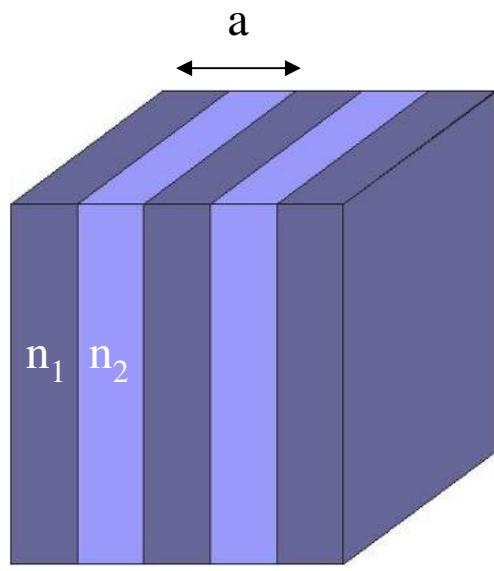
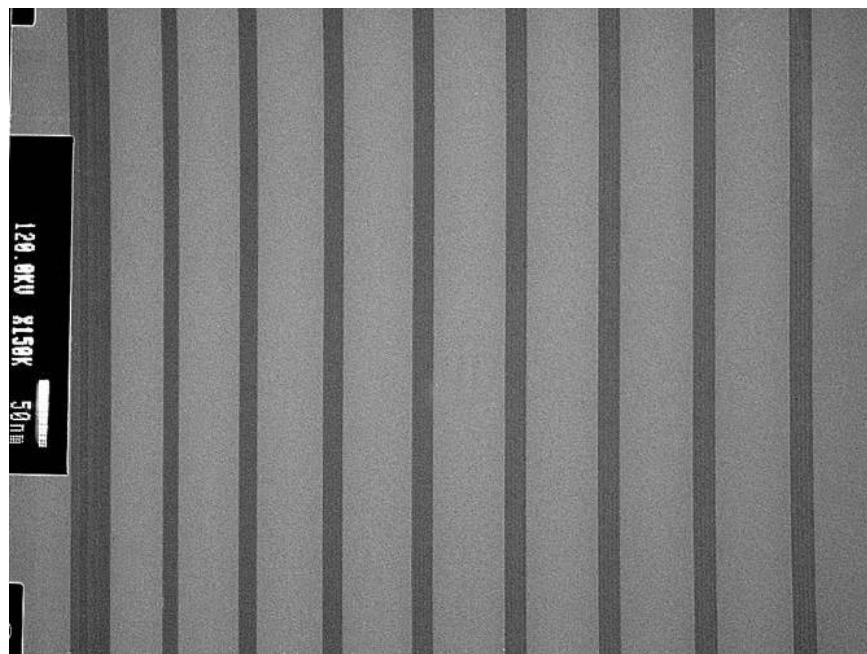


PhC in 1D

1D



\rightarrow
z



Slab: sistema 1D non periodico

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{||} \cdot \vec{\rho}} \vec{u}_{n,k_z}(z)$$

$$\vec{k} = \vec{k}_{||} + \vec{k}_z \quad |\vec{k}| = \frac{\omega n}{c}$$

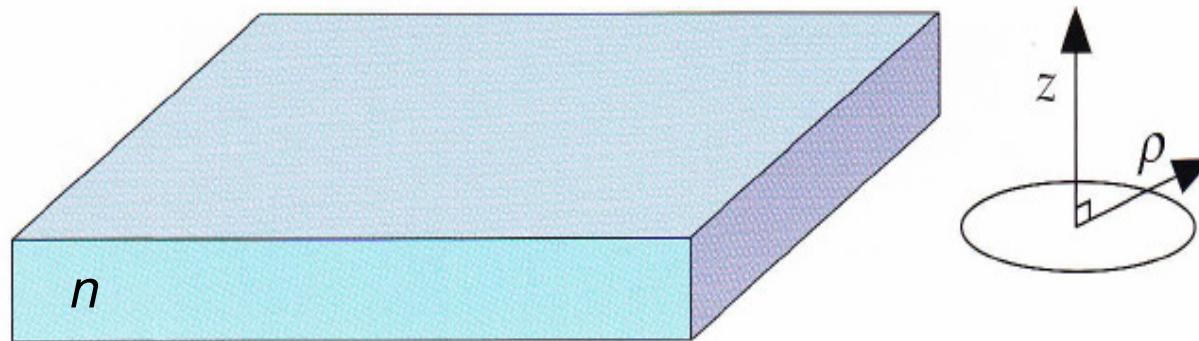
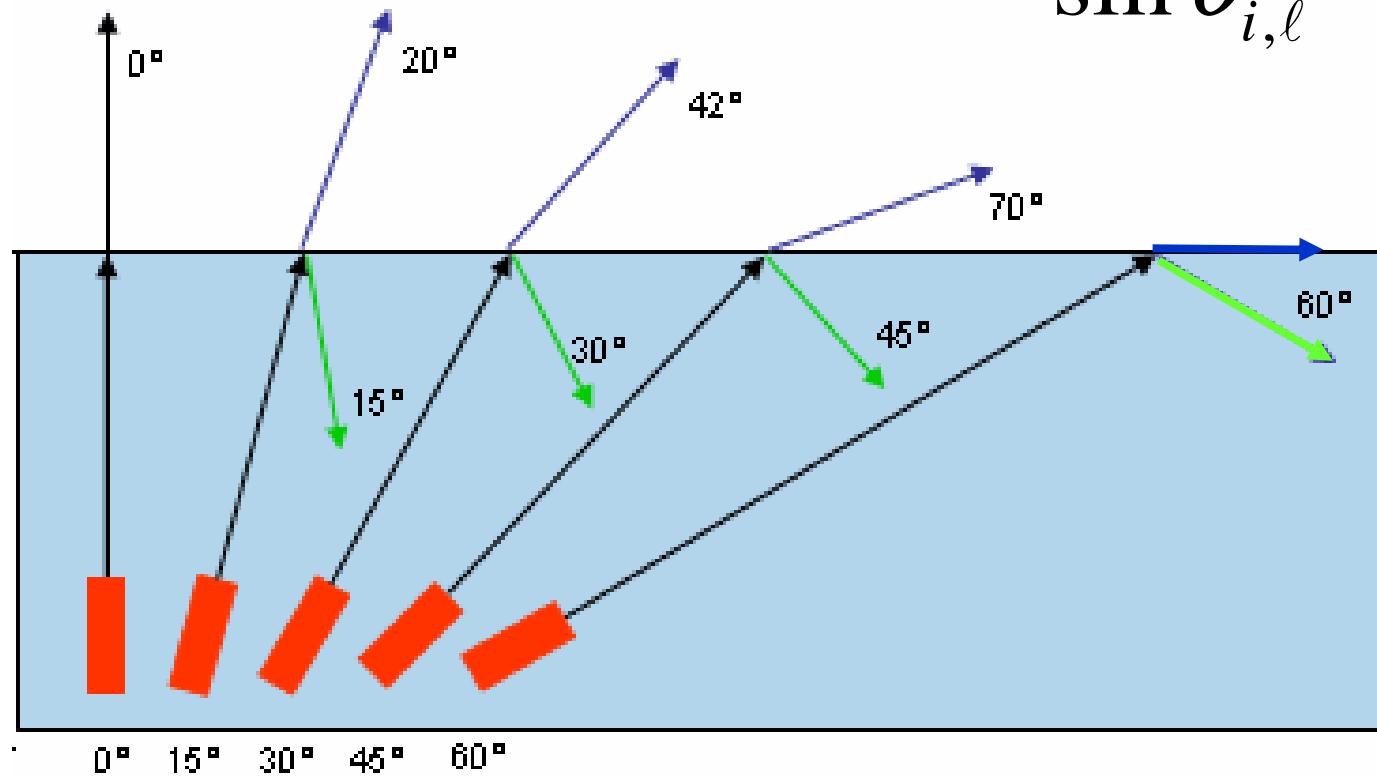


Figure 2: A plane of glass. If the glass extends much farther in the x and y directions than in the z direction, we may consider this system to be one-dimensional: the dielectric function $\epsilon(\mathbf{r})$ varies in the z direction, but has no dependence on the in-plane coordinate ρ .

Rifrazione da mezzo denso $n_1 > n_2$

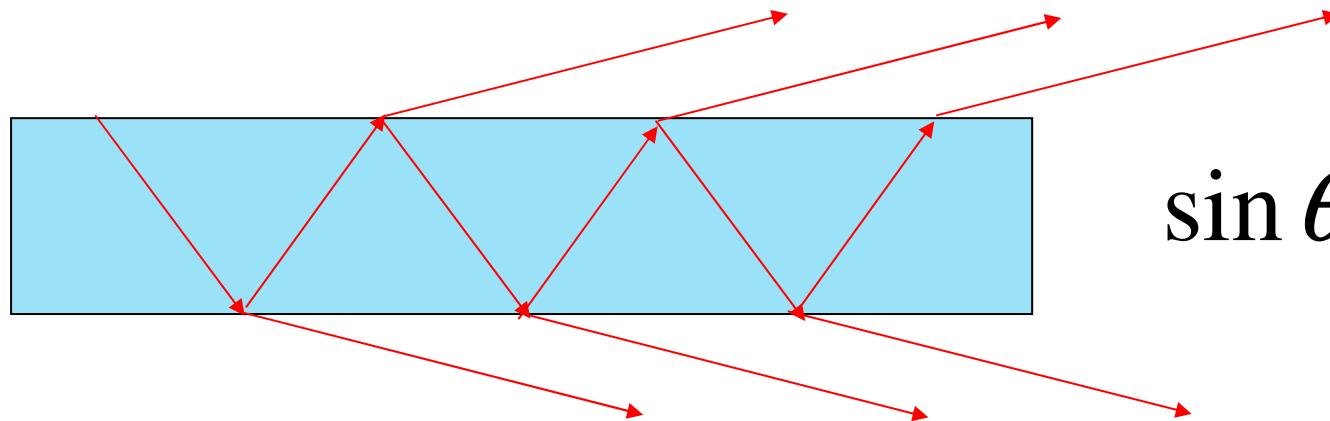
$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\sin \theta_{i,\ell} = \frac{n_2}{n_1}$$



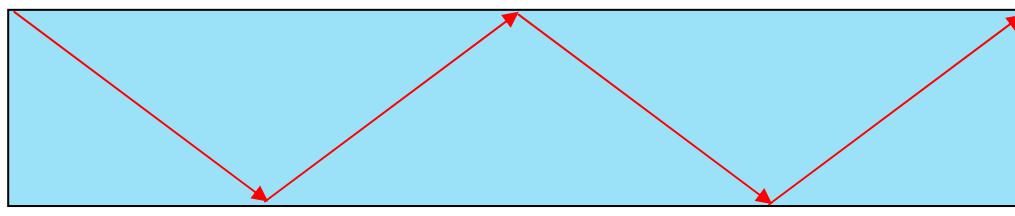
Sistema 1D non periodico

Entro l'angolo limite



$$\sin \theta_i < \frac{n_2}{n_1}$$

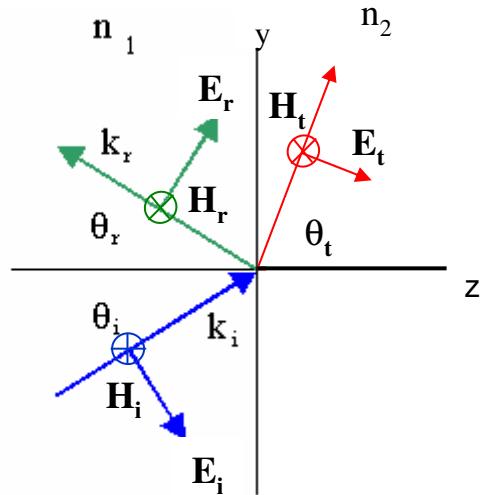
Oltre l'angolo limite



$$\sin \theta_i > \frac{n_2}{n_1}$$

Relazioni di Fresnel

$$n_1 > n_2$$



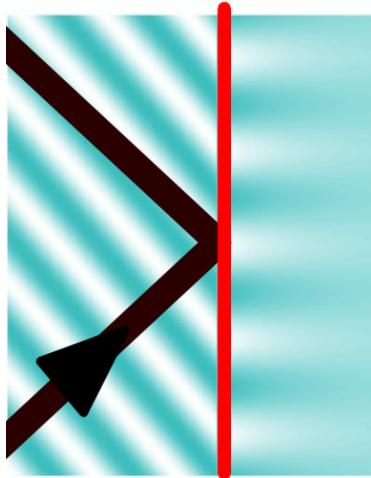
$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i < 1$$

$$\begin{cases} E_r = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_i \\ E_t = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_i \end{cases}$$

$$\begin{cases} E_r = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} E_i \\ E_t = \frac{2n_1 \cos \theta_i}{n_2 \cos \theta_i + n_1 \cos \theta_t} E_i \end{cases}$$

Oltre l'angolo limite: onde evanescenti

$$n = \frac{n_2}{n_1} < 1$$



$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i > 1$$

$$\vec{E}_t(\vec{r}, t) = \vec{E}_t e^{j(\vec{k}_{t,\parallel} \cdot \vec{r}_\parallel - \omega_i t)} e^{-\beta z}$$

$$\cos \theta_t = ja = j\sqrt{(n \sin \theta_i)^2 - 1}$$

$$\begin{cases} E_r = \frac{\cos \theta_i - jna}{\cos \theta_i + jna} E_i = r_\perp E_i \\ E_t = \frac{2 \cos \theta_i}{\cos \theta_i + jna} E_i = t_\perp E_i \end{cases}$$

$$\begin{cases} E_r = \frac{n \cos \theta_i - ja}{n \cos \theta_i + ja} E_i = r_\parallel E_i \\ E_t = \frac{2n \cos \theta_i}{n \cos \theta_i + ja} E_i = t_\parallel E_i \end{cases}$$

$$|r_\perp| = |r_\parallel| = 1 \quad \text{Riflessione totale}$$

Sistema 1D (non periodico)

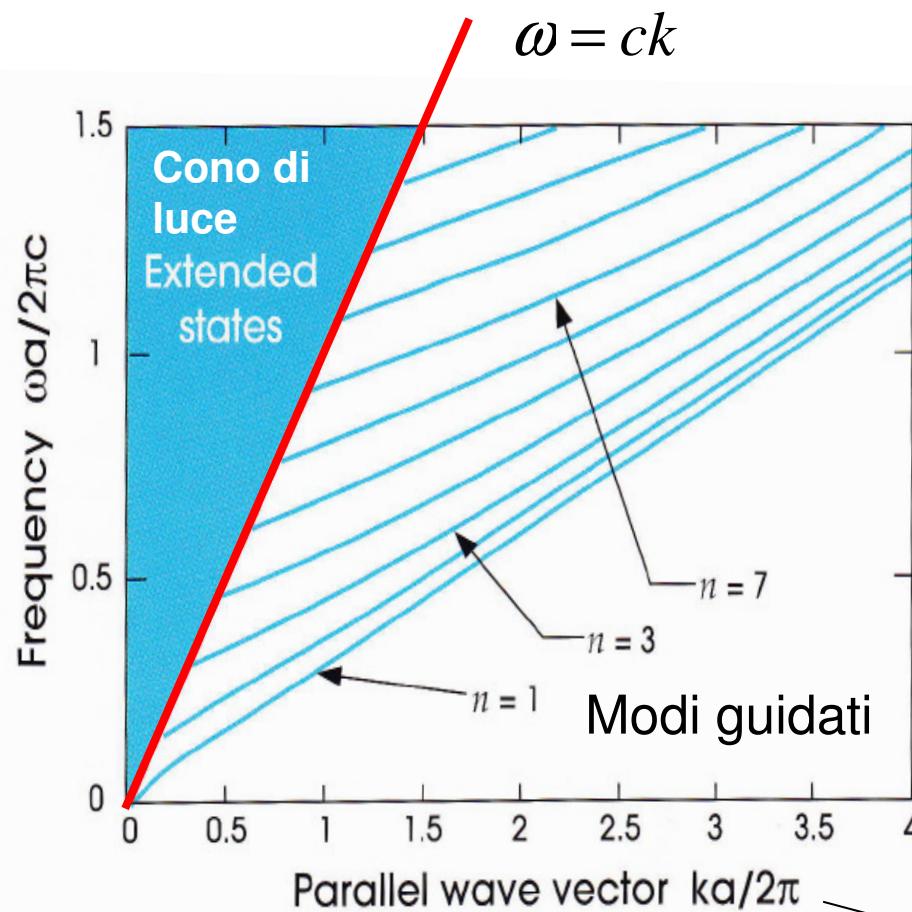
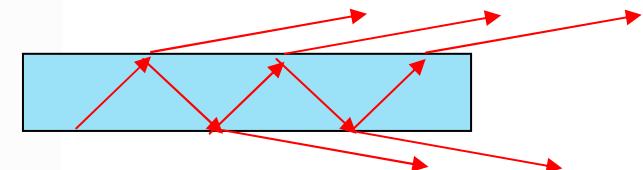


Figure 3: Harmonic mode frequencies for a plane of glass of thickness a and $\epsilon = 11.4$.

Blue lines correspond to modes that are localized in the glass. The shaded blue region is a continuum of states that extend into both the glass and the air around it. The red line is the **light line** $\omega = ck$. This plot shows modes of only one polarization, for which \mathbf{H} is perpendicular to both the z and k directions.

Entro il cono di luce



Oltre il cono di luce

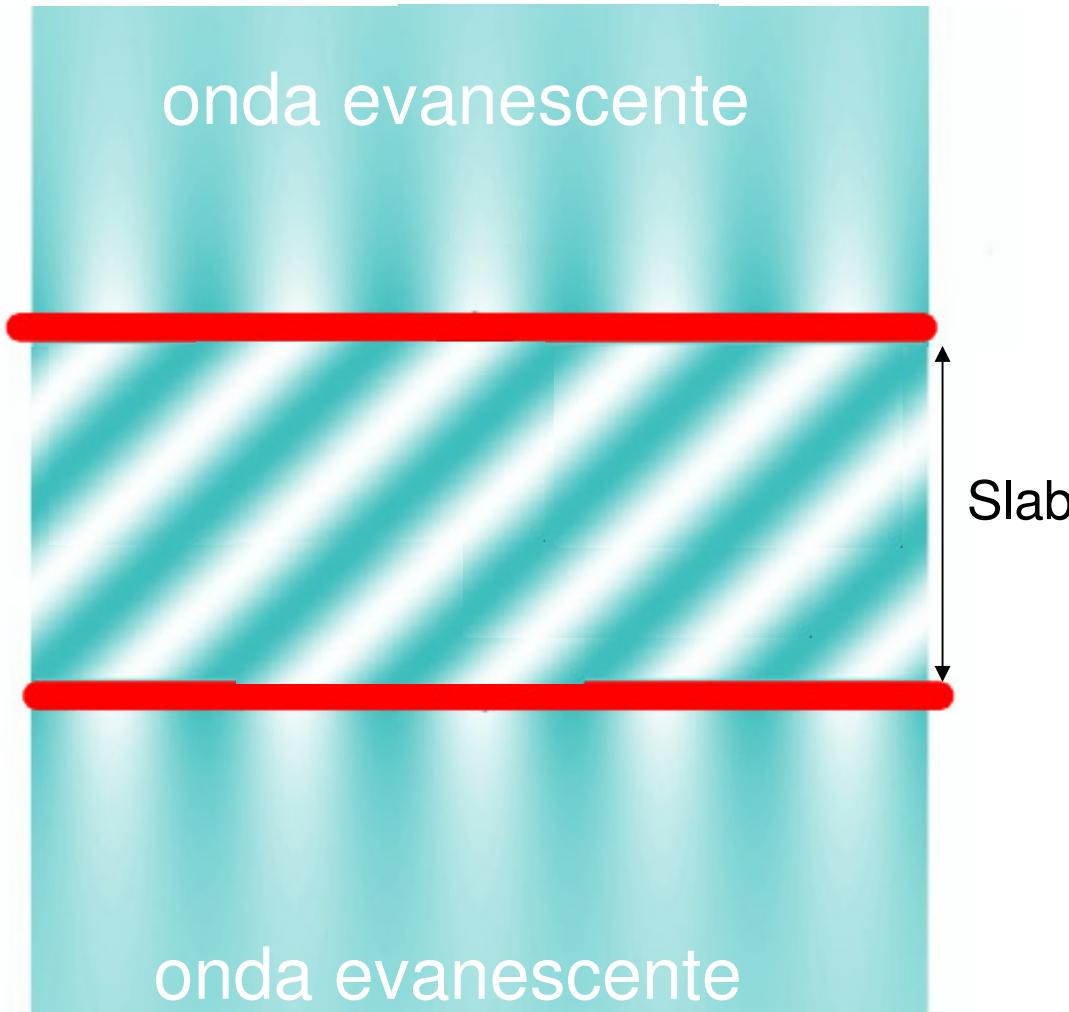


Segue da legge di scala

Rappresentazione modi guidati: Confinamento 1D della luce

$$\sin \theta_i > \frac{n_2}{n_1}$$

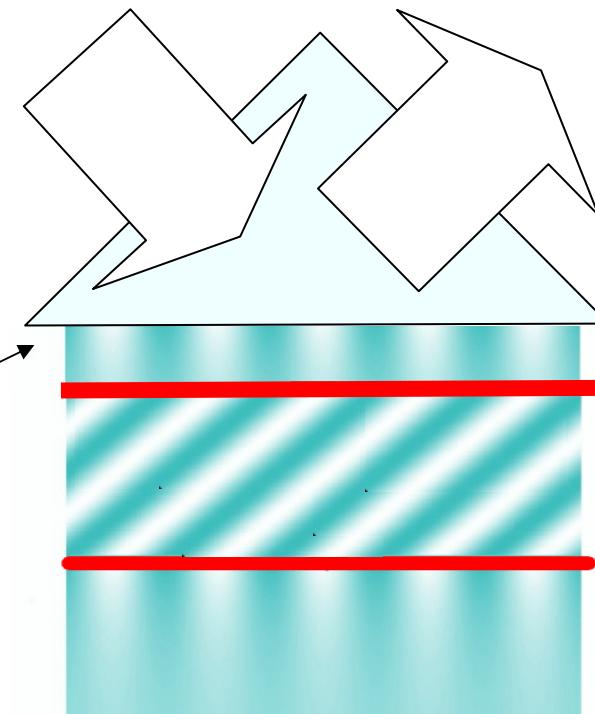
Complicato
eccitarli
dall'esterno,
ma i modi
guidati
esistono



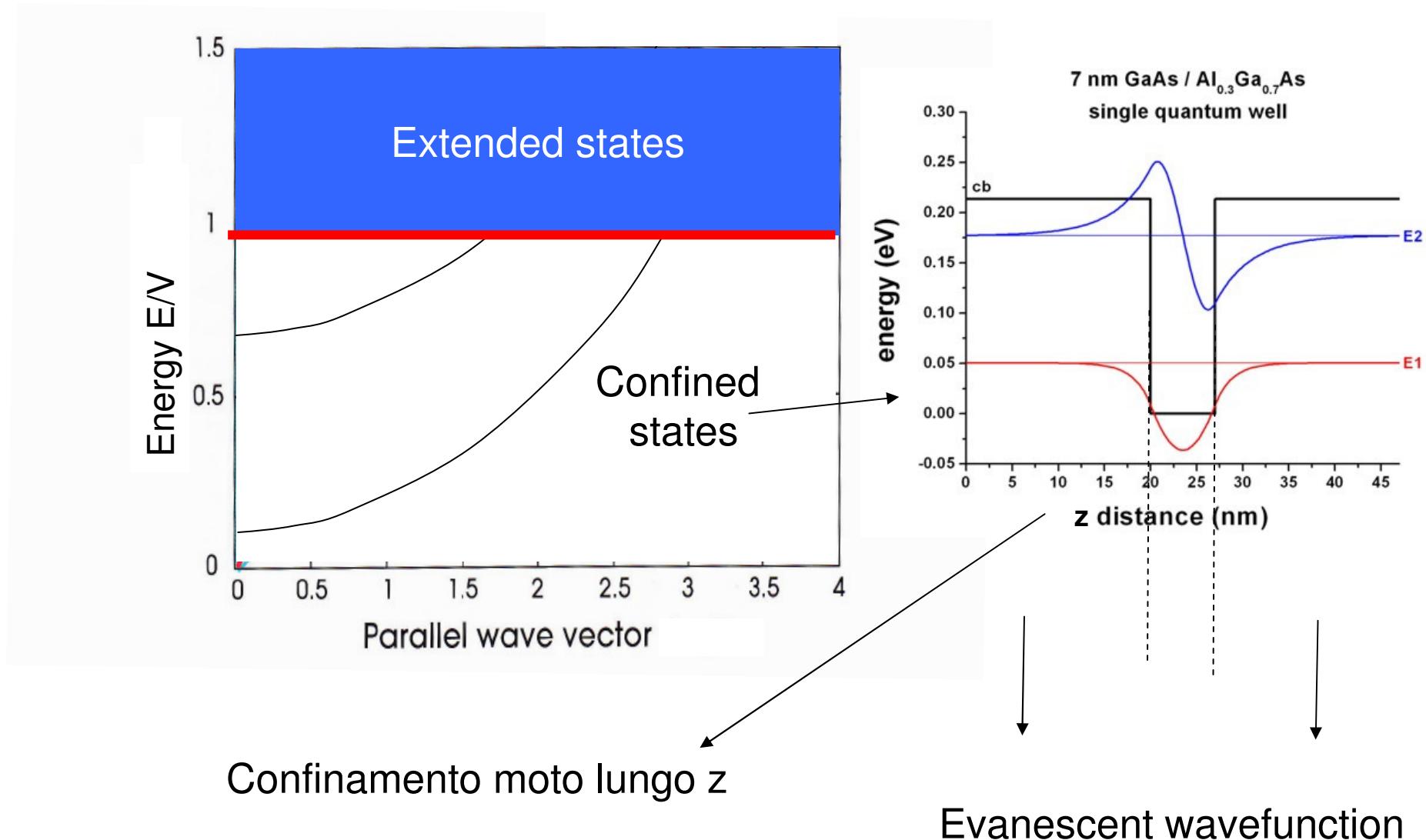
Eccitazione modi guidati:

i modi guidati possono essere eccitati ad esempio con prisma a riflessione totale interna

Frustrated total internal reflection



Analogia con QWell

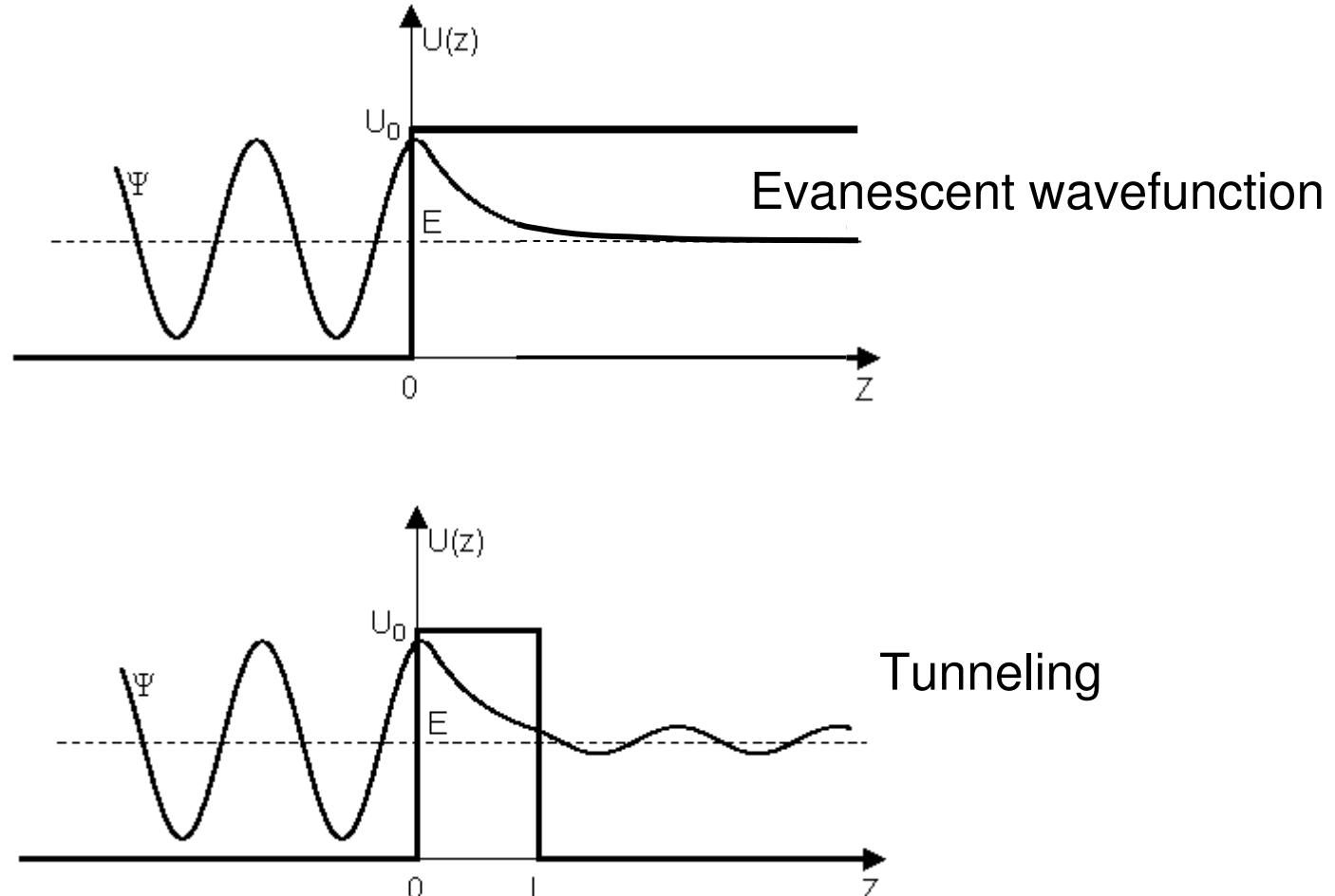


Analogia con MQ

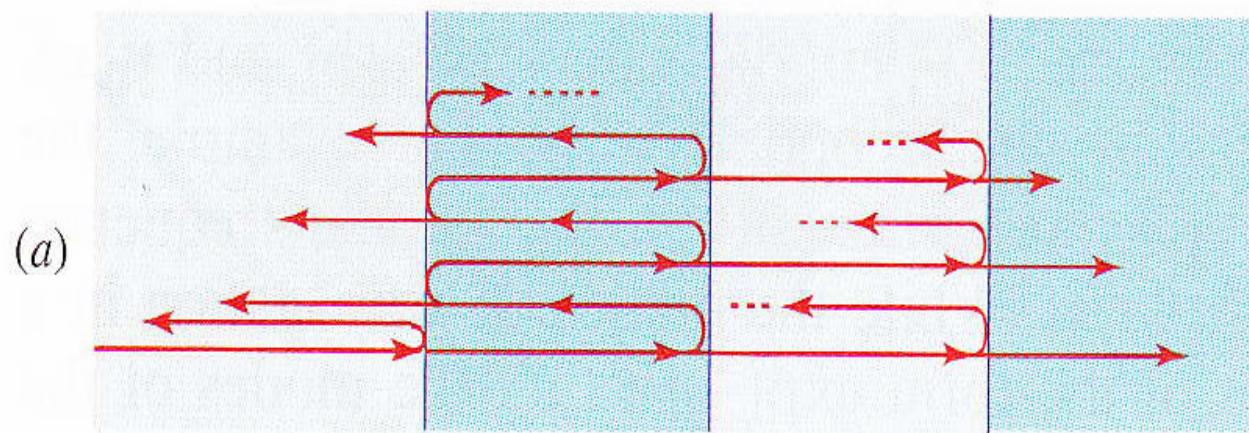
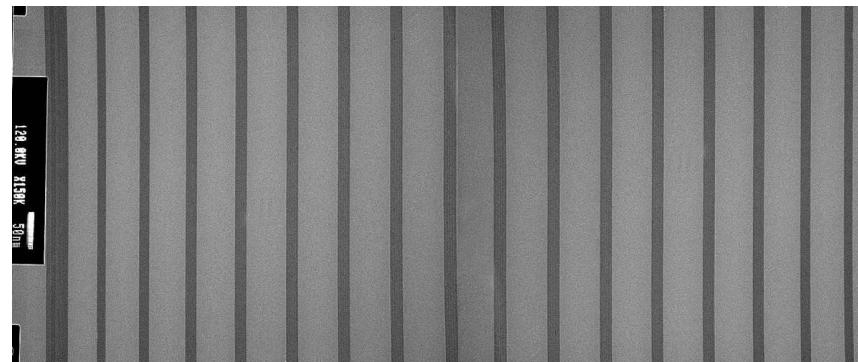
Photonics

Total internal reflection

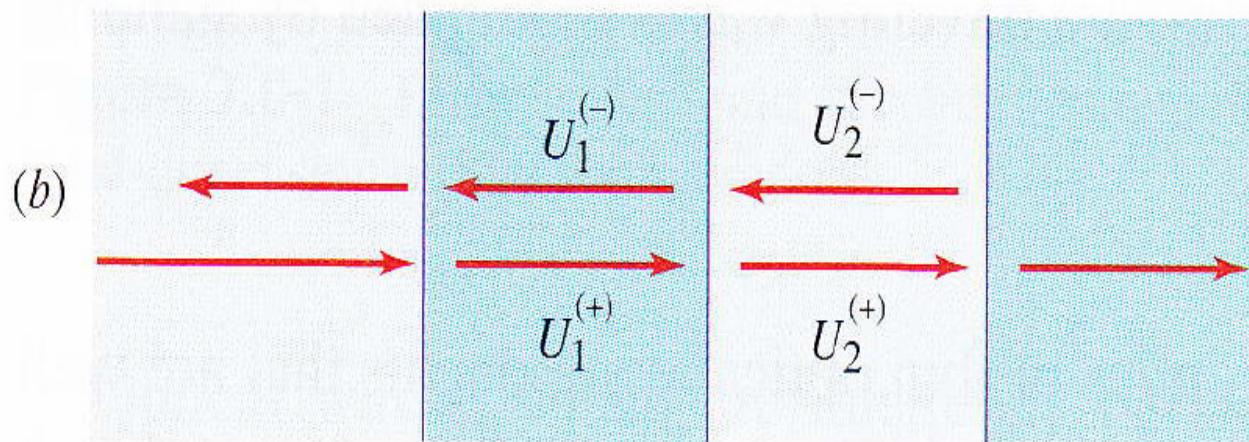
FTIR



Sistema 1D periodico



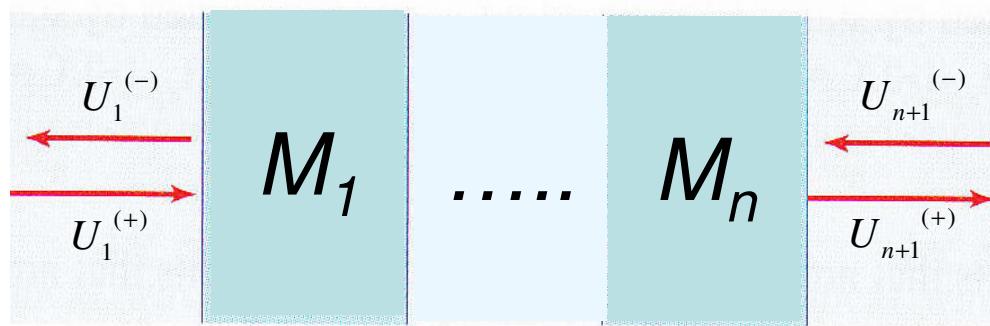
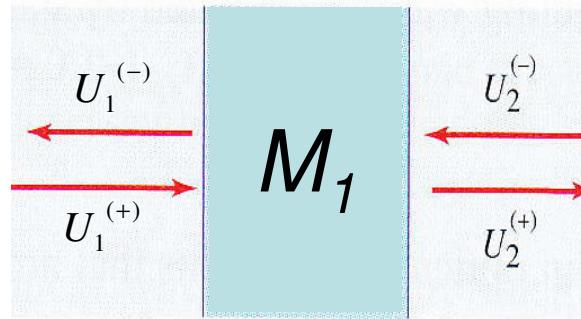
Visione
cinetica



Visione
statica

Metodo matrici M

$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

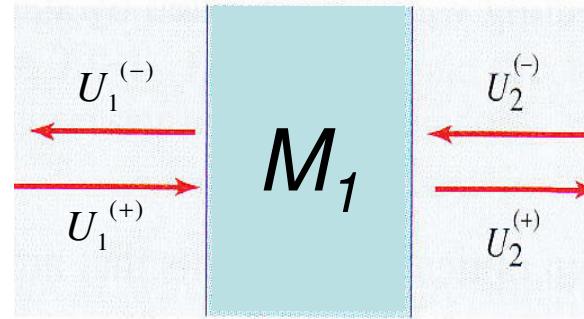


$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix} ; \begin{bmatrix} U_3^{(+)} \\ U_3^{(-)} \end{bmatrix} = M_2 \begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} ; \dots ; \begin{bmatrix} U_{n+1}^{(+)} \\ U_{n+1}^{(-)} \end{bmatrix} = M_n \begin{bmatrix} U_n^{(+)} \\ U_n^{(-)} \end{bmatrix}$$

$$\begin{bmatrix} U_{n+1}^{(+)} \\ U_{n+1}^{(-)} \end{bmatrix} = M_n \dots M_2 M_1 \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix} = M \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

Mezzo simmetrico e privo di perdite

$$M_1 = \begin{bmatrix} 1 & r \\ \frac{t^*}{r^*} & \frac{t}{t^*} \\ r^* & 1 \\ \frac{t^*}{t} & \frac{t}{t^*} \end{bmatrix}$$

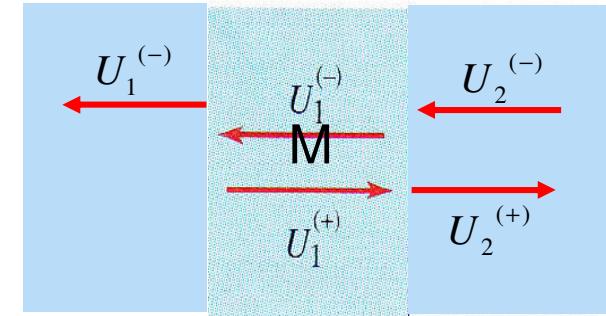


$$\frac{r}{t} = -\frac{r^*}{t^*} \quad \arg\{t\} - \arg\{r\} = \pm \frac{\pi}{2}$$

Fase associata a r e t
differisce di $\pi/2$

Significato di t e r

$$U_1^{(+)} = 0$$



$$U_2^{(+)} = \frac{r}{t} U_1^{(-)}$$

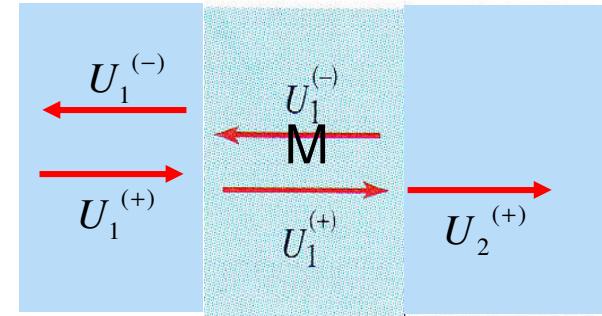
$$U_2^{(-)} = \frac{1}{t} U_1^{(-)}$$

$$U_1^{(-)} = t U_2^{(-)} \quad U_2^{(+)} = r U_2^{(-)}$$

Vera anche per sistema non simmetrico!

Significato di t e r

$$U_2^{(-)} = 0$$



$$U_2^{(+)} = \frac{1}{t^*} U_1^{(+)} + \frac{r}{t} U_1^{(-)} \quad \Rightarrow \quad U_2^{(+)} = \left(\frac{1}{t^*} - \frac{|r|^2}{t^*} \right) U_1^{(+)} = \frac{1 - |r|^2}{t^*} U_1^{(+)}$$

$$0 = \frac{r^*}{t^*} U_1^{(+)} + \frac{1}{t} U_1^{(-)} \quad \Rightarrow \quad U_1^{(-)} = r U_1^{(+)}$$

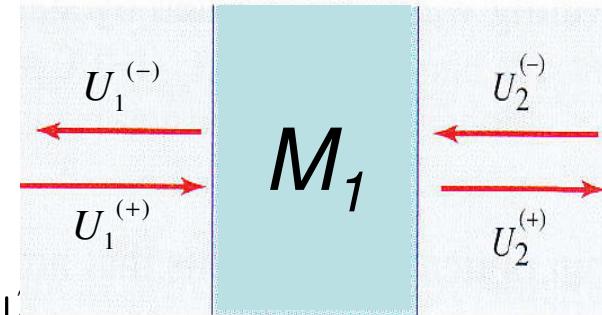
Essendo $\frac{r}{t} = -\frac{r^*}{t^*}$ e se $|r|^2 + |t|^2 = 1$

$$U_2^{(+)} = t U_1^{(+)} \quad U_1^{(-)} = r U_1^{(+)}$$

Vera solo per sistema simmetrico

Verifica conservazione flusso

$$|U_2^{(+)}|^2 - |U_2^{(-)}|^2 = |U_1^{(+)}|^2 - |U_1^{(-)}|^2 =$$



$$= \left| \frac{1}{t^*} U_2^{(+)} + \frac{r}{t} U_2^{(-)} \right|^2 - \left| \frac{r^*}{t^*} U_2^{(+)} + \frac{1}{t} U_2^{(-)} \right|^2 =$$

$$= \left| \frac{1}{t^*} U_2^{(+)} \right|^2 + \left| \frac{r}{t} U_2^{(-)} \right|^2 - \left| \frac{r^*}{t^*} U_2^{(+)} \right|^2 - \left| \frac{1}{t} U_2^{(-)} \right|^2$$

$$\left(\frac{1}{|t|^2} - \left| \frac{r}{t} \right|^2 \right) |U_2^{(+)}|^2 - \left(\frac{1}{|t|^2} - \left| \frac{r^*}{t^*} \right|^2 \right) |U_2^{(-)}|^2$$

\Rightarrow

$$1 - |r|^2 = |t|^2$$

A proposito ricordiamo che la conservazione in generale è:

$$|\vec{S}| = |\vec{E} \times \vec{H}^*| = \left| \vec{E} \times \left(\frac{\vec{k} \times \vec{E}^*}{\mu\omega} \right) \right|$$

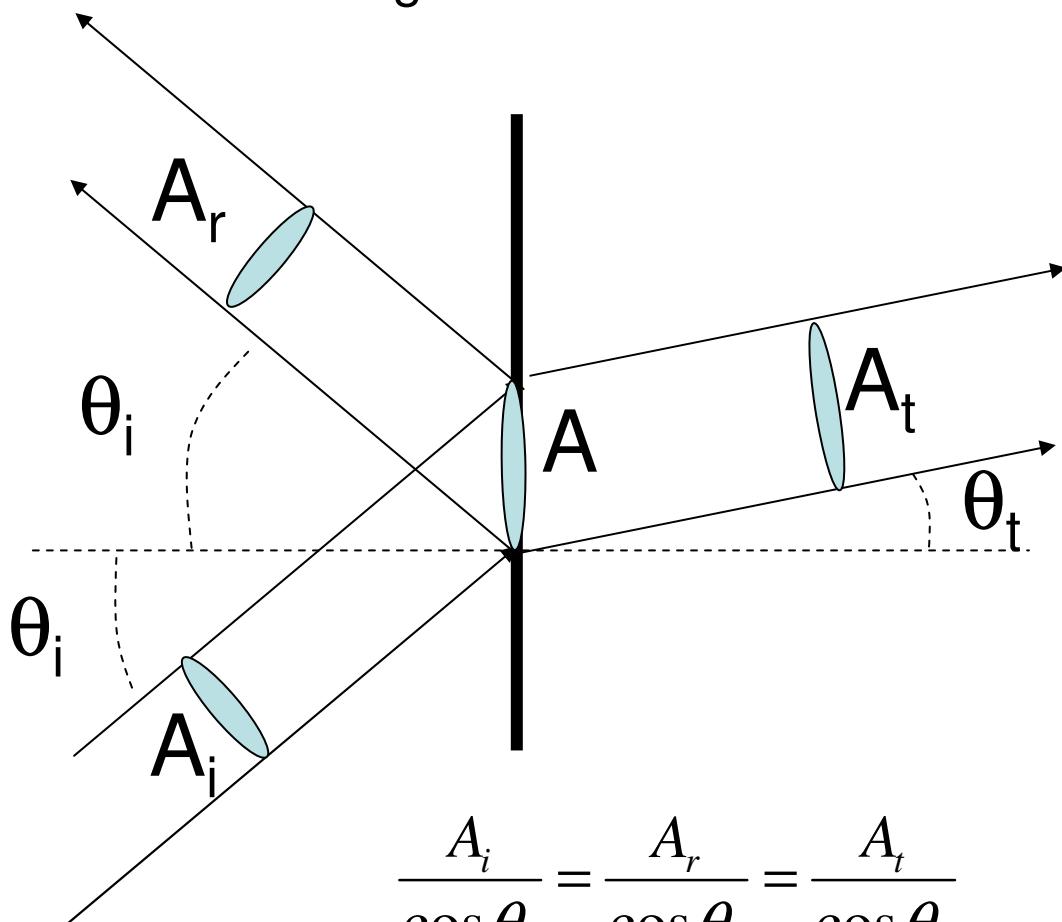
$$= \frac{n}{\mu c} |\vec{E}|^2 = \frac{\epsilon c}{n} |\vec{E}|^2 = n c |\vec{E}|^2$$

Conservazione energia

$$|\vec{S}_i| A_i = |\vec{S}_r| A_r + |\vec{S}_t| A_t$$

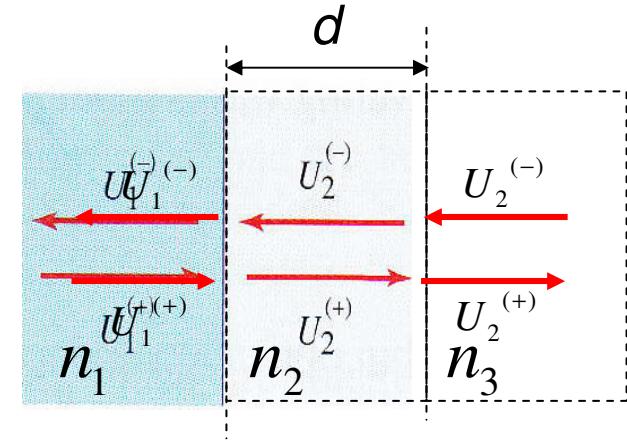
$$n_1 |\vec{E}_i|^2 A_i = n_1 |\vec{E}_r|^2 A_r + n_2 |\vec{E}_t|^2 A_t$$

$$1 = |r|^2 + |t|^2 \frac{n_2}{n_1} \frac{\cos \theta_i}{\cos \theta_t}$$



$$\frac{A_i}{\cos \theta_i} = \frac{A_r}{\cos \theta_r} = \frac{A_t}{\cos \theta_t}$$

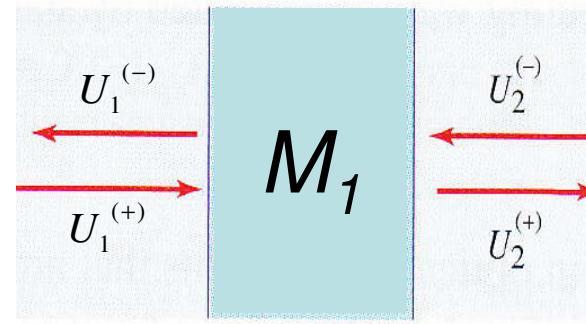
Conservazione energia in sistema asimmetrico



$$1 - |r|^2 = \frac{n_3}{n_1} |t|^2$$

Mezzo simmetrico e privo di perdite

$$M_1 = \begin{bmatrix} 1 & r \\ \bar{t^*} & \bar{t} \\ \bar{r^*} & 1 \\ \frac{1}{t^*} & \frac{r}{t} \end{bmatrix}$$



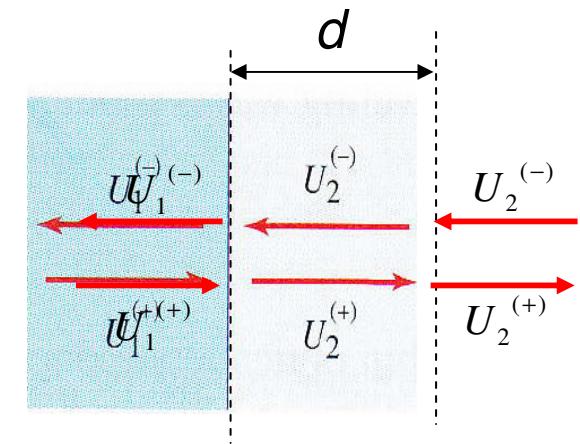
$$\det M_1 = \frac{1}{|t|^2} - \frac{|r|^2}{|t|^2} = 1$$

Trasformazione unitaria

Propagazione attraverso un mezzo omogeneo

$$U^{(+)}(z) = E^{(+)} e^{i(kz - \omega t)} \quad U^{(-)}(z) = E^{(-)} e^{i(-kz - \omega t)}$$

$$\begin{aligned} U_2^{(+)} &= E^{(+)}(z+d) & U_2^{(-)} &= E^{(-)}(z+d) \\ U_1^{(+)} &= E^{(+)}(z) & U_1^{(-)} &= E^{(-)}(z) \\ U_2^{(+)} &= U_1^{(+)} e^{ikd} & U_2^{(-)} &= U_1^{(-)} e^{-ikd} \end{aligned}$$



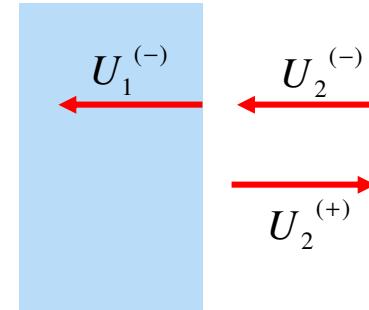
Ricordando

$$U_2^{(+)} = t U_1^{(+)} \rightarrow t = e^{ikd} \rightarrow 1/t^* = e^{ikd}$$

$$M = \begin{bmatrix} \exp(j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix} \quad \varphi = \frac{2\pi}{\lambda} nd$$

Singola interfaccia dielettrica
Sistema asimmetrico, vale

$$U_1^{(-)} = t U_2^{(-)} \quad U_2^{(+)} = r U_2^{(-)}$$



con t e r definiti da dx verso sx M

Relazioni di Fresnel ad incidenza normale da dx verso sx

$$\begin{cases} E_r = \frac{n_2 - n_1}{n_1 + n_2} E_i = r E_i \\ E_t = \frac{2n_2}{n_1 + n_2} E_i = t E_i \end{cases}$$

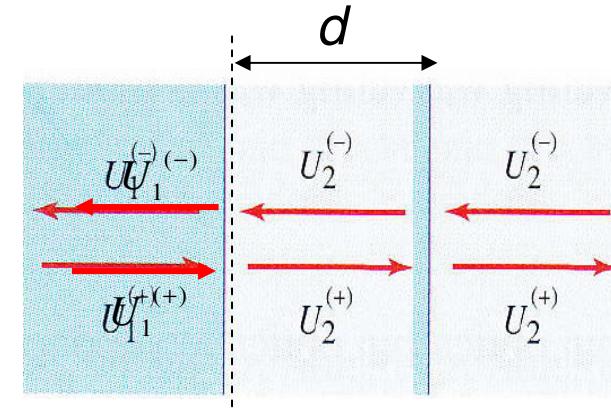
→

$$M = \begin{bmatrix} \frac{n_2 + n_1}{2n_2} & \frac{n_2 - n_1}{2n_2} \\ \frac{n_2 - n_1}{2n_2} & \frac{n_2 + n_1}{2n_2} \end{bmatrix}$$

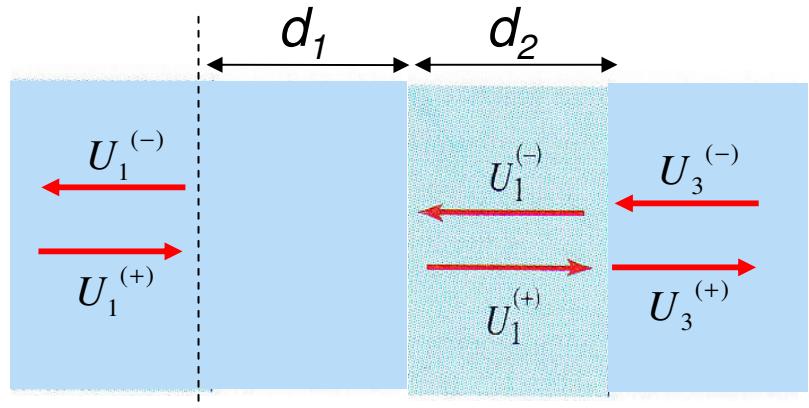
Propagazione attraverso un mezzo omogeneo
seguita da una interfaccia dielettrica

$$M = \begin{bmatrix} \frac{n_2 + n_1}{2n_2} & \frac{n_2 - n_1}{2n_2} \\ \frac{n_2 - n_1}{2n_2} & \frac{n_2 + n_1}{2n_2} \end{bmatrix} \begin{bmatrix} \exp(j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{n_2 + n_1}{2n_2} \exp(j\varphi) & \frac{n_2 - n_1}{2n_2} \exp(-j\varphi) \\ \frac{n_2 - n_1}{2n_2} \exp(j\varphi) & \frac{n_2 + n_1}{2n_2} \exp(-j\varphi) \end{bmatrix} \quad \varphi = \frac{2\pi}{\lambda} nd$$



Propagazione attraverso un mezzo omogeneo seguita da una slab dielettrica



$$M = \begin{bmatrix} \frac{n_2 + n_1}{2n_2} \exp(j\varphi_2) & \frac{n_1 - n_2}{2n_2} \exp(-j\varphi_2) \\ \frac{n_2 - n_1}{2n_2} \exp(j\varphi_2) & \frac{n_2 + n_1}{2n_2} \exp(-j\varphi_2) \end{bmatrix} ; \quad \varphi_i = \frac{2\pi}{\lambda} n_i d_i$$

$$\times \begin{bmatrix} \frac{n_2 + n_1}{2n_1} \exp(j\varphi_1) & \frac{n_2 - n_1}{2n_1} \exp(-j\varphi_1) \\ \frac{n_2 - n_1}{2n_1} \exp(j\varphi_1) & \frac{n_2 + n_1}{2n_1} \exp(-j\varphi_1) \end{bmatrix} = \begin{bmatrix} 1 & r \\ \frac{r^*}{t^*} & \frac{t}{t^*} \\ \frac{r}{t^*} & \frac{1}{t} \end{bmatrix}$$

Propagazione attraverso un mezzo omogeneo
seguita da una slab dielettrica

$$\begin{aligned} \frac{1}{t^*} &= \frac{(n_2 + n_1)^2}{4n_2 n_1} \exp j(\varphi_2 + \varphi_1) - \frac{(n_2 - n_1)^2}{4n_2 n_1} \exp j(-\varphi_2 + \varphi_1) = \\ &= \frac{\exp(j\varphi_1)}{4n_2 n_1} [(n_2 + n_1)^2 \exp(j\varphi_2) - (n_2 - n_1)^2 \exp(-j\varphi_2)] \end{aligned}$$

$$\begin{aligned} t &= \frac{4n_2 n_1 \exp(j\varphi_1)}{[(n_2 + n_1)^2 \exp(-j\varphi_2) - (n_2 - n_1)^2 \exp(j\varphi_2)]} = \\ &= \frac{4n_2 n_1 \exp(j\varphi_1)}{[-2j(n_2^2 + n_1^2) \sin(\varphi_2) + 4n_2 n_1 \cos(\varphi_2)]} \end{aligned}$$

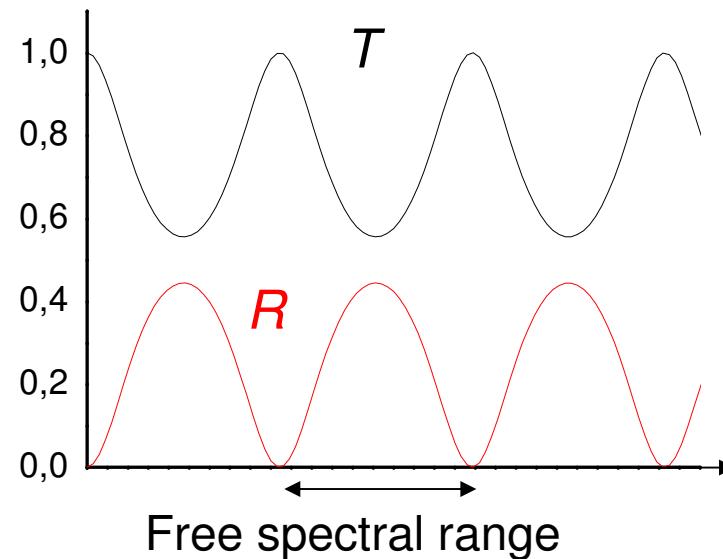
Propagazione attraverso un mezzo omogeneo
seguita da una slab dielettrica

$$t = \exp(j\varphi_1) \frac{2n_2 n_1}{-j(n_2^2 + n_1^2)\sin(\varphi_2) + 2(n_2 n_1)\cos(\varphi_2)}$$

$$T = |t|^2 = \frac{1}{1 + \frac{(n_2^2 - n_1^2)^2}{4n_2^2 n_1^2} \sin^2(\varphi_2)} \quad \varphi_2 = \frac{2\pi}{\lambda} n_2 d_2$$

$$R = 1 - T$$

Formula di Airy



$$\frac{n_2}{n_1} = 2.2$$

Interferenza pellicole



Antireflection coating (in realtà sono multilayer)

$$T = |t|^2 = \frac{1}{1 + \frac{(n_2^2 - n_1^2)^2}{4n_2^2 n_1^2} \sin^2(\varphi_2)}$$

$$\varphi_2 = \frac{\pi}{2} \quad d_2 = \frac{\lambda}{4n_2} \quad T = 1$$



Without Anti reflection

With Anti reflection

$$\varphi_2 = \frac{2\pi}{\lambda} n_2 d_2$$

