

# **Quasi cristalli**



W H A T   I S . . .

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# a Quasicrystal?

*Marjorie Senechal*

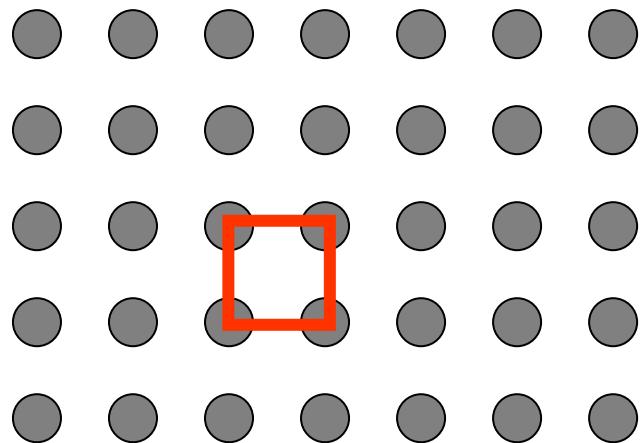
The long answer is: no one is sure. But the short answer is straightforward: a quasicrystal is a crystal with forbidden symmetry.

# Cristalli

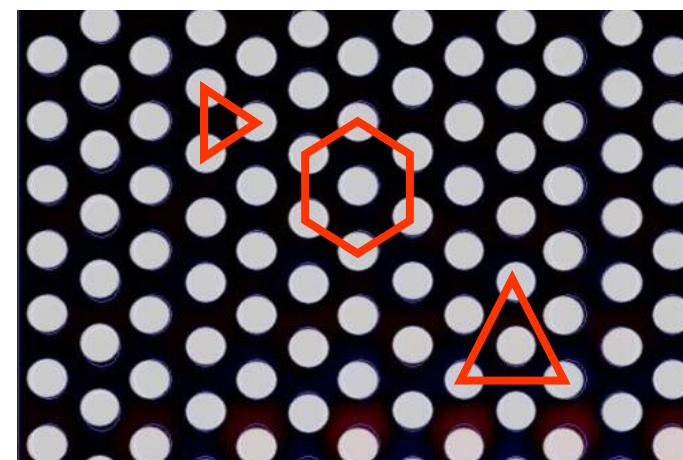
- 1) Invarianza traslazionale
- 2) Simmetria di rotazione

- 3) Riempimento completo
- 4) Sharp spots in X diffraction

Nel piano:



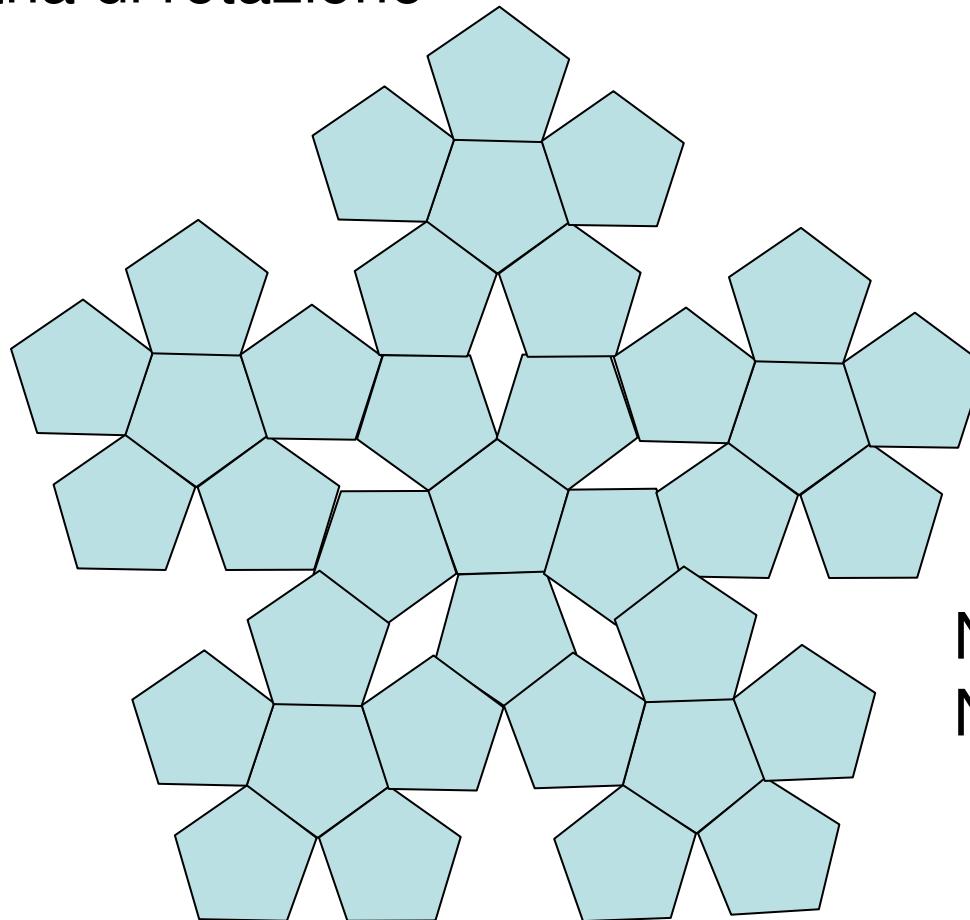
Reticolo quadrato  
Four (two) fold



Reticolo triangolare (esagonale)  
Six (three) fold

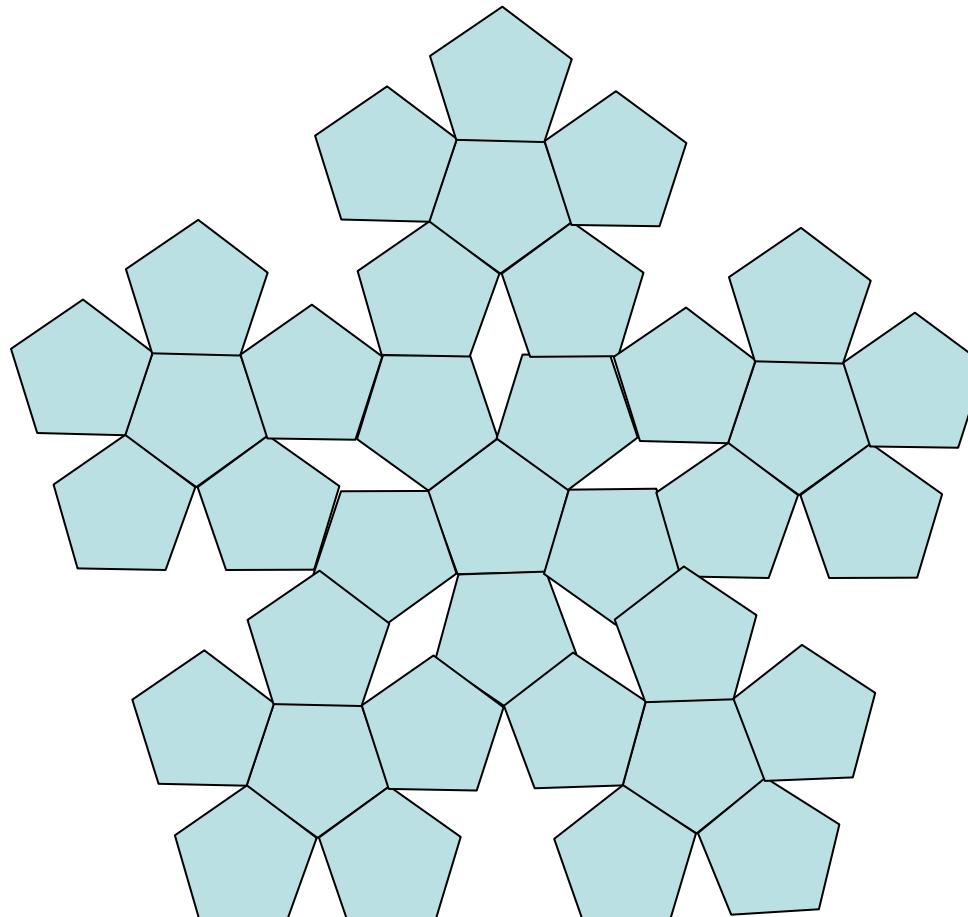
# Five fold case (cristallo pentagonale)

Simmetria di rotazione



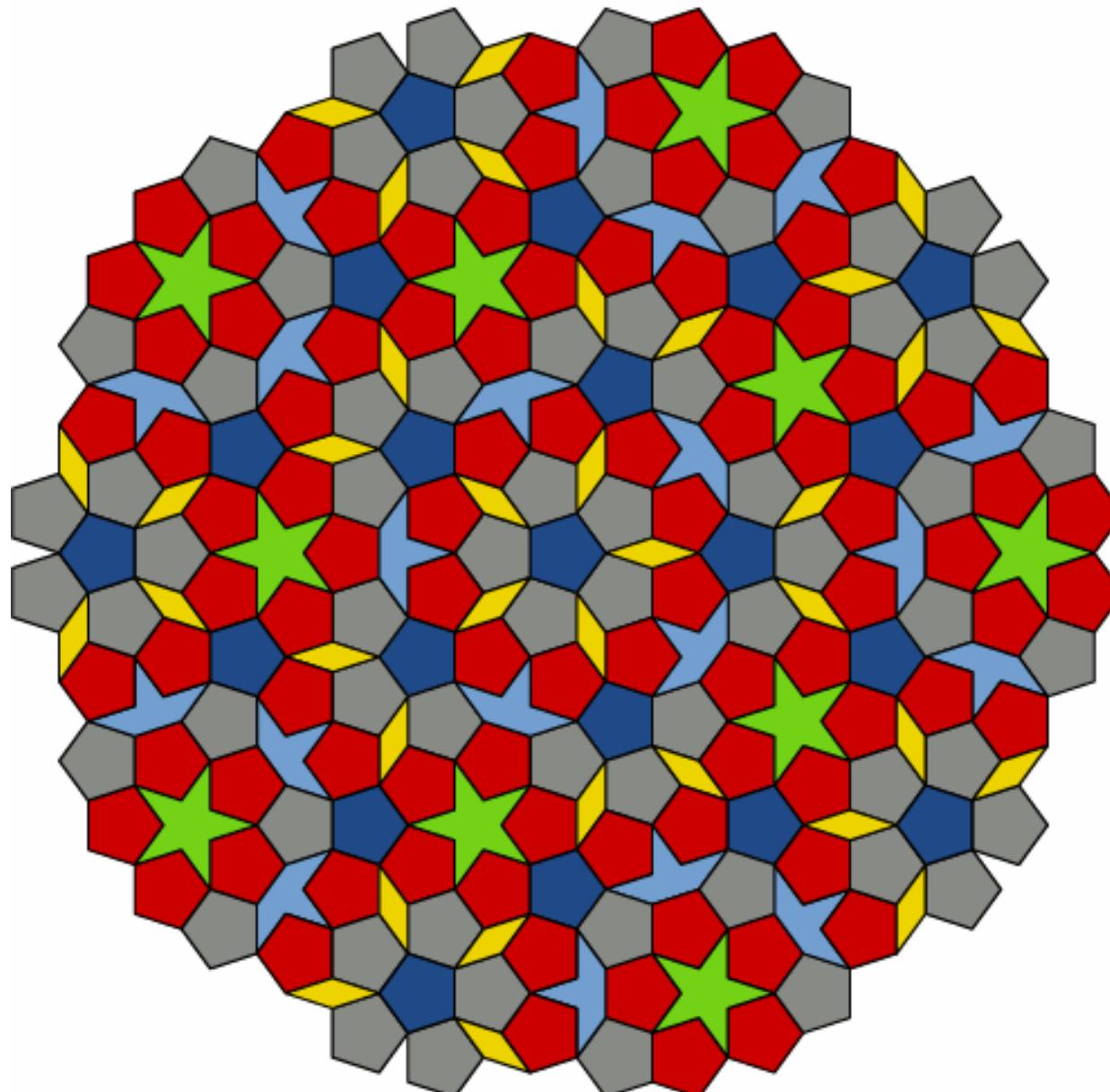
No traslazione  
No riempimento

Esistono simmetrie (di rotazione)  
che non ammettono simmetrie di traslazione



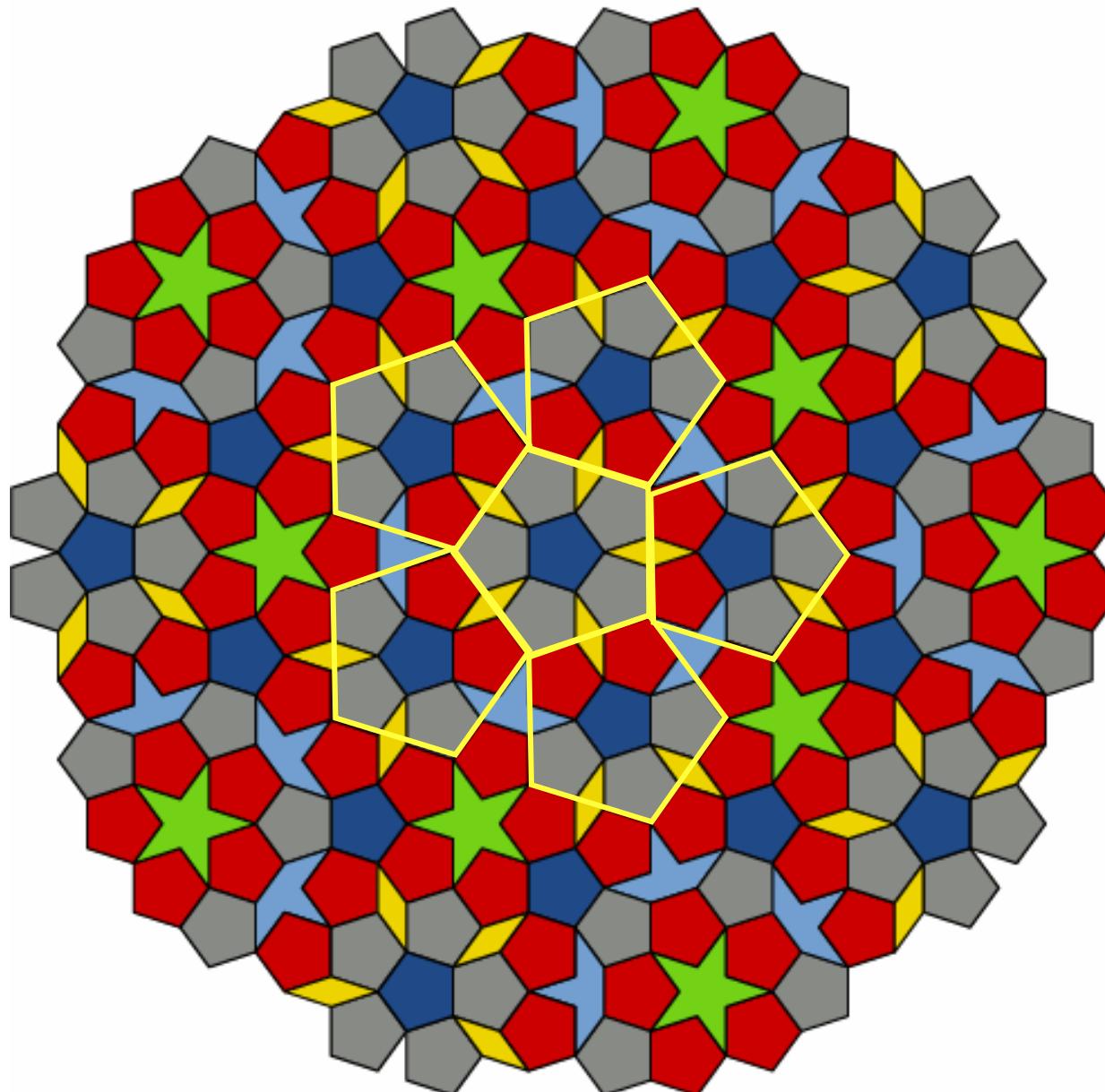
Pero' il riempimento del piano puo' essere fatto con simmetria "fivefold"

4 elementi

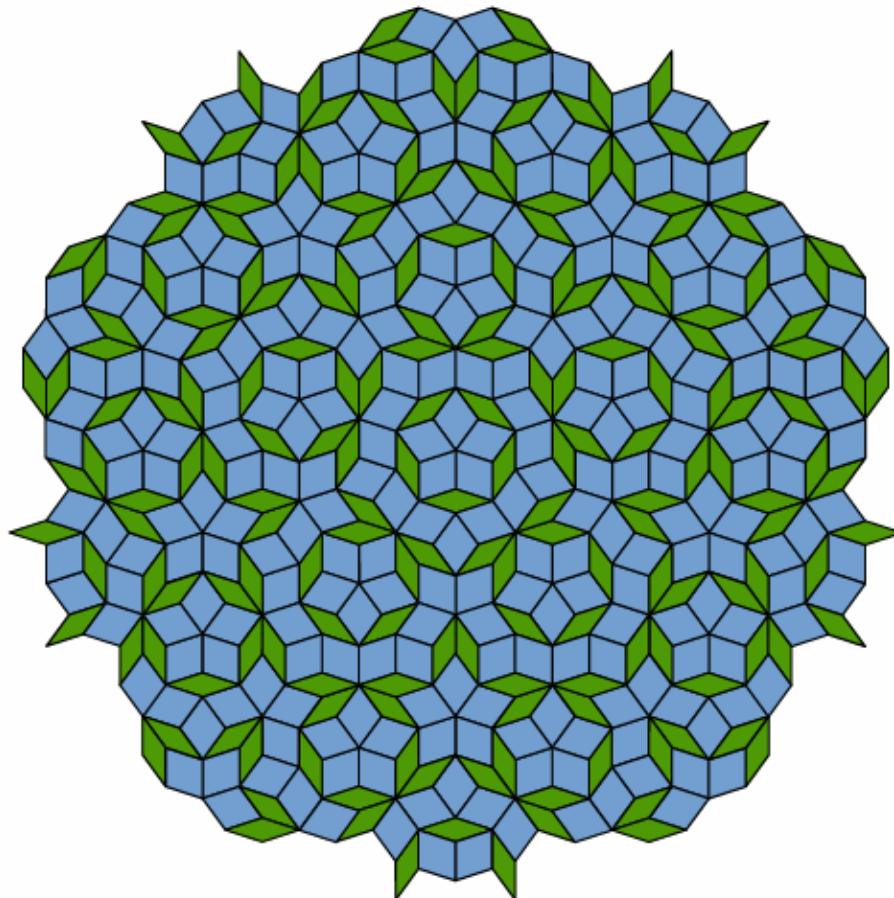


Pero' il riempimento del piano puo' essere fatto con simmetria "fivefold"

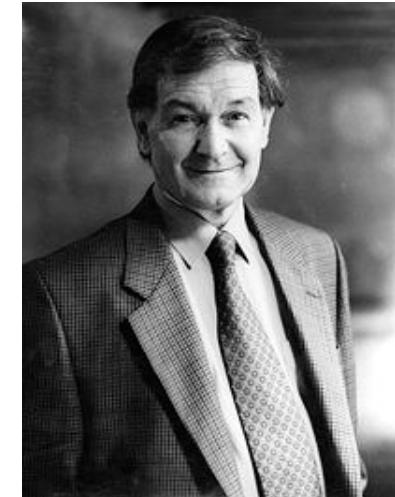
4 elementi



# Penrose tiling (1974)



2 elementi

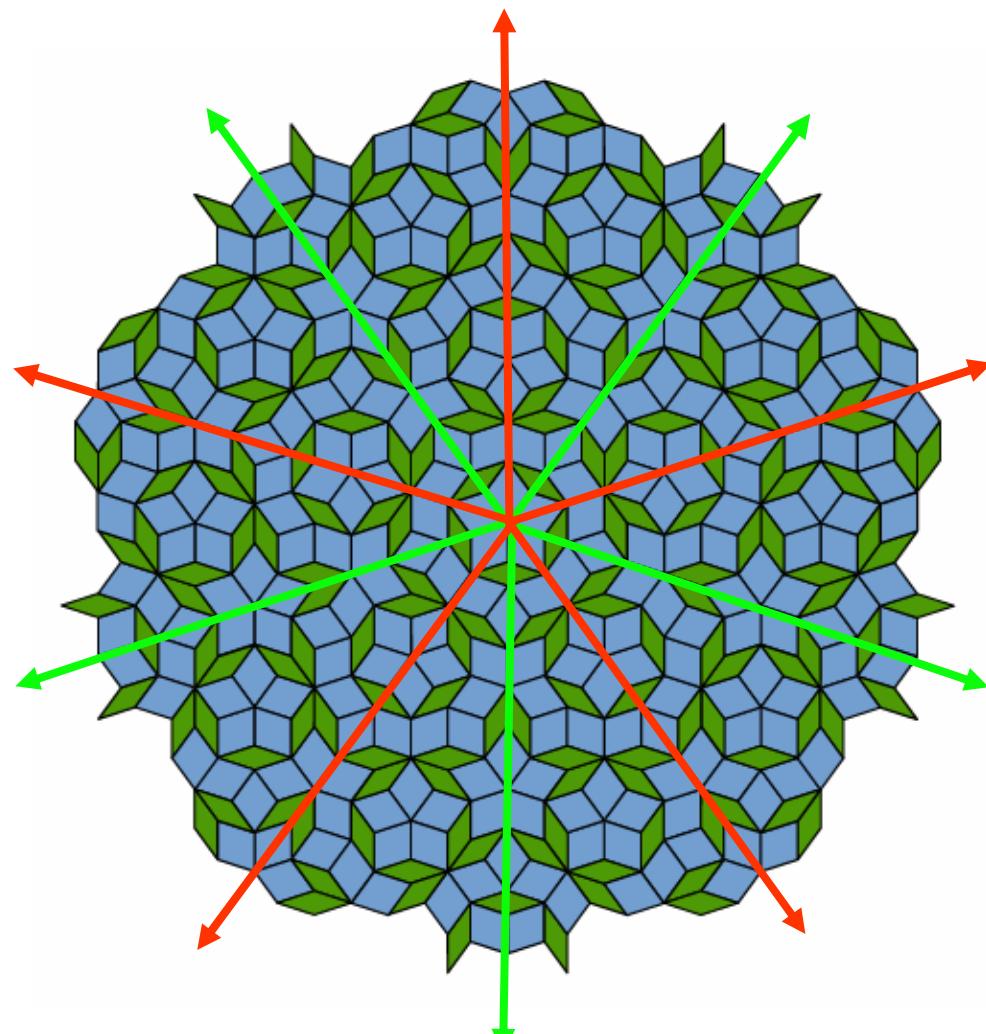


*Sir Roger Penrose*

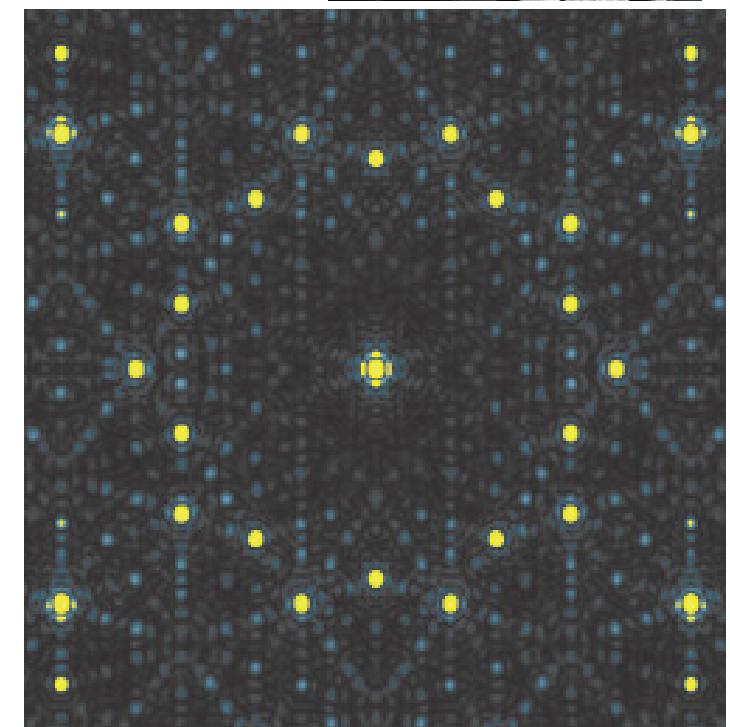
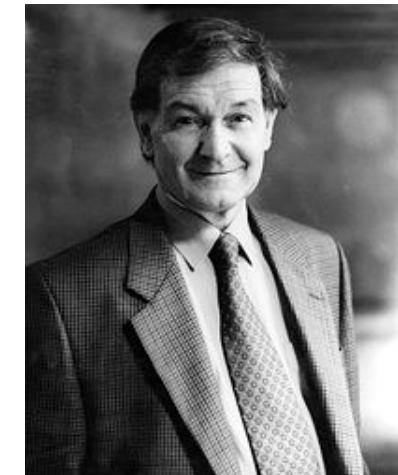
E' possibile riempire ol piano con simmetria five fold partendo da due figure geometriche e definendo una procedura di suddivisione e iterazione. Questa è legata alla sezione aurea e alla successione di Fibonacci

Penrose R., "Role of aesthetics in pure and applied research ", Bull. Inst. Maths. Appl. 10 (1974) 266

# Penrose tiling



*fivefold symmetry*



*Bragg diffraction*

# Definizione ufficiale

In 1992, the International Union for Crystallography's newly-formed Commission on Aperiodic Crystals decreed a *crystal* to be

***“any solid having an essentially discrete diffraction diagram.”***

In the special case that

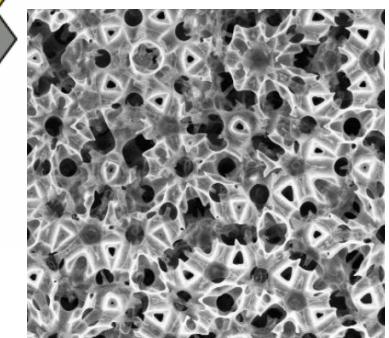
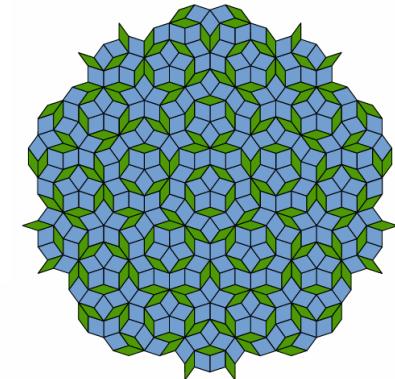
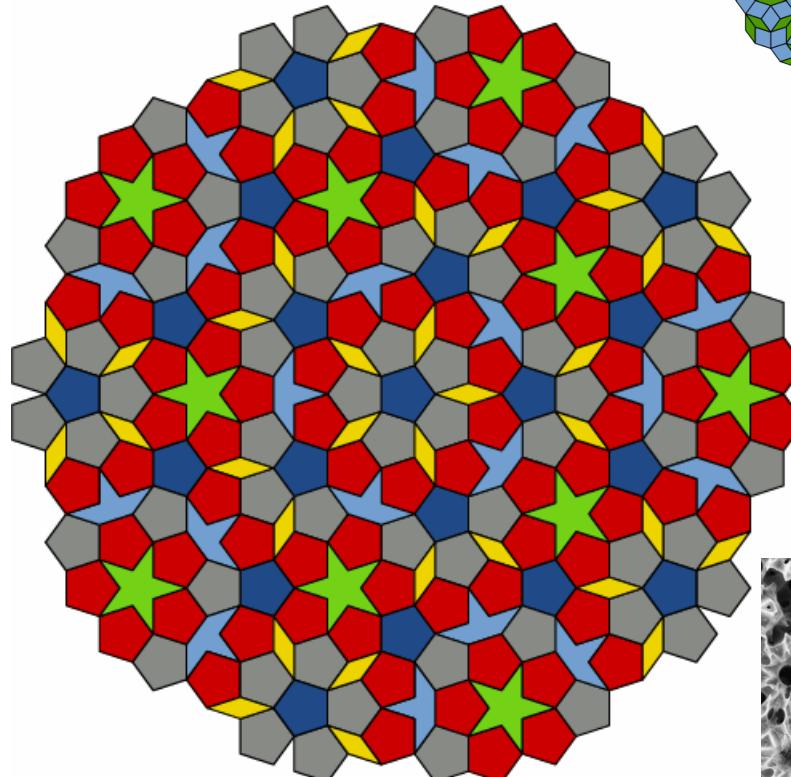
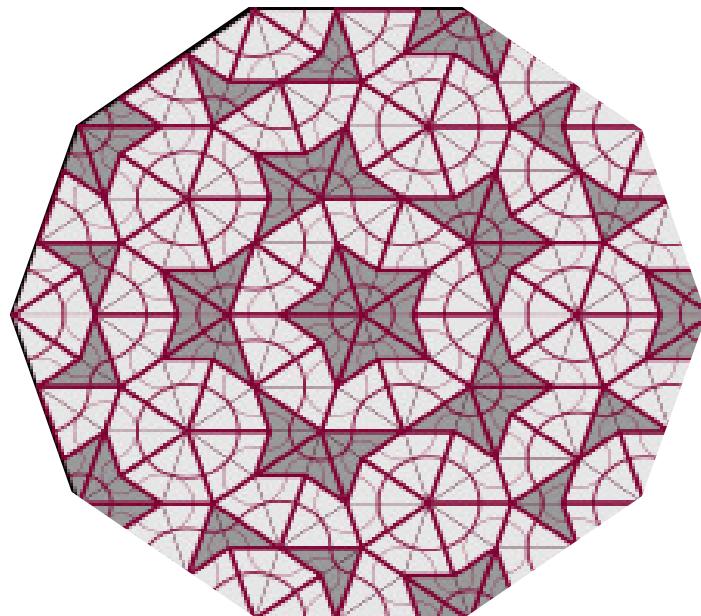
***“three dimensional lattice periodicity can be considered to be absent”***

the crystal is *aperiodic*

<http://www.iucr.org/iucr-top/iucr/cac.html>

# Proprietà quasi cristallo

1. Non periodico, ma determina “complete filling”
2. Ogni regione appare infinite volte
3. Ordine a lungo raggio



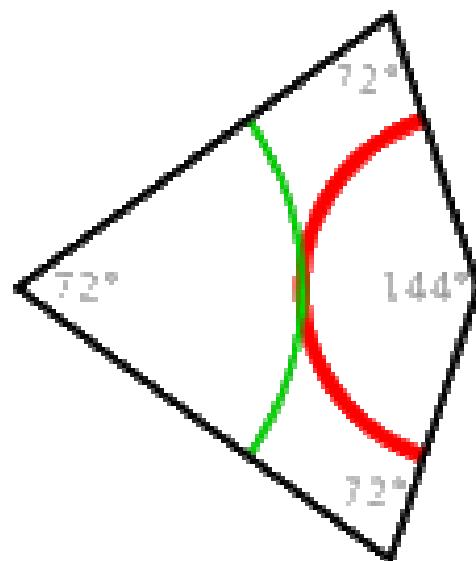
4. Si costruisce per ricorrenza
5. Diffrazione X produce Bragg pattern
6. PhC QC ha band gap anche con basso mismatch dielettrico

# Costruzione di un quasi cristallo in 2D

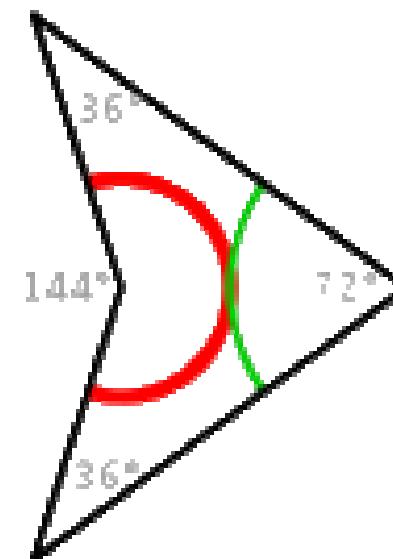
## Esempio di ricorrenza

Due strutture di base

Kite

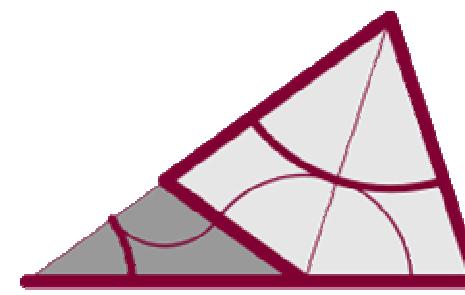
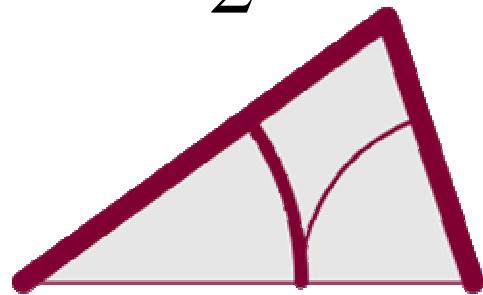


Dart

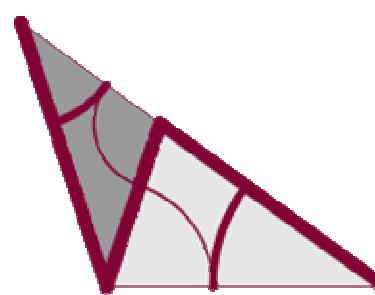
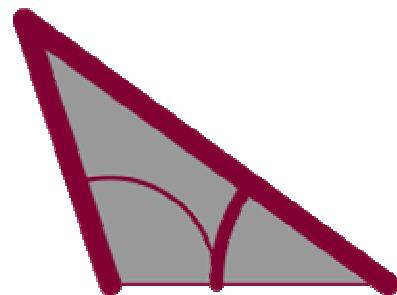


# Ricorrenze: Deflation

a)  $\frac{1}{2} Kite = \frac{1}{2} Dart + 1Kite$

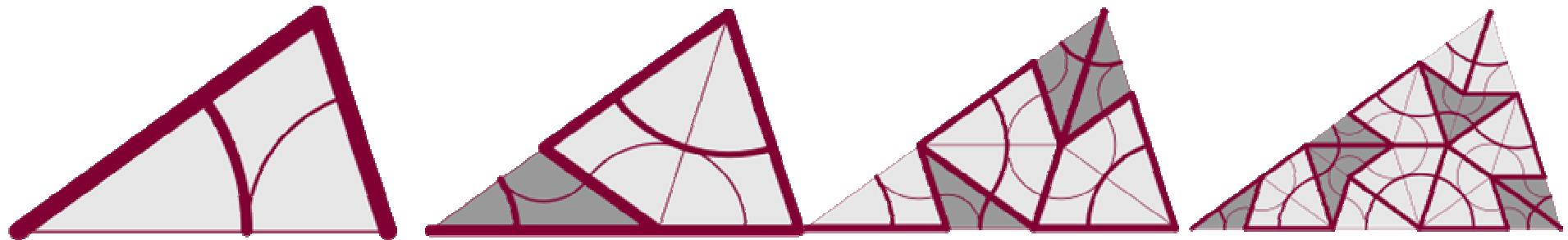


b)  $\frac{1}{2} Dart = \frac{1}{2} Dart + \frac{1}{2} Kite$

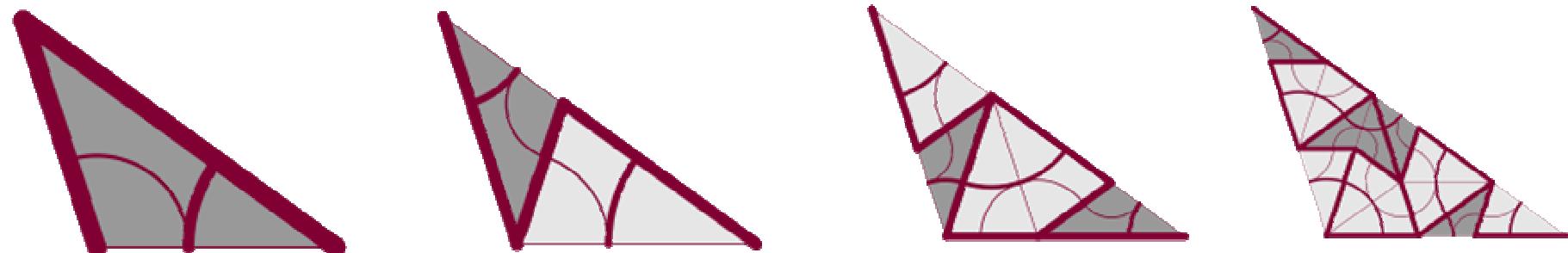


# Deflation

$$\frac{1}{2} Kite = \frac{1}{2} Dart + 1Kite$$

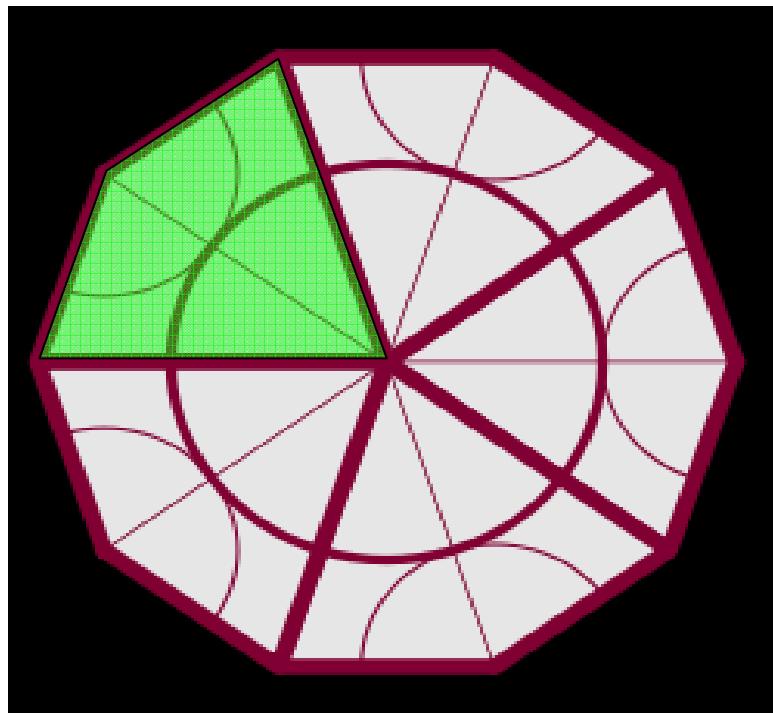


$$\frac{1}{2} Dart = \frac{1}{2} Dart + \frac{1}{2} Kite$$

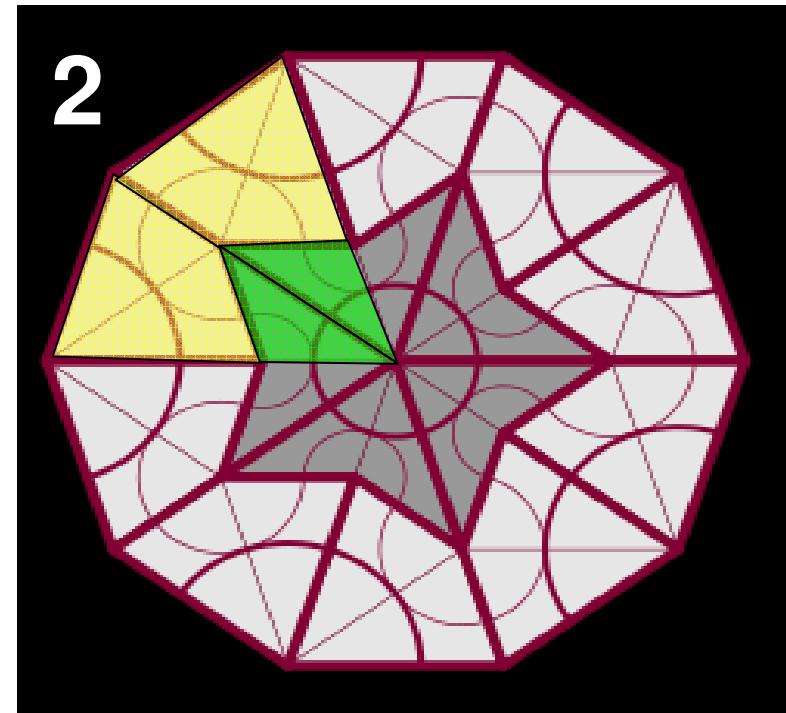


# Costruiamo il SUN

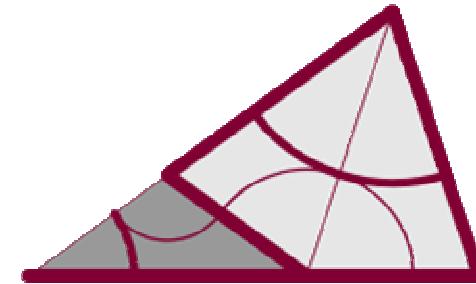
Tiling: 1 kite  $\rightarrow$  2 kite + 1 dart



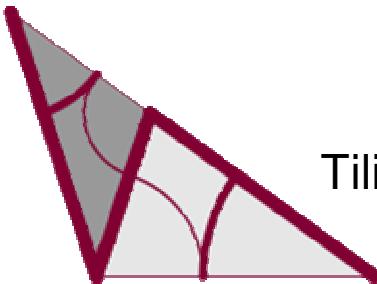
5 kites



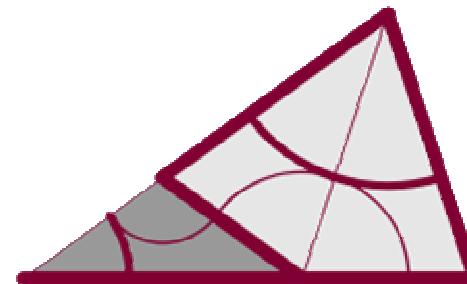
10 kites+5 darts



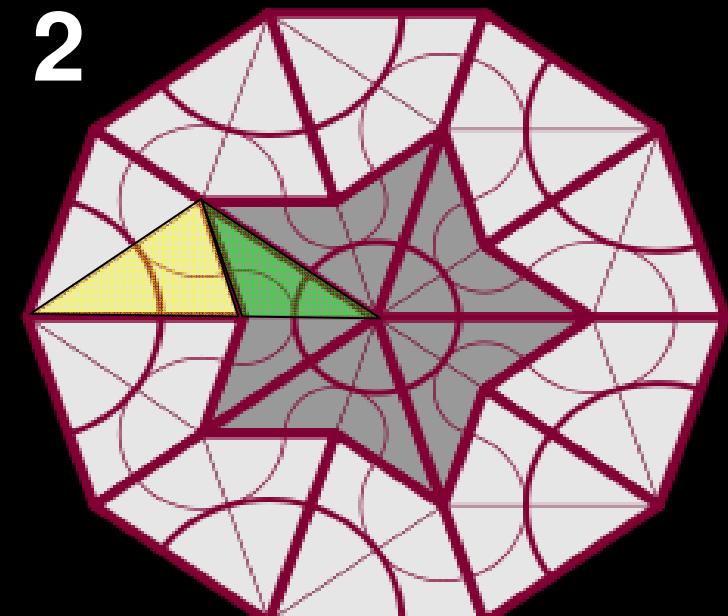
SUN



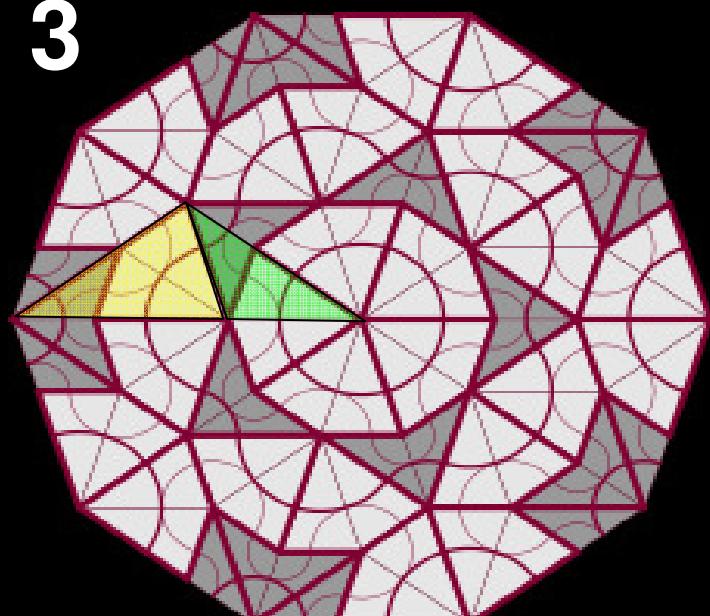
Tiling: 1 kite  $\rightarrow$  2 kite + 1 dart  
1 dart  $\rightarrow$  1 kite + 1 dart



2

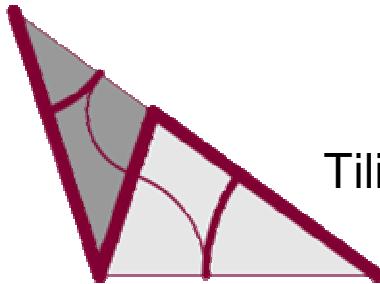


3

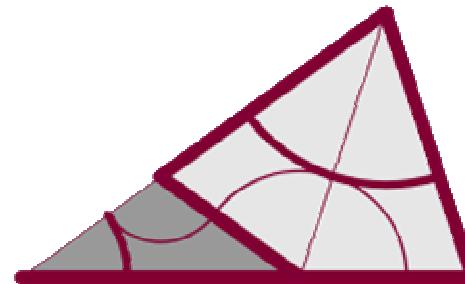


10 kites + 5 darts

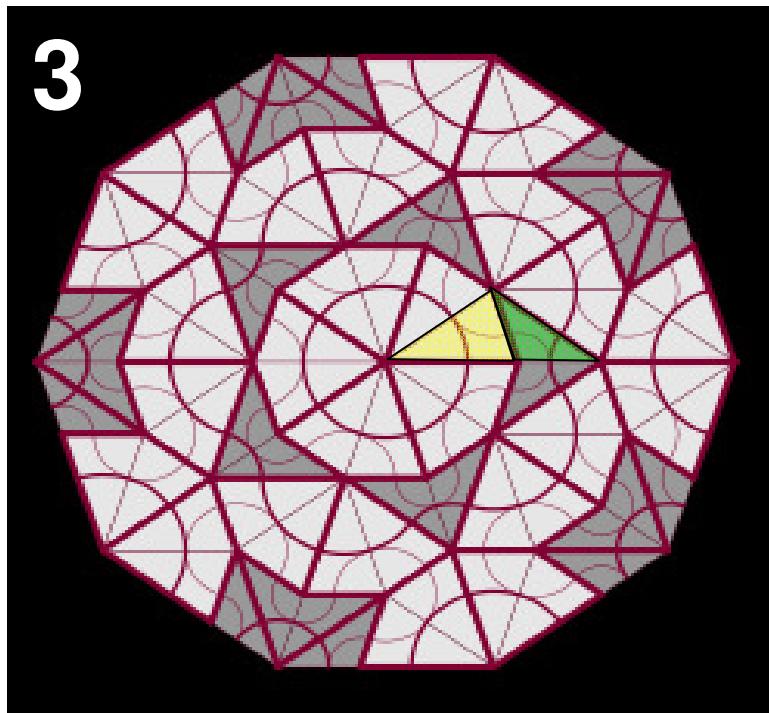
*SUN*



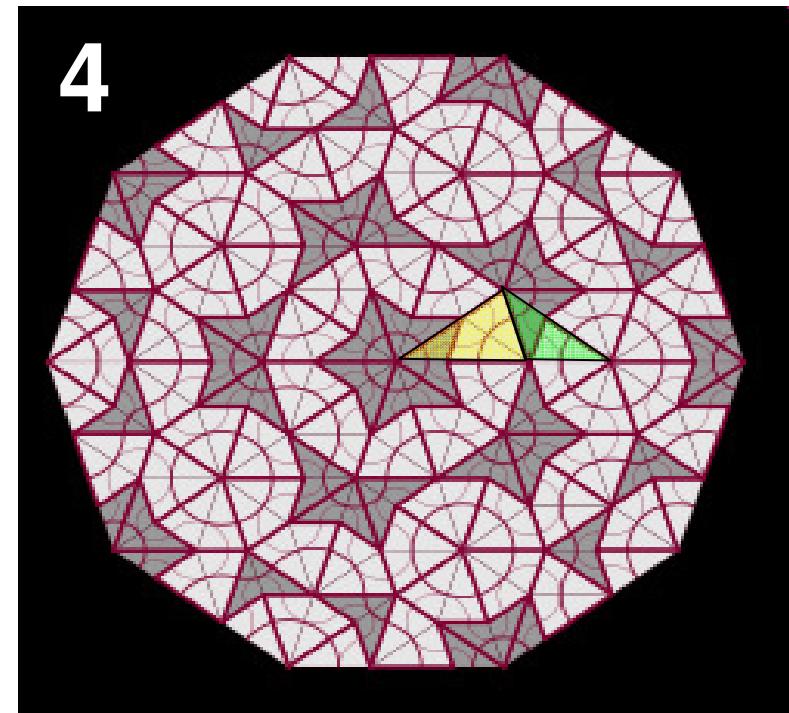
Tiling: 1 kite  $\rightarrow$  2 kite + 1 dart  
1 dart  $\rightarrow$  1 kite + 1 dart



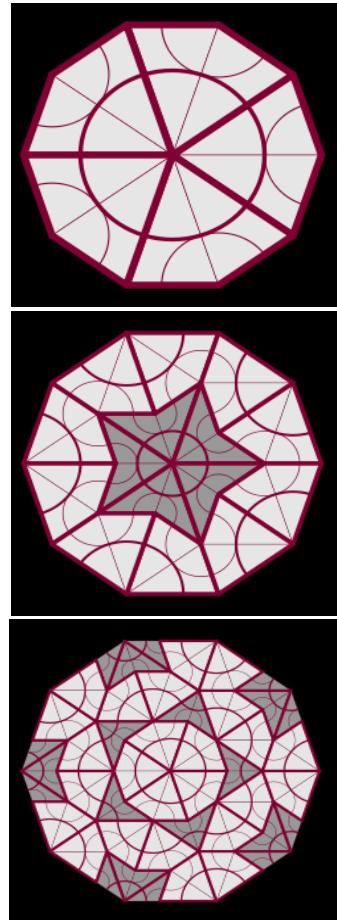
3



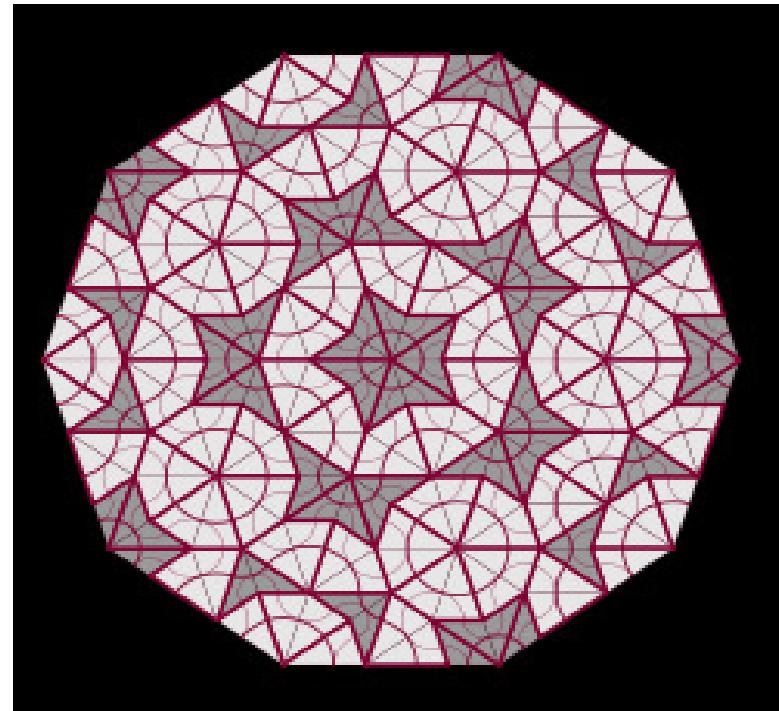
4



SUN



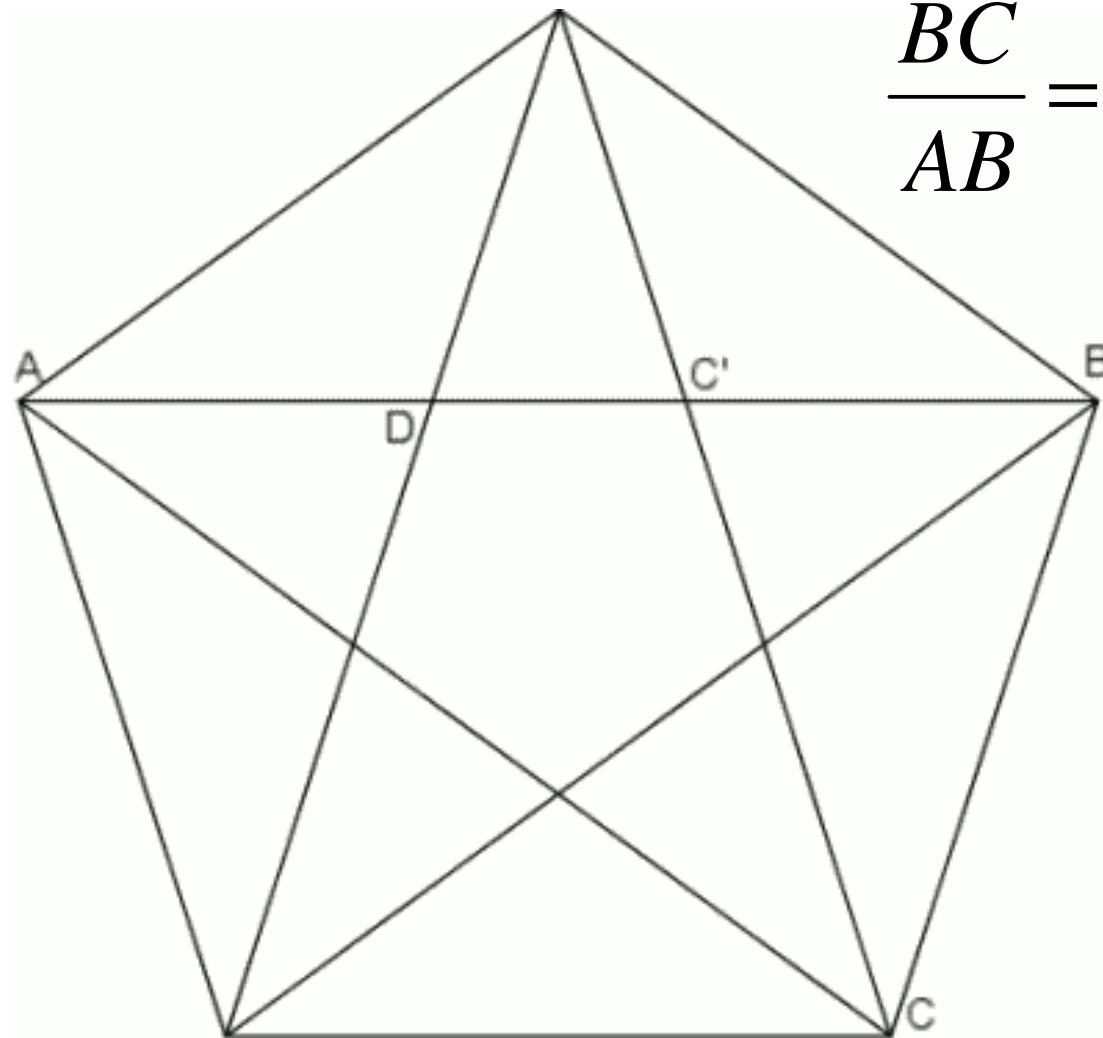
SELF SIMILARITY



kites e darts si ripetono con frequenze il cui rapporto è

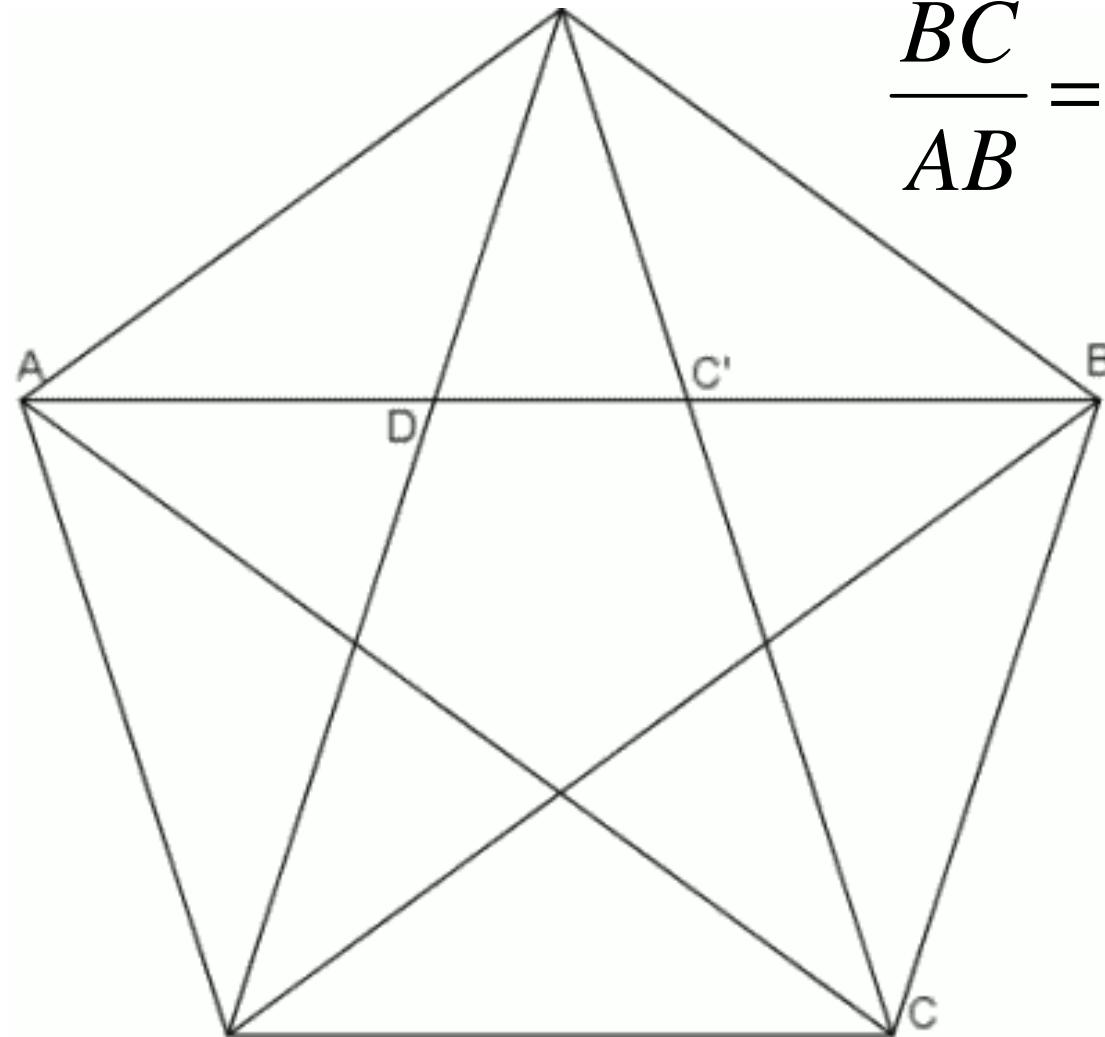
la sezione aurea       $\varphi = \frac{1 + \sqrt{5}}{2} = 1.618\dots$

# Sezione aurea



$$\frac{BC}{AB} = \frac{AB}{BD} = \varphi = \frac{1 + \sqrt{5}}{2}$$

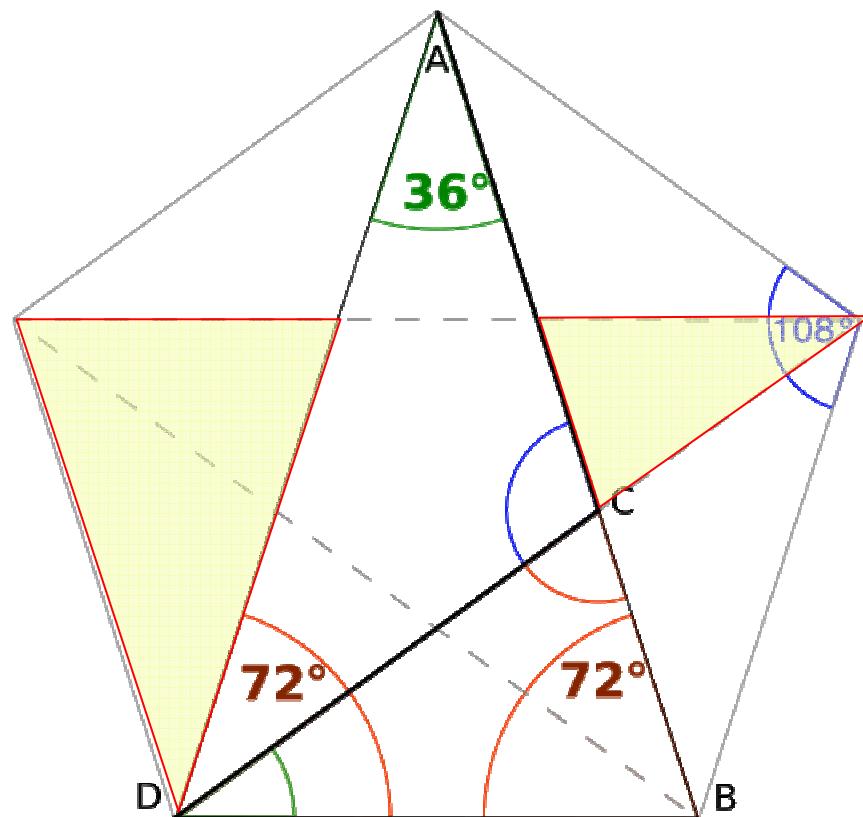
# Sezione aurea



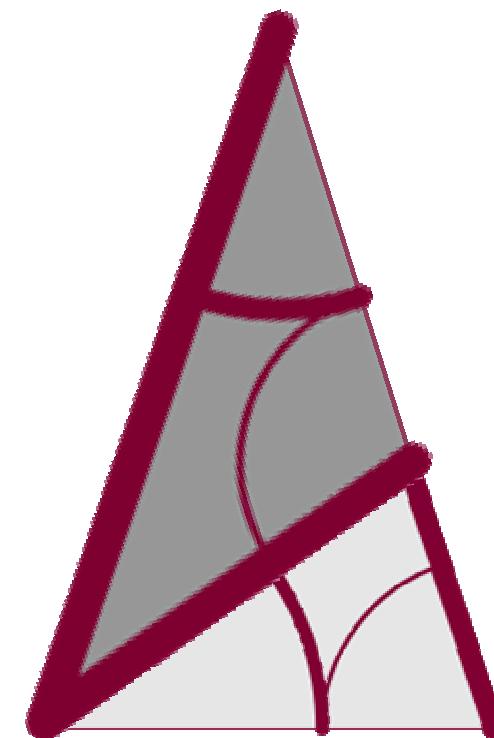
$$\frac{BC}{AB} = \frac{AB}{BD} = \varphi = \frac{1 + \sqrt{5}}{2}$$

# Sezione aurea

$$\varphi = \frac{1 + \sqrt{5}}{2}$$



Triangolo aureo



Kites and Darts

# Sezione aurea in algebra

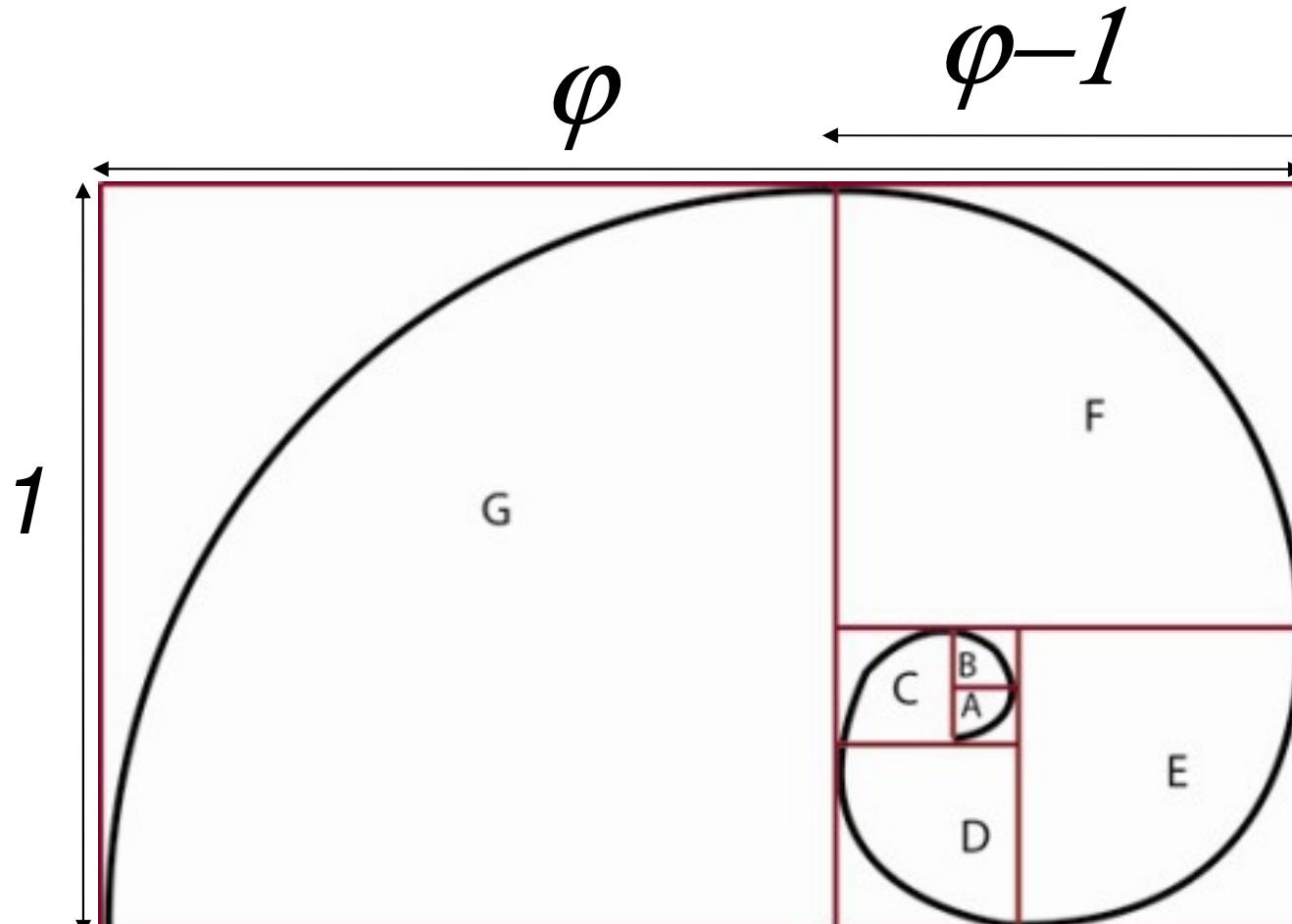
$$\varphi = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

$$\varphi = \frac{1}{\varphi - 1}$$

Frazione continua

# Sezione aurea in geometria

$$\varphi = \frac{1 + \sqrt{5}}{2}$$



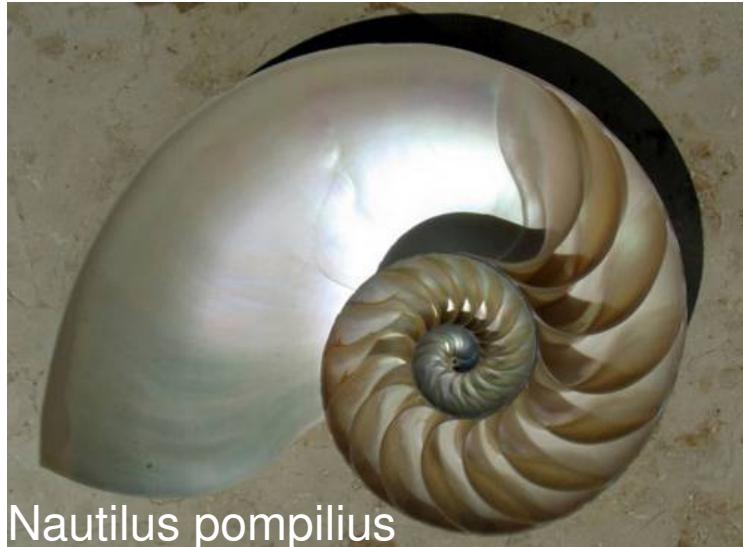
Rettangolo aureo

Spirale aurea

$$\varphi = \frac{1}{\varphi - 1}$$

$$r = e^{\varphi\theta}$$

# Sezione aurea in natura



Spirale aurea

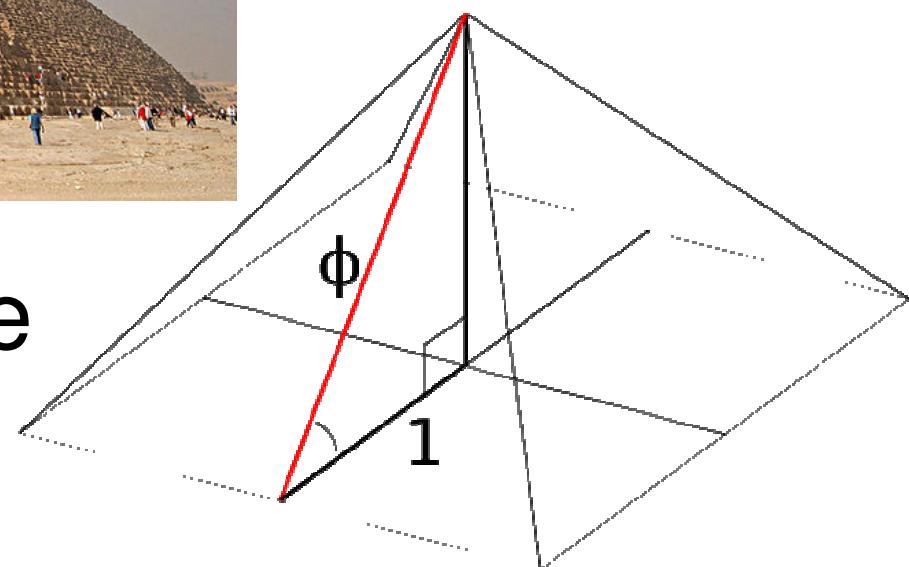
$$r = e^{\varphi\theta}$$



# Sezione aurea in architettura



Piramide di Cheope





## Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture

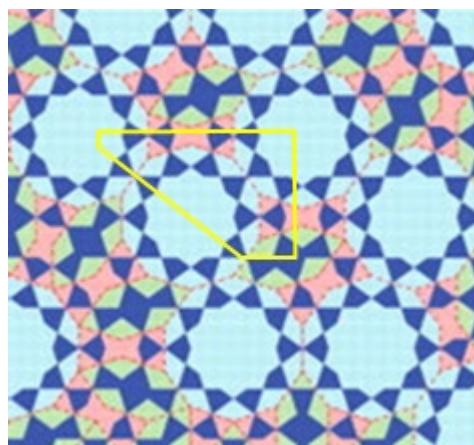
Peter J. Lu, *et al.*

Science **315**, 1106 (2007);

DOI: 10.1126/science.1135491

# Quasi cristalli in arte

Darb-i Imam shrine (1453 C.E., Isfahan, Iran)



## Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture

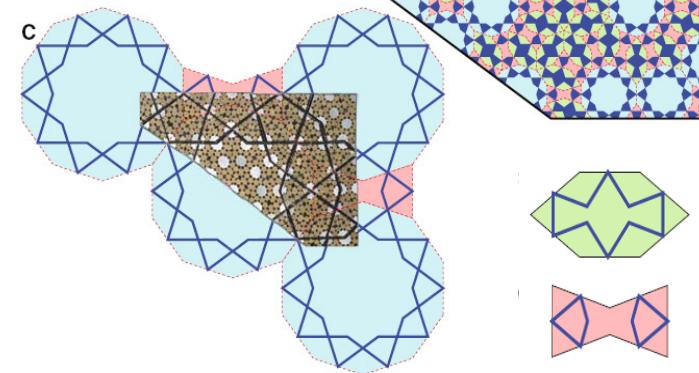
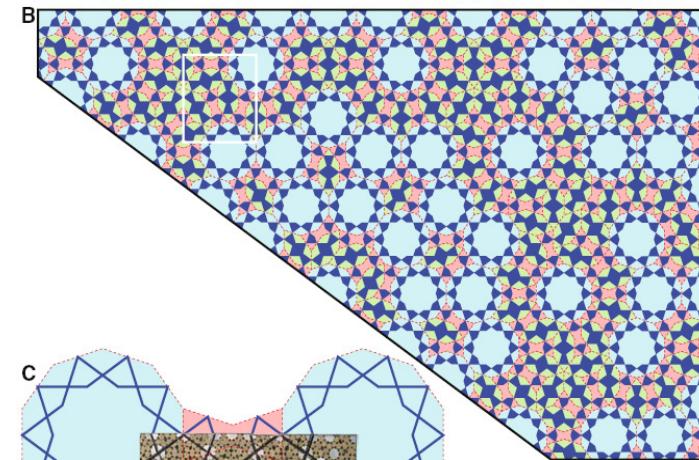
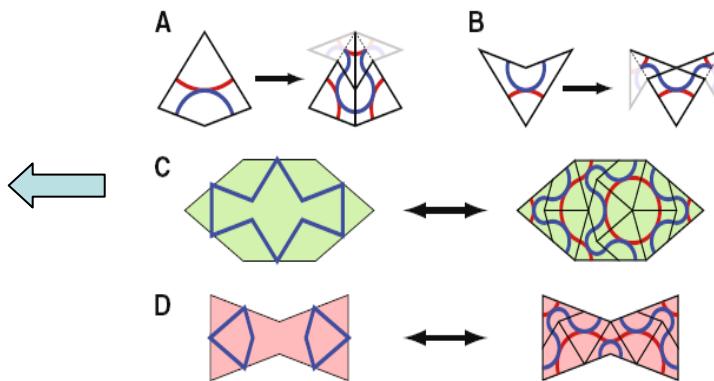
Peter J. Lu, et al.  
*Science* **315**, 1106 (2007);  
DOI: 10.1126/science.1135

# Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture

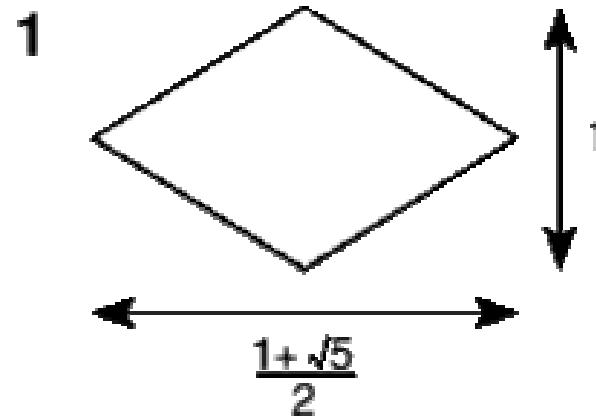
Peter J. Lu<sup>1\*</sup> and Paul J. Steinhardt<sup>2</sup>

The conventional view holds that girih (geometric star-and-polygon, or strapwork) patterns in medieval Islamic architecture were conceived by their designers as a network of zigzagging lines, where the lines were drafted directly with a straightedge and a compass. We show that by 1200 C.E. a conceptual breakthrough occurred in which girih patterns were reconceived as tessellations of a special set of equilateral polygons ("girih tiles") decorated with lines. These tiles enabled the creation of increasingly complex periodic girih patterns, and by the 15th century, the tessellation approach was combined with self-similar transformations to construct nearly perfect quasi-crystalline Penrose patterns, five centuries before their discovery in the West.

Kites &  
Darts

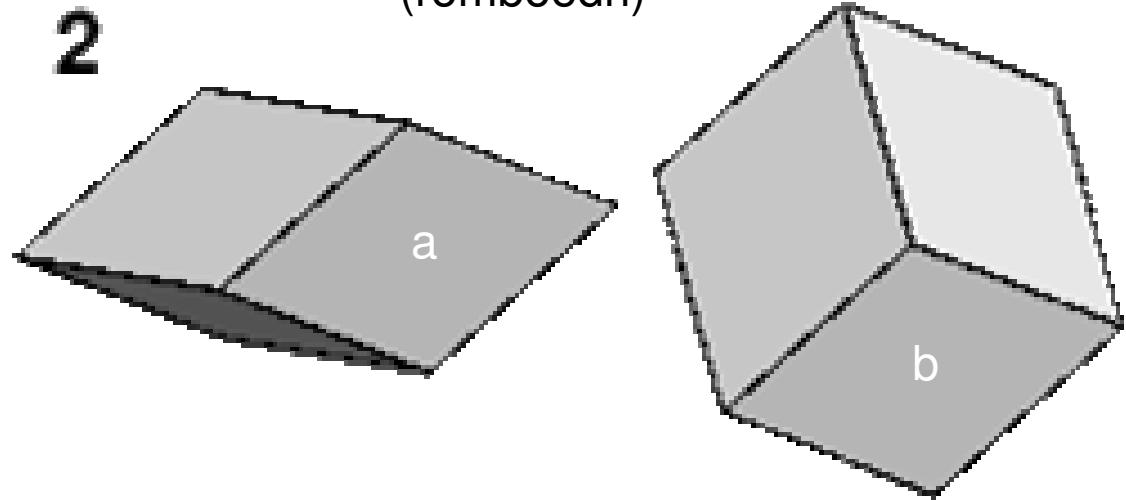


# Ricorrenza: Icosahederal Quasi Crystal in 3D



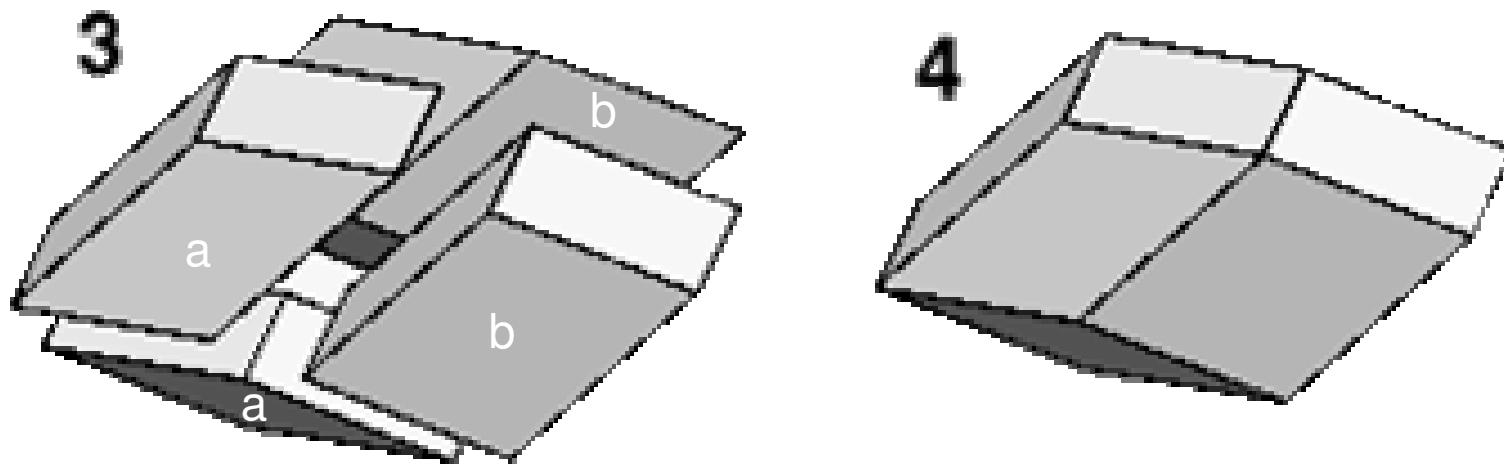
Rombo aureo

2 rhombic hexahedrons  
(romboedri)



$$\varphi = \frac{1+\sqrt{5}}{2} = 1.618\dots = \text{Sezione aurea}$$

# Ricorrenza: Icosaherdal Quasi Crystal in 3D

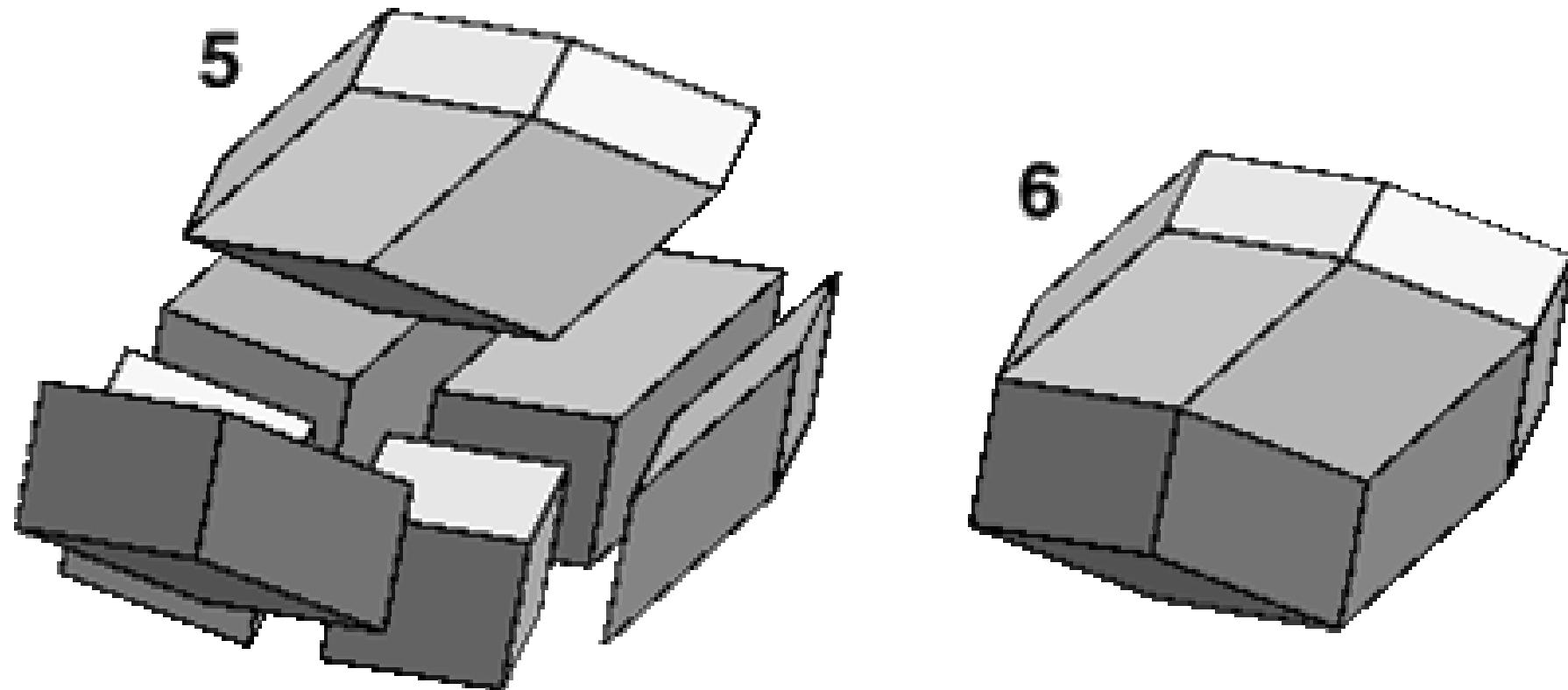


2 oblate rhombic hexahedrons +  
2 prolate rhombic hexahedrons



Bilinski's rhombic  
dodecahedron

# Ricorrenza: Icosaherdal Quasi Crystal in 3D

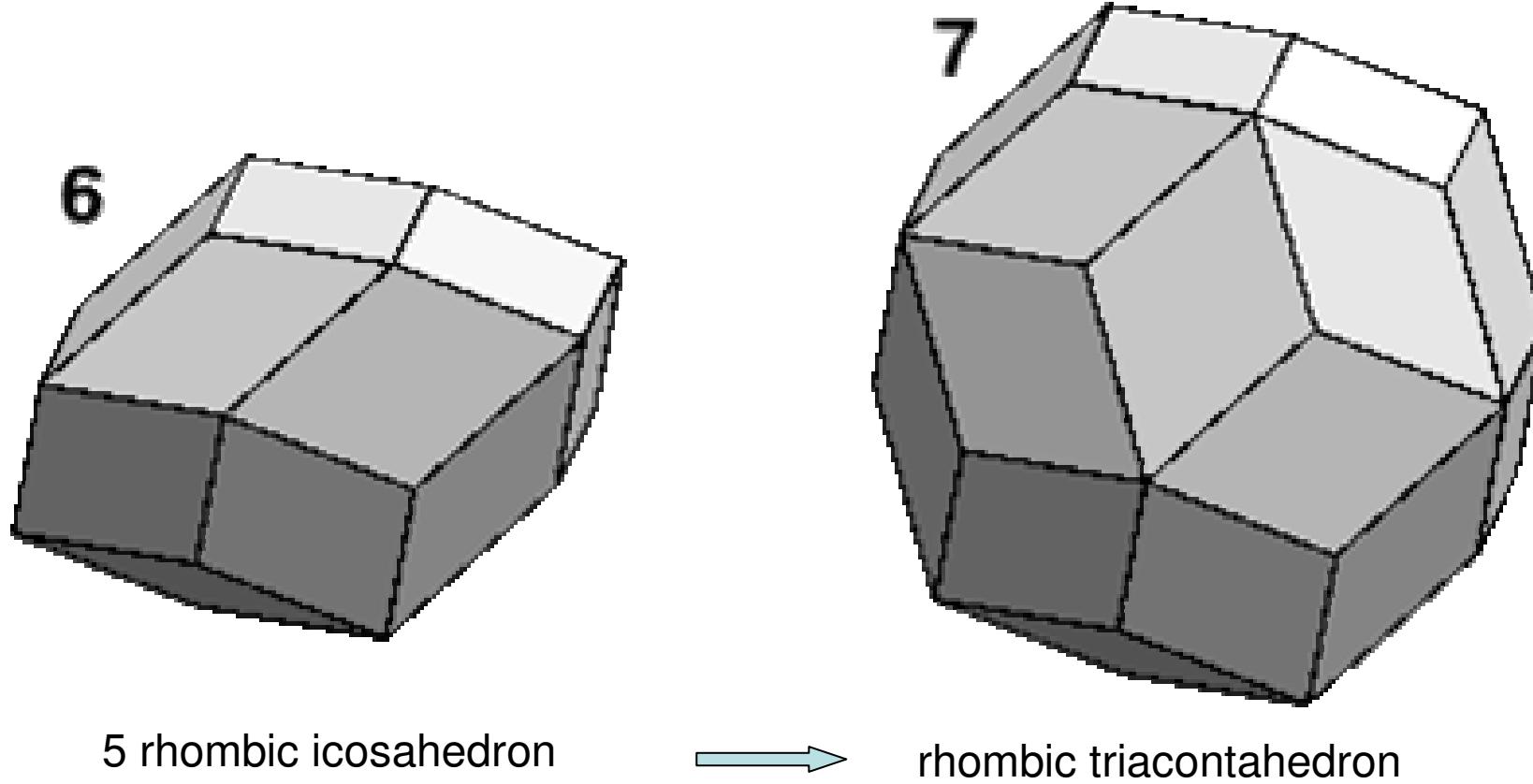


1 Bilinski's rhombic dodecahedron +  
3 oblate rhombic hexahedrons +  
3 prolate rhombic hexahedrons

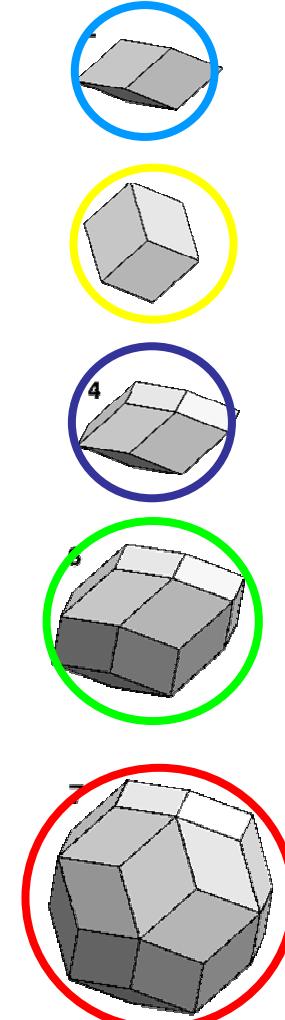
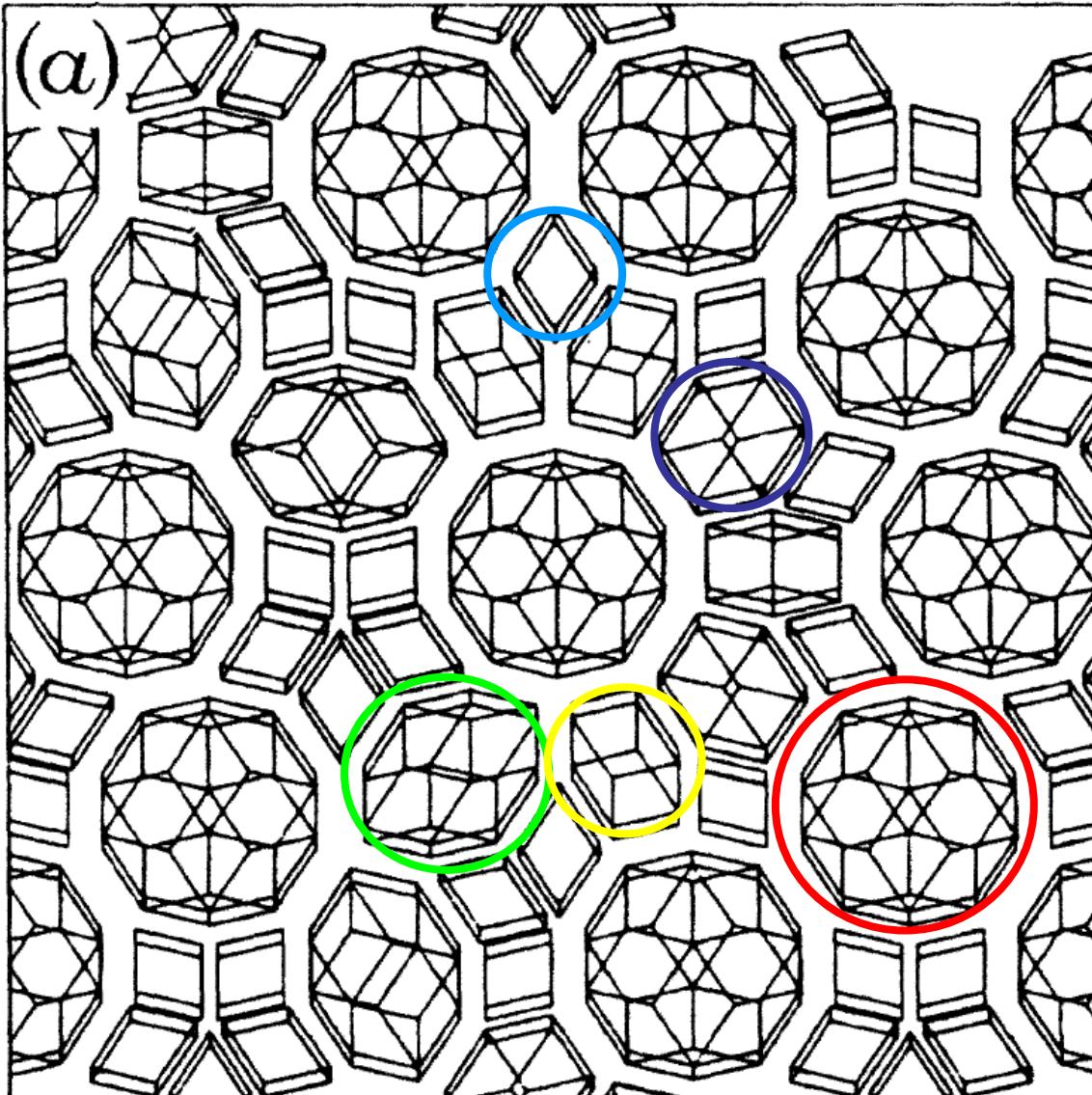


rhombic icosahedron

# Ricorrenza: Icosahederal Quasi Crystal in 3D



# Close packing: Icosaherdal Quasi Crystal



# *Prima evidenza sperimentale $Al_{0.9}Mn_{0.1}$ after annealing*

VOLUME 53, NUMBER 20

PHYSICAL REVIEW LETTERS

12 NOVEMBER 1984

## Metallic Phase with Long-Range Orientational Order and No Translational Symmetry

D. Shechtman and I. Blech

*Department of Materials Engineering, Israel Institute of Technology—Technion, 3200 Haifa, Israel*

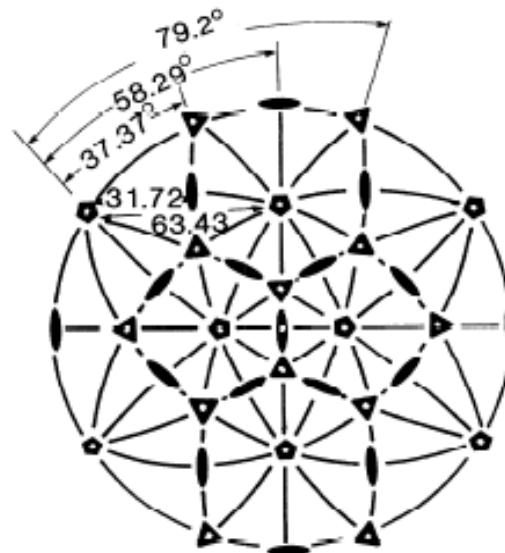


FIG. 1. Stereographic projection of the symmetry elements of the icosahedral group  $m\bar{3}\bar{5}$ .

Icosahedral order is inconsistent with translational symmetry

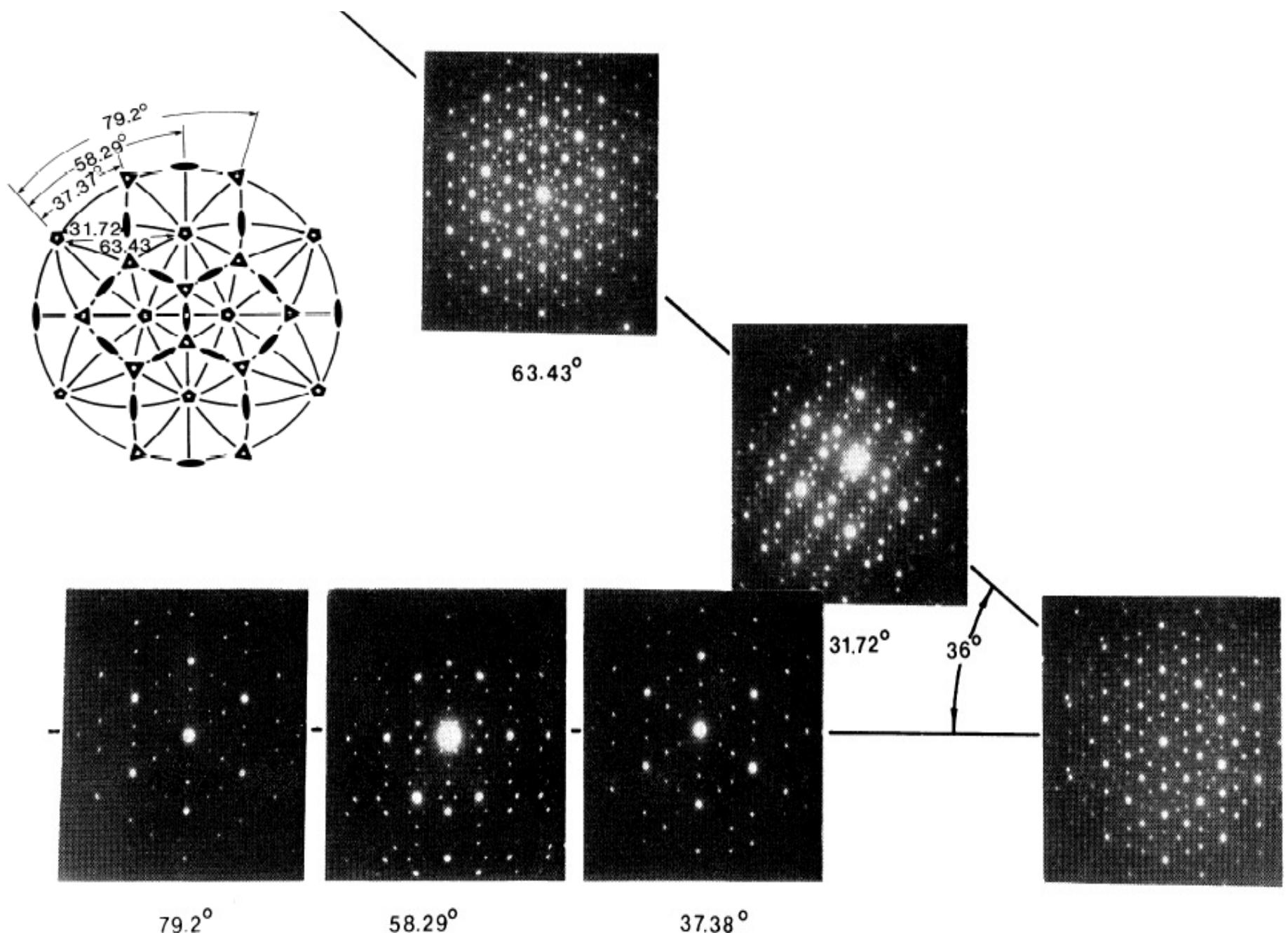


FIG. 2. Selected-area electron diffraction patterns taken from a single grain of the icosahedral phase. Rotations match those in Fig. 1.

# Primo quasi cristallo in natura



Natural Quasicrystals  
Luca Bindi, et al.  
*Science* **324**, 1306 (2009);  
DOI: 10.1126/science.1170827

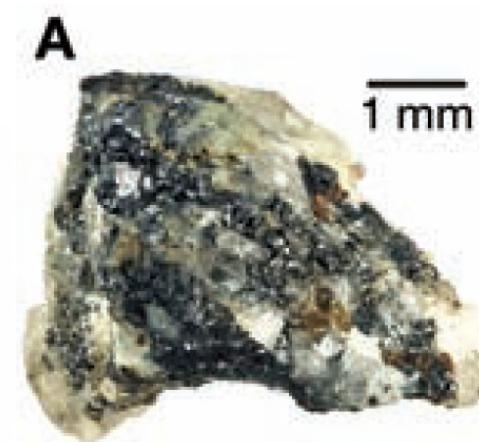
Museo di Storia Naturale, Sezione di Mineralogia, Università  
degli Studi di Firenze, Firenze I-50121, Italy.

REPORTS

## Natural Quasicrystals

Luca Bindi,<sup>1</sup> Paul J. Steinhardt,<sup>2\*</sup> Nan Yao,<sup>3</sup> Peter J. Lu<sup>4</sup>

Quasicrystals are solids whose atomic arrangements have symmetries that are forbidden for periodic crystals, including configurations with fivefold symmetry. All examples identified to date have been synthesized in the laboratory under controlled conditions. Here we present evidence of a naturally occurring icosahedral quasicrystal that includes six distinct fivefold symmetry axes. The mineral, an alloy of aluminum, copper, and iron, occurs as micrometer-sized grains associated with crystalline khatyrkite and cupalite in samples reported to have come from the Koryak Mountains in Russia. The results suggest that quasicrystals can form and remain stable under geologic conditions, although there remain open questions as to how this mineral formed naturally.



khatyrkite-bearing  
sample  
khatyrkite ( $\text{CuAl}_2$ )

## HRTEM

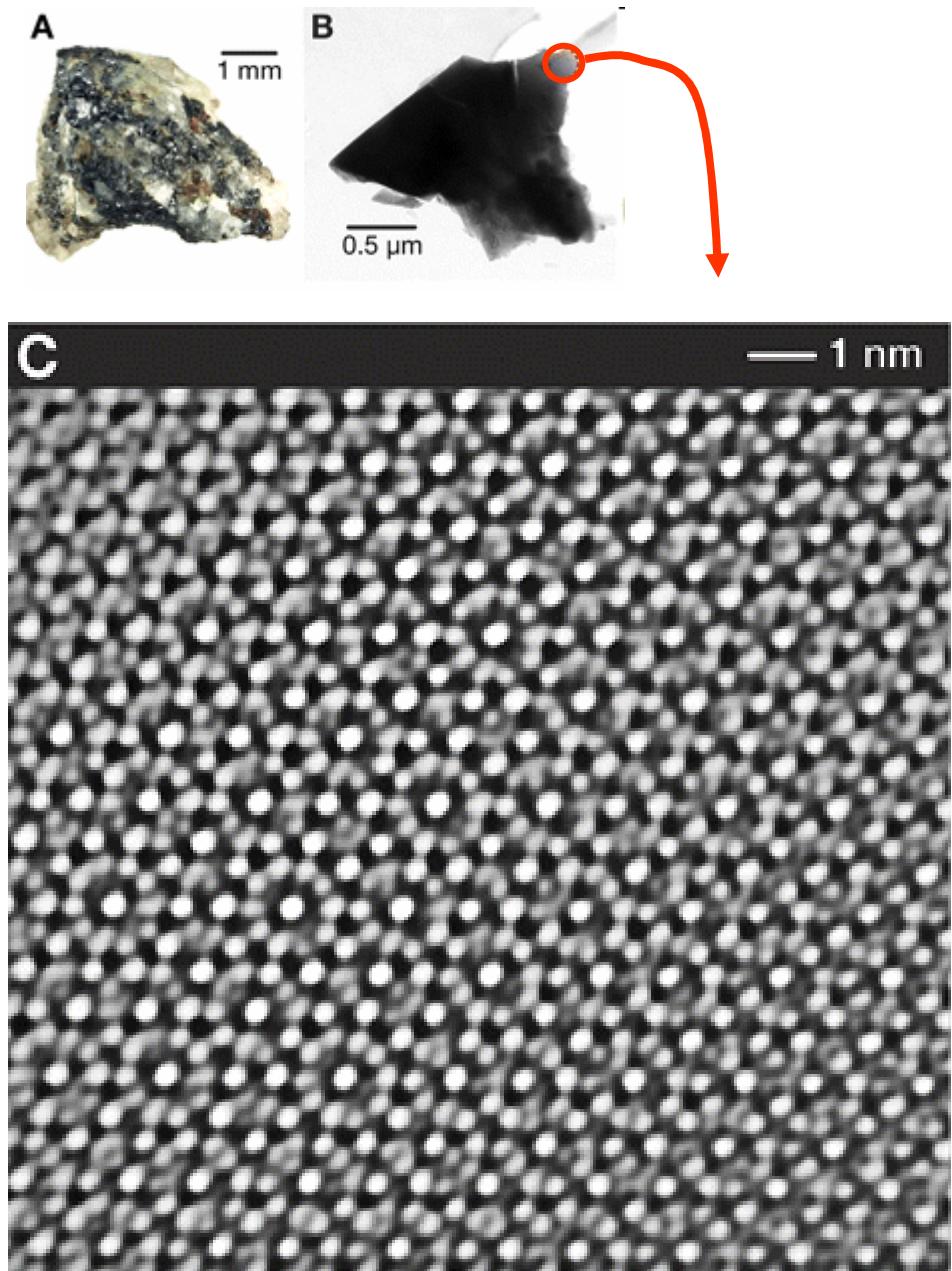


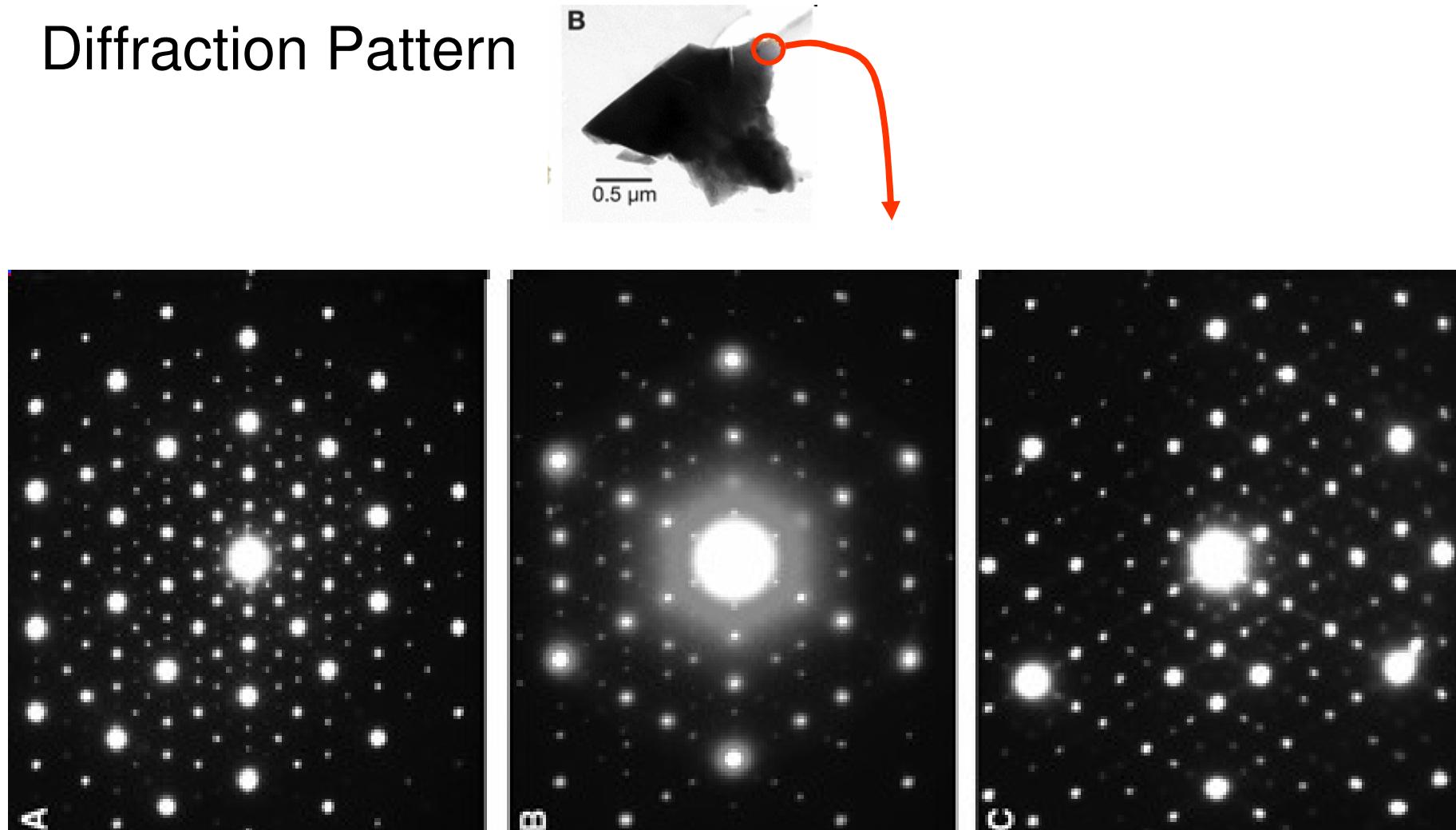
Fig. 1 (A) The original khatyrkite-bearing sample used in the study. The lighter-colored material on the exterior contains a mixture of spinel, augite, and olivine. The dark material consists predominantly of khatyrkite ( $\text{CuAl}_2$ ) and cupalite ( $\text{CuAl}$ ) but also includes granules, like the one in (B), with composition  $\text{Al}_{63}\text{Cu}_{24}\text{Fe}_{13}$ . The diffraction patterns in Fig. 4 were obtained from the thin region of this granule indicated by the red dashed circle, an area 0.1  $\mu\text{m}$  across. (C) The inverted Fourier transform of the HRTEM image taken from a subregion about 15 nm across displays a homogeneous, quasiperiodically ordered, fivefold symmetric, real space pattern characteristic of quasicrystals.

Granulo di



QUASI CRISTALLO

# Diffraction Pattern

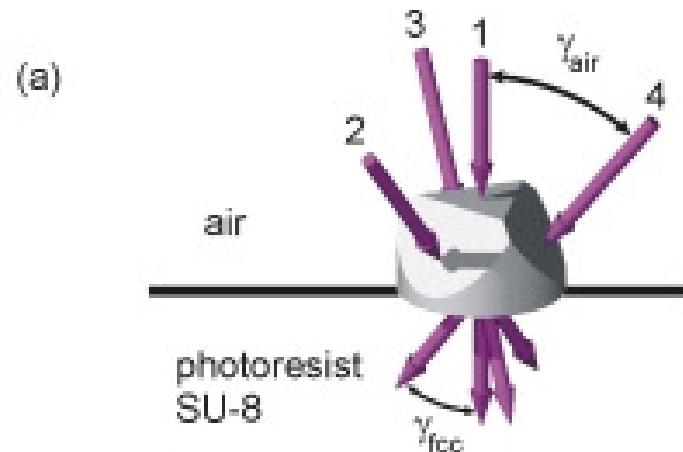


**Fig. 4.** The fivefold (**A**), threefold (**B**), and twofold (**C**) diffraction patterns obtained from a region (red dashed circle) of the granule in [Fig. 1B](#) match those predicted for a FCI quasicrystal, as do the angles that separate the symmetry axes.

# Quasi cristalli fotonici

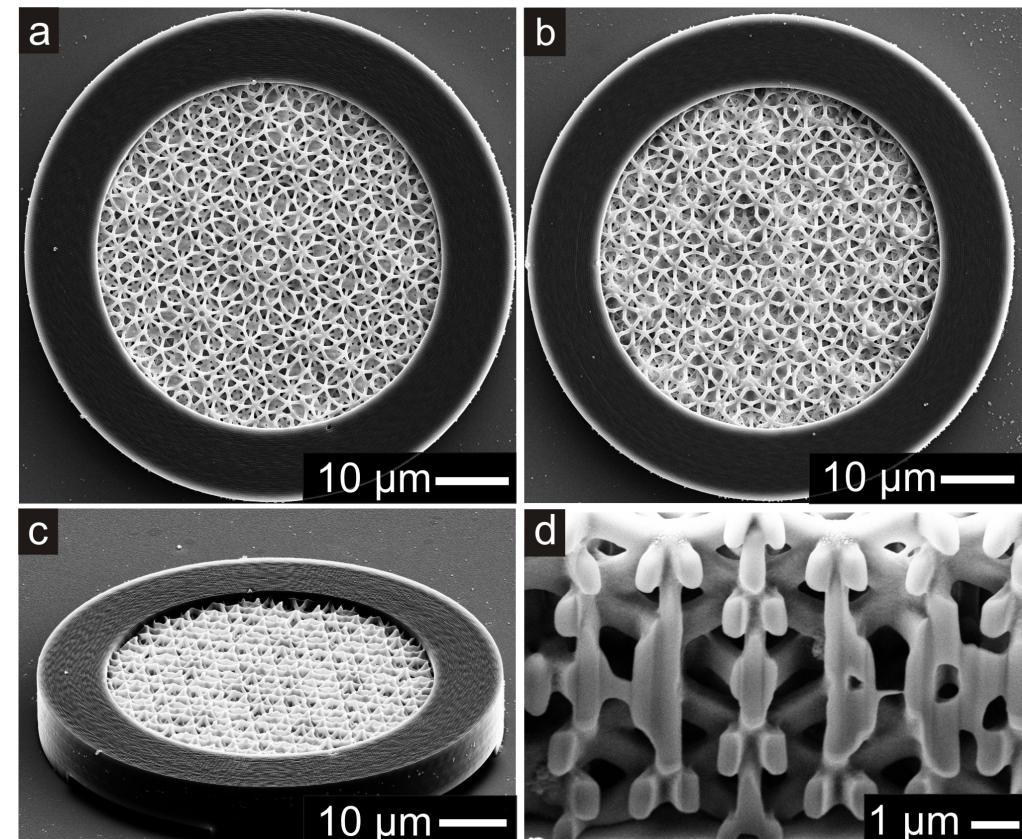
Photonic crystals are based on the fact that photons Bragg-scatter from a medium with a periodically modulated refractive index. Multiple scattering at frequencies near the Bragg condition prevents propagation in these directions, producing a ‘stop gap’. Overlap of the stop gaps in all directions yields a complete photonic bandgap and traps the light. Intuitively, the complete overlap occurs more readily in more isotropic structures. Quasicrystals have long-range quasiperiodic order and higher point group symmetries, so photons Bragg-scatter along a more spherically symmetric set of directions.

# Direct laser writing



Photonic  
QuasiCrystal

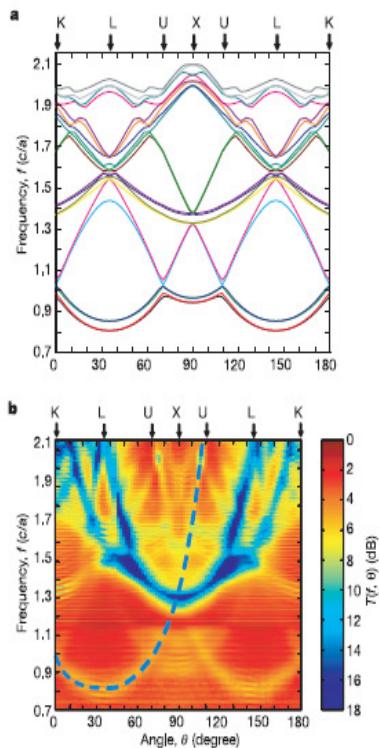
Interference pattern of several  
light beams inside photo resist



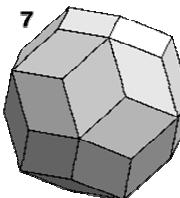
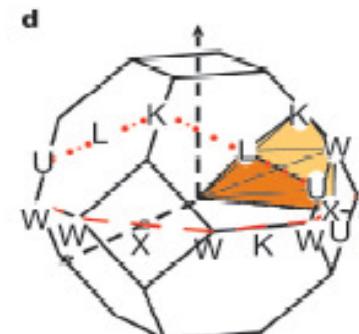
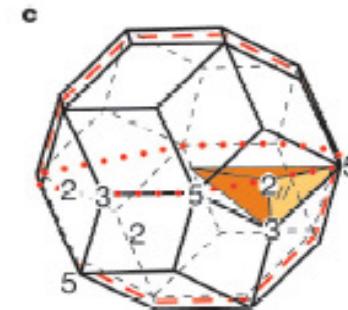
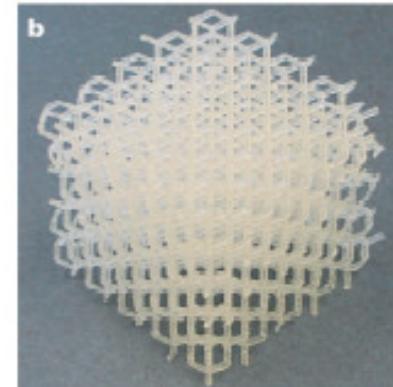
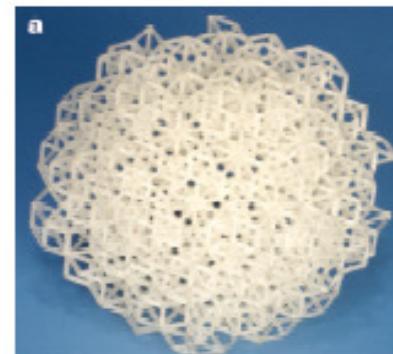
Group Wegener, Univ Karlsruhe

## Experimental measurement of the photonic properties of icosahedral quasicrystals

Q1

Weining Man<sup>1,2</sup>, Mischa Megens<sup>3</sup>, Paul J. Steinhardt<sup>1</sup> & P. M. Chaikin<sup>1,2,4</sup>

**Figure 3 |** Comparison of calculated bands and measured transmission for a diamond structure. **a**, Calculated dispersion relation  $f$  on the boundary of the first Brillouin zone versus  $\theta$ , for the diamond structure along the dotted curve in Fig. 1d. **b**,  $T(f, \theta)$  for the sample rotation along the same curve. There is excellent agreement at the photonic gap centre frequencies.



**Figure 1 | Experimental photonic structures and their Brillouin zones.** **a**, Stereolithographically produced icosahedral quasicrystal with 1-cm-long rods. **b**, Diamond structure with 1-cm-long rods. **c**, Triacontahedron, one of several possible effective Brillouin zones with icosahedral symmetry. **d**, Brillouin zone for the f.c.c./diamond structure.

# Esempio 2D fotonico: Complete BAND GAP

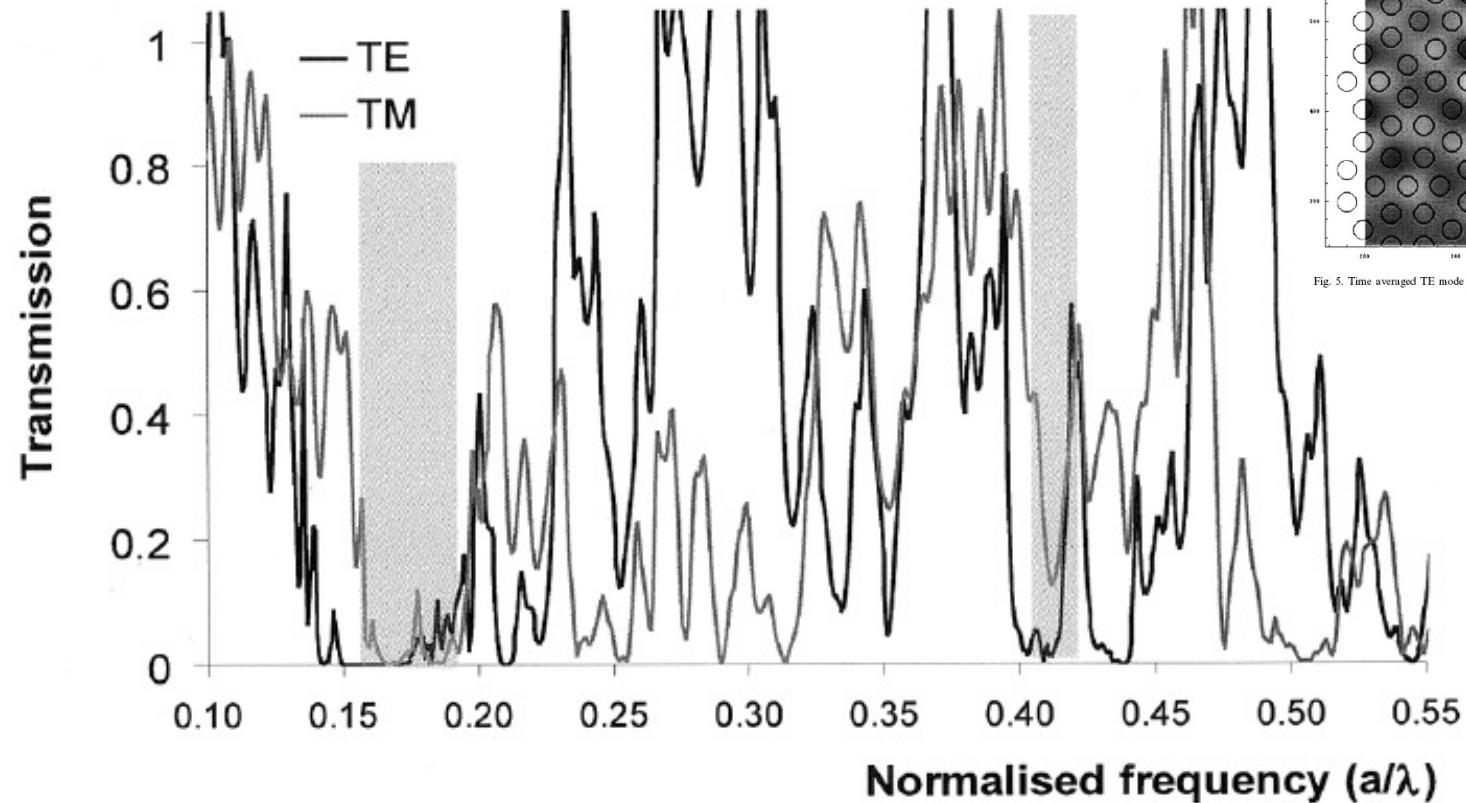


Fig. 4. Superimposed TE and TM transmission plots for the photonic quasicrystal in the  $\Gamma J$  propagation directions.

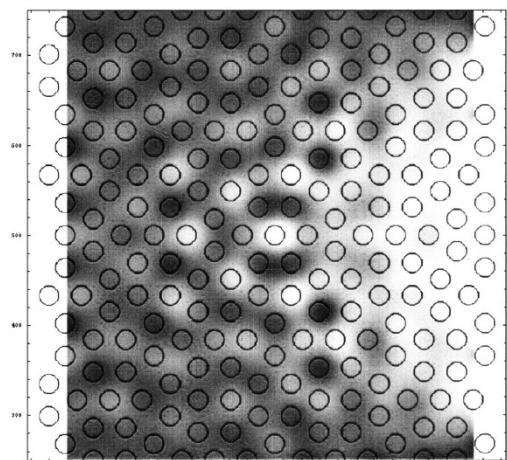
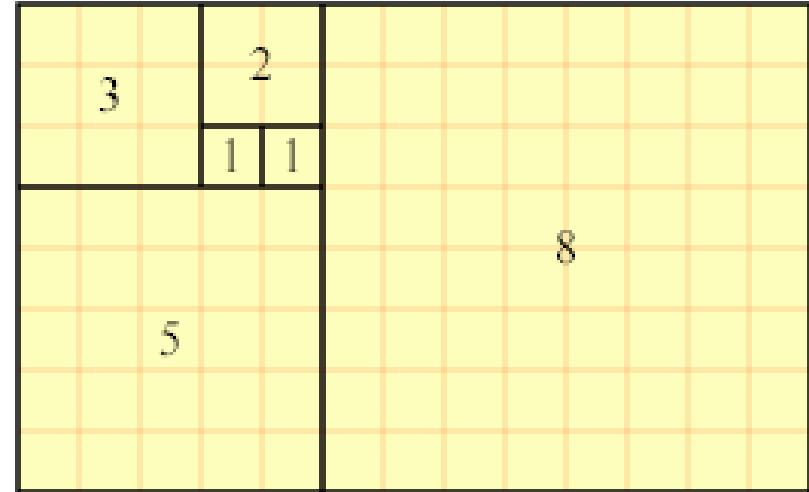


Fig. 5. Time averaged TE mode profile in photonic quasicrystal for  $\lambda = 1100$  nm residing in the PBG.

# **Quasi cristalli fotonici**

## **1D**

# Leonardo da Pisa (Fibonacci)



$$F_0 = 1 \quad F_1 = 1$$

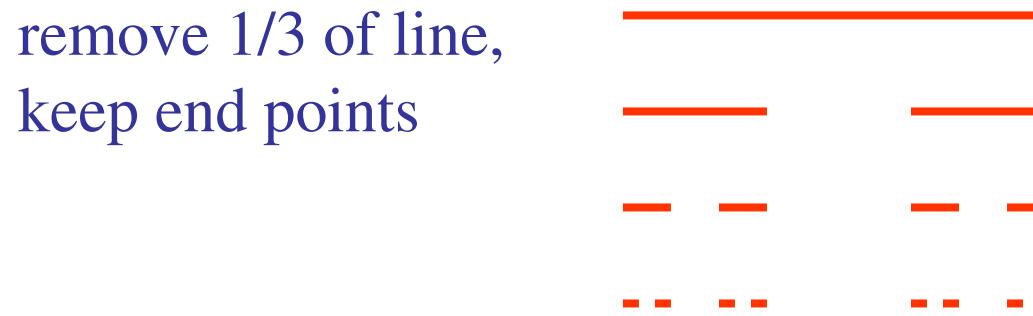
$$F_{n+1} = F_{n-1} + F_n$$

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,.....

$$F_{n+1} / F_n \xrightarrow{n \rightarrow \infty} \varphi = 1.618\cdots \quad \text{Sezione aurea}$$

# Cantor set

- Fibonacci spectrum is a self-similar Cantor set



Total length removed in limit to infinite order?

$$1/3 + 2/3 * 1/3 + 4/9 * 1/3 + \dots = 1/3 \sum_{n=0}^{\infty} (2/3)^n = 1/3 \frac{1}{1 - 2/3} = 1/3 * 3 = 1$$

We have removed 1!

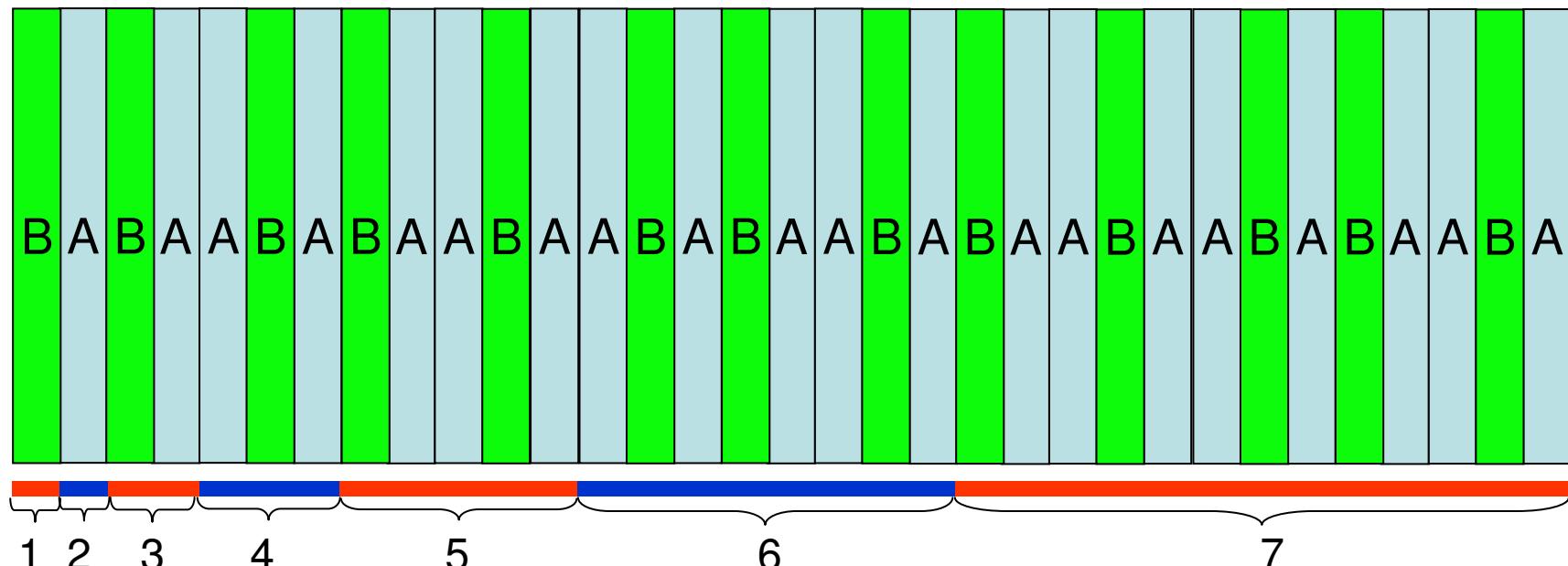
Infinite number of points, yet length zero. Lebesgue measure = 0

# Fibonacci 1D QuasiCrystal

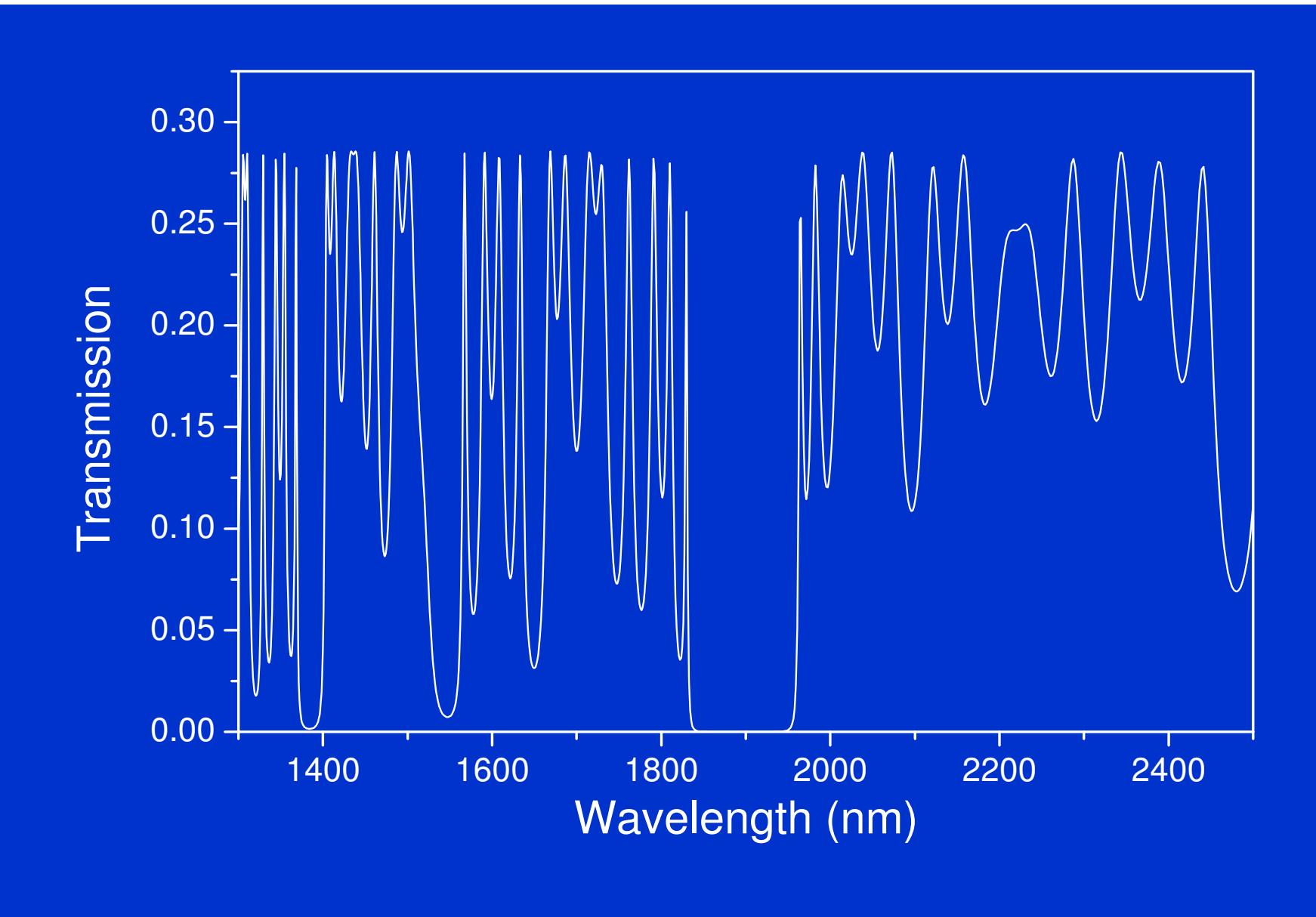
$$F_0 = \{B\} \quad F_1 = \{A\}$$

$$F_{n+1} = \{F_{n-1} F_n\}$$

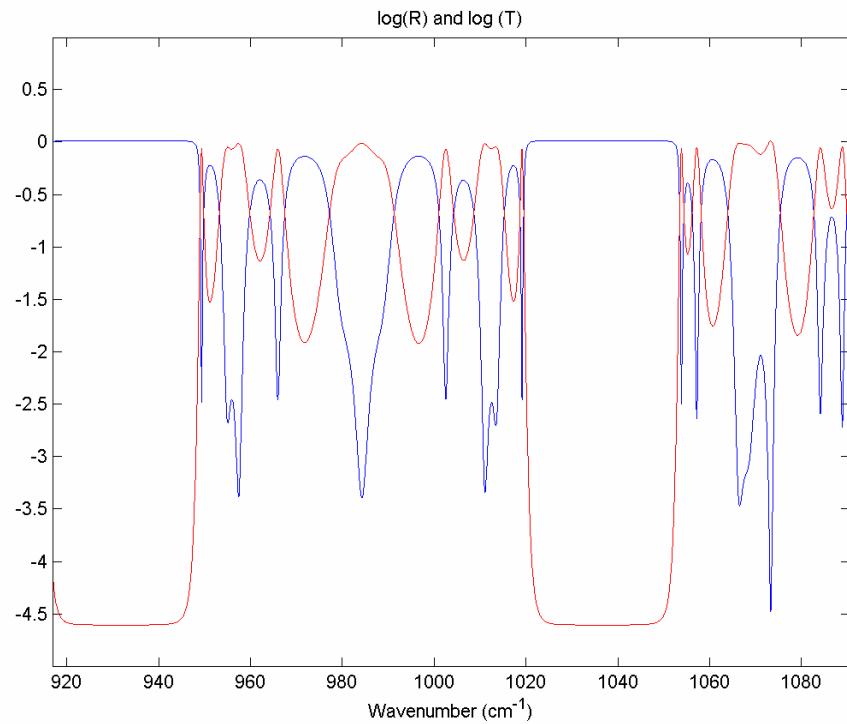
Layer A: 157 nm, 69% porosity, n = 1.6  
Layer B: 105 nm, 47% porosity, n = 2.2



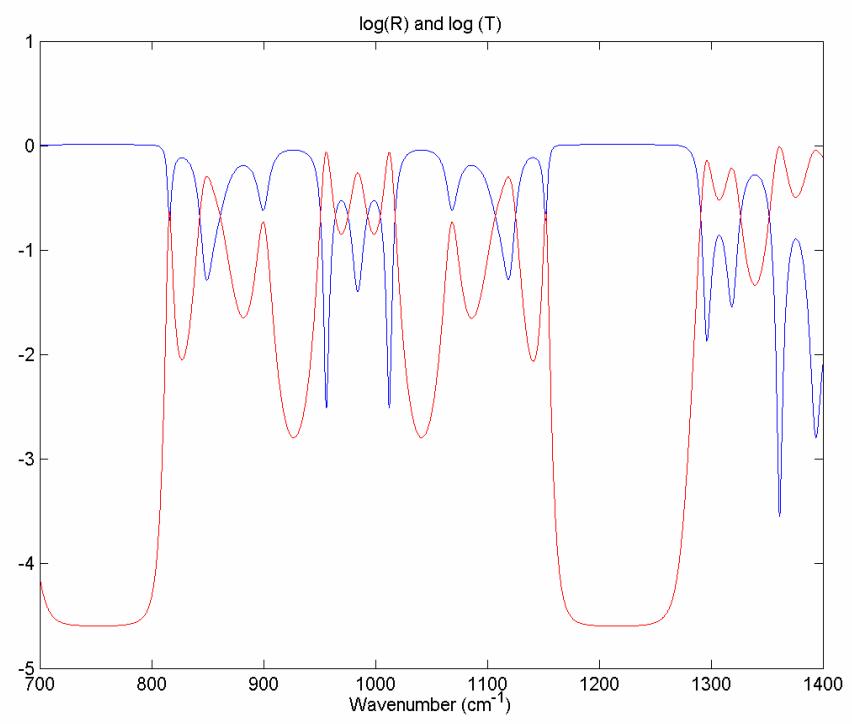
# *Fibonacci band gaps*



# Effetto della finitezza della successione



12th order

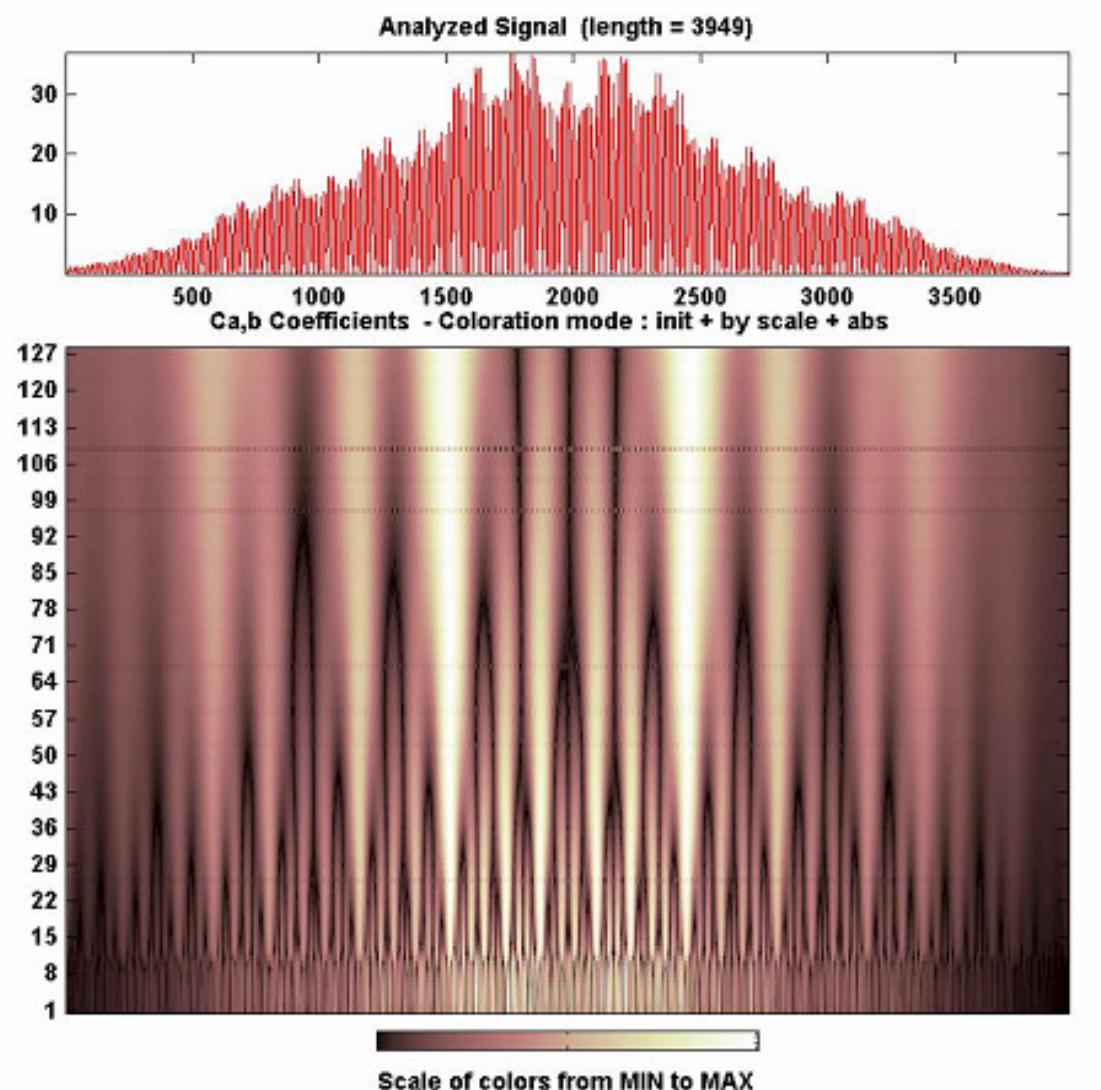


9th order

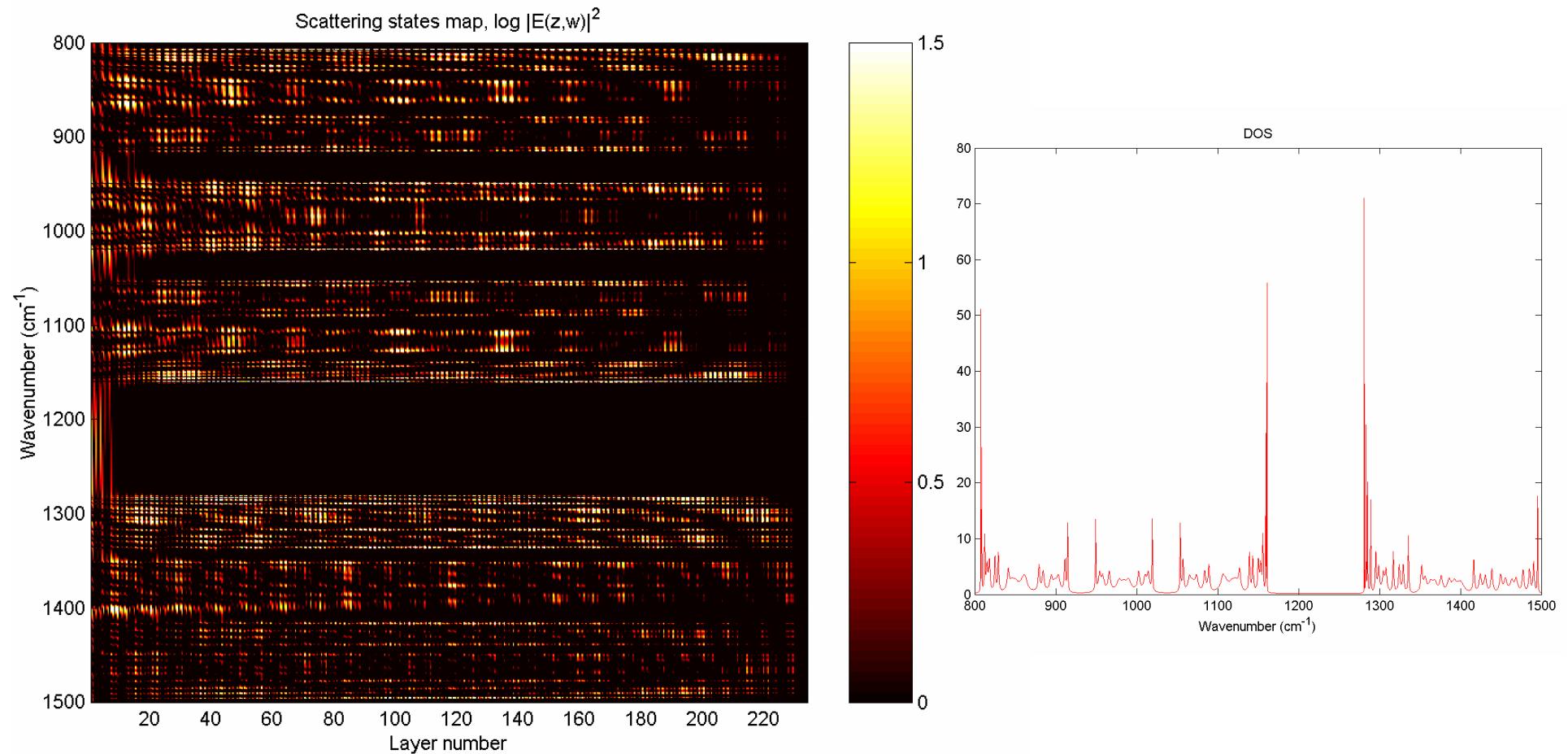
**Self-similarity in spectra**

# Self similar mode structure

Wavelet analysis on 15<sup>th</sup> order Fibonacci



# Fibonacci states map (12th order)



## Light Transport through the Band-Edge States of Fibonacci Quasicrystals

Luca Dal Negro,<sup>1,\*</sup> Claudio J. Oton,<sup>1</sup> Zeno Gaburro,<sup>1</sup> Lorenzo Pavesi,<sup>1</sup> Patrick Johnson,<sup>2</sup> Ad Lagendijk,<sup>2,3</sup> Roberto Righini,<sup>4</sup> Marcello Colocci,<sup>4</sup> and Diederik S. Wiersma<sup>4,†</sup>

<sup>1</sup>*INFM and Department of Physics, University of Trento, I-38050, Povo, Trento, Italy*

<sup>2</sup>*Van der Waals-Zeeman Institute, University of Amsterdam, 1018 XE Amsterdam, The Netherlands*

<sup>3</sup>*Department of Applied Physics & MESA+ Research Institute, University of Twente, Enschede, The Netherlands*

<sup>4</sup>*INFM and European Laboratory for Non-Linear Spectroscopy, 50019 Sesto-Fiorentino (Florence), Italy*

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### Fibonacci ( $N=233$ ) transmission spectrum

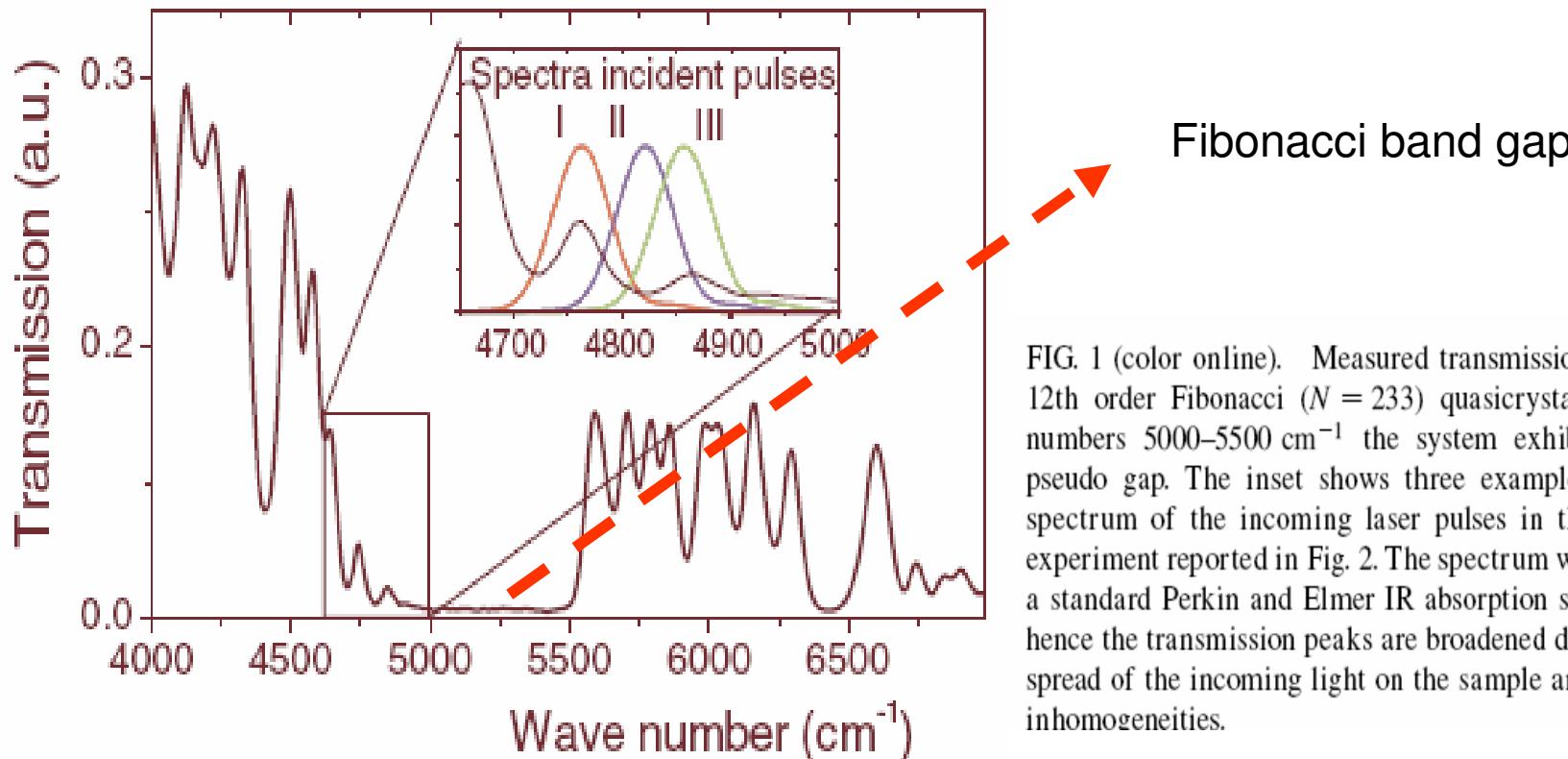


FIG. 1 (color online). Measured transmission spectrum of a 12th order Fibonacci ( $N = 233$ ) quasicrystal. Around wave numbers  $5000\text{--}5500 \text{ cm}^{-1}$  the system exhibits a Fibonacci pseudo gap. The inset shows three examples of the power spectrum of the incoming laser pulses in the time-resolved experiment reported in Fig. 2. The spectrum was recorded with a standard Perkin and Elmer IR absorption spectrometer, and hence the transmission peaks are broadened due to the angular spread of the incoming light on the sample and lateral sample inhomogeneities.

# Propagazione sugli stati di band edge

