

Fotonica 1D

Struttura a bande

Riprendiamo

Sistema con N periodi

$$R_N = \frac{R\Psi_N^2}{(1-R) + \Psi_N^2 R}$$

$$\Psi_N = \frac{\sin N\Phi}{\sin \Phi} \quad \cos \Phi = \operatorname{Re} \left\{ \frac{1}{t} \right\}$$

$$\operatorname{Re} \left\{ \frac{1}{t} \right\} = \operatorname{Re} \left\{ \frac{e^{-i\vartheta}}{|t|} \right\} = \frac{\cos \vartheta}{|t|}$$

$$-\frac{1}{|t|} \leq \operatorname{Re} \left\{ \frac{1}{t} \right\} \leq \frac{1}{|t|} \quad \frac{1}{|t|} \geq 1$$

$$|\cos \Phi| > 1$$



Stop band

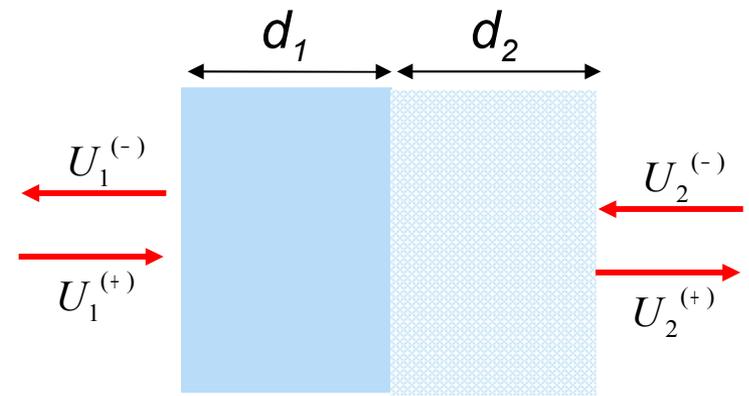
99.99% riflessione

$$|\cos \Phi| \leq 1$$



Leaky modes

Trasmissione dell'elemento singolo



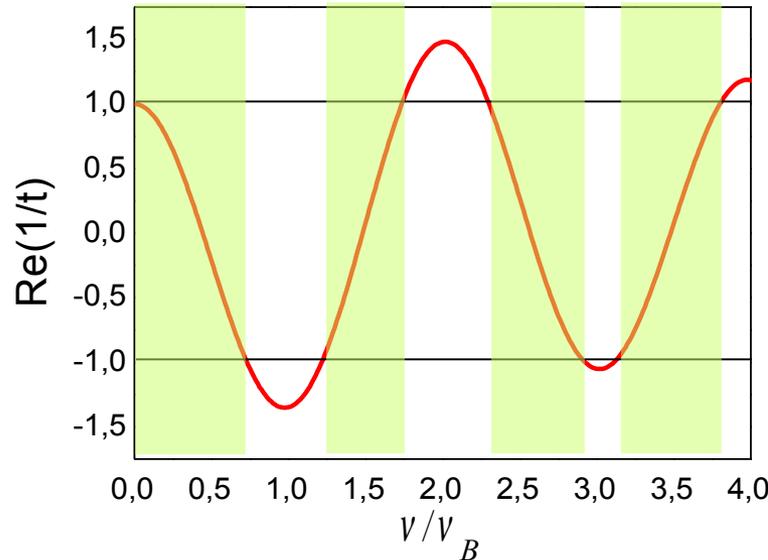
$$\frac{1}{t^*} = \frac{\exp(j\varphi_1)}{4n_2n_1} \left[(n_2 + n_1)^2 \exp(j\varphi_2) - (n_2 - n_1)^2 \exp(-j\varphi_2) \right]$$

$$\operatorname{Re}\left\{\frac{1}{t}\right\} = \left[\frac{(n_2 + n_1)^2}{4n_2n_1} \cos(\varphi_2 + \varphi_1) - \frac{(n_2 - n_1)^2}{4n_2n_1} \cos(\varphi_2 - \varphi_1) \right]$$

$$\varphi_i = \frac{2\pi\nu}{c} n_i d_i$$

$$V_B = \frac{c}{2(n_1 d_1 + n_2 d_2)}$$

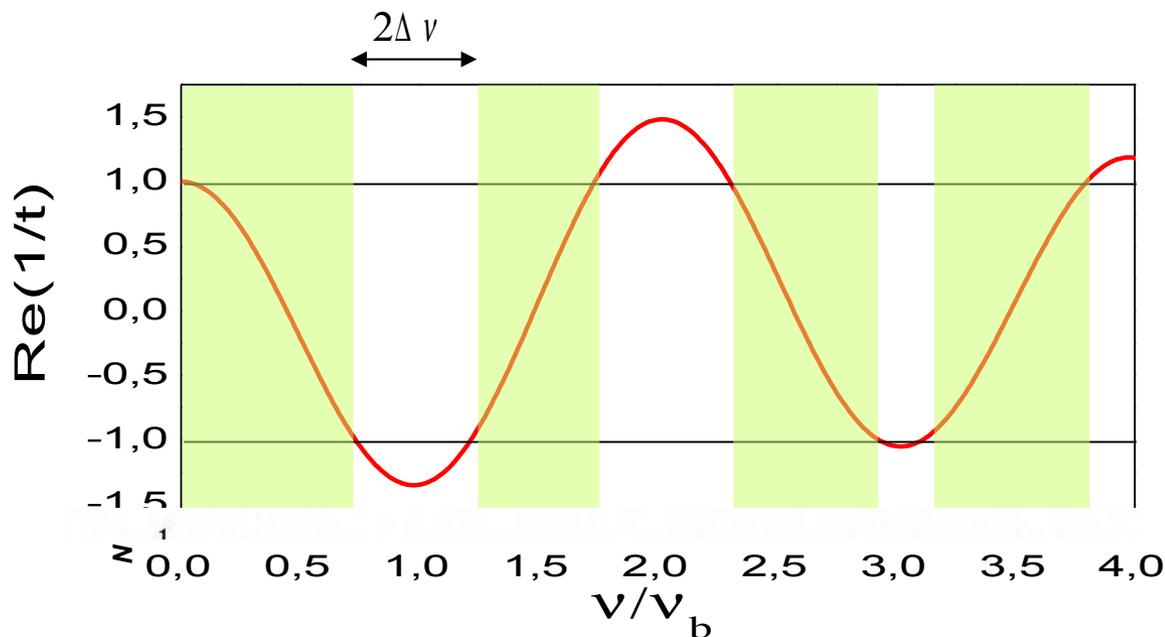
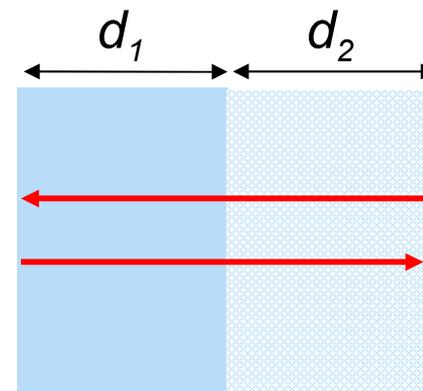
$$\varphi_1 + \varphi_2 = \frac{\nu}{V_B} \pi$$



$$\begin{aligned} d_1 &= d_2 \\ n_1 &= 1.5 \\ n_2 &= 3.5 \end{aligned}$$

Trasmissione dell'elemento singolo

$$\Delta \varphi_{round\ trip} = 2(\varphi_1 + \varphi_2)$$



$$d_1 = d_2$$

$$n_1 = 1.5$$

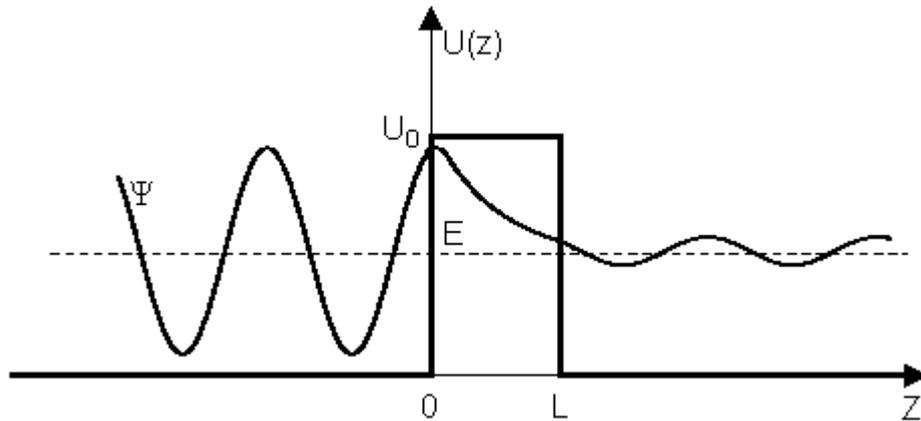
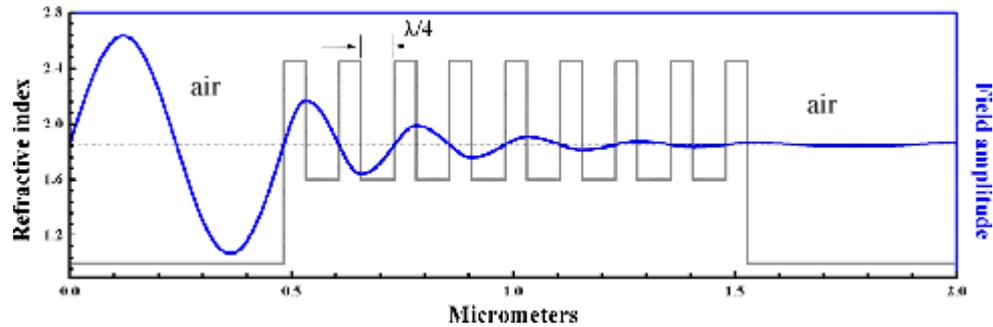
$$n_2 = 3.5$$

$$N = 10$$

$$@v_B \Delta \varphi_{round\ trip} = 2\pi$$

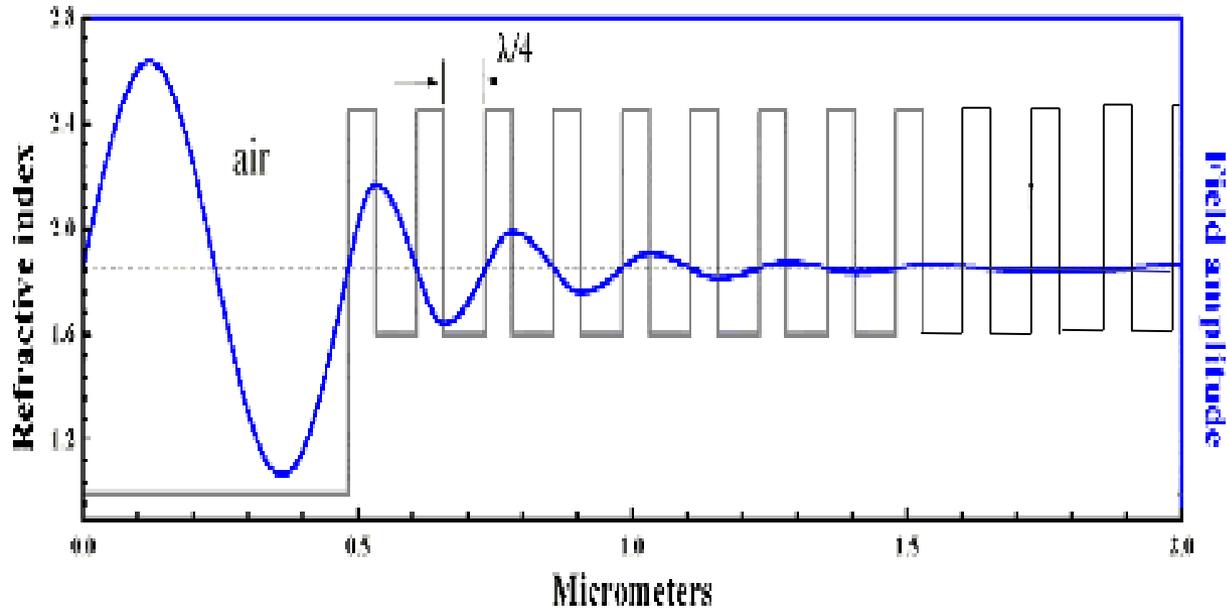
$$Max R_N$$

Bragg mirrors: finite 1D PhC

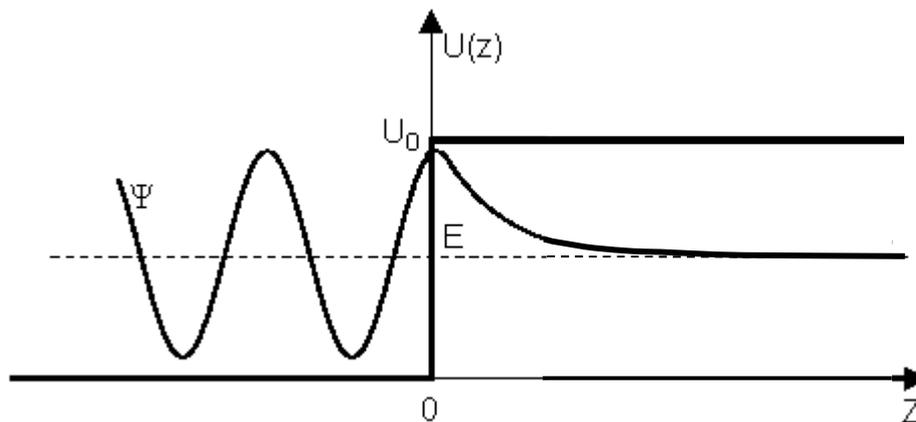


$$T=1-R$$

Infinite Bragg: 1d PhC



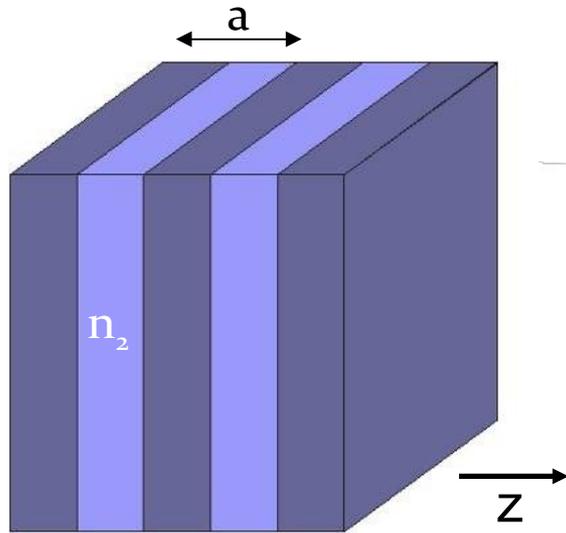
Evanescent field



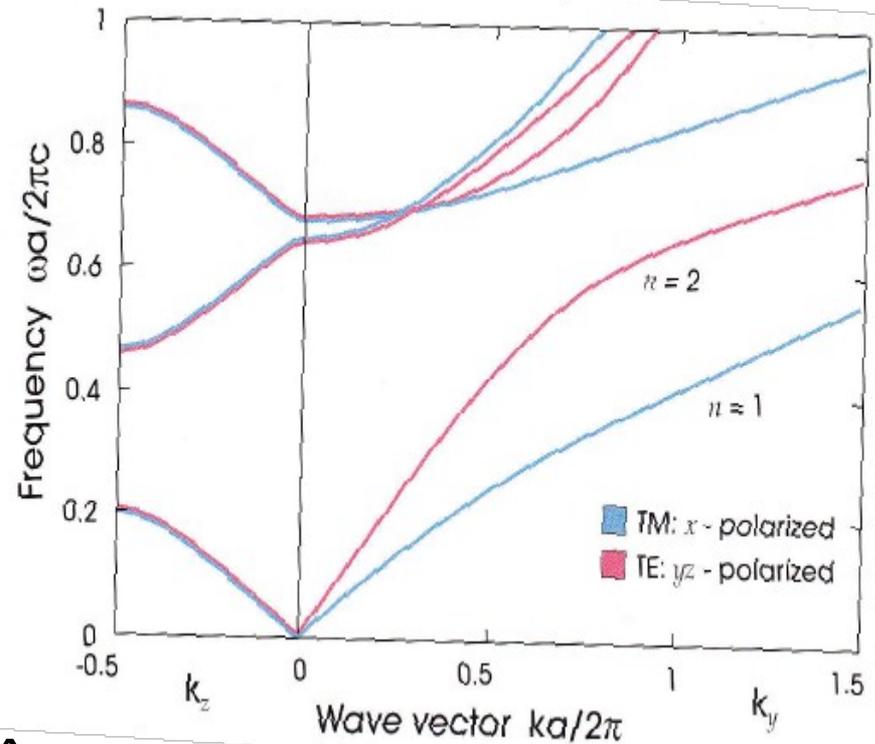
Evanescent wavefunction

PhC in 1D

1D



$$\omega = \omega_n(\vec{k}) \quad \text{FBZ} \quad -\frac{\pi}{a} \leq |\vec{k}_z| \leq \frac{\pi}{a}$$



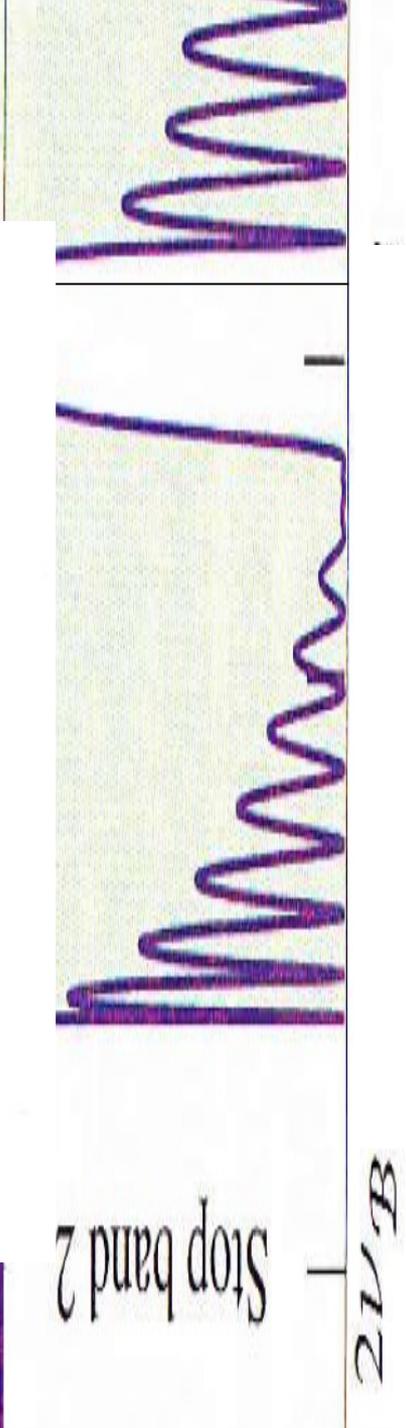
$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{||} \cdot \vec{\rho}} \vec{u}_{n,k_z}(z)$$

$$\vec{k} = \vec{k}_{||} + \vec{k}_z$$

$$\vec{\rho} = x\hat{x} + y\hat{y}$$

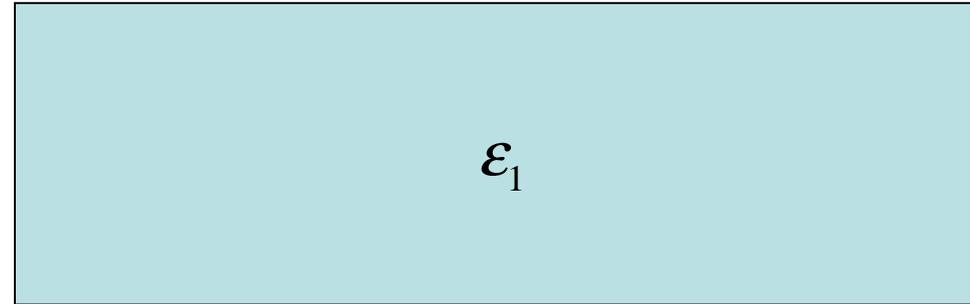
Infinite Bragg

n

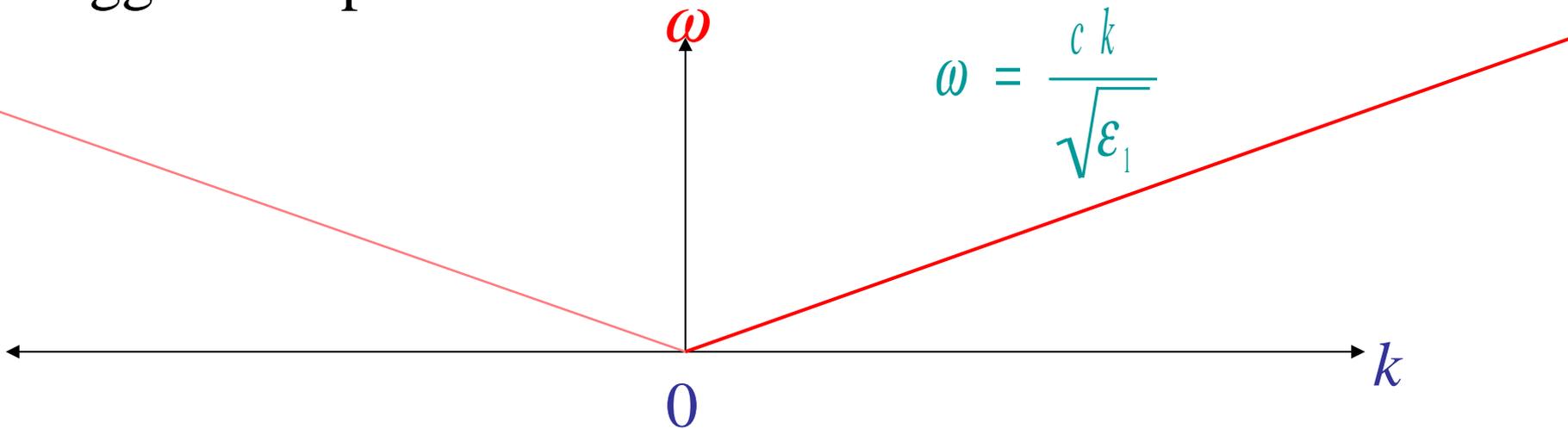


Origine del band gap

Mezzo uniforme



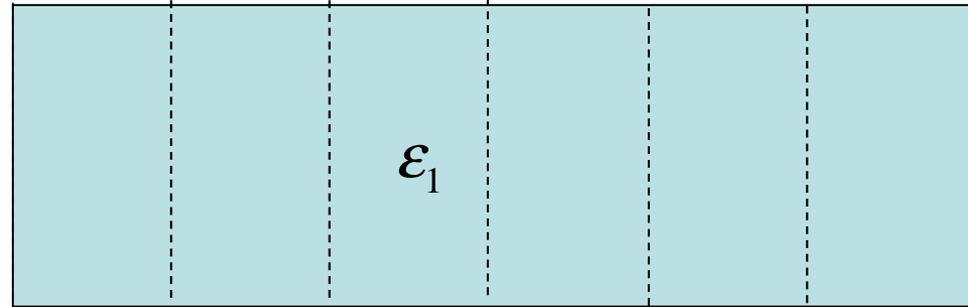
Legge di dispersione



Origine del band gap

[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

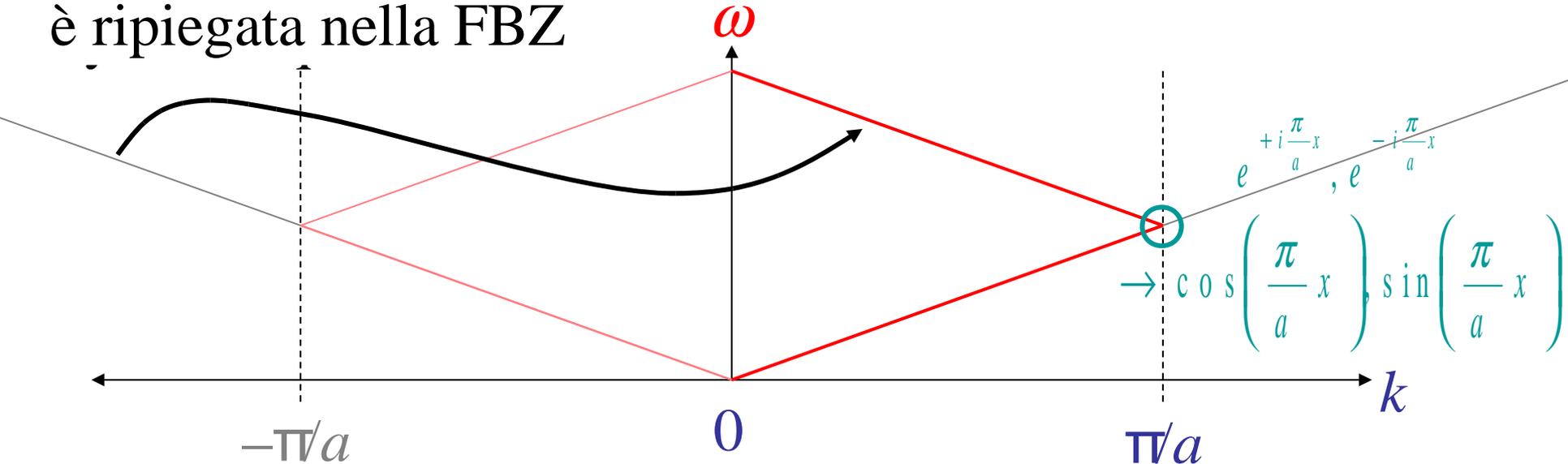
Trattiamolo
come periodico



a

$$\epsilon(x) = \epsilon(x+a)$$

Legge di dispersione
è ripiegata nella FBZ



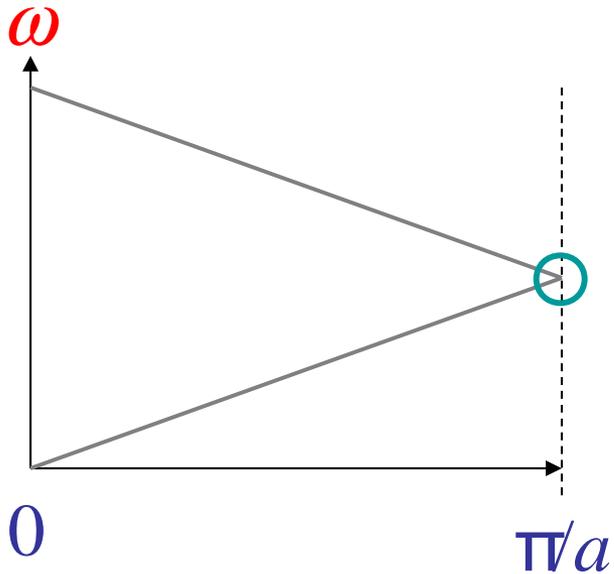
Origine del band gap

Trattiamolo
come periodico

Gli stati degeneri a
bordo zona sono
riscrivibili come

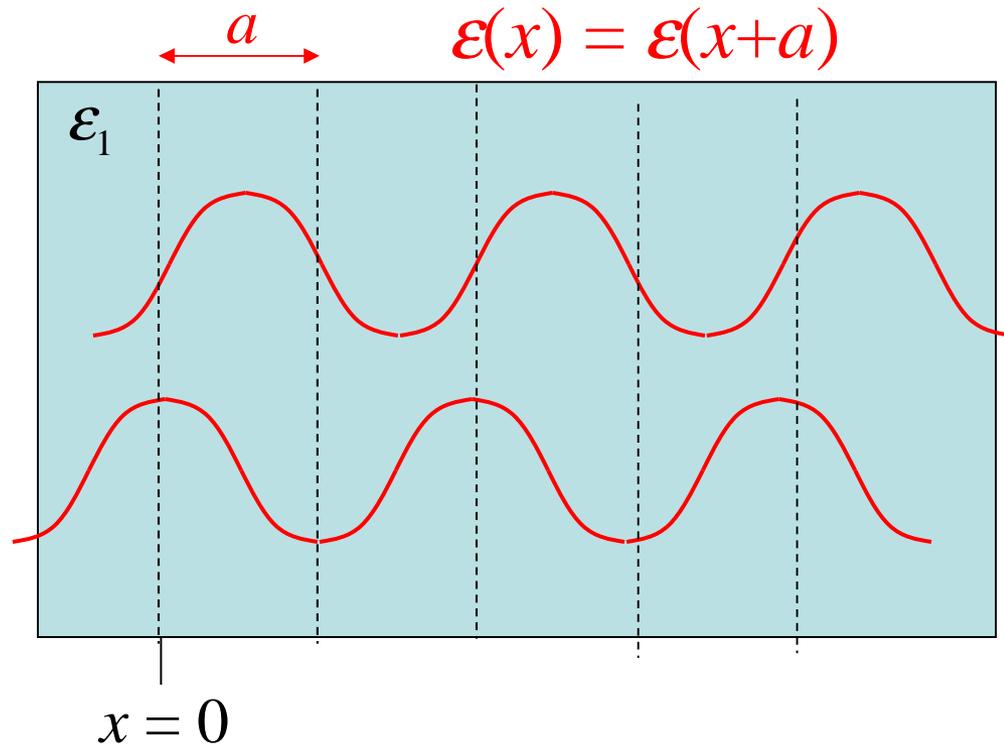
$$E_1(x) = A \sin\left(\frac{\pi}{a} x\right)$$

$$E_2(x) = A \cos\left(\frac{\pi}{a} x\right)$$



$E_1(x)$

$E_2(x)$



Tutti i sistemi 1d hanno gap

Aggiungiamo una piccola anisotropia

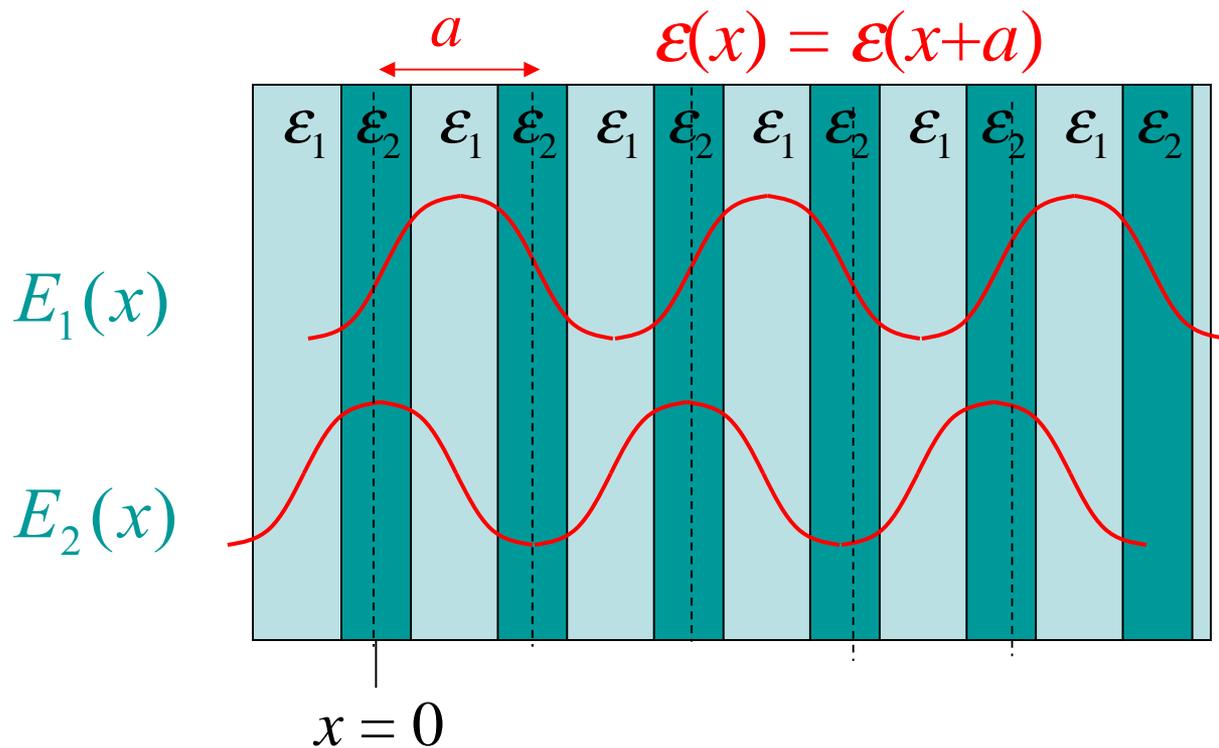
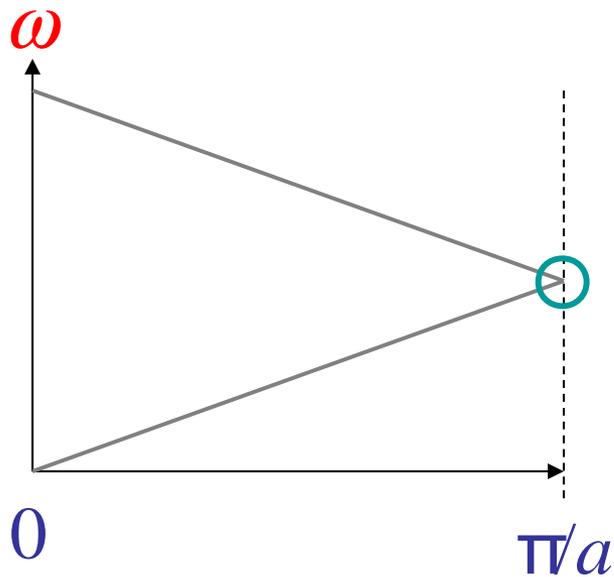
$$\epsilon_2 = \epsilon_1 + \Delta\epsilon$$

Stato con ventre in 1

$$E_1(x) = A \sin\left(\frac{\pi}{a} x\right)$$

Stato con ventre in 2

$$E_2(x) = A \cos\left(\frac{\pi}{a} x\right)$$



Principio variazionale

Principio variazionale:

gli autostati minimizzano il funzionale energia, quindi i modi fotonici di più bassa frequenza

hanno ampiezza concentrata nella regione ad alto dielettrico. Inoltre un dato modo in generale conterrà più nodi rispetto a un modo di minore frequenza.

In (MQ) le funzioni d'onda di più bassa energia hanno ampiezza concentrata nelle regioni a potenziale minore. Vale anche in MQ la "legge dei nodi".

$$U_f(\vec{H}) \equiv \frac{\int d^3r |\vec{\nabla} \times \vec{E}|^2}{\int d^3r \epsilon(\vec{r}) |\vec{E}|^2}$$

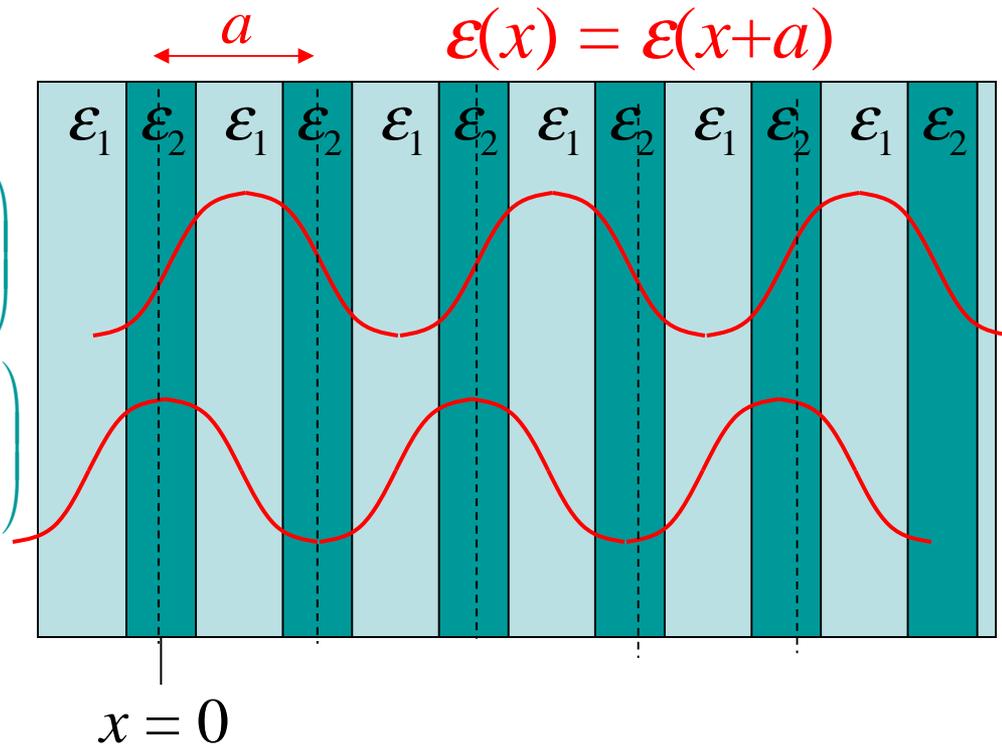
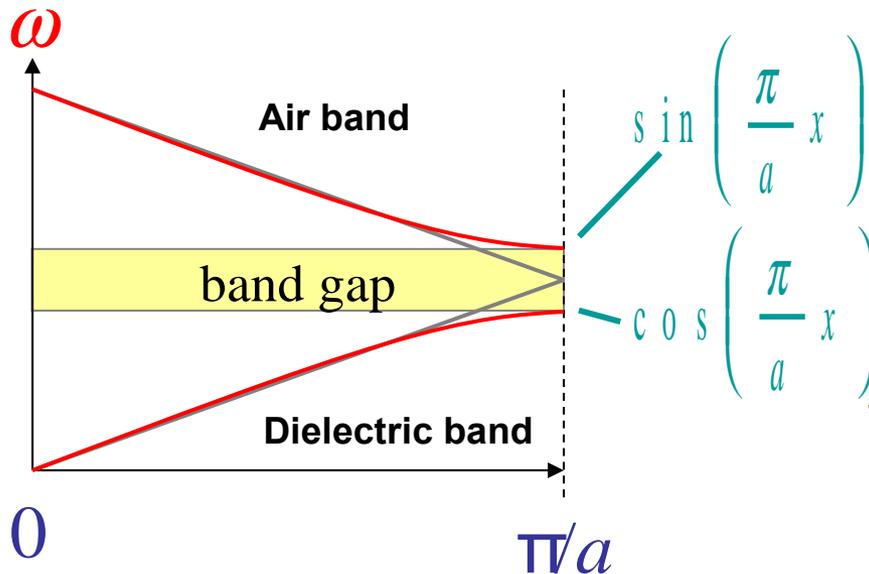
$$\langle \hat{H} \rangle \equiv \frac{\int d^3r \psi^* \hat{H} \psi}{\int d^3r |\psi|^2}$$

Origine del band gap

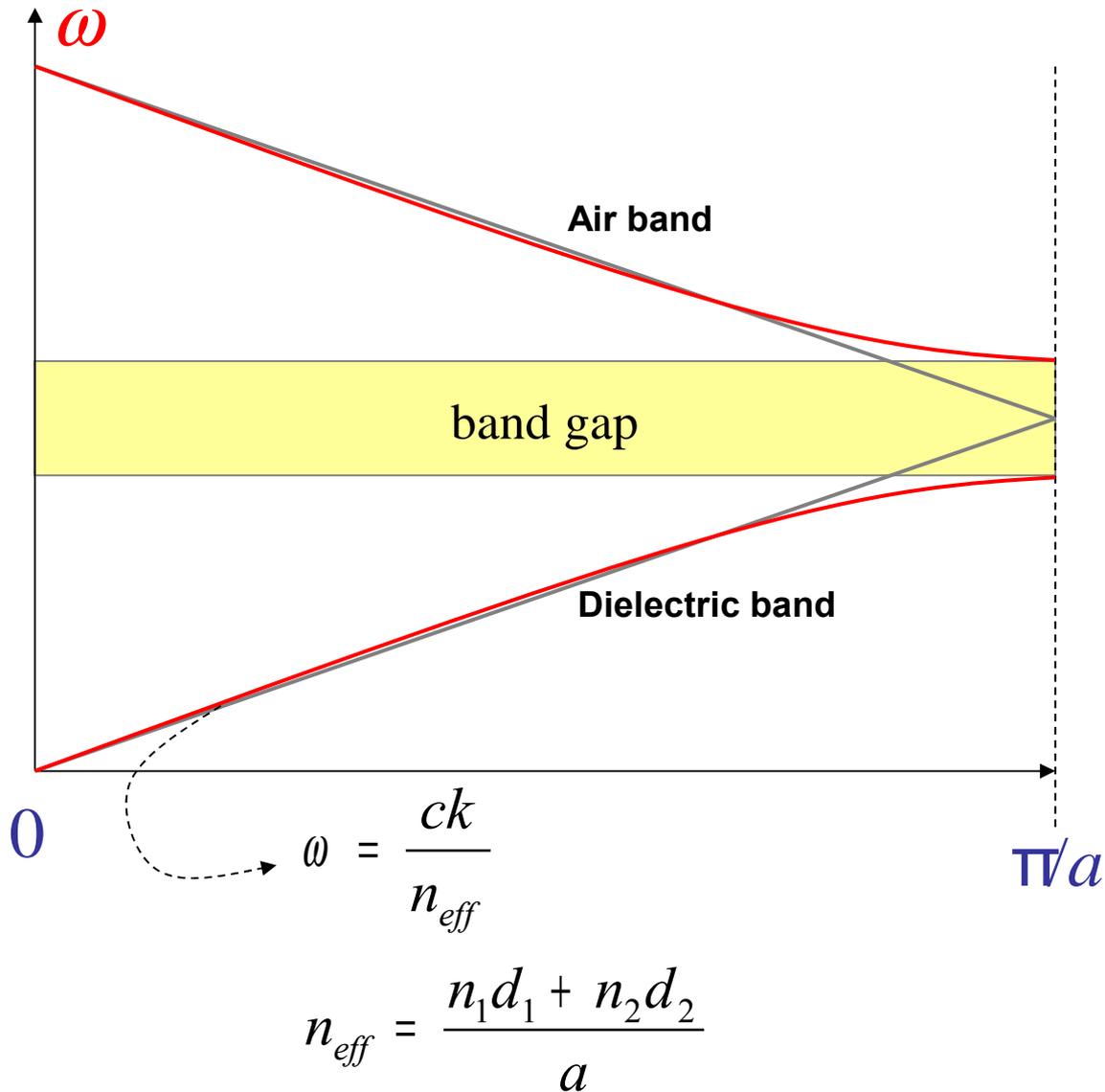
Aggiungiamo una piccola anisotropia

$$\epsilon_2 = \epsilon_1 + \Delta\epsilon$$

Splitting della degenerazione:
state concentrated in **higher index** (ϵ_2)
has **lower frequency**

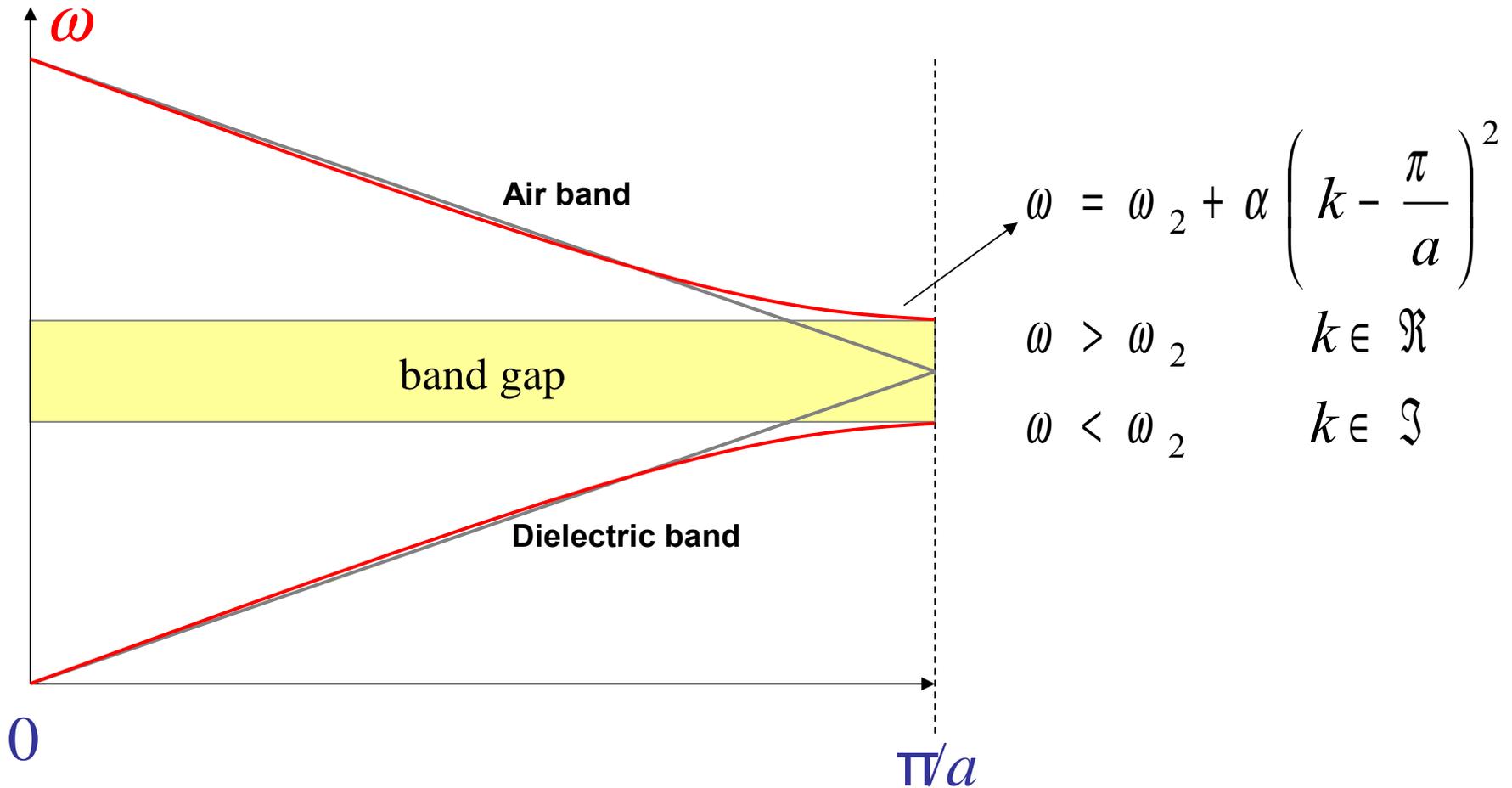


Valore del mid gap



$$\begin{aligned} \omega_{midgap} &= \frac{ck_M}{n_{eff}} = \\ &= \frac{c\pi}{n_1 d_1 + n_2 d_2} = \\ &= 2\pi \nu_B \end{aligned}$$

Stati nel band gap



Nel band gap onde evanescenti

Ingegnerizzazione del band gap

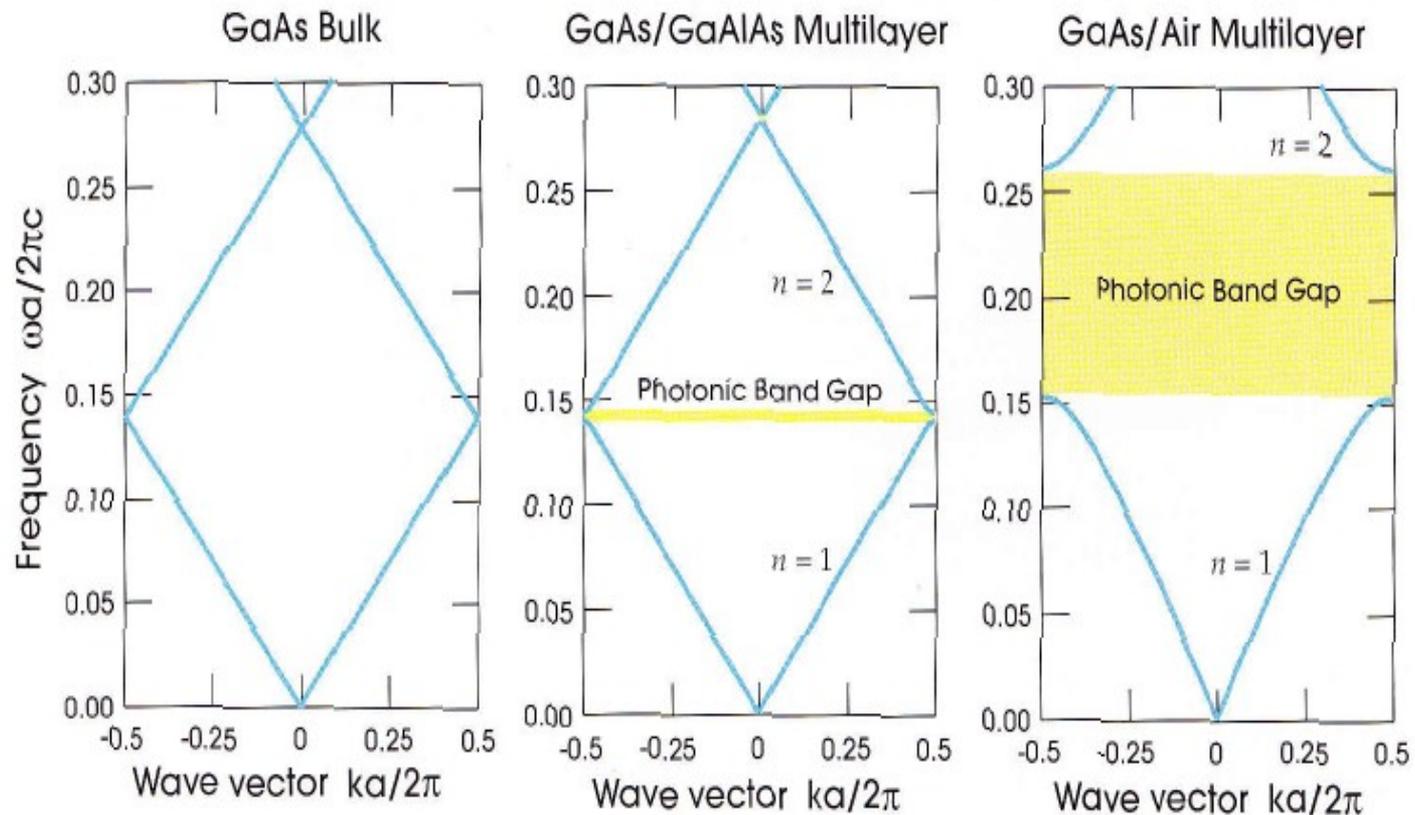


Figure 2: The photonic band structures for on-axis propagation, as computed for three different multilayer films. In all three cases, each layer has a width $0.5a$. *Left:* every layer has the same dielectric constant $\epsilon = 13$. *Center:* layers alternate between ϵ of 13 and 12. *Right:* layers alternate between ϵ of 13 and 1.

Gap/mid gap: quarter wave stack $\varphi_1 - \varphi_2 = 0$

$$\operatorname{Re}\left\{\frac{1}{t}\right\} = \left[\frac{(n_2 + n_1)^2}{4n_2n_1} \cos\left(\pi \frac{v}{v_B}\right) - \frac{(n_2 - n_1)^2}{4n_2n_1} \cos(\varphi_2 - \varphi_1) \right]$$

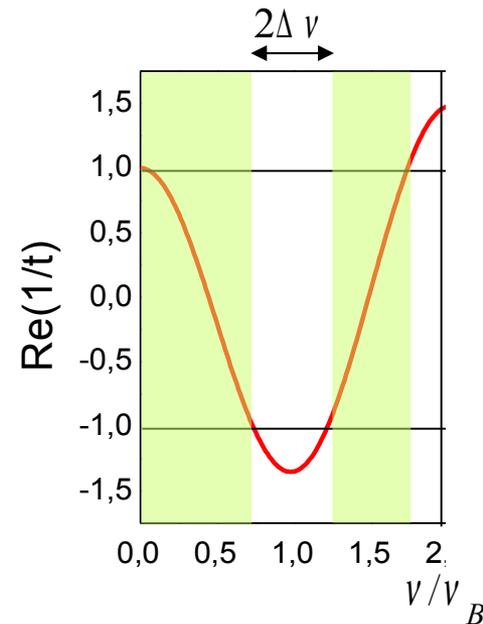
$$\left[\frac{(n_2 + n_1)^2}{4n_2n_1} \cos\left(\pi \frac{(v_B \pm \Delta v)}{v_B}\right) - \frac{(n_2 - n_1)^2}{4n_2n_1} \right] = -1$$

$$\cos\left(\pi \frac{(v_B \pm \Delta v)}{v_B}\right) = \frac{-4n_2n_1 + (n_2 - n_1)^2}{(n_2 + n_1)^2} = 1 - \frac{8n_2n_1}{(n_2 + n_1)^2}$$

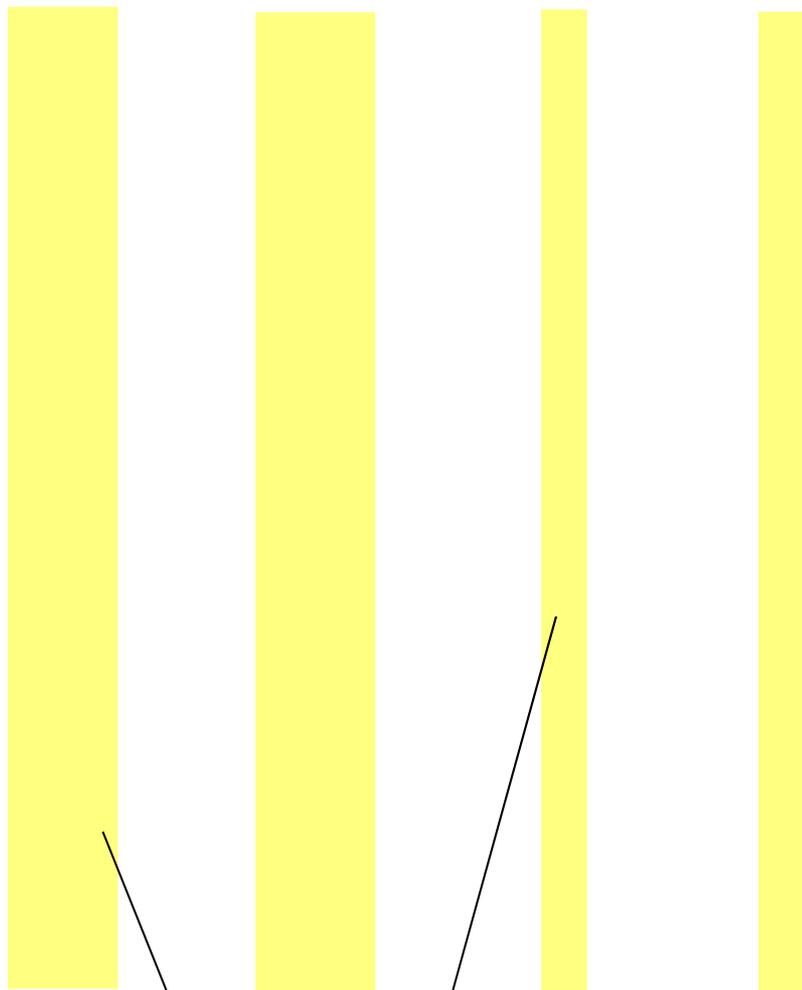
$$\cos\left(\pi \frac{\Delta v}{v_B}\right) = \frac{8n_2n_1}{(n_2 + n_1)^2} - 1 = 1 - 2 \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2}$$

$$\sin^2\left(\pi \frac{\Delta v}{2v_B}\right) = \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2}$$

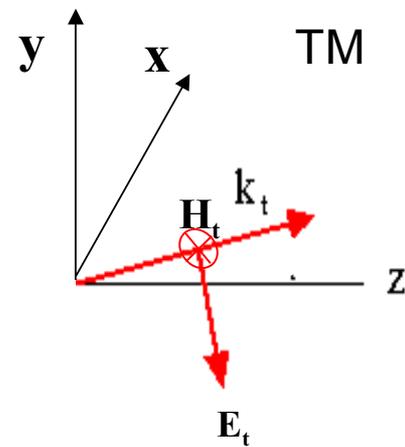
$$\frac{2\Delta v}{v_B} = \frac{4}{\pi} \arcsin\left(\frac{|n_2 - n_1|}{(n_2 + n_1)}\right)$$



Incidenza obliqua



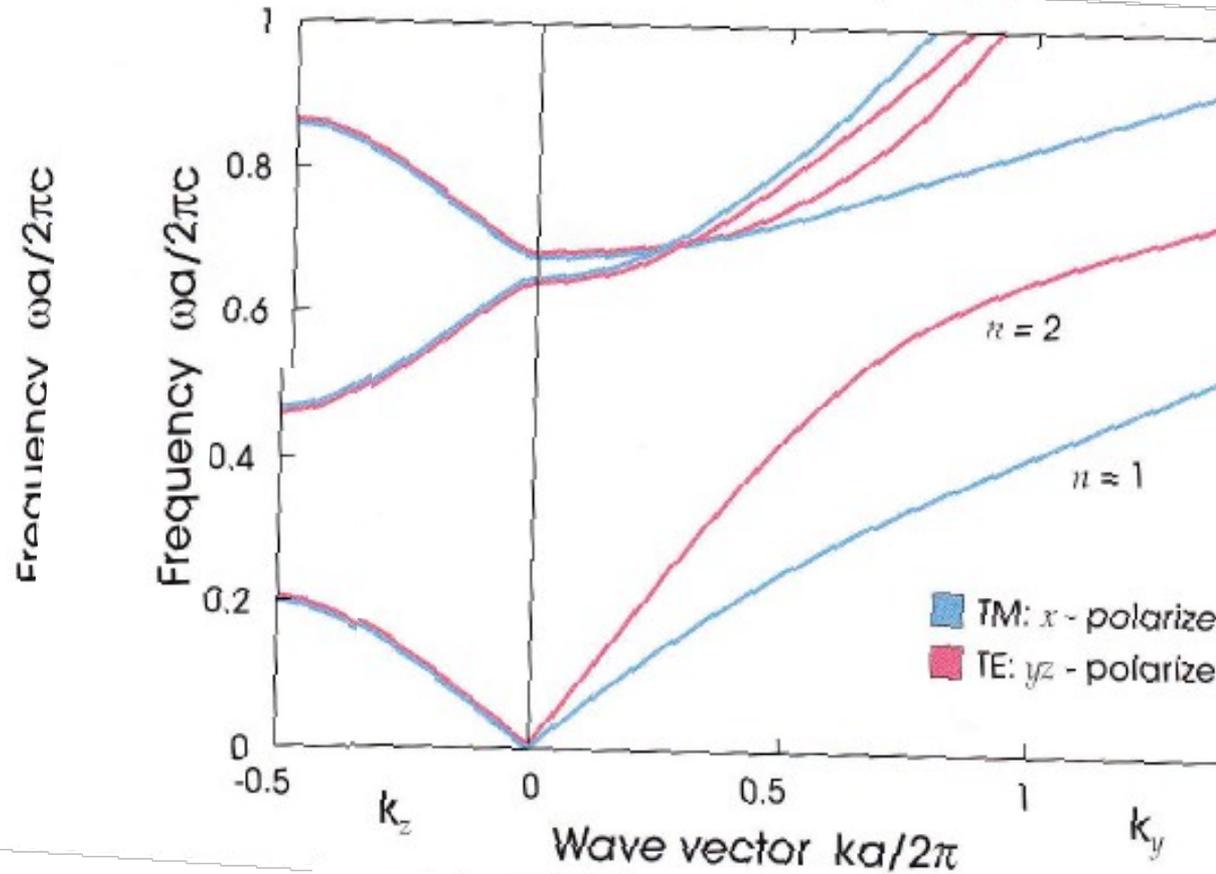
Perdita gap



Near Brewster angle

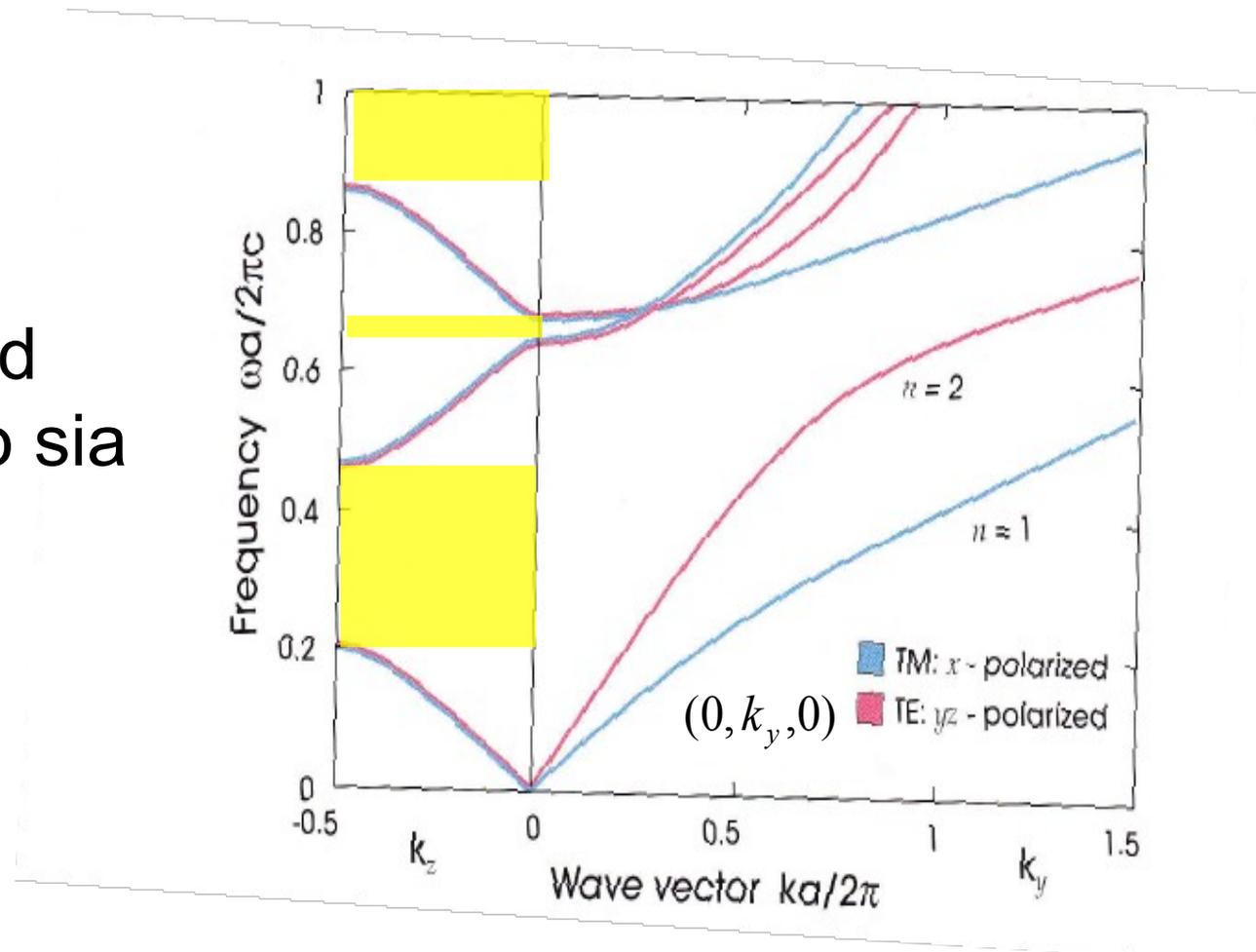
Struttura a bande per propagazione nel piano

Non c'è gap
per TM e TE



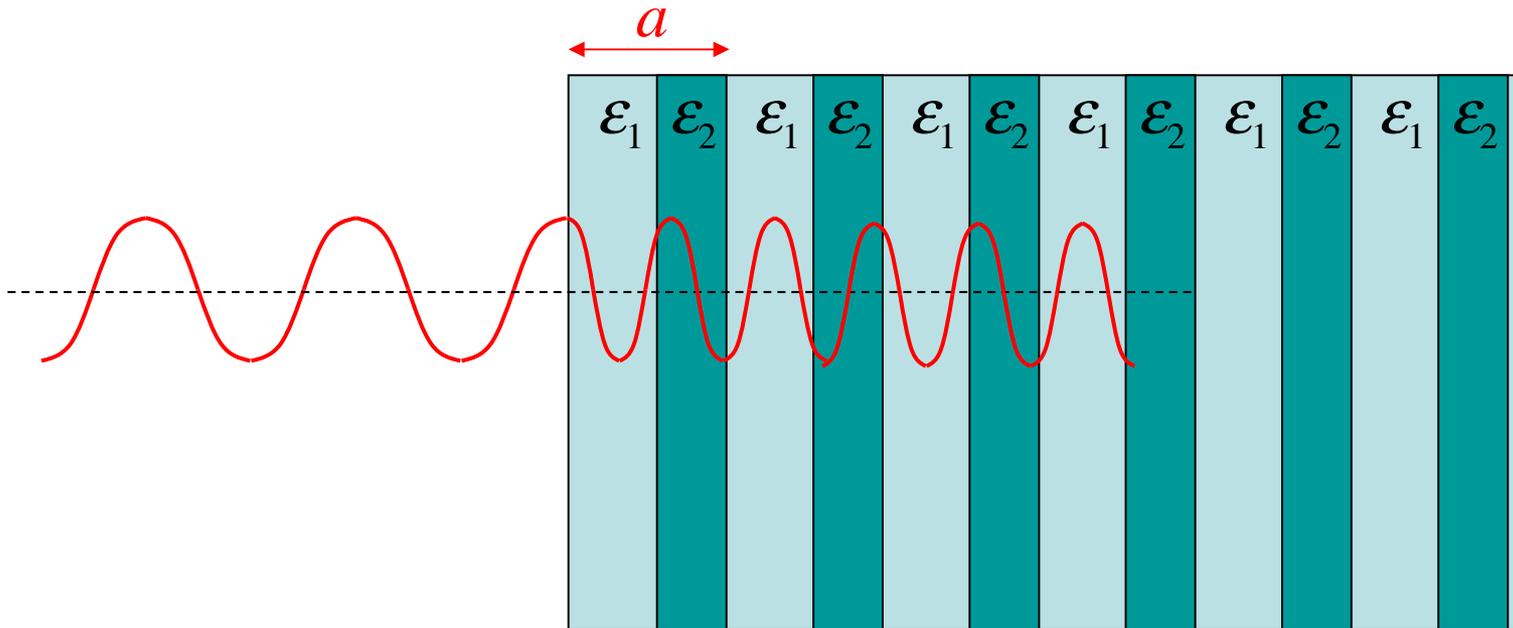
Struttura a bande per propagazione nel piano

Assenza band gap completo sia in TM e TE



Tipologia dei modi

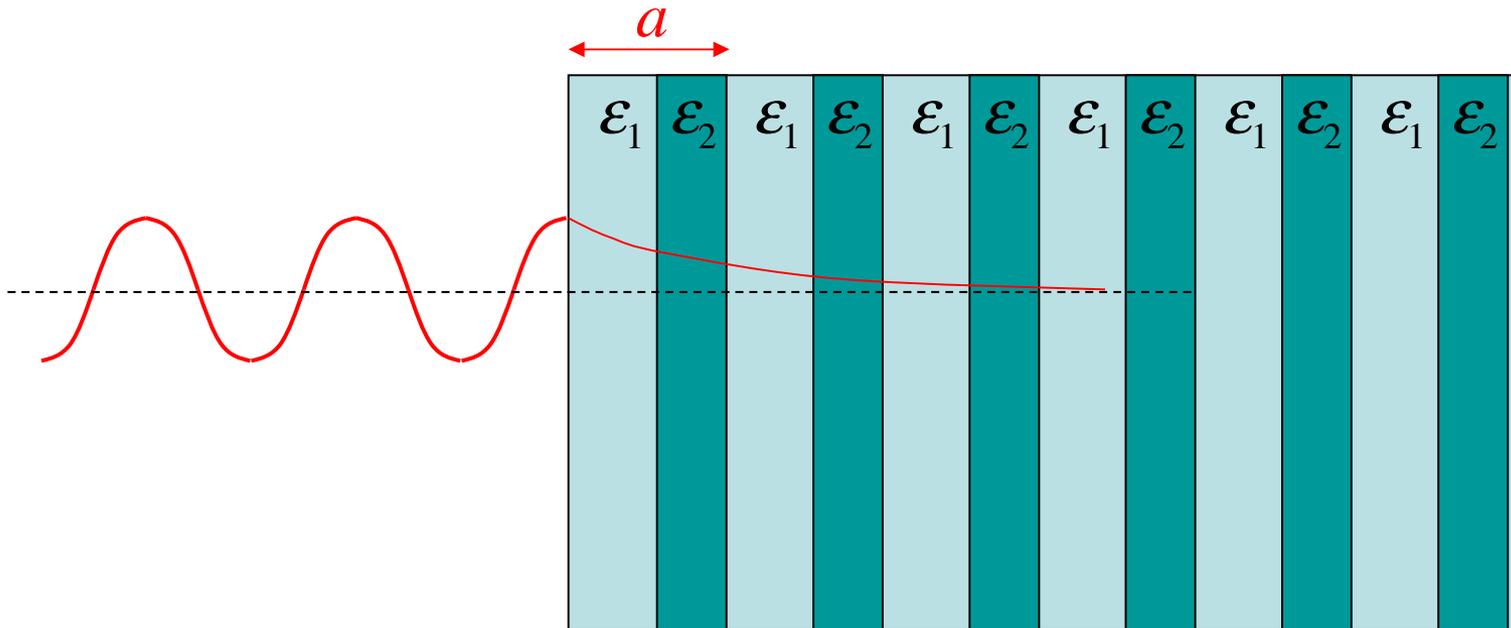
Modi EE Extended-Extended



All'interno di una banda e dentro il cono di luce

Origine del band gap

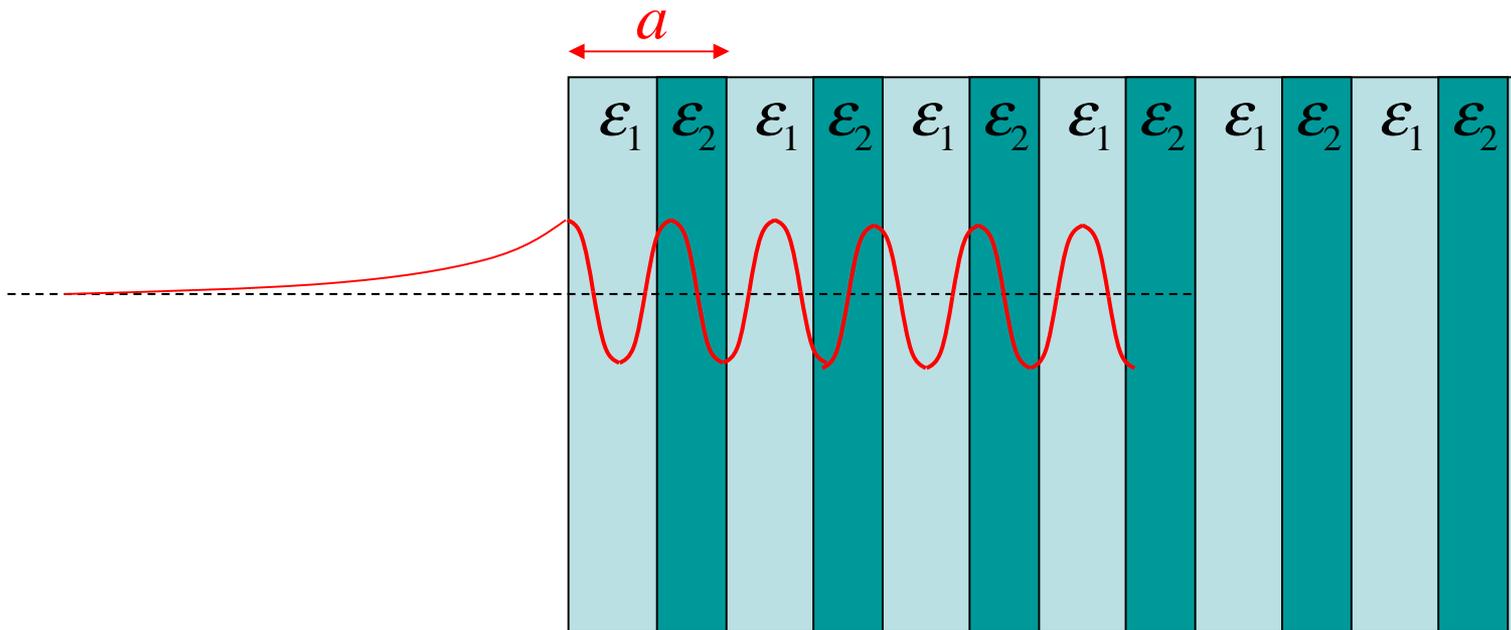
Modi ED Extended-Decay



All'interno di un gap e dentro il cono di luce

Tipologia dei modi

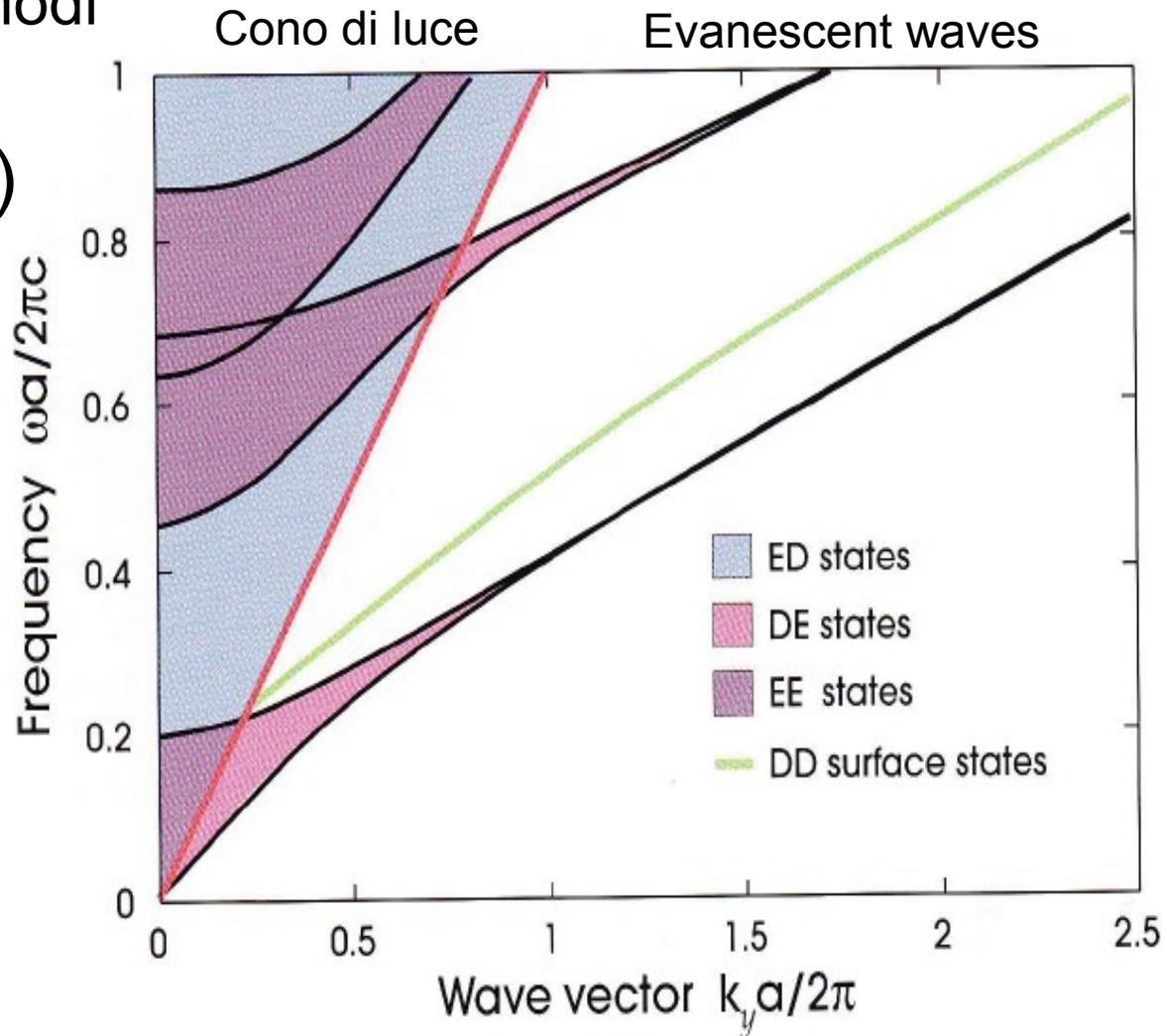
Modi DE Decay-Extended



All'interno di una banda e oltre il cono di luce

Tipologia dei modi

Modi E_x (TM)



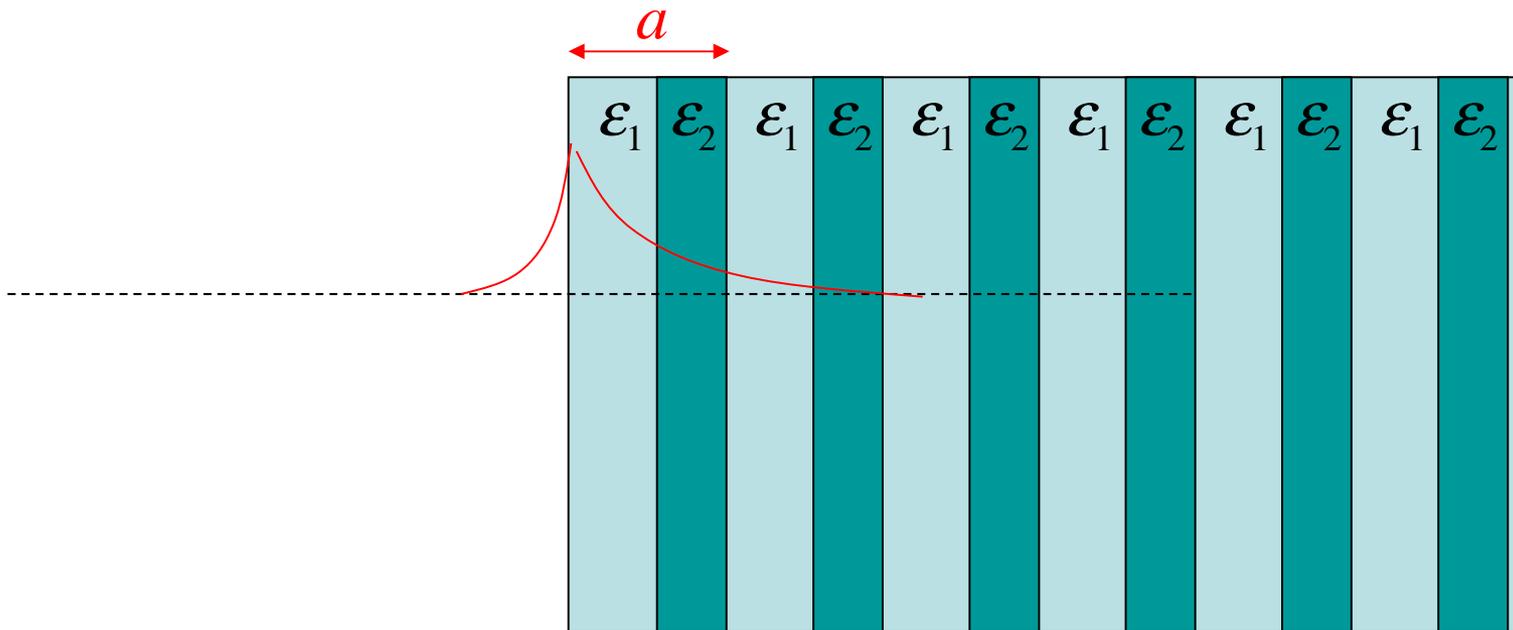
LEGENDA

ED=Extended in air, Decay in PhC
EE=Extended in air, Extended in PhC

DE=Decay in air, Extended in PhC
DD=Decay in air, Decay in PhC

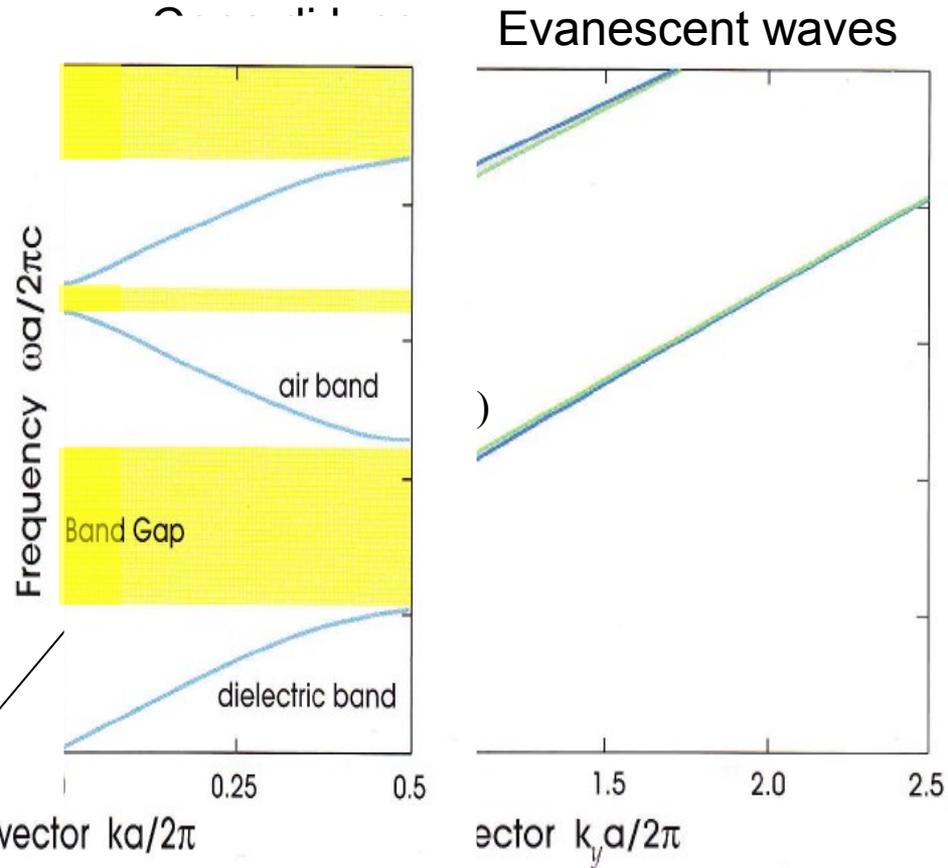
Tipologia dei modi

Modi DD Decay-Decay



Stati di interfaccia

Modi E_x (TE)



k_z

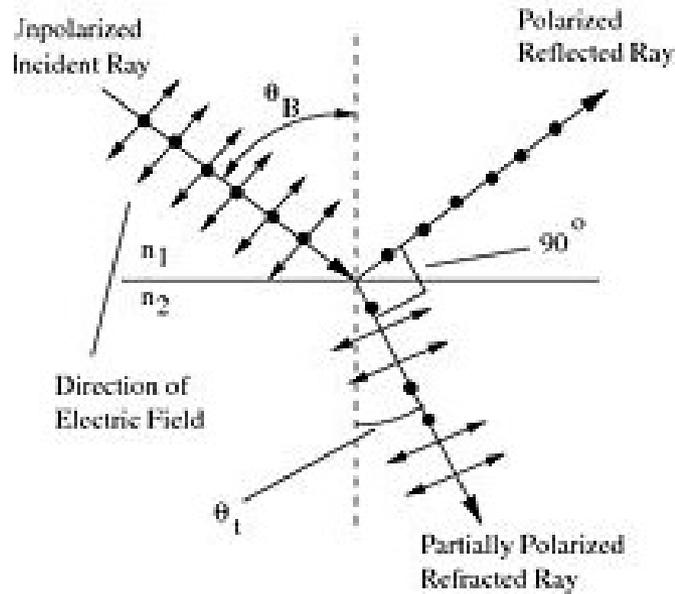
Wave vector $ka/2\pi$

Wave vector $k_y a/2\pi$

Gap entro il cono di luce

Figure 10: Two superimposed x -polarized band structures of a multilayer film, showing how the bandwidths vary with k_y . The blue lines refer to bands along $(0, k_y, 0)$, while the green lines beside them refer to the same bands along $(0, k_y, \pi/a)$. The regions in between are shaded gray to indicate where the continuum of bands for intermediate k_z would lie. Only modes with electric field oriented along the x direction are shown. The straight red line is the **light line** $\omega = ck_y$. The layered material is the same as the one described in the caption of figure 5.

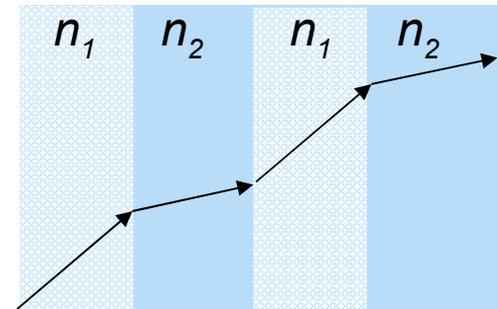
Angolo di Brewster



$$E_r = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} E_i$$

$$\theta_B + \theta_t = \frac{\pi}{2}$$

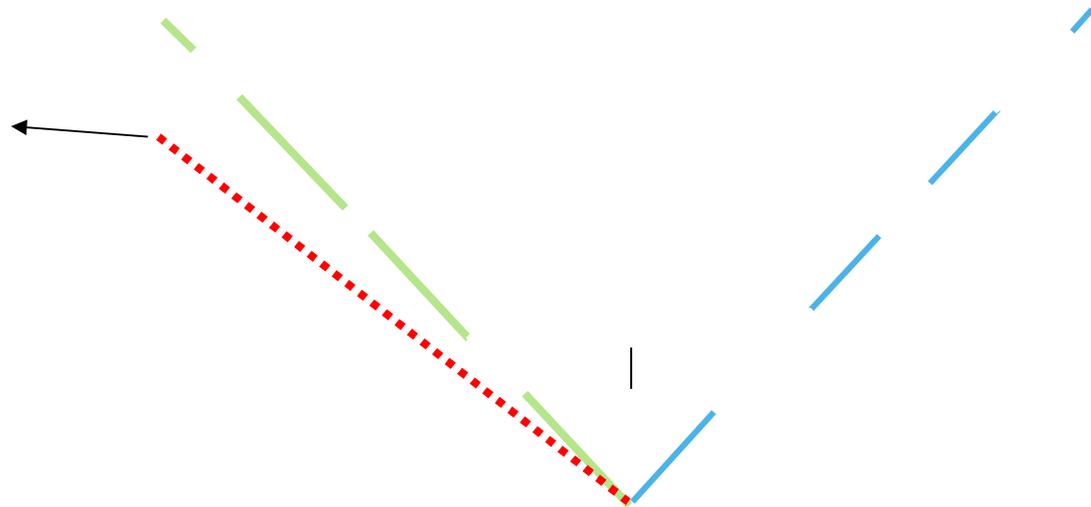
$$\tan \theta_B = \frac{n_2}{n_1}$$



Onda TM non è riflessa

Angolo di Brewster

Tutta la linea che
descrive Brewster
deve stare fuori dal
band gap

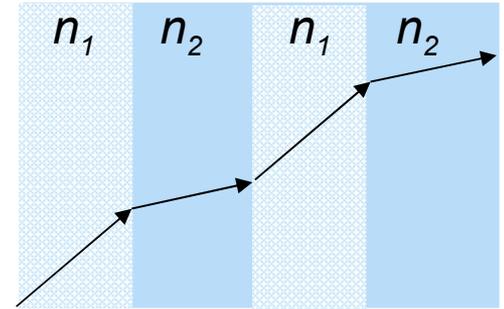


$$\tan \theta_B = \frac{n_2}{n_1}$$

Figure 15: Extended modes (shaded regions) for off-axis propagation vectors $(0, k_y, k_z)$ in a quarter-wave stack with ϵ of 13 and 2. The right side (blue) indicates modes with \mathbf{E} fields polarized in the x direction (TM or s -polarized), similar to figure 10. The left side (green) indicates modes with fields polarized in the yz incidence plane (TE or p -polarized). The straight red line is the **light line** $\omega = ck$, above which extended modes exist in air. In yellow

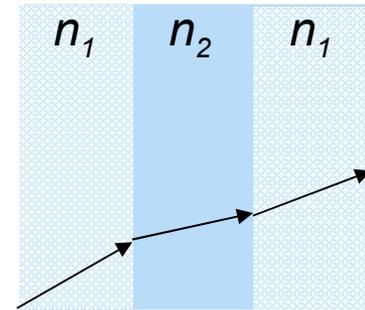
Angolo di Brewster

$$\tan \theta_{1,B} = \frac{n_2}{n_1}$$



Cono di luce

$$\sin \theta_{1,L} = \frac{1}{n_1}$$



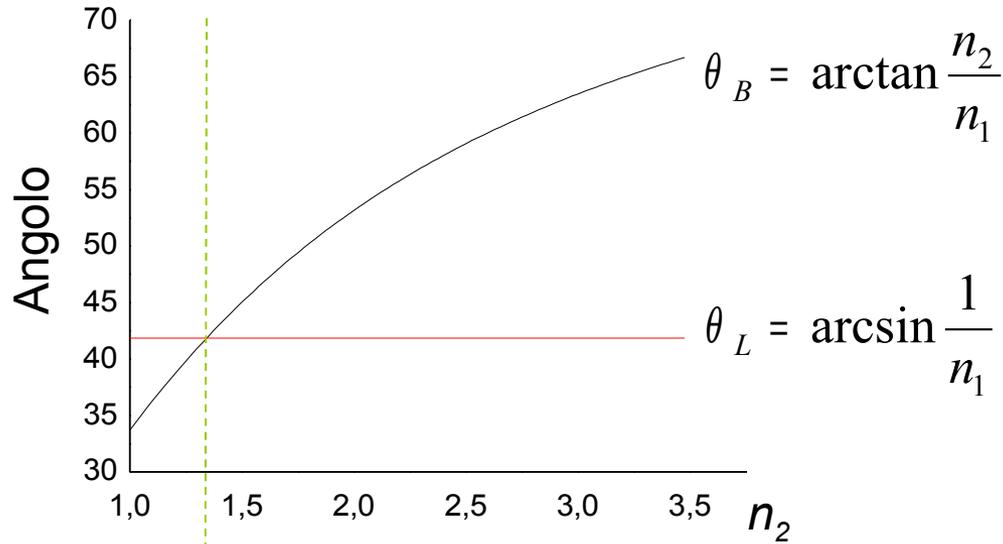
$$\theta_{1,L} > \theta_{1,B}$$

Onda esterna TM può propagarsi a Brewster

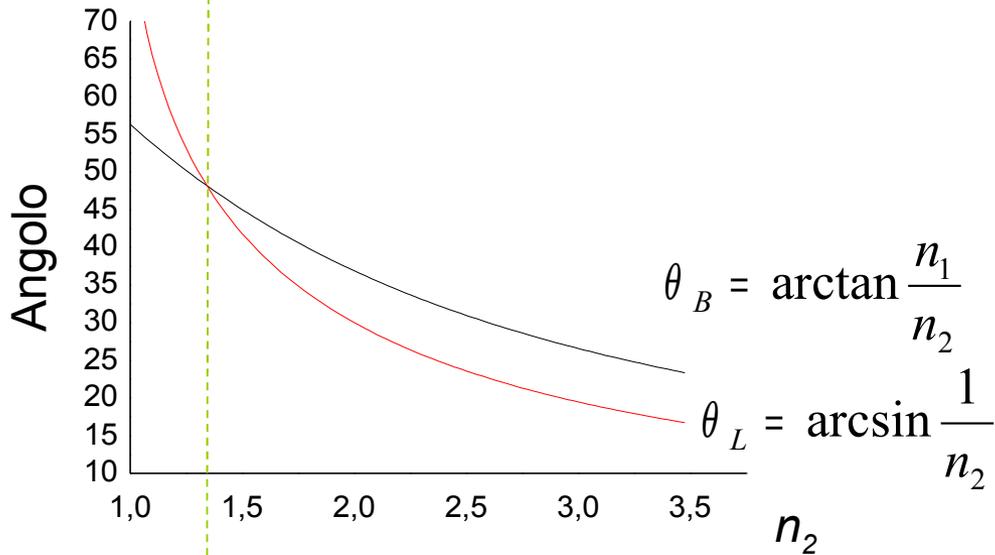
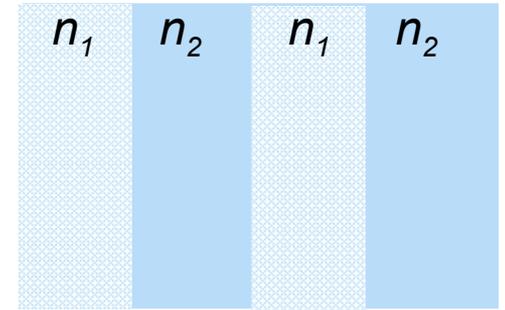
$$\theta_{1,B} > \theta_{1,L}$$

Onda esterna TM non può propagarsi a Brewster

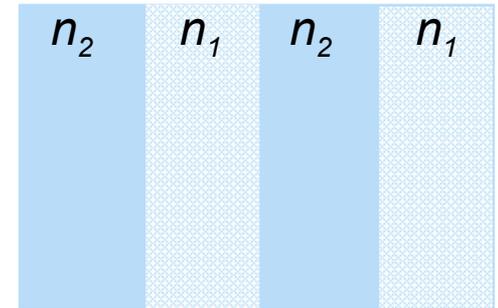
Confronto angolo Brewster vs angolo limite ($n_1=1.5$)



Air



Air

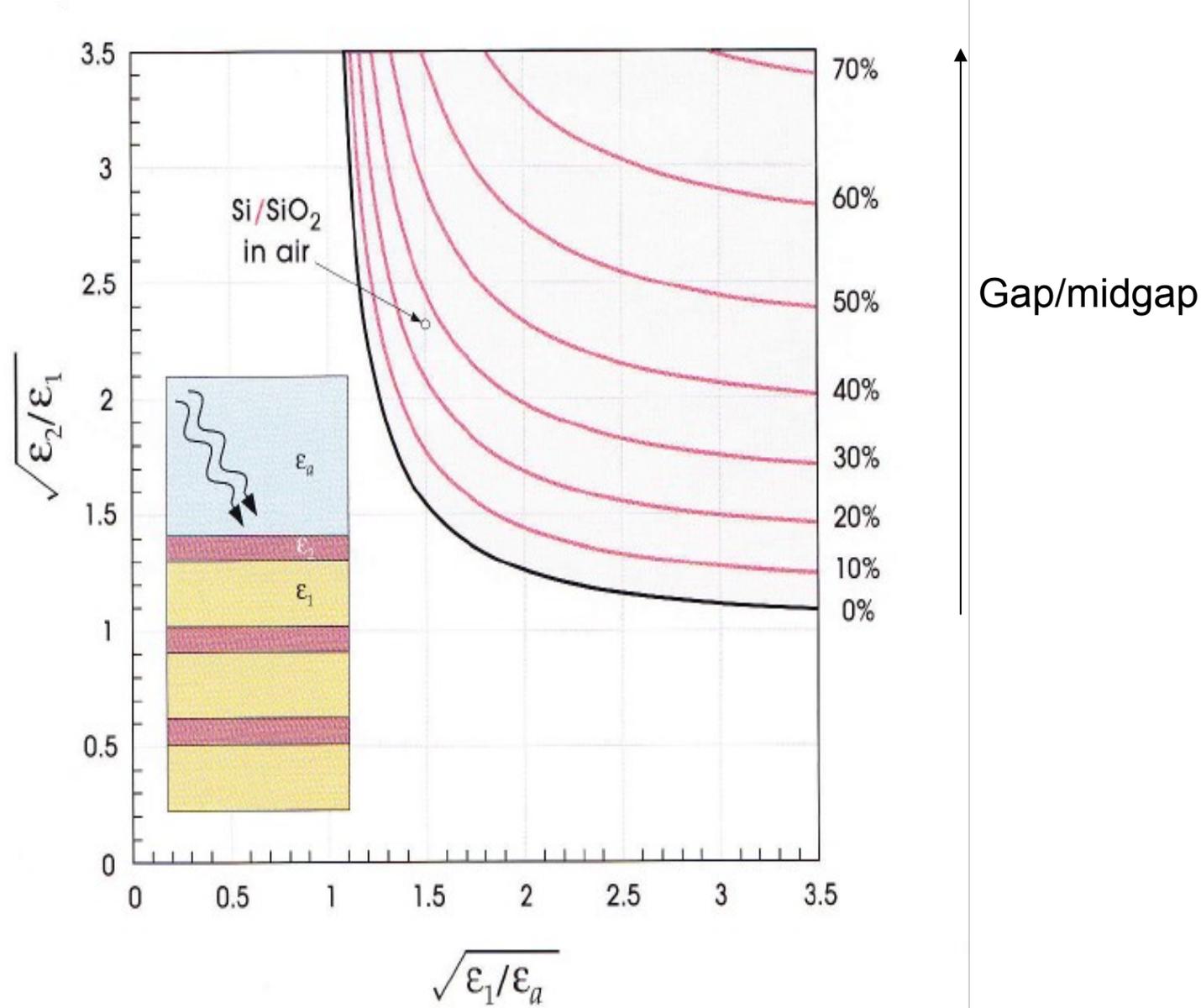


$$\frac{1}{n_1^2} + \frac{1}{n_2^2} = 1$$

Specchio Omnidirezionale

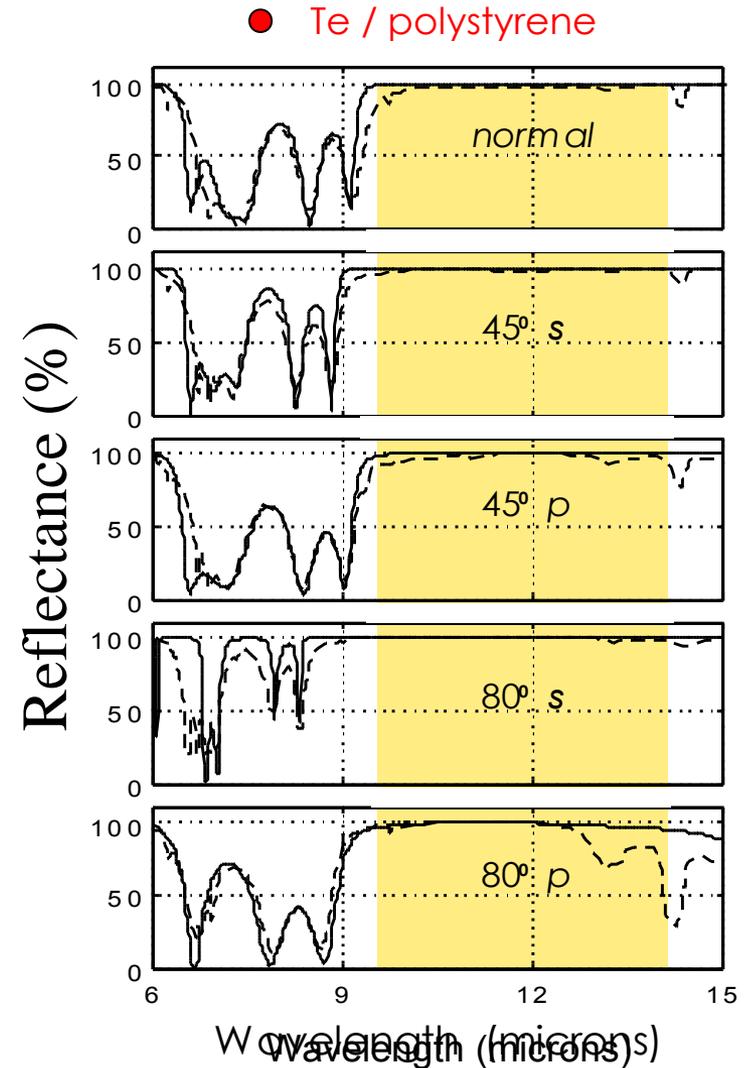
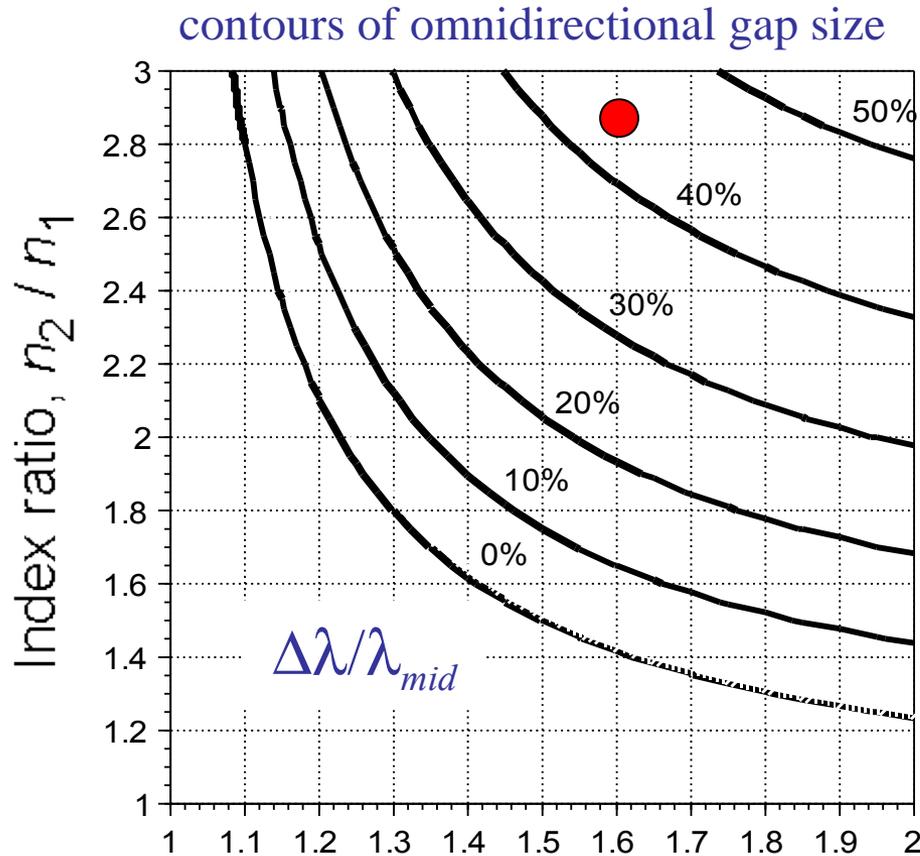
----- gap

Specchio Omnidirezionale



Omnidirectional Mirrors in Practice

[Y. Fink *et al*, *Science* **282**, 1679 (1998)]



Effetti del disordine

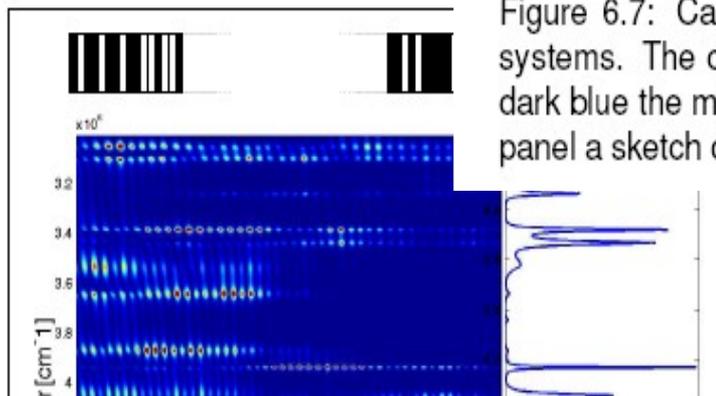
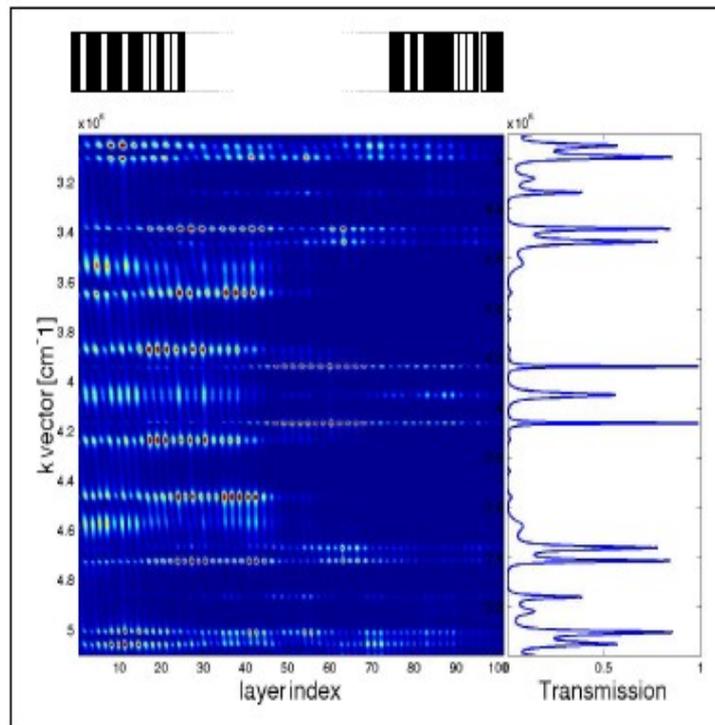
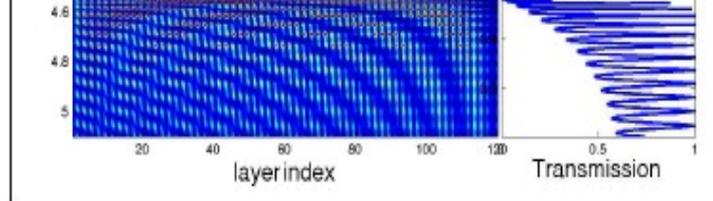


Figure 6.7: Calculated scattering state map for 1D ordered (top) and disordered (bottom) systems. The colorbar refers to the intensity of the field, being red the maximum value and dark blue the minimum. On the right the transmission spectrum is shown. On the top of each panel a sketch of the multilayer structure is shown (dark layers have higher refractive index).