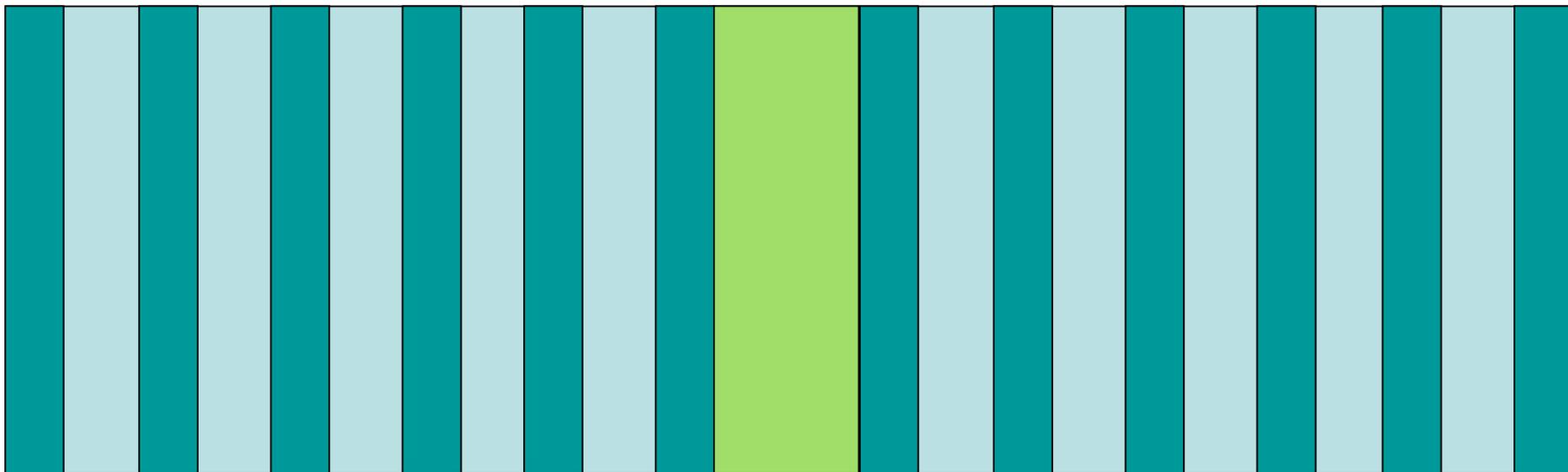


Fotonica 1D

Microcavità

Difetti in PhC

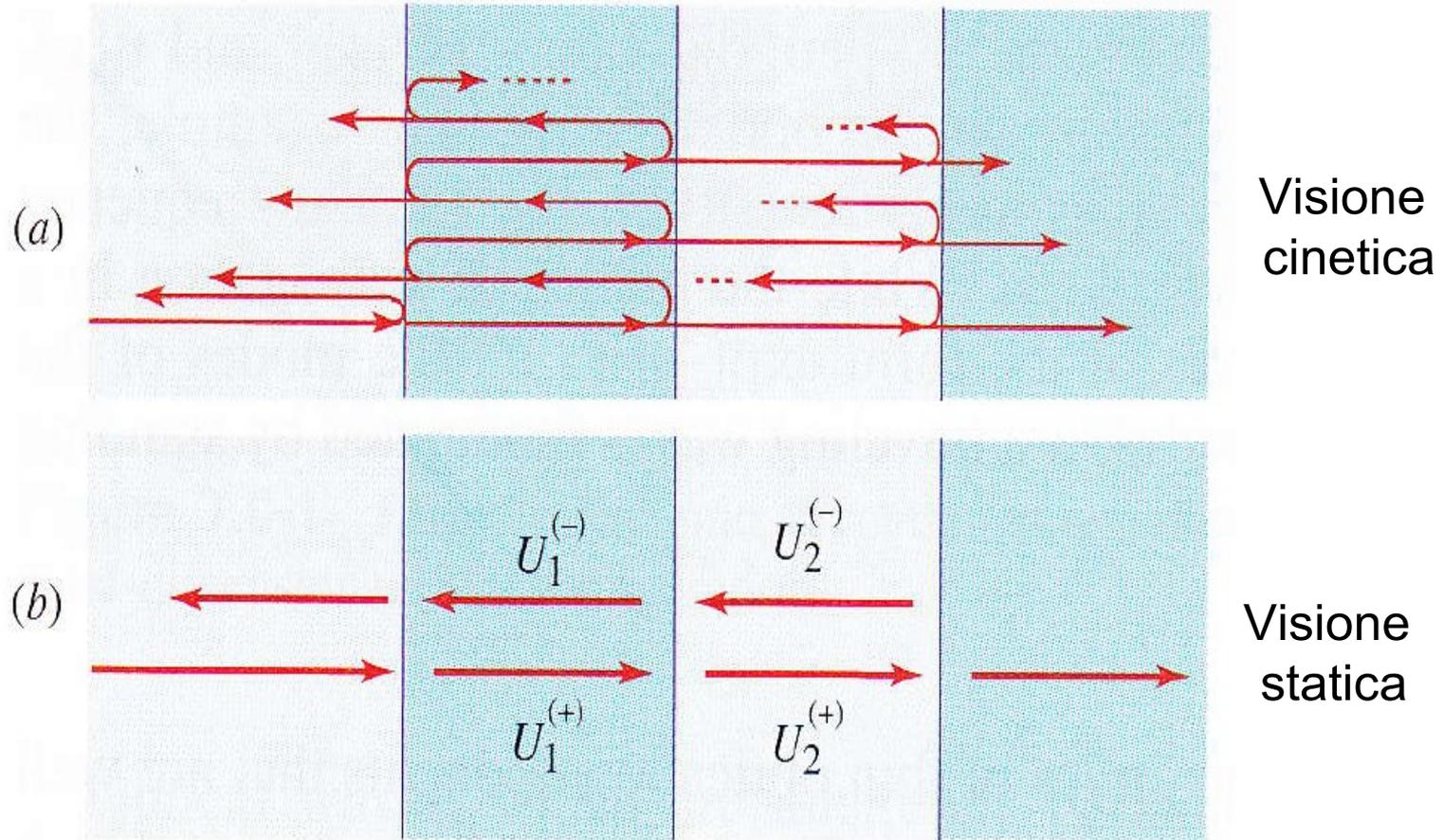
l



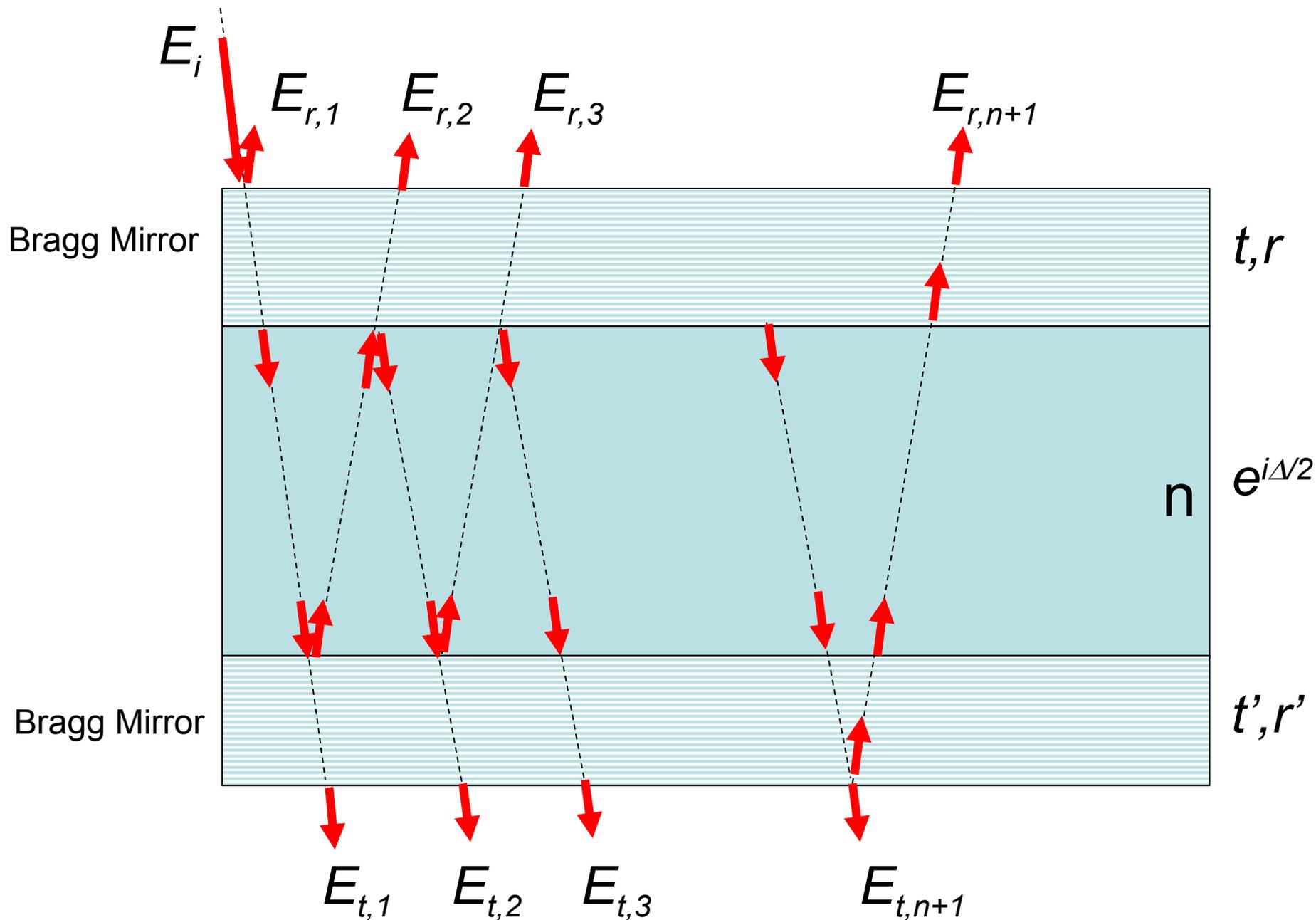
BM

BM

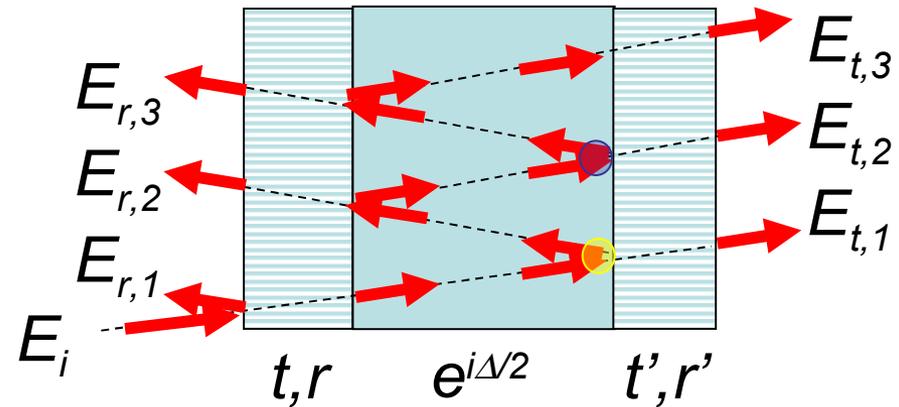
Risoluzione problema



Risoluzione cinetica



Risoluzione cinetica



$$E_{r,1} = rE_i$$

$$E_{t,1} = t' \left(\underline{te^{i\Delta/2} E_i} \right)$$

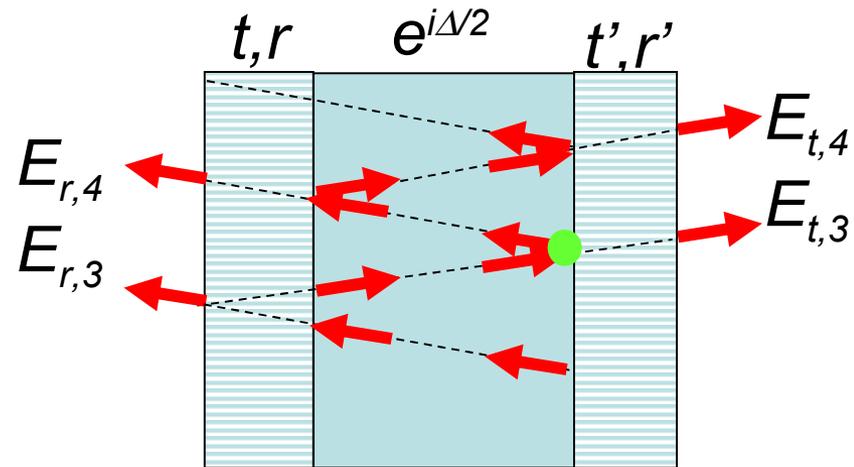
$$E_{r,2} = t \left[e^{i\Delta/2} r' \left(\underline{te^{i\Delta/2} E_i} \right) \right] = \left(t^2 r' e^{i\Delta} E_i \right)$$

$$E_{t,2} = t' \left\{ \underline{e^{i\Delta/2} r \left[e^{i\Delta/2} r' \left(\underline{te^{i\Delta/2} E_i} \right) \right]} \right\} = rr' e^{i\Delta} \left(t' te^{i\Delta/2} E_i \right)$$

$$E_{r,3} = t \left[e^{i\Delta/2} r' \left(\underline{trr' e^{i\Delta} e^{i\Delta/2} E_i} \right) \right] = rr' e^{i\Delta} \left(t^2 r' e^{i\Delta} E_i \right)$$

$$E_{t,3} = t' \left\{ \underline{e^{i\Delta/2} r \left[e^{i\Delta/2} r' \left(\underline{trr' e^{i\Delta} e^{i\Delta/2} E_i} \right) \right]} \right\} = r^2 r'^2 e^{i2\Delta} \left(tt' e^{i\Delta/2} E_i \right)$$

Risoluzione cinetica



$$E_{r,3} = t \left[e^{i\Delta/2} r' \left(trr' e^{i\Delta} e^{i\Delta/2} E_i \right) \right] = rr' e^{i\Delta} \left(t^2 r' e^{i\Delta} E_i \right)$$

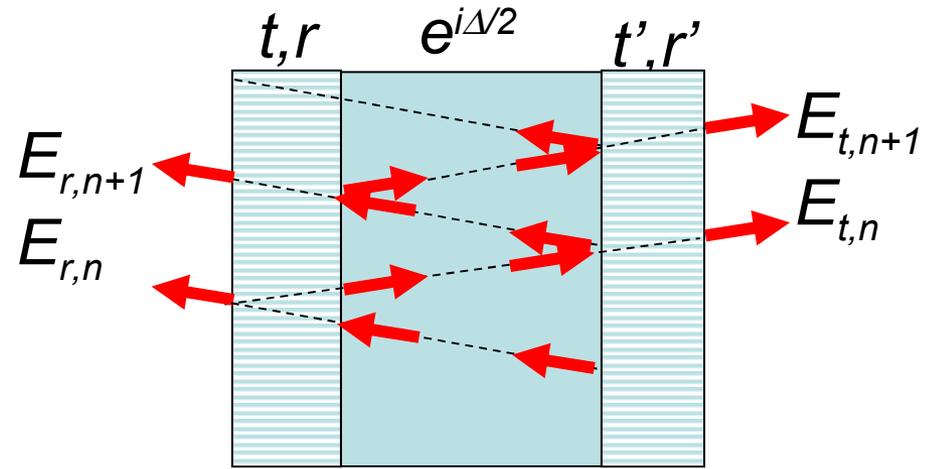
$$E_{t,3} = t' \left\{ e^{i\Delta/2} r \left[e^{i\Delta/2} r' \left(trr' e^{i\Delta} e^{i\Delta/2} E_i \right) \right] \right\} = r^2 r'^2 e^{i2\Delta} \left(tt' e^{i\Delta/2} E_i \right)$$

$$E_{r,4} = t \left[e^{i\Delta/2} r' \left(tr^2 r'^2 e^{i2\Delta} e^{i\Delta/2} E_i \right) \right] = r^2 r'^2 e^{i2\Delta} \left(t^2 r' e^{i\Delta} E_i \right)$$

$$E_{t,4} = t' \left\{ e^{i\Delta/2} r \left[e^{i\Delta/2} r' \left(tr^2 r'^2 e^{i2\Delta} e^{i\Delta/2} E_i \right) \right] \right\} = r^3 r'^3 e^{i3\Delta} \left(t' t e^{i\Delta/2} E_i \right)$$

Risoluzione cinetica

$$\begin{cases} E_{r,1} &= rE_i \\ E_{t,1} &= t'(te^{i\Delta/2} E_i) \end{cases}$$



$$\begin{cases} E_{r,2} &= (t^2 r' e^{i\Delta} E_i) \\ E_{t,2} &= rr' e^{i\Delta} (t' te^{i\Delta/2} E_i) \end{cases}$$

$$\begin{cases} E_{r,3} &= rr' e^{i\Delta} (t^2 r' e^{i\Delta} E_i) \\ E_{t,3} &= r^2 r'^2 e^{i2\Delta} (tt' e^{i\Delta/2} E_i) \end{cases} \quad \begin{cases} E_{r,4} &= r^2 r'^2 e^{i2\Delta} (t^2 r' e^{i\Delta} E_i) \\ E_{t,4} &= r^3 r'^3 e^{i3\Delta} (t' te^{i\Delta/2} E_i) \end{cases}$$

⋮

$$\begin{cases} E_{r,n} &= r^{n-2} r'^{n-2} e^{i(n-2)\Delta} (t^2 r' e^{i\Delta} E_i) & n \neq 1 \\ E_{t,n} &= r^{n-1} r'^{n-1} e^{i(n-1)\Delta} (t' te^{i\Delta/2} E_i) & \forall n \end{cases}$$

Risoluzione cinetica

$$E_{r,n} = r^{n-2} r'^{n-2} e^{i(n-2)\Delta} \left(t^2 r' e^{i\Delta} E_i \right) \quad n \neq 1$$

$$E_{t,n} = r^{n-1} r'^{n-1} e^{i(n-1)\Delta} \left(t' t e^{i\Delta/2} E_i \right) \quad \forall n$$

$$E_t = \sum_{n=1}^{\infty} E_{t,n} =$$

$$= \left(\sum_{n=1}^{\infty} r^{n-1} r'^{n-1} e^{i(n-1)\Delta} \right) \left(t' t e^{i\Delta/2} \right) E_i =$$

$$= \left[\sum_{n=0}^{\infty} r^n r'^n e^{i(n)\Delta} \right] \left(t' t e^{i\Delta/2} \right) E_i = \left[\frac{t' t e^{i\Delta/2}}{1 - r r' e^{i\Delta}} \right] E_i$$

Calcolo Trasmissione

$$t = t' = |t|e^{i\varphi_t} \quad r = r' = |r|e^{i\varphi_r} \quad \text{Specchi identici}$$

$$E_t = \left[\frac{|t|^2 e^{i(2\varphi_t + \Delta/2)}}{1 - |r|^2 e^{i(2\varphi_r + \Delta)}} \right] E_i = t_C E_i$$

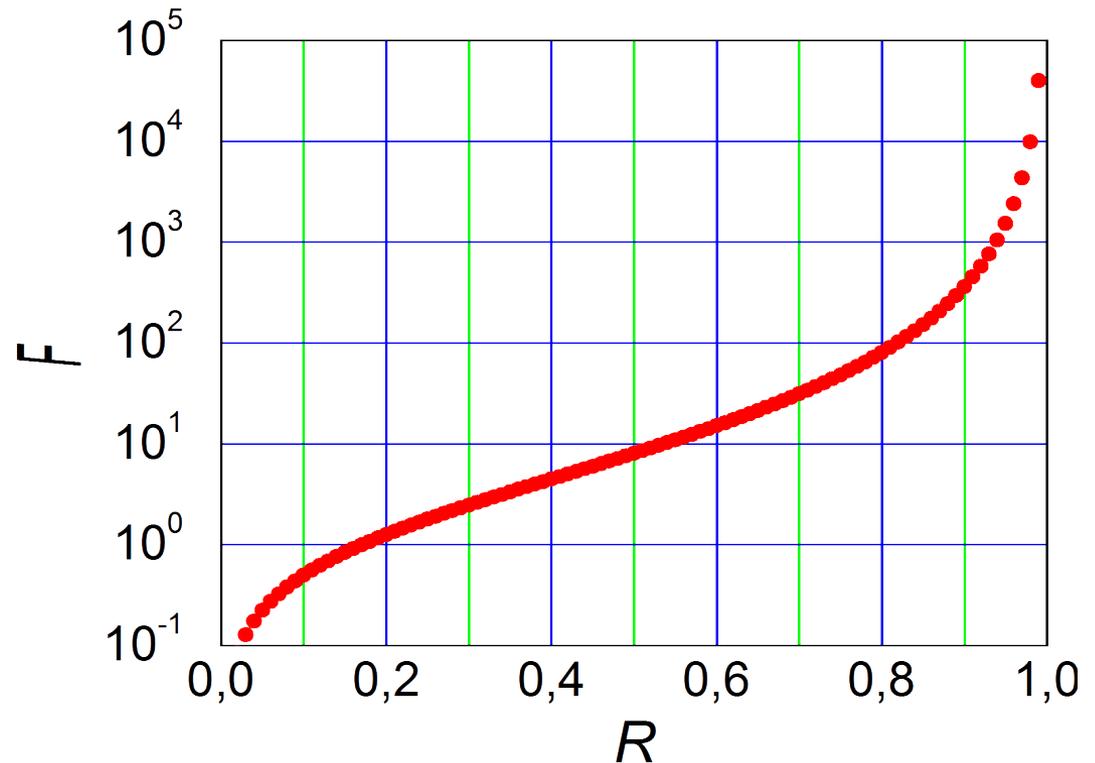
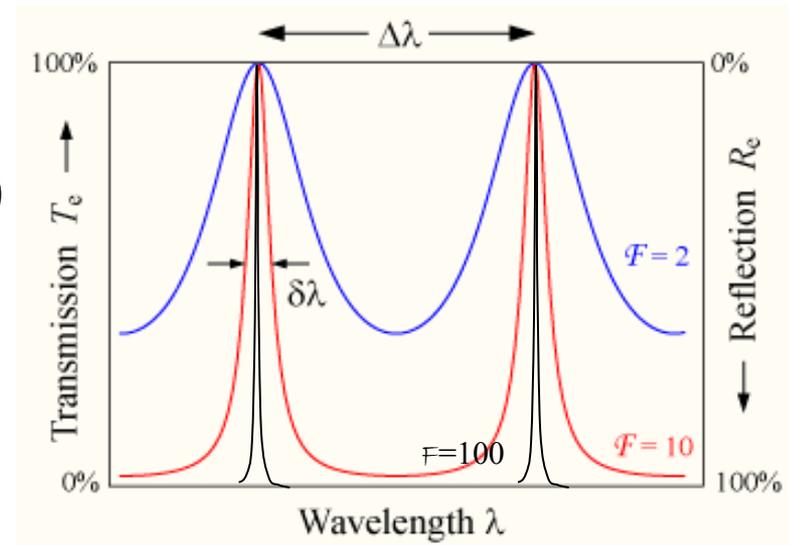
$$\begin{aligned} T_C = |t_C|^2 &= \frac{T^2}{(1 - R \cos(2\varphi_r + \Delta))^2 + R^2 \sin^2(2\varphi_r + \Delta)} = \\ &= \frac{T^2}{1 + R^2 - 2R \cos(2\varphi_r + \Delta)} = \frac{T^2}{1 + R^2 - 2R[1 - 2\sin^2(\varphi_r + \Delta/2)]} = \\ &= \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(\varphi_r + \Delta/2)} = \frac{1}{1 + \frac{4R}{(1 - R)^2} \sin^2(\varphi_r + \Delta/2)} \end{aligned}$$

Formula di Airy

$T_C = 1$ anche se $T = 1 - R \rightarrow 0$

$$T_C = \frac{1}{1 + F \sin^2(\phi_r + \Delta / 2)}$$

$$F = \frac{4R}{(1 - R)^2}$$

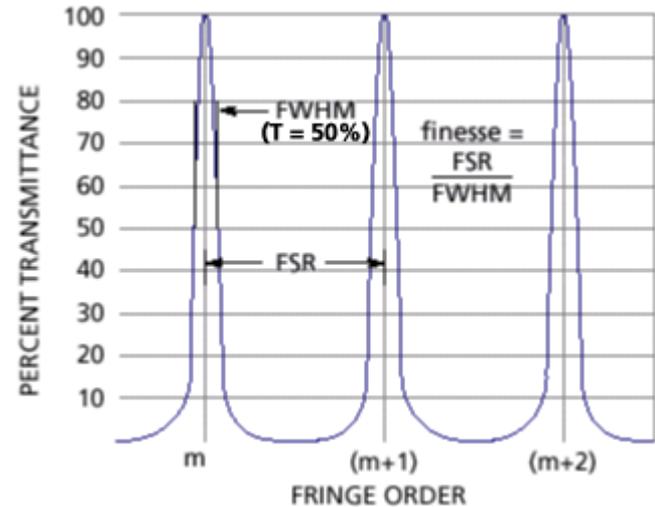


$$T_C = \frac{1}{1 + F \sin^2(\varphi_r + \Delta / 2)}$$

$$\frac{\Delta}{2} = \frac{2\pi n}{\lambda} \ell = \frac{\omega}{c} n\ell$$

Posizione picco $(\varphi_r + \frac{\omega_m}{c} n\ell) = m\pi$

HWHM picco $F \sin^2(\varphi_r + \frac{\omega_m + \delta\omega}{c} n\ell) = 1$



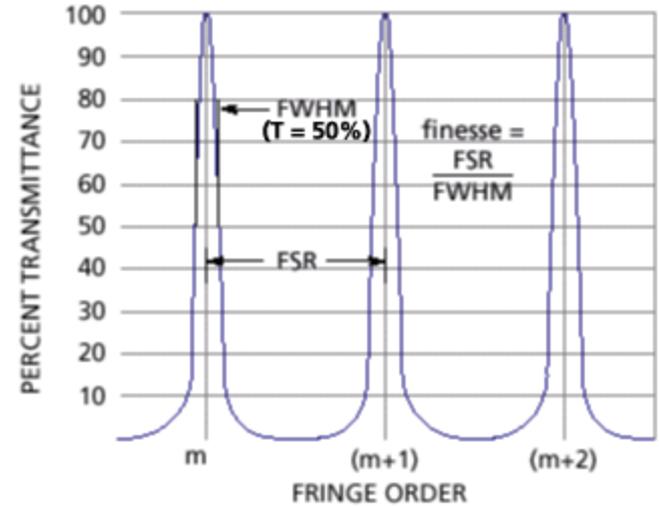
$$F \left(\frac{\delta\omega}{c} n\ell \right)^2 = 1 \Rightarrow \boxed{FWHM = \frac{2c}{n\ell} \frac{1}{\sqrt{F}}}$$

$$T_C = \frac{1}{1 + F \sin^2(\varphi_r + \Delta / 2)}$$

$$(\varphi_r + \Delta_m / 2) = m\pi$$

$$\text{Se } \varphi_r = 0 \Rightarrow n\ell = m \frac{\lambda_m}{2} \quad e \quad \omega_m = \frac{m\pi c}{n\ell}$$

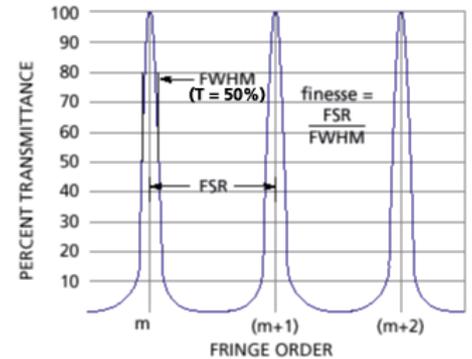
$$\Delta_{m+1} - \Delta_m = \pi$$



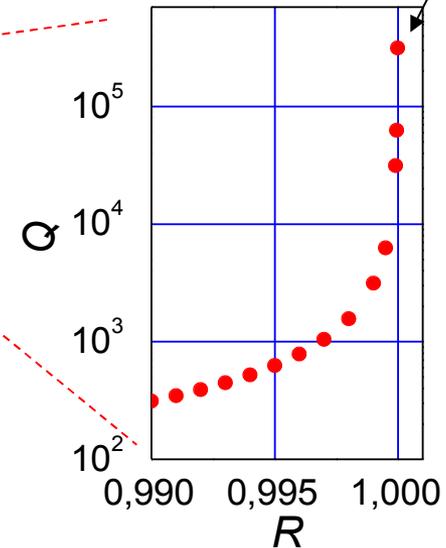
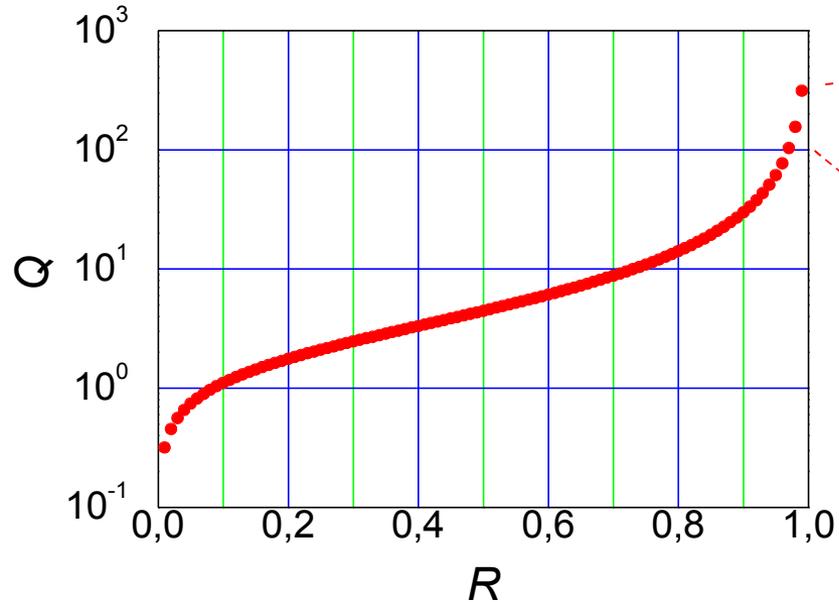
$$\omega_{m+1} - \omega_m = \frac{\pi c}{n\ell} = \text{Free Spectral Range}$$

$$Finesse = \frac{FSR}{FWHM} = \frac{\pi \sqrt{F}}{2}$$

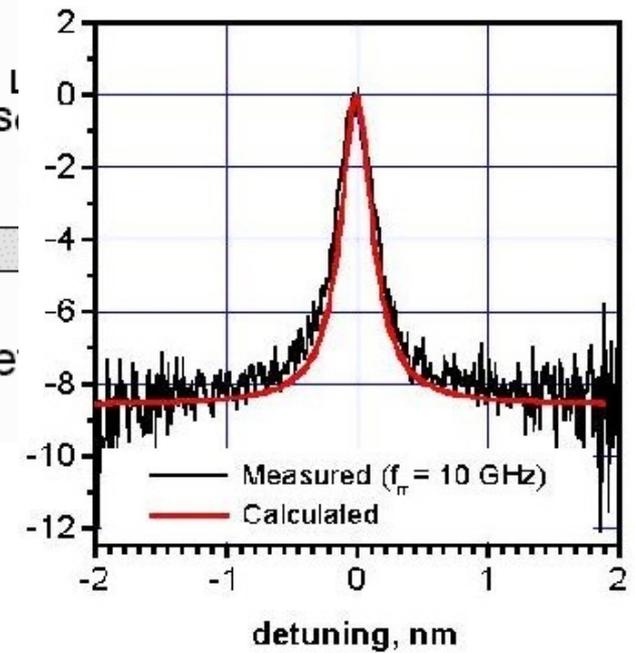
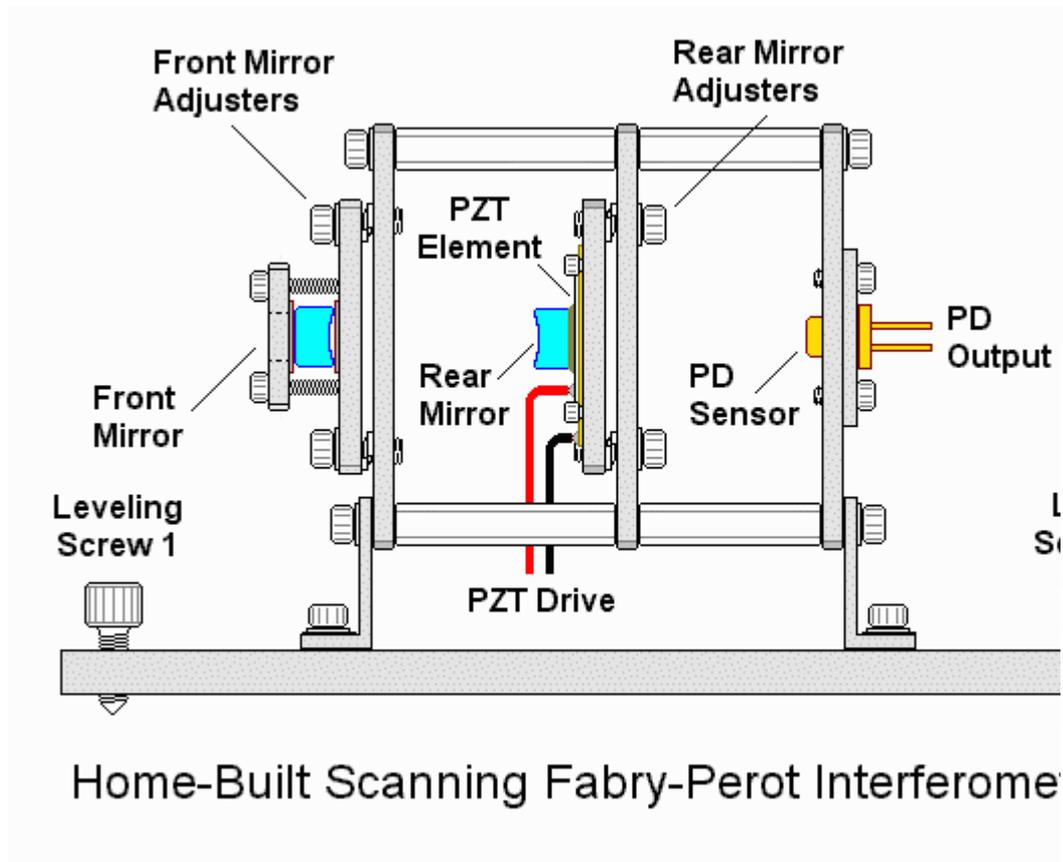
$$Quality\ factor\ Q_m = \frac{\omega_m}{FWHM} = \frac{m\pi \sqrt{F}}{2}$$



R=0.99999



Interferometro Fabry-Perot



Microcavità $\ell = \frac{\lambda}{2n}$ con λ nel gap

$$T_C = \frac{1}{1 + F \sin^2(\varphi_r + \Delta / 2)}$$

$$(\varphi_r + \Delta_m / 2) = m\pi$$

$$\text{Se } \varphi_r = 0 \Rightarrow \lambda_m = \frac{2n\ell}{m}$$

$$\lambda_1 = 2n\ell = 800nm$$

$$\lambda_2 = n\ell = 400nm$$



Free Spectral Range = 400nm \Rightarrow

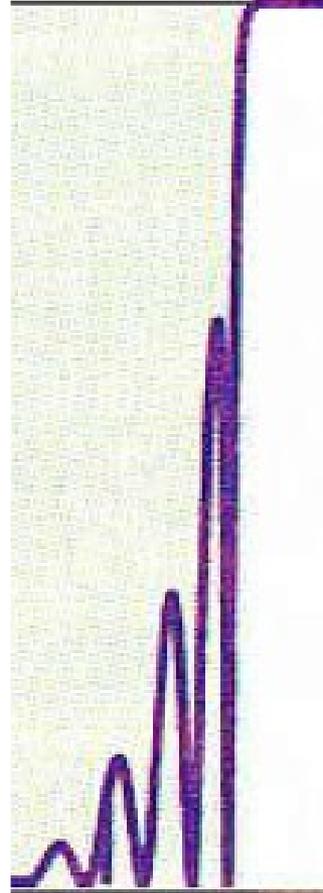
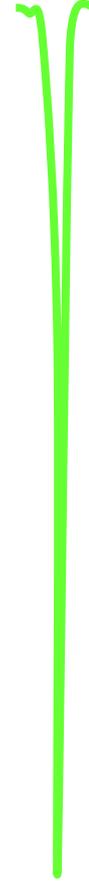
Singolo modo

Microcavità $l = \frac{\lambda}{2}$ con λ nel gap

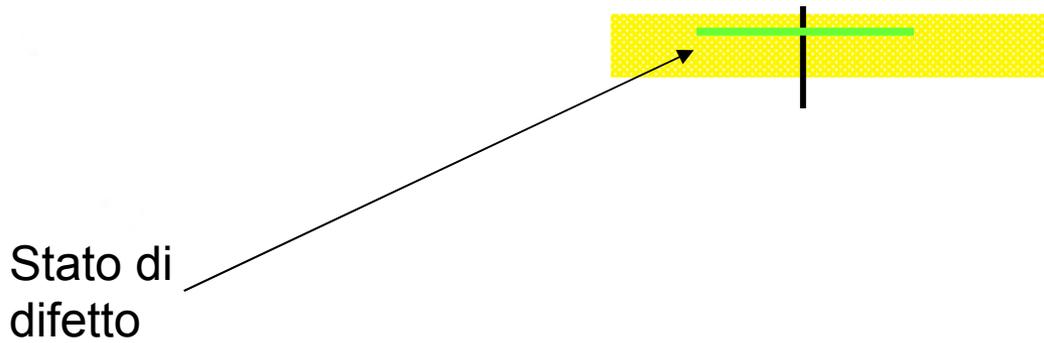
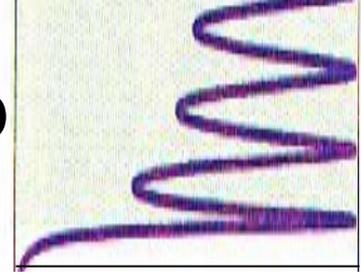
Profondo dip nella stop band

1

2

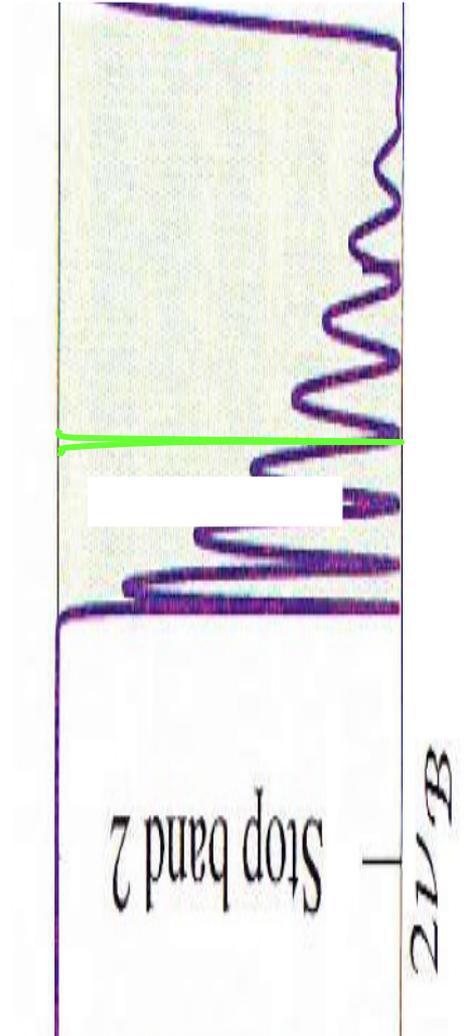


Microcavità $l = \frac{\lambda}{2}$ con λ nel gap



Stato di difetto

Aumentare Q: difetto al centro di un largo band gap

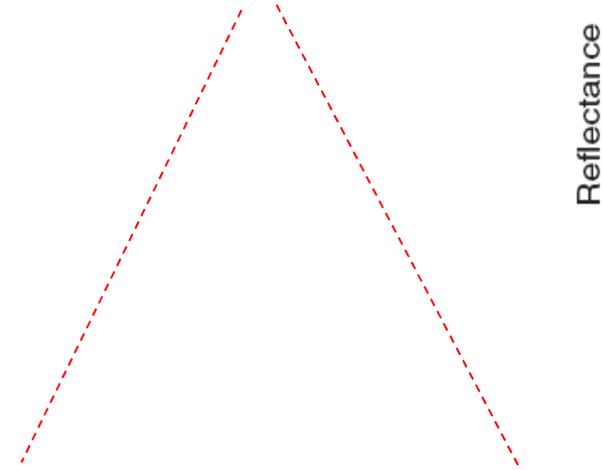
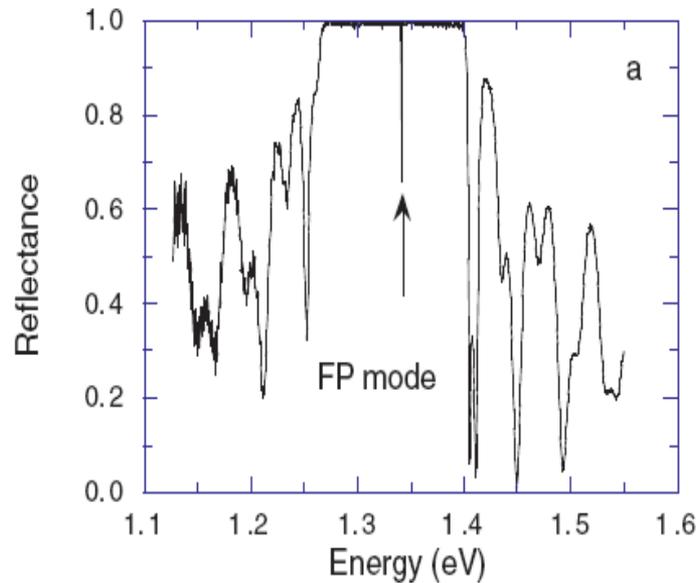
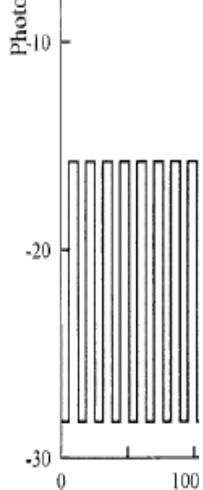
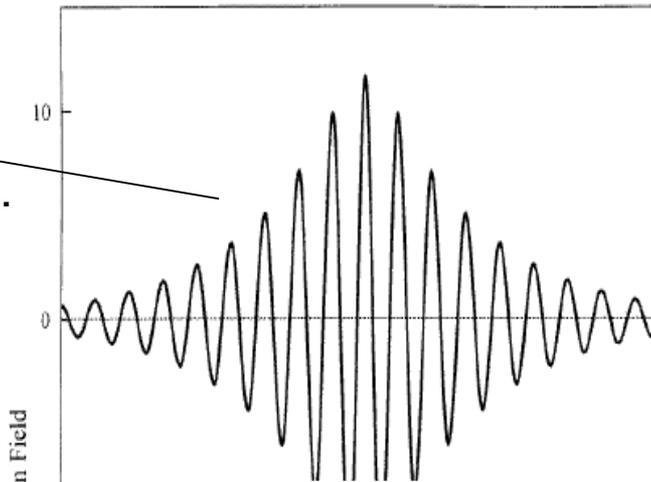


Microcavità $l = \frac{\lambda}{2}$ con λ nel gap

Dati Exp.

Distribuzione modo

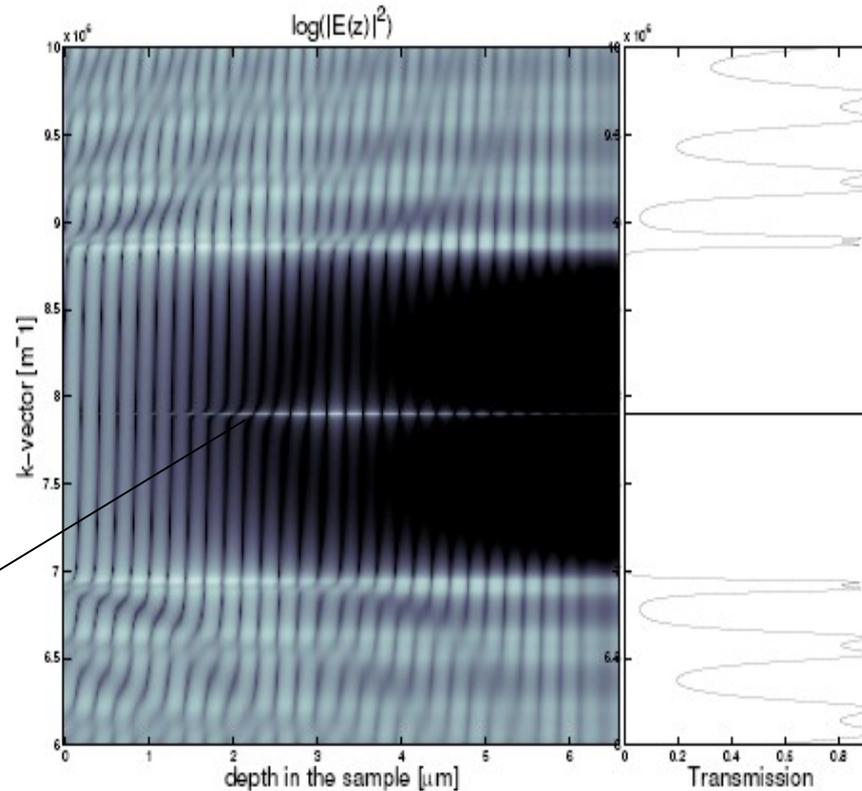
Modo evanesc.



Scattering state map:

Scattering state map:

Electric field distribution of the waves along their propagation direction at different frequencies



Difetto al centro
del gap
massimizza Q.

Figure 6.10: (left panel) Scattering state map of a single microcavity made of an extra A layer, surrounded by two Bragg mirrors made of 30 periods AB . Darker areas have a lower intensity. In the middle of the bandgap, the single microcavity is visible. (right panel) The transmissivity of the structure. The two refractive indices are $A = 1.58$ and $B = 2.21$ and the central wavelength is $\lambda = 800$ nm.

Dipendenza angolare

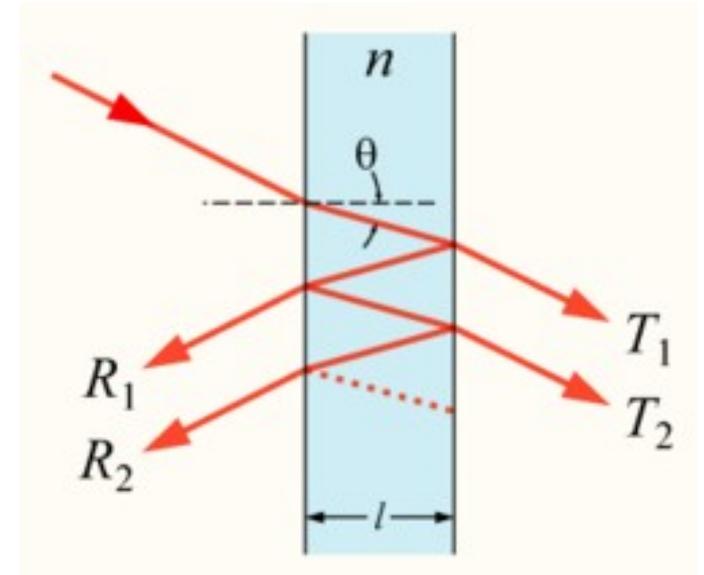
$$T_C(\theta) = \frac{1}{1 + F \sin^2(\varphi_r + \Delta(\theta)/2)}$$

$$\frac{\Delta(\theta)}{2} = k_z \ell = \frac{\omega(\theta)}{c} n \ell \cos \theta$$

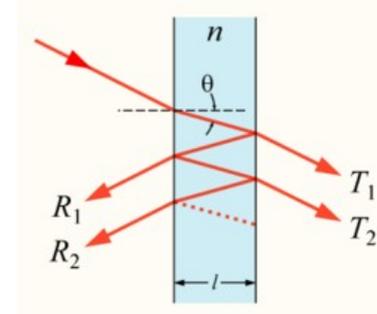
$$\text{Posizione picco} \quad \left(\varphi_r + \frac{\omega_m(\theta)}{c} n \ell \cos \theta \right) = m\pi$$

$$\omega_m(\theta) = \frac{\omega_m(0)}{\cos \theta} \approx \omega_m(0) \left(1 + \frac{\theta^2}{2} \right)$$

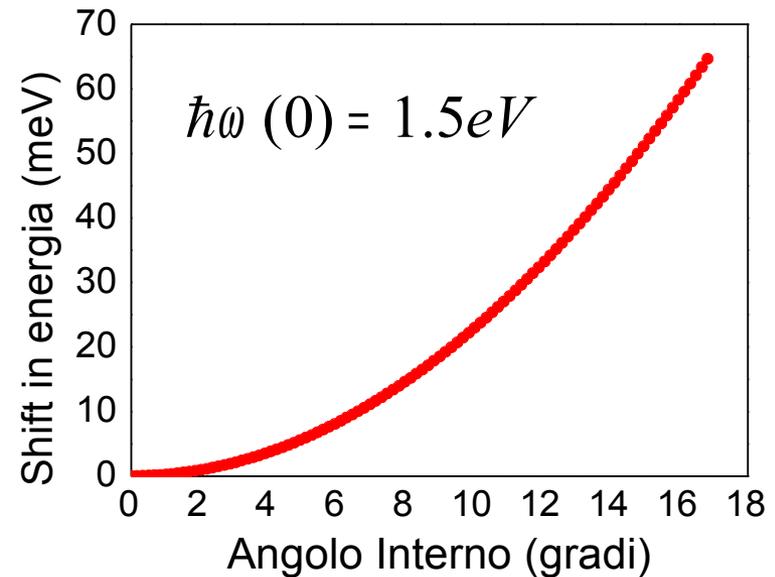
Shift verso il blu



Dipendenza angolare



$$\begin{aligned} \hbar\omega(\theta) &\approx \hbar\omega(0) + \hbar\omega(0) \frac{\theta^2}{2} = \\ &\approx \hbar\omega(0) + \hbar\omega(0) \frac{\sin^2 \theta}{2} = \\ &= \hbar\omega(0) + \hbar \frac{ck \sin^2 \theta}{n} = \\ &= \hbar\omega(0) + \frac{\hbar ck_{//}^2}{2nk} \equiv \hbar\omega(0) + \frac{\hbar^2 k_{//}^2}{2m_{ph}} \end{aligned}$$



$$m_{ph} = \frac{nk\hbar}{c} = \frac{n^2}{c^2} \hbar\omega$$

$$\frac{m_{ph}}{m_{el}} \approx \frac{1.5 eV}{0.5 MeV} = 3 \cdot 10^{-6}$$

Struttura a bande

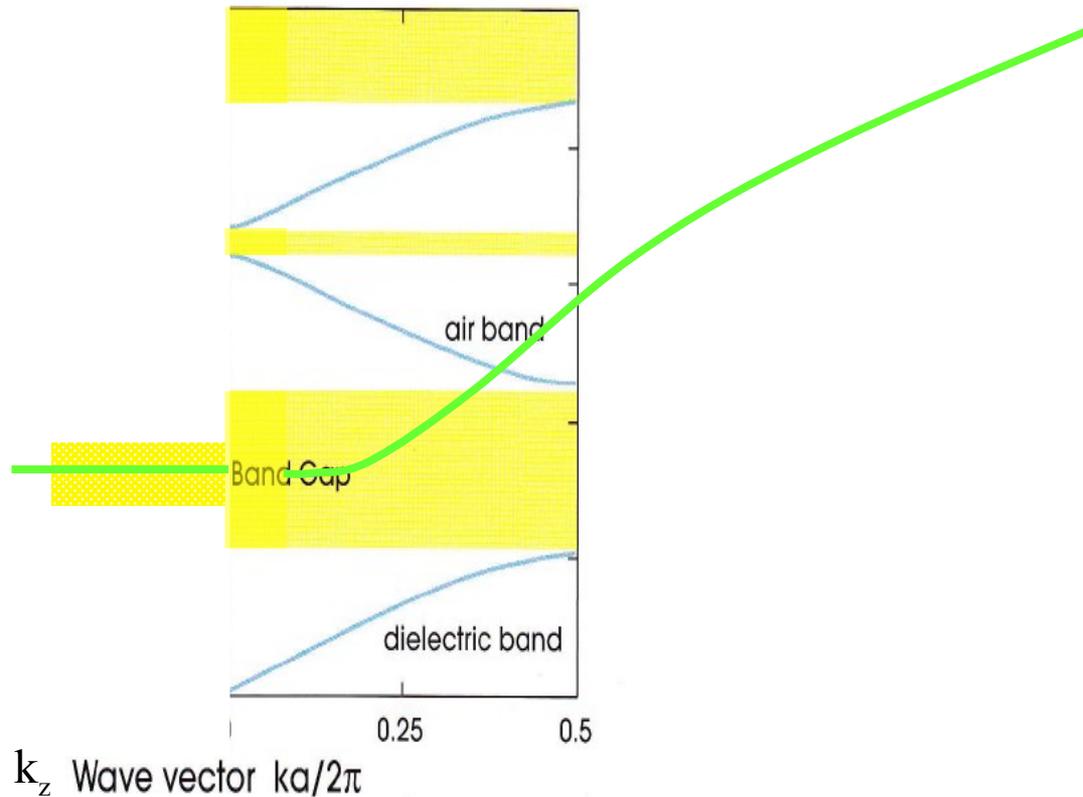
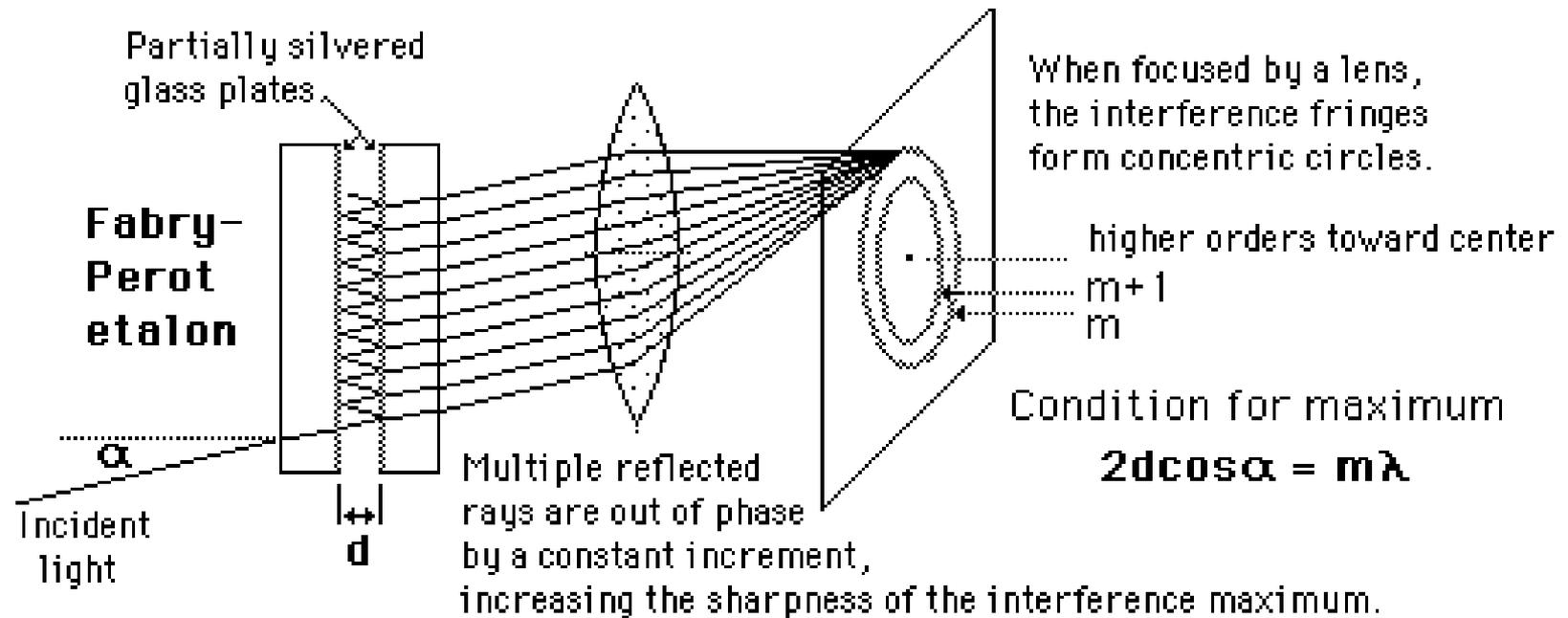
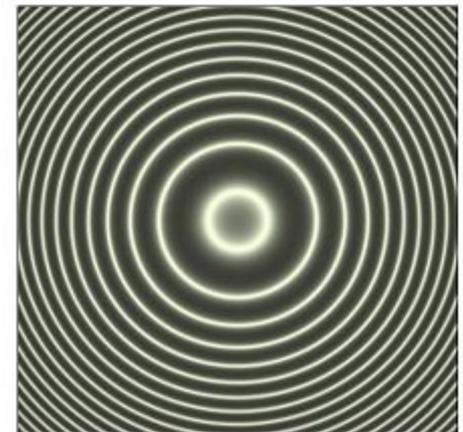


Figure 10: Two superimposed x -polarized band structures of a multilayer film, showing how the bandwidths vary with k_y . The blue lines refer to bands along $(0, k_y, 0)$, while the red lines beside them refer to the same bands along $(0, k_y, \pi/a)$. The regions in between are shaded gray to indicate where the continuum of bands for intermediate k_z would lie. Only modes with electric field oriented along the x direction are shown. The straight red line is the **light line** $\omega = ck$. The layered material is the same as the one described in the caption.

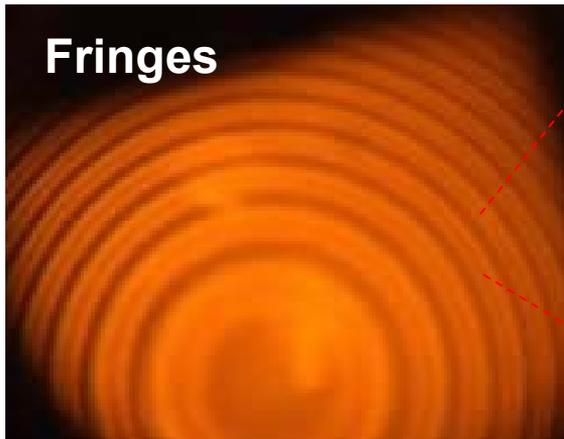
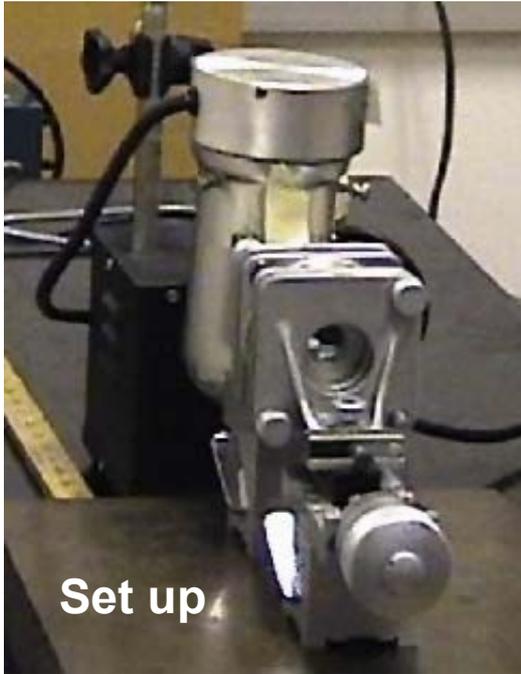
Misura di lambda dalle frange di interferenza ($d \gg \lambda$)



Franghe di interferenza
per luce monocromatica



Esempio di spettroscopia atomica



zoom



Sodium doublet

