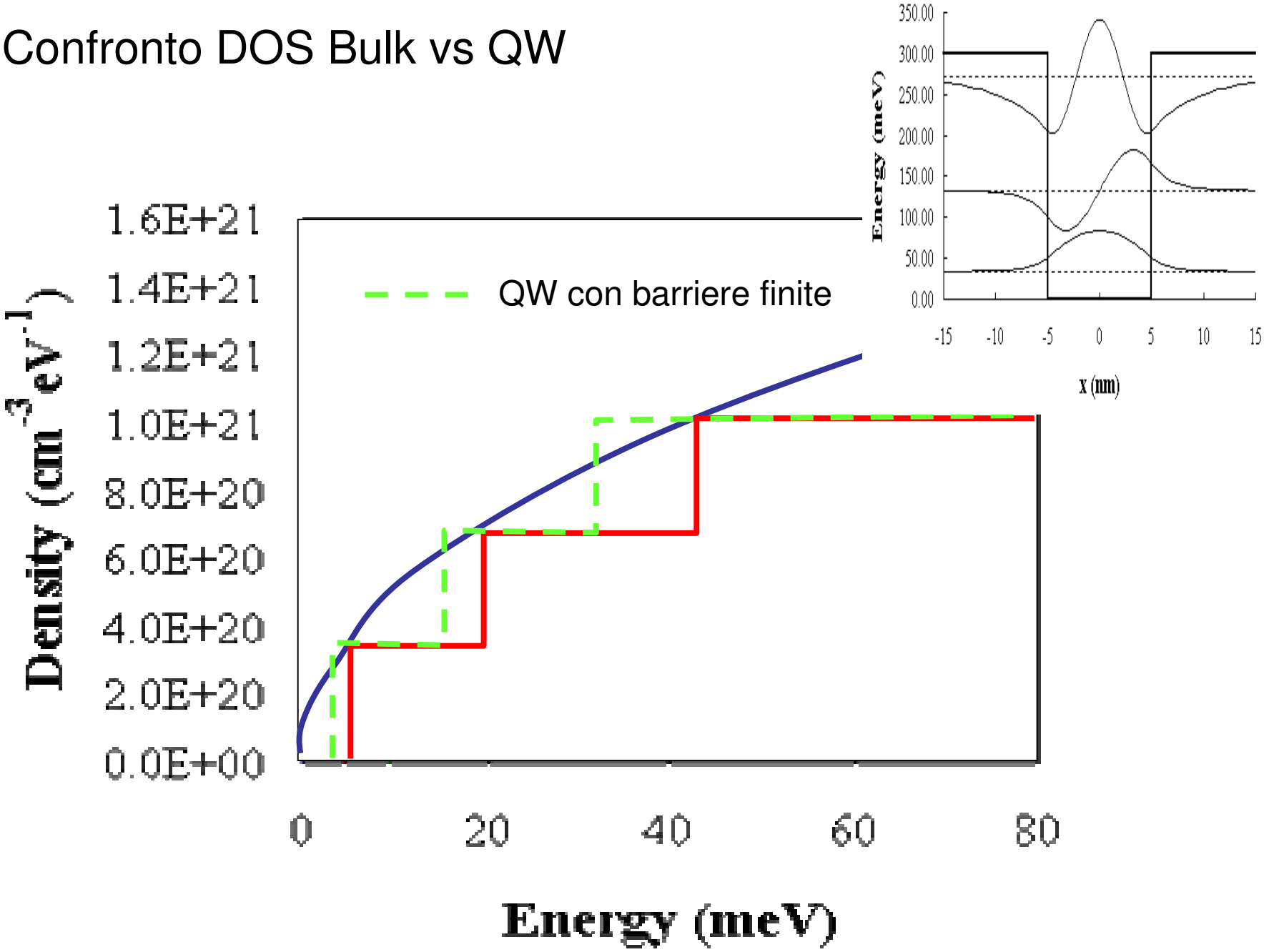
$$E_n = \frac{1}{2m^*}\left(\frac{\hbar\pi n}{L}\right)^2$$

$$\Delta E = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 (2n+1) > KT = \frac{\hbar^2}{2m^*}\left(\frac{2\pi}{\lambda_{DeBroglie}}\right)^2$$

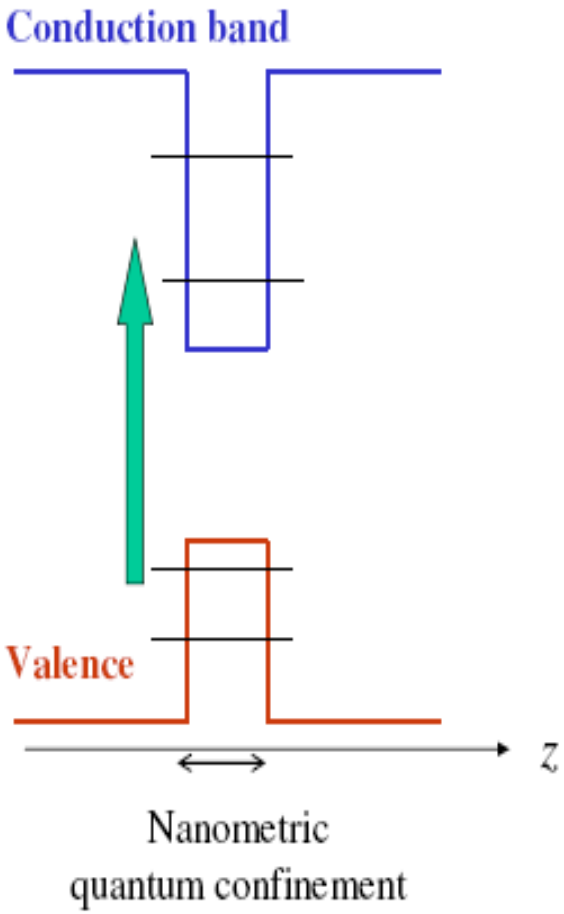
$$\left(\frac{1}{L}\right)^2 (2n+1) > 4\left(\frac{1}{\lambda_{DeBroglie}}\right)^2$$

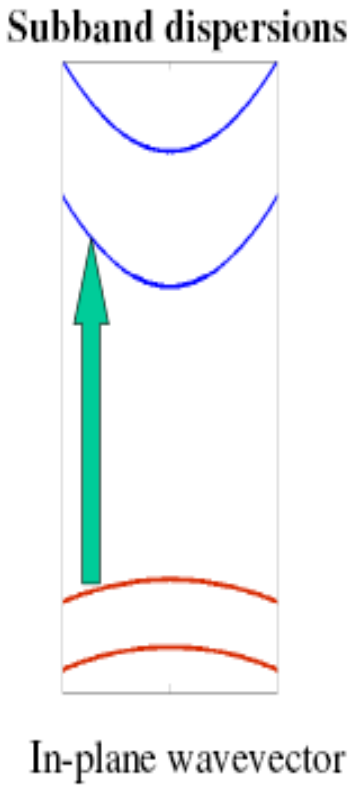
$$\boxed{L < \lambda_{DeBroglie}}$$

Confronto DOS Bulk vs QW



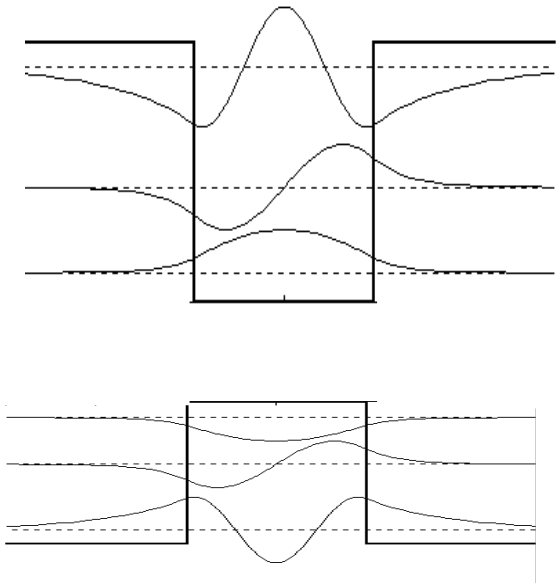
Interband optical transitions





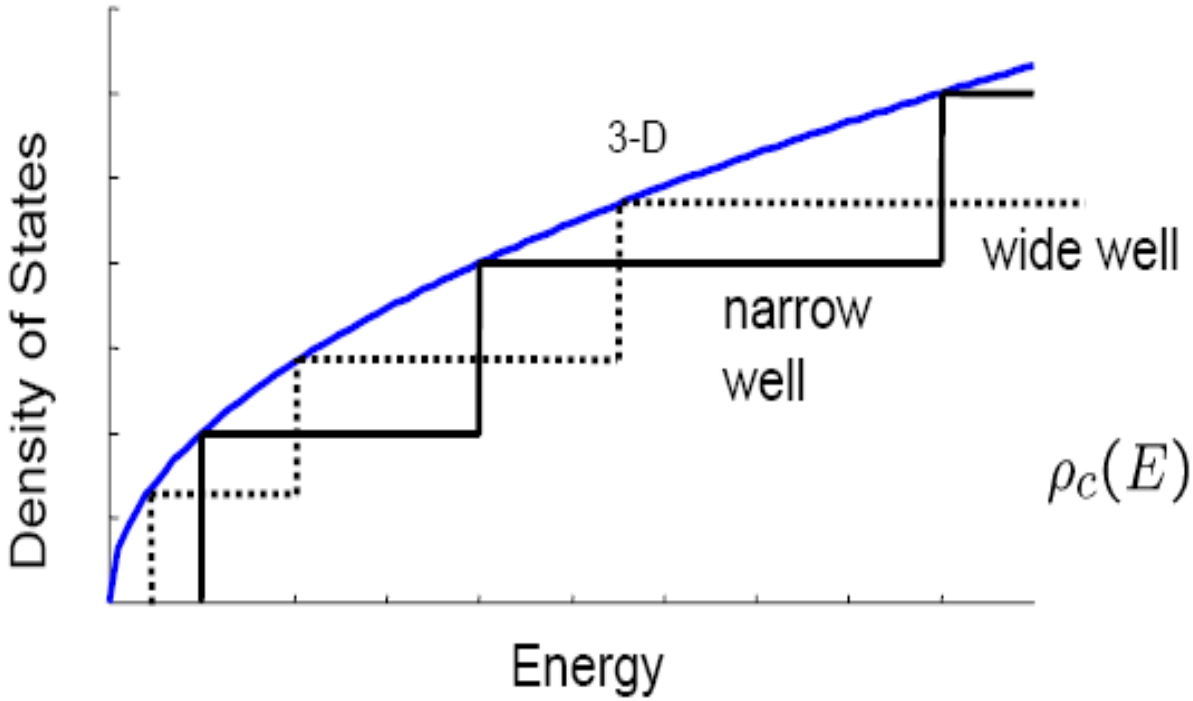
$k_{//}$ si deve conservare

$\Delta n=0$



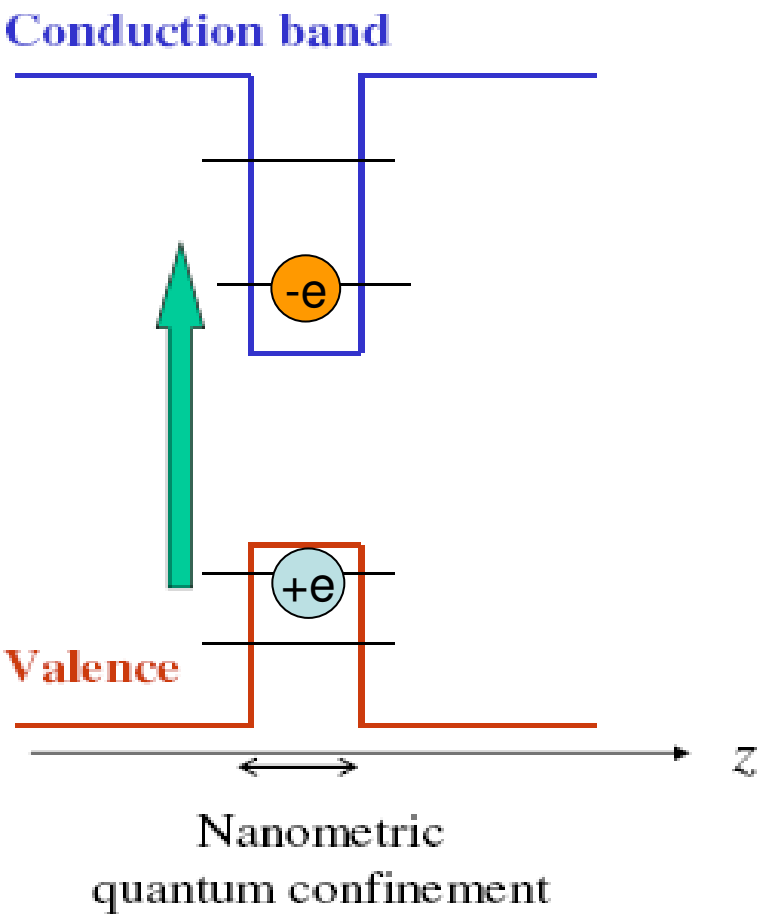
Assorbimento, Emissione: joint DOS

$$\alpha(\omega) = \frac{\pi\, q^2 c \mu_0}{m_0^2 \omega\, n} |p_{cv}|^2 \, | < F_c | F_v > |^2 \; \rho_r(\hbar\omega - E_{c1} - E_{v1})$$

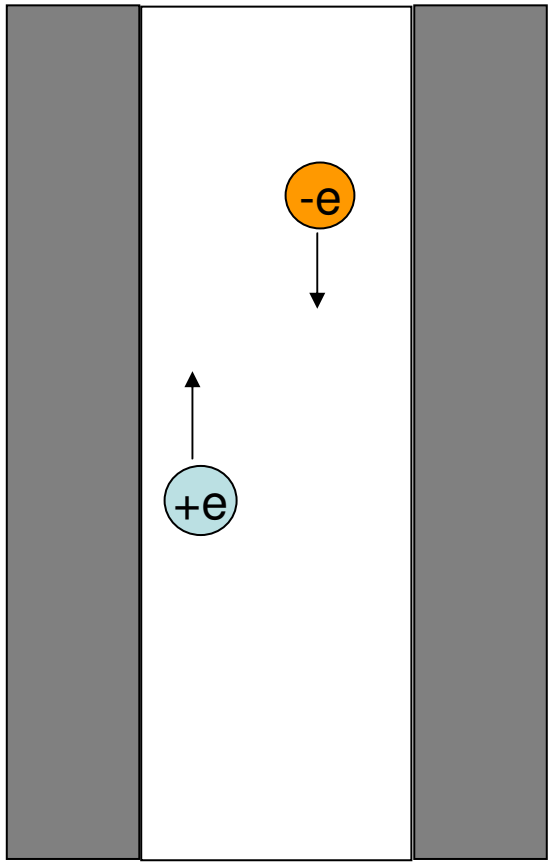


$$\rho_c(E) = \frac{m^*}{\pi \hbar^2 L_{QW}} \quad \text{(2-D)}$$

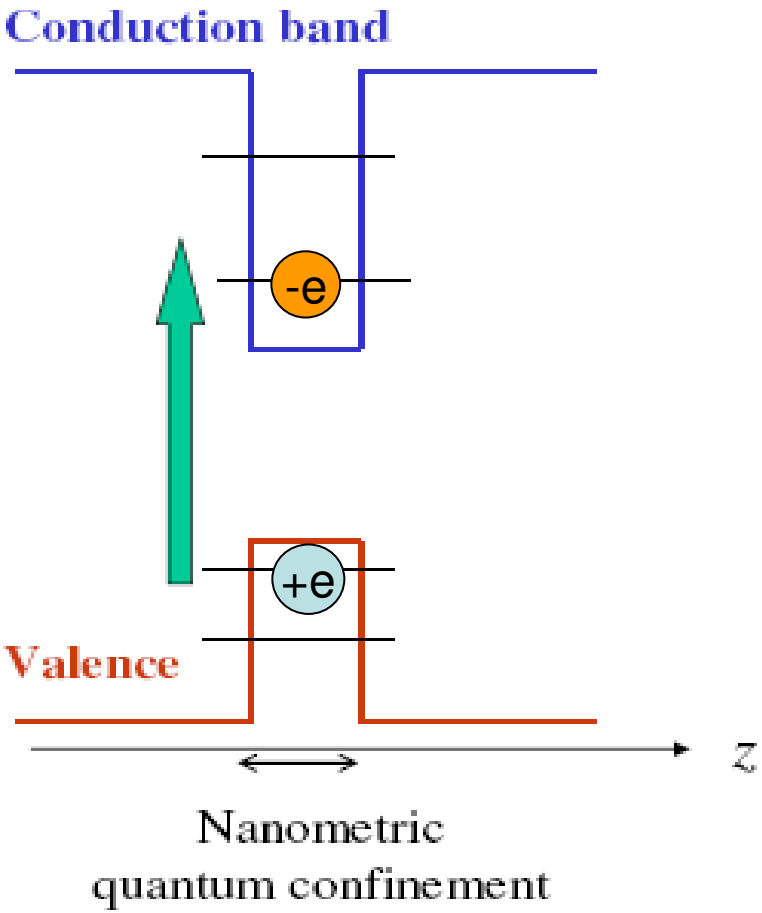
Assorbimento, creazione di elettrone e lacuna



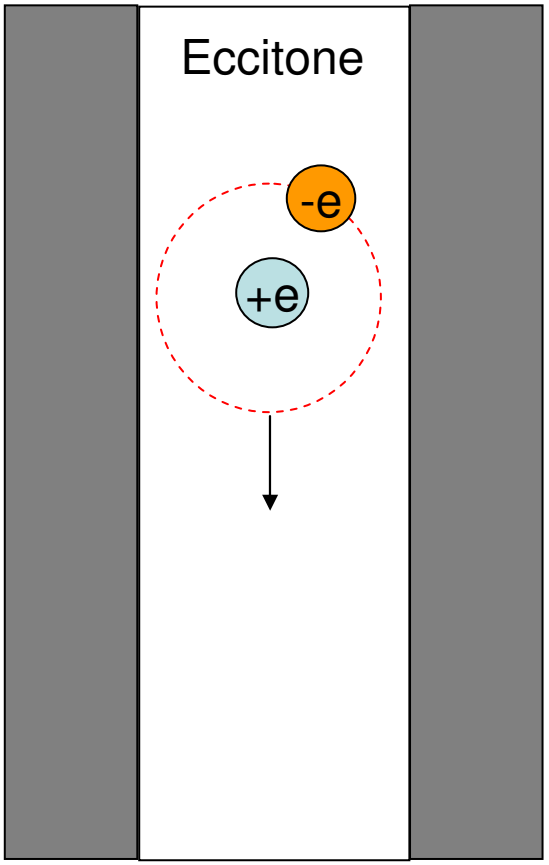
Nel piano elettrone e lacuna sono liberi di muoversi



Elettrone e lacuna si attraggono e possono formare un eccitone

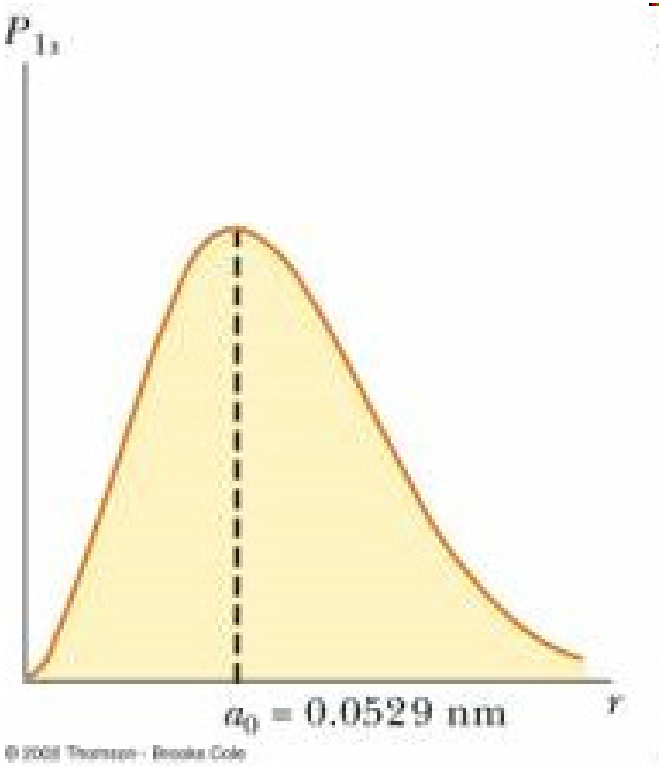


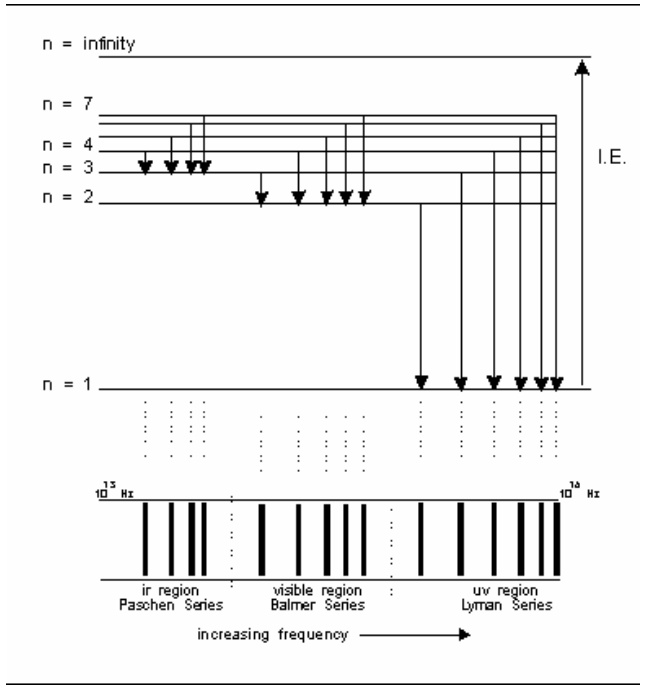
Nel piano l'eccitone
è libero di muoversi



Atomo di idrogeno

$$Ry = \frac{m_e e^4}{8\varepsilon_o^2 h^2} = 13.6056923(12)\ eV$$





$$a_o = \frac{4\pi\varepsilon_o \hbar^2}{m_e e^2} \longrightarrow a_o^* = \frac{4\pi\varepsilon_o \varepsilon^* \hbar^2}{m_e^* e^2} = a_o \frac{\varepsilon^* m_e}{m_e^*}$$

$$Ry = \frac{m_e e^4}{8\varepsilon_o^2 h^2} \longrightarrow Ry^* = \frac{m_e^* e^4}{8(\varepsilon_o \varepsilon^*)^2 h^2} = Ry \frac{m_e^*}{m_e (\varepsilon^*)^2}$$

2-D and 3-D Excitons

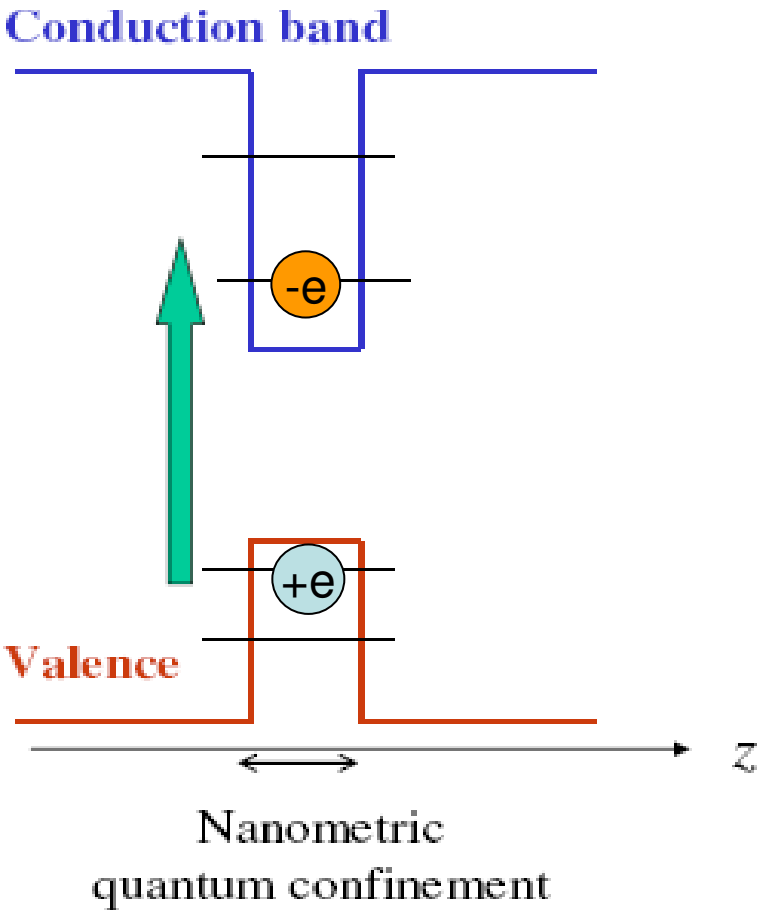
- Energy levels: bound states and continuum

$$E_B = -\frac{Ry}{n^2} \qquad \text{(3 D)}$$

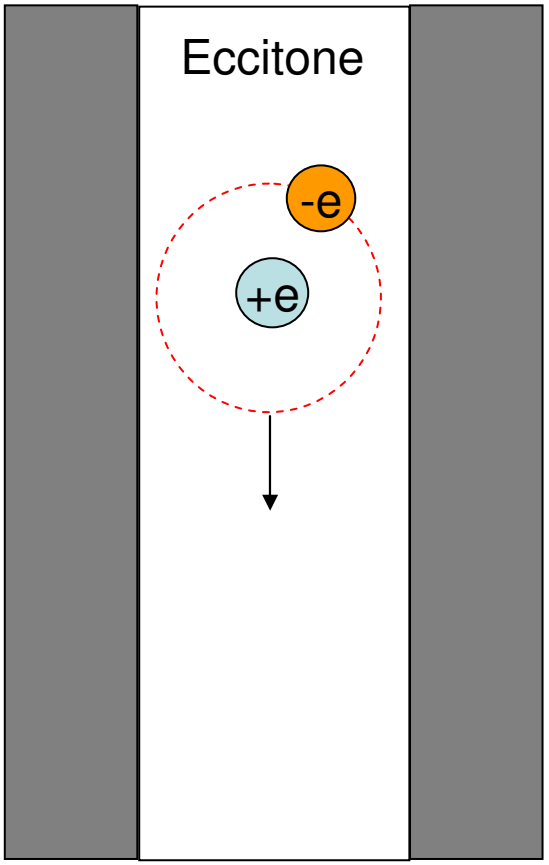
$$E_B = -\frac{Ry}{(n-1/2)^2} \qquad \text{(2 D hydrogen atom)}$$

- Ionization energy for ground state exciton in 2D case is 4 times larger than it is in 3D case. Therefore in QW, it's easier to form bound exciton states.

Elettrone e lacuna si attraggono e possono formare un eccitone



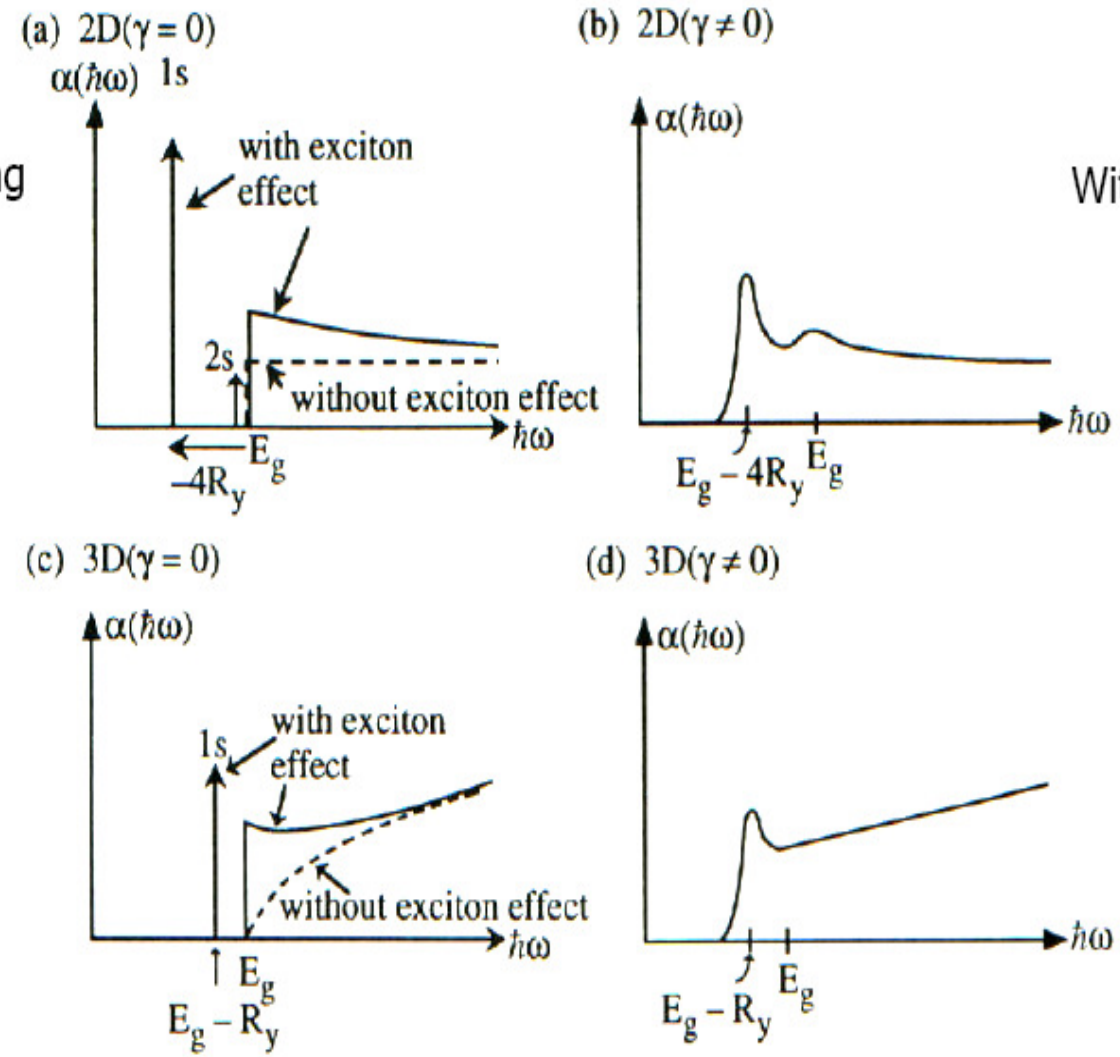
Nel piano l'eccitone
è libero di muoversi



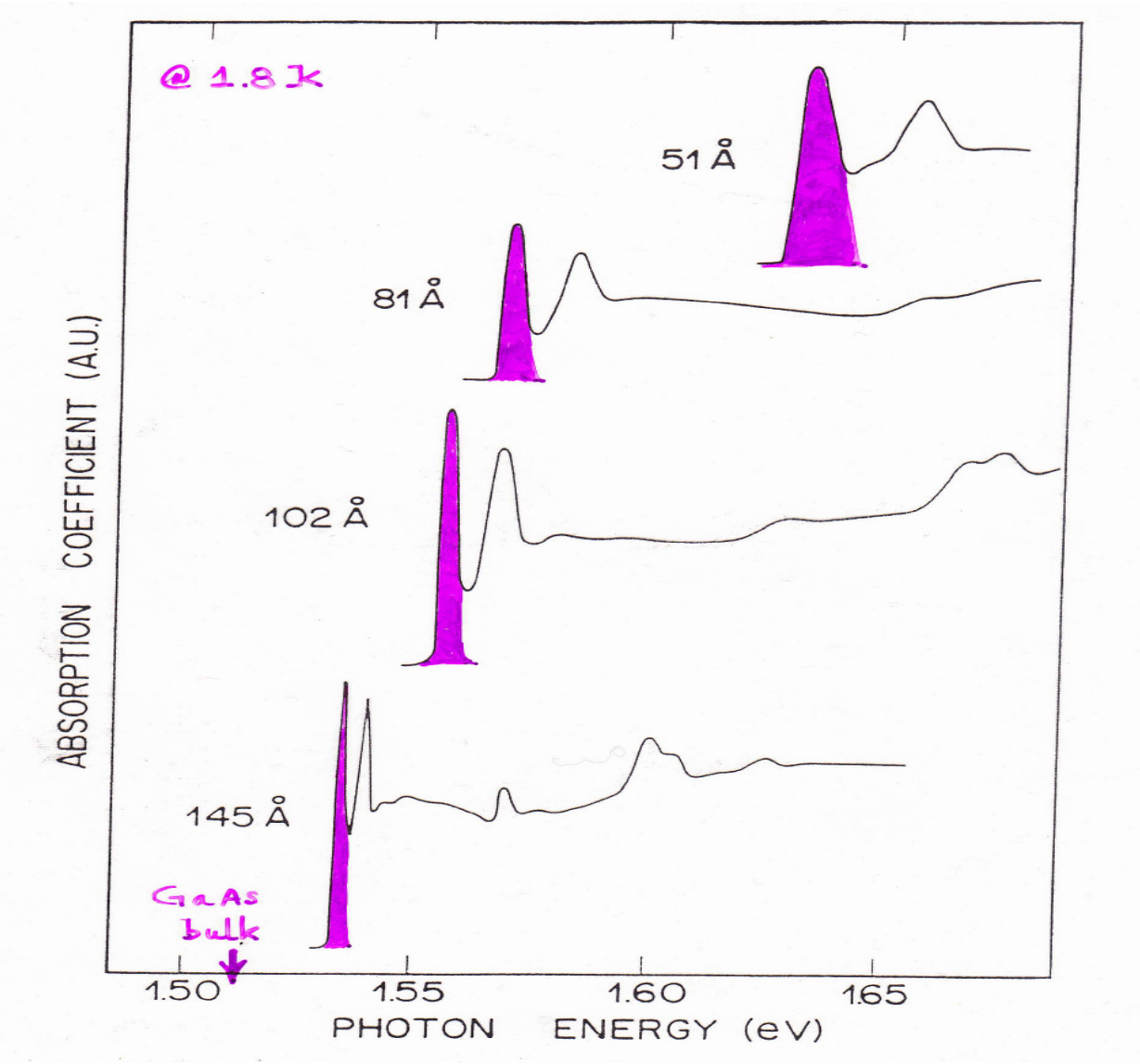
Absorption Including Coulomb Interactions

Without damping

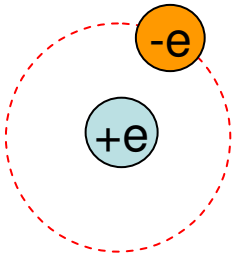
With damping



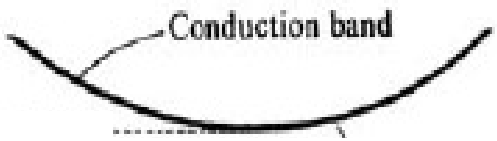
Tunabilità dovuta al confinamento

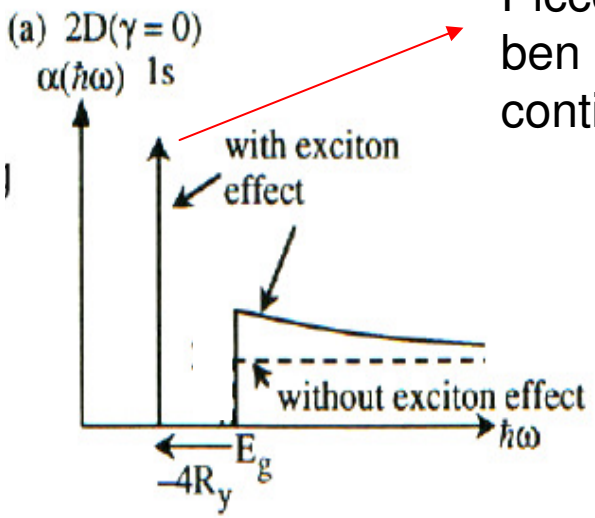


Sommario eccitone

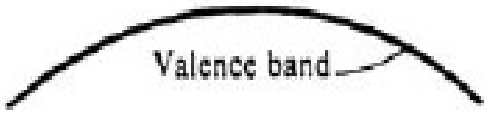


Spin intero





Picco di assorbimento ben separato dal continuo e-h



1. Stato isolato nel gap
2. Transizione tunabile
3. Statistica bosonica