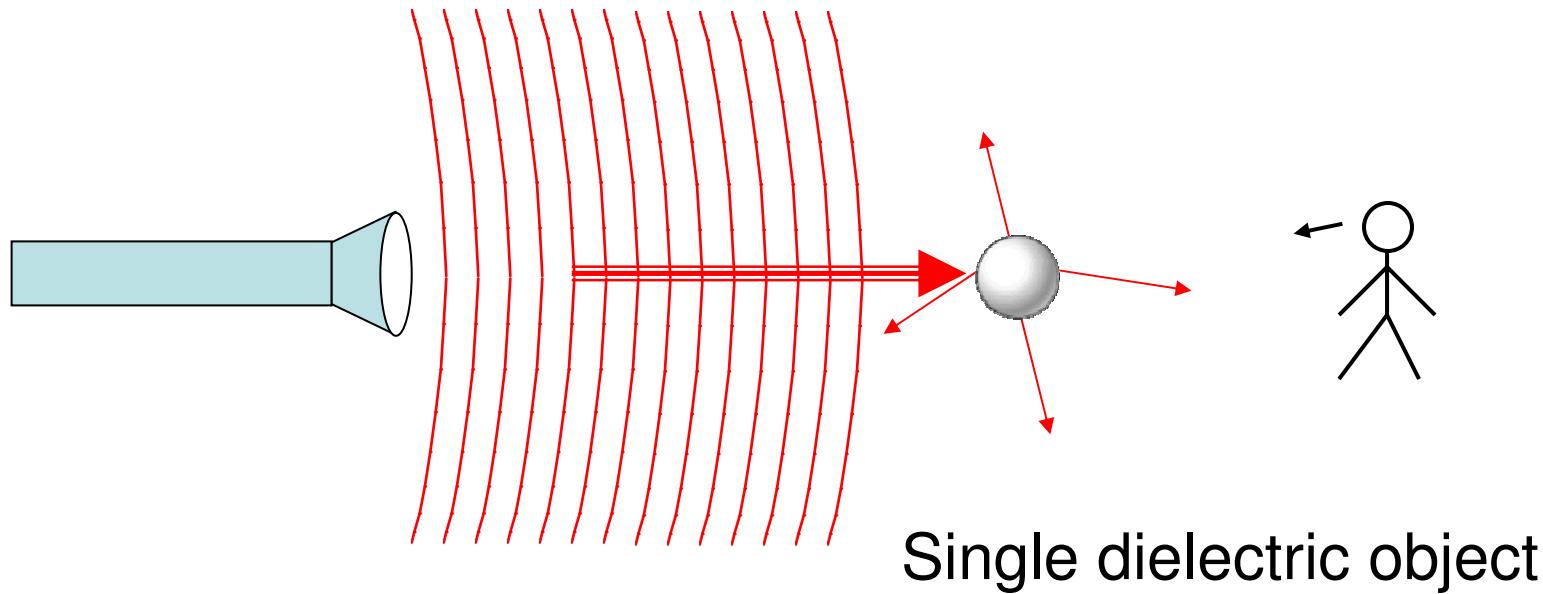
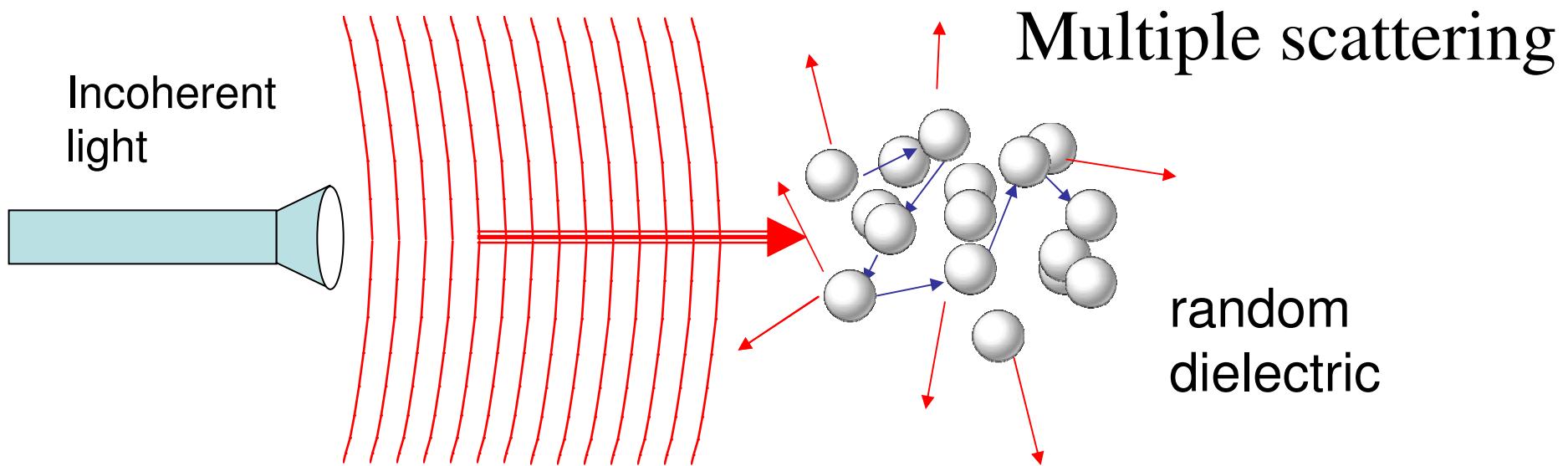
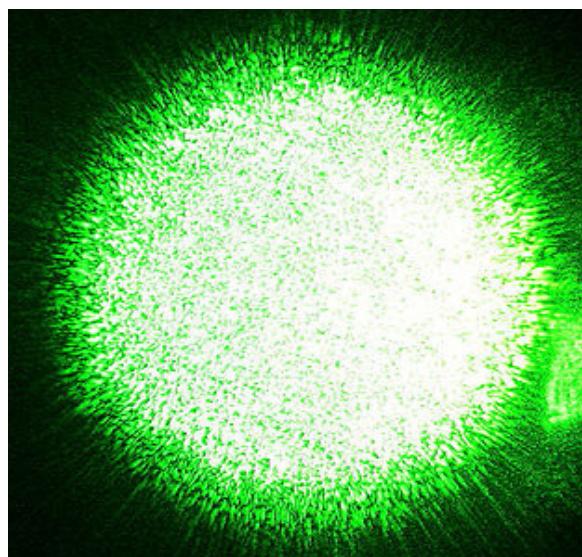
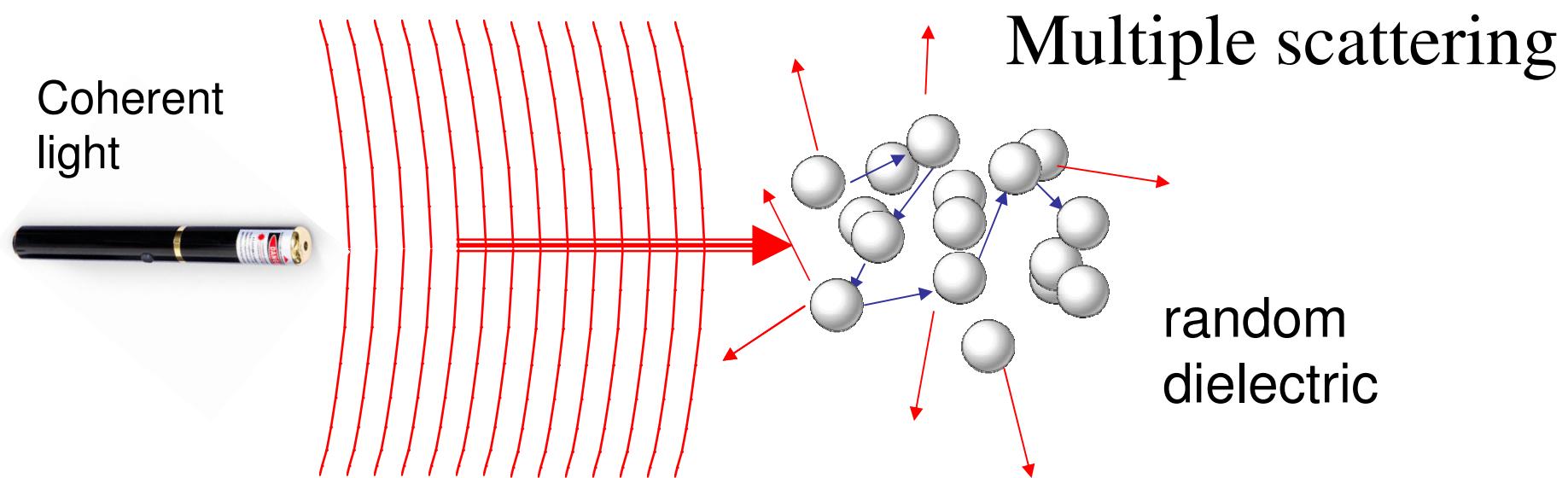


Single scattering

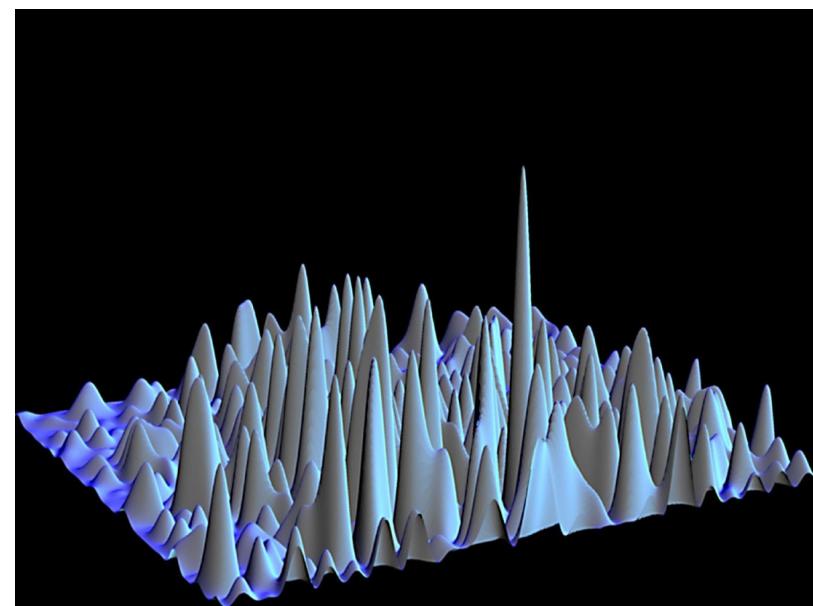


Frequency and phase are maintained

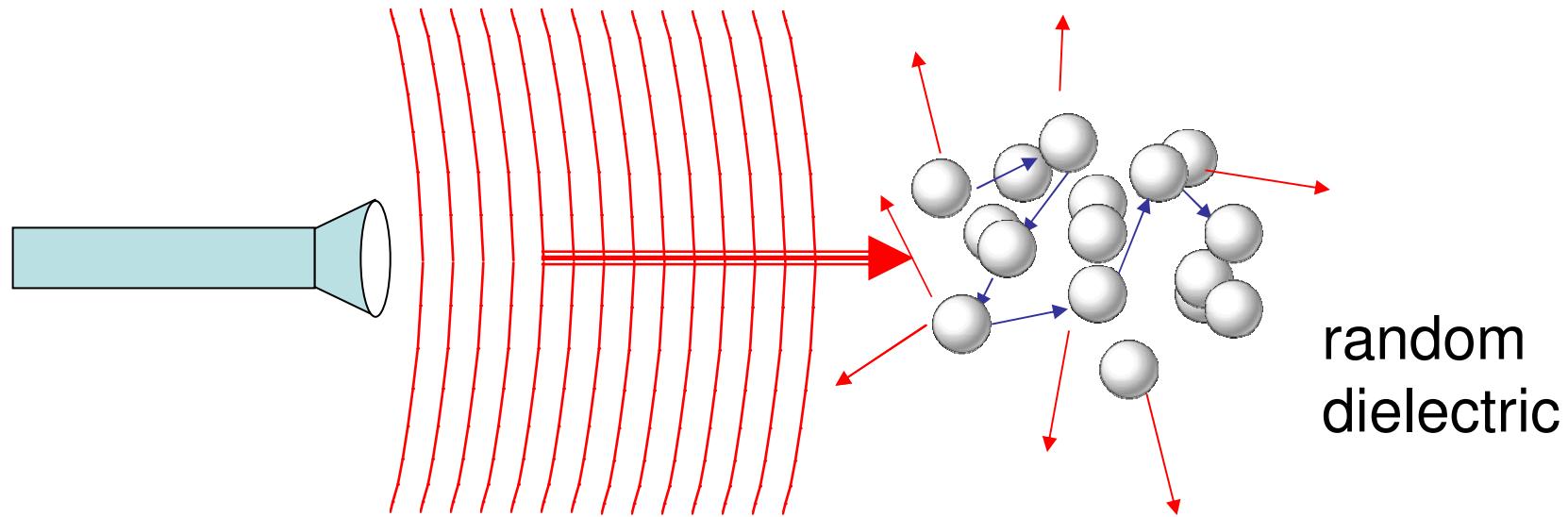




*phase is
maintained*
↓
interference
Speckle
pattern



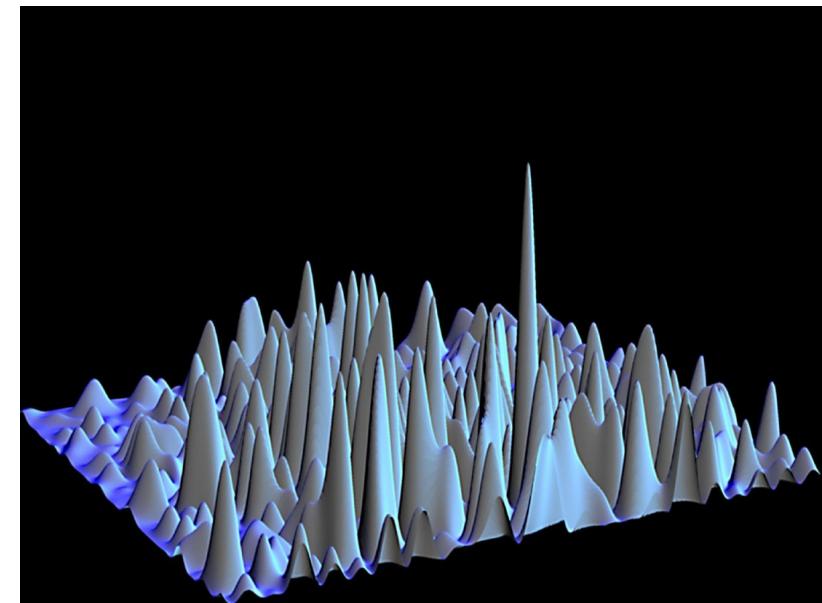
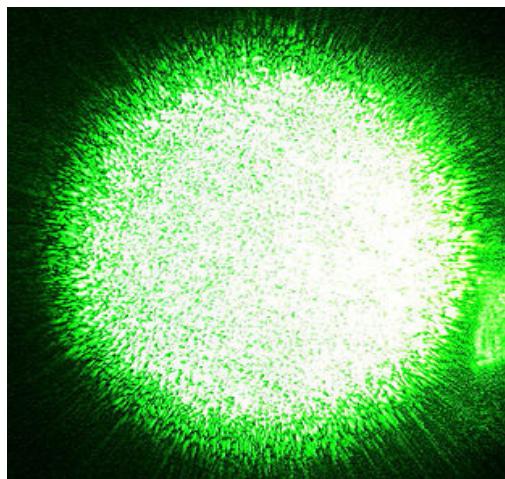
Multiple scattering



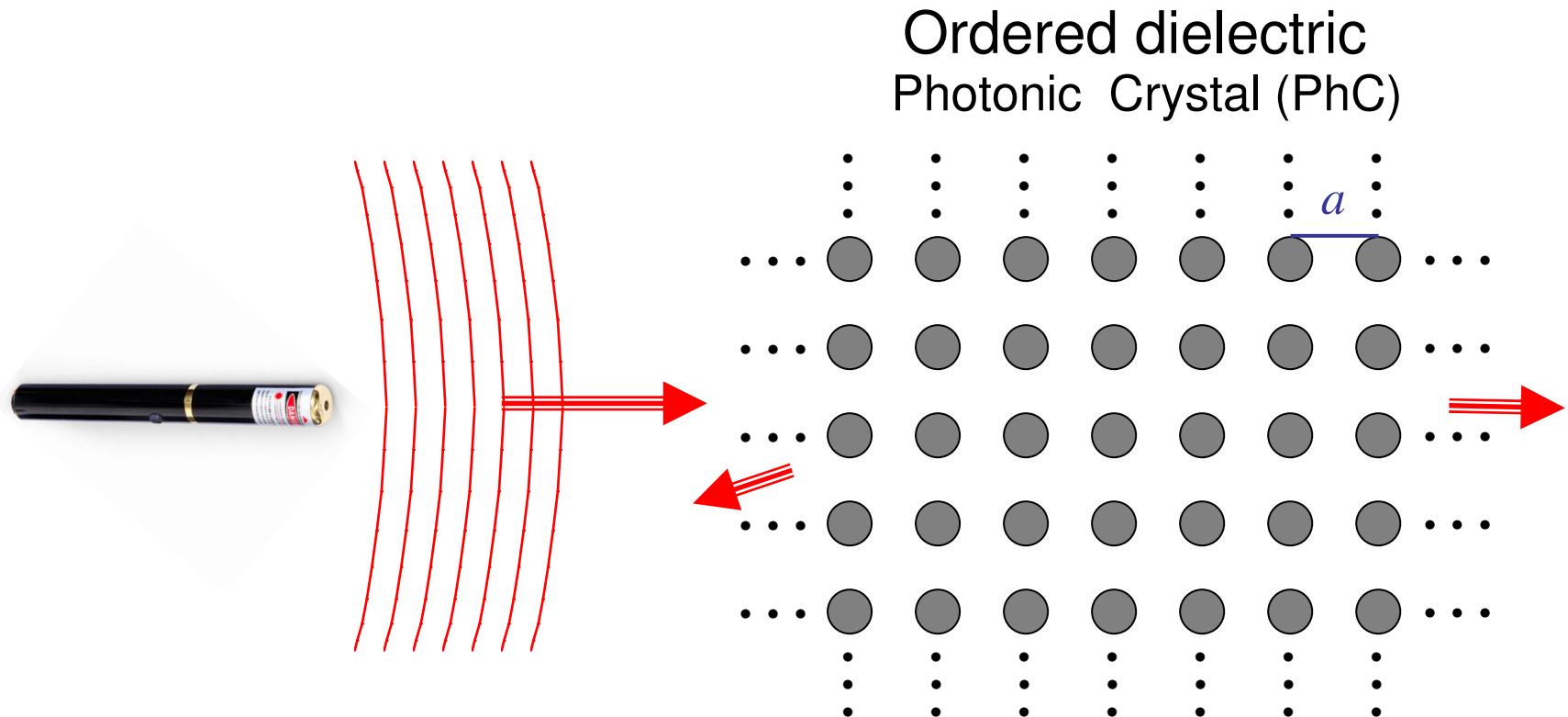
*phase is
maintained*

↓
interference

Speckle
pattern



Bragg scattering



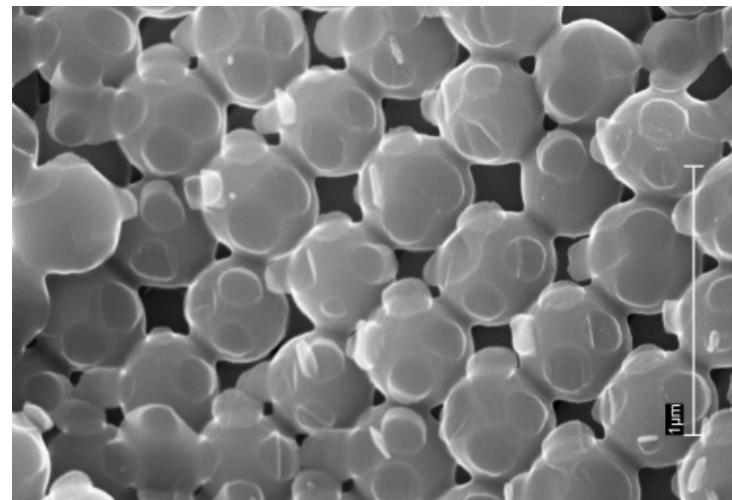
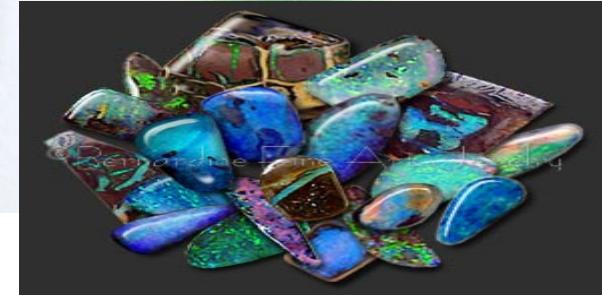
Usually only reflection and refraction

...but for some λ ($\sim 2a$), no light could propagate: a PhC band gap

for most λ , beams propagate through crystal without scattering (scattering cancels coherently)

PhC in natura

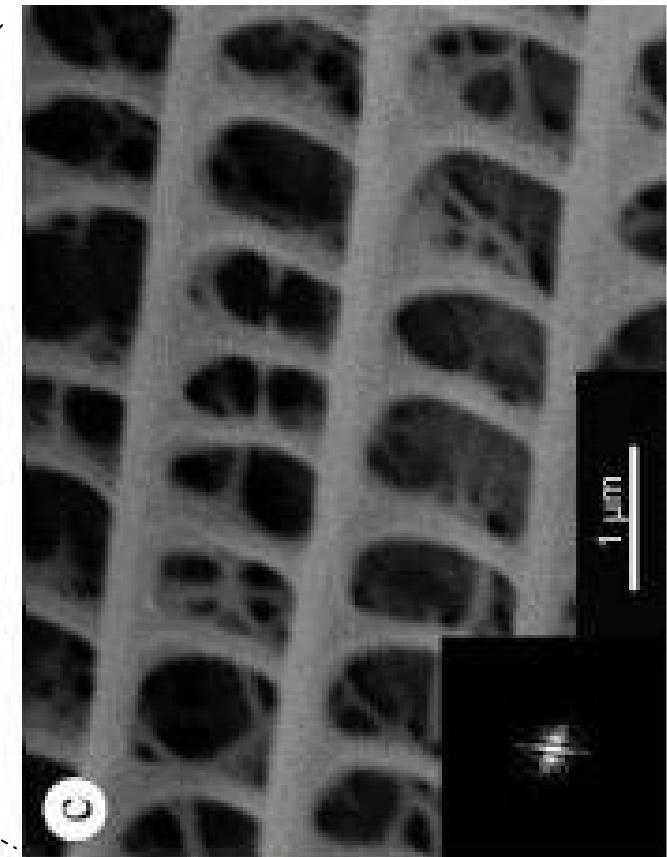
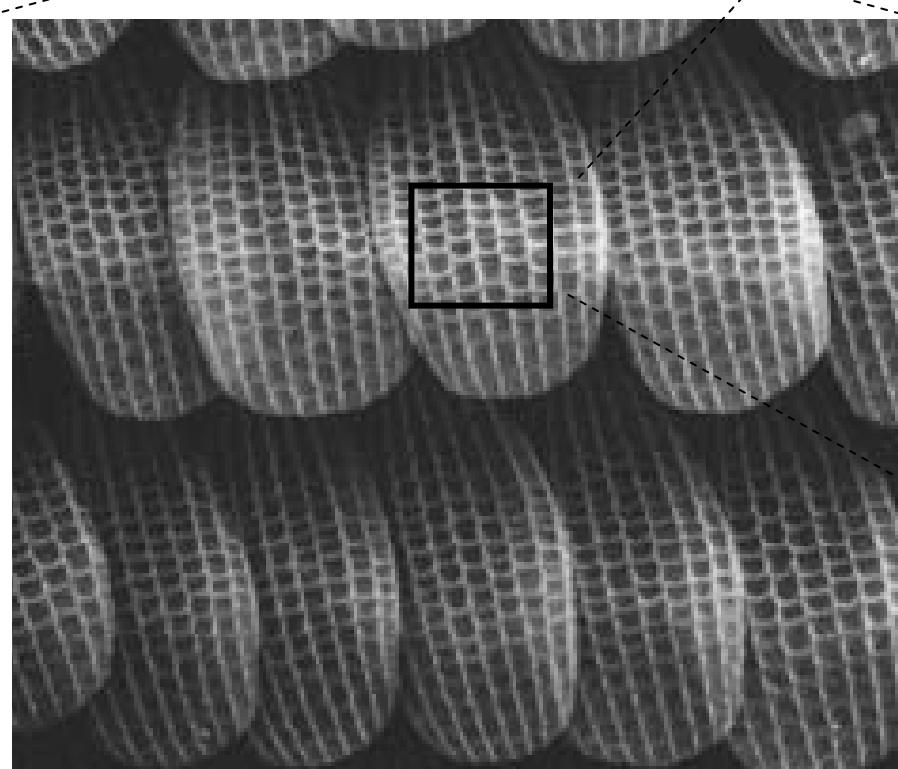
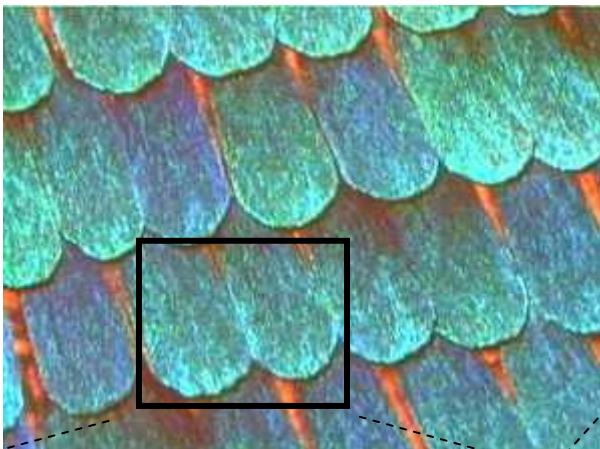
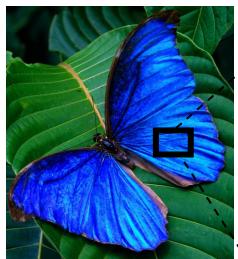
OPALI



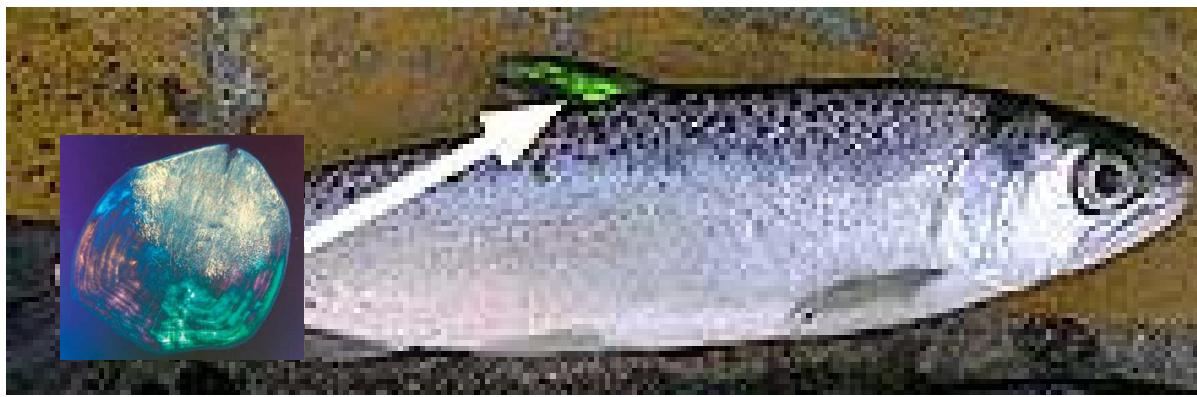
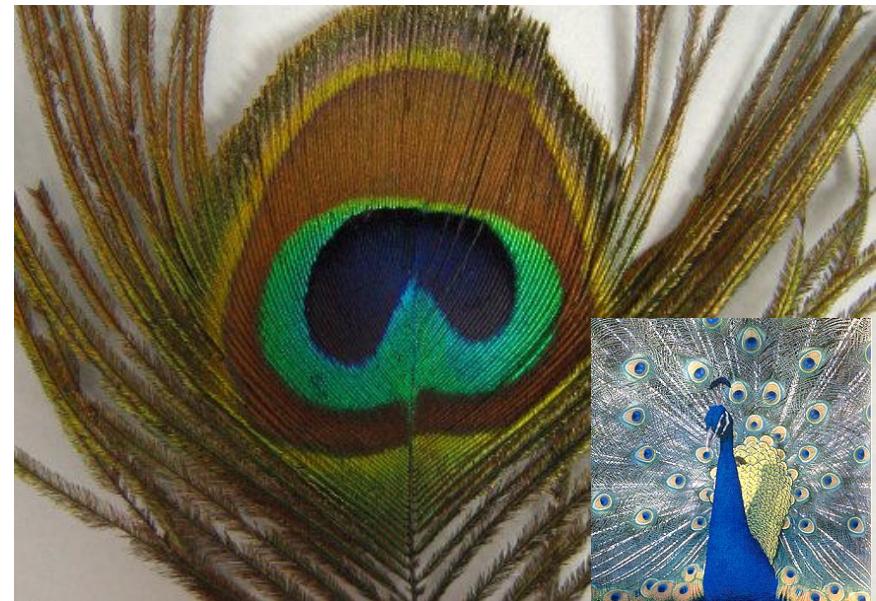
PhC in biologia



PhC in biologia



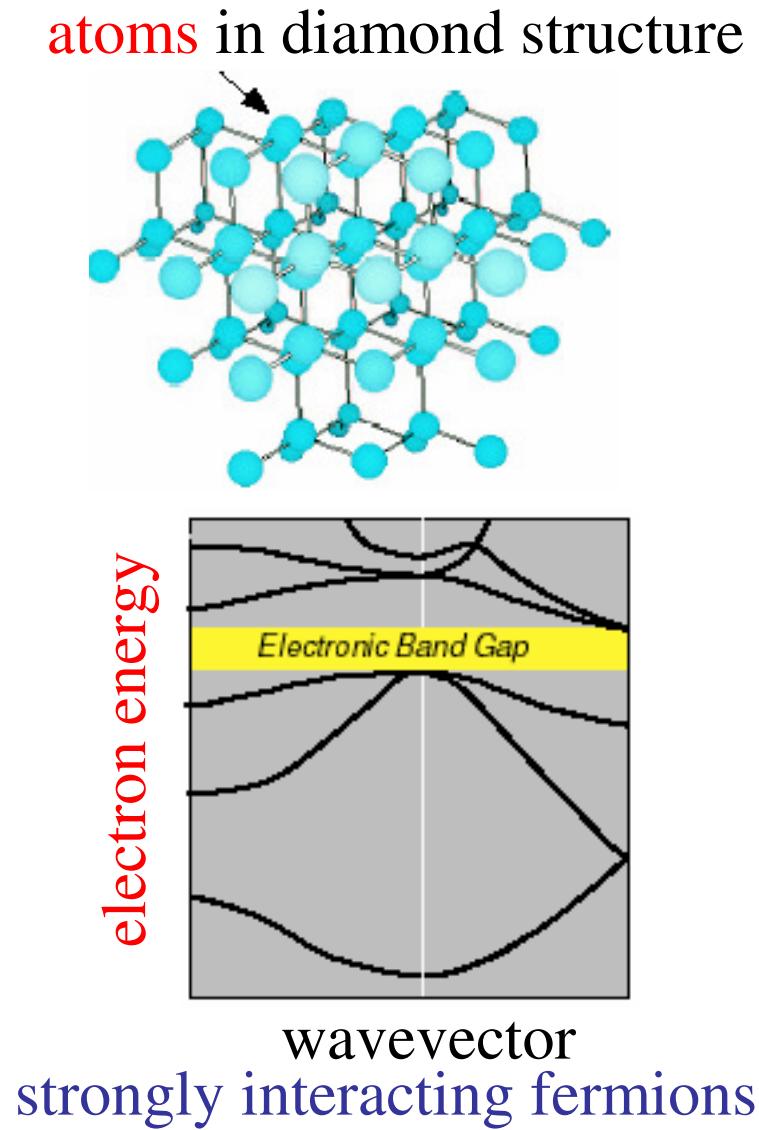
PhC in biologia



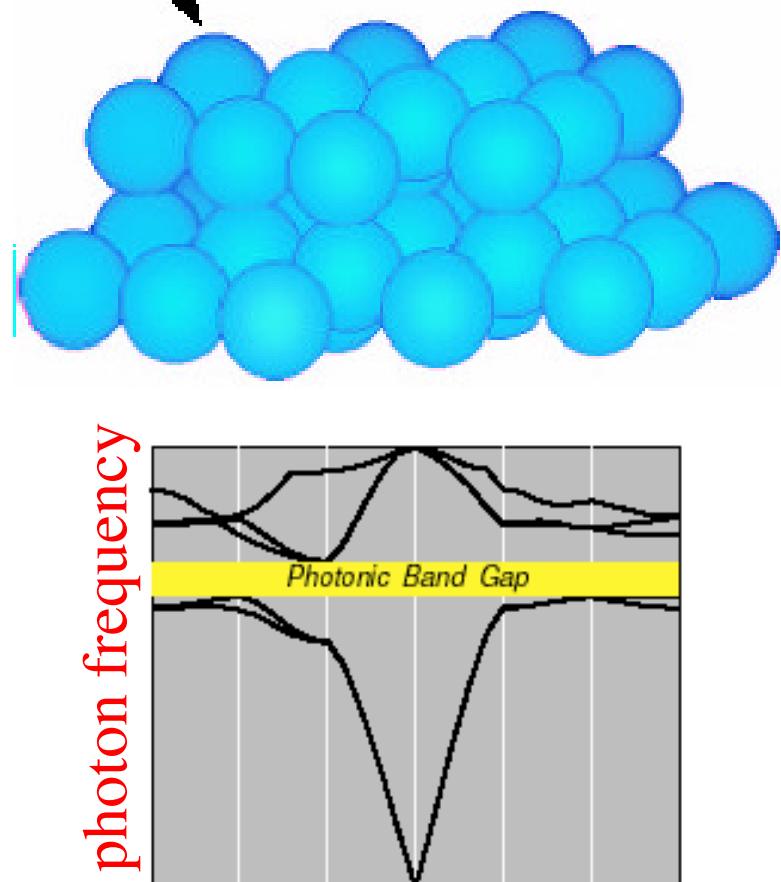
Idea di base: analogia con cristalli

Periodic Medium

Bloch waves: Band Diagram



dielectric spheres, diamond lattice



weakly-interacting bosons

Equazioni Maxwell semplificate nella materia

$$\vec{\nabla} \cdot \vec{D} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} = \epsilon_o \frac{\partial \vec{E}}{\partial t}$$

$$\vec{D} = \epsilon_o \vec{E} + \vec{P} = \epsilon_o \epsilon(\vec{r}) \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M} = \frac{\vec{B}}{\mu}$$

Equazioni onde monocromatiche

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\vec{\nabla} \cdot \epsilon(\vec{r}) \vec{E} = 0$$

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H} \right) = \frac{\omega^2}{c^2} \vec{H}$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

Problema autovalori con
vincolo

$$\vec{E}(\vec{r}) = \frac{i}{\omega \epsilon_o \epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r})$$

Equazioni onde

$$\hat{\Theta} \vec{H}(\vec{r}) = \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}) \right)$$

Master equation

Schroedinger equation

$$\hat{\Theta} \vec{H}(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r})$$

$$\hat{H} \psi = E \psi$$

Linearità

autovalore

$$\hat{\Theta} (\alpha \vec{H}_1(\vec{r}) + \beta \vec{H}_2(\vec{r})) = \alpha \hat{\Theta} \vec{H}_1(\vec{r}) + \beta \hat{\Theta} \vec{H}_2(\vec{r})$$

Hermitianità (ci riconduce alle proprietà della MQ)

Norma:

$$(\vec{F}, \vec{G}) \equiv \int d^3r \vec{F}^*(\vec{r}) \cdot \vec{G}(\vec{r})$$

$$(\vec{F}, \vec{F}) \geq 0$$

$$\forall \vec{F}' \neq 0 \quad \exists \vec{F} = \frac{\vec{F}'}{\sqrt{(\vec{F}', \vec{F}')}} \quad e \quad (\vec{F}, \vec{F}) = 1$$

$\hat{\Xi}$ è Hermitiano se

$$(\vec{F}, \hat{\Xi} \vec{G}) = (\hat{\Xi} \vec{F}, \vec{G})$$

Dimostriamo che $\hat{\Theta}$ è Hermitiano

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3r \underbrace{\vec{F}^*(\vec{r})}_{\vec{B}} \cdot \left[\vec{\nabla} \times \left(\underbrace{\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r})}_{\vec{A}} \right) \right]$$

Usando $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\begin{aligned} & \vec{F}^*(\vec{r}) \cdot \left[\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \right] = \\ &= \vec{\nabla} \cdot \left[\left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \times \vec{F}^*(\vec{r}) \right] + \left[\left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right) \cdot \vec{\nabla} \times \vec{F}^*(\vec{r}) \right] \end{aligned}$$

Trascurando il termine di superficie

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3r \vec{\nabla} \times \vec{F}^*(\vec{r}) \cdot \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{G}(\vec{r}) \right)$$

Dimostriamo che $\hat{\Theta}$ è Hermitiano

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3r \underbrace{\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r})}_{\vec{B}} \cdot \underbrace{(\vec{\nabla} \times \vec{G}(\vec{r}))}_{\vec{A}}$$

Usando $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\begin{aligned} & \frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \cdot (\vec{\nabla} \times \vec{G}(\vec{r})) = \\ & \vec{\nabla} \cdot \left(\vec{G}(\vec{r}) \times \frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \right) + \vec{G}(\vec{r}) \cdot \vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}^*(\vec{r}) \right) \end{aligned}$$

Trascurando il termine di superficie e per ϵ reale

$$(\vec{F}, \hat{\Theta} \vec{G}) = \int d^3r \left[\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{F}(\vec{r}) \right) \right]^* \cdot \vec{G}(\vec{r}) = (\hat{\Theta} \vec{F}, \vec{G}) \quad \therefore$$

Perché H e non E ?

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \epsilon(r) \vec{E}$$

$$\frac{1}{\epsilon(r)} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\omega^2}{c^2} \vec{E}$$

$$\hat{\Sigma} = \frac{1}{\epsilon(r)} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) \quad \hat{\Sigma} \text{ non è Hermitiano}$$

$$\vec{\nabla} \cdot \vec{E} \neq 0 \quad \vec{E} \text{ non è trasverso}$$

Proprietà generali

$$\hat{\Theta} \vec{H}(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r})$$

$$(\vec{H}(\vec{r}), \hat{\Theta} \vec{H}(\vec{r})) = \frac{\omega^2}{c^2} (\vec{H}(\vec{r}), \vec{H}(\vec{r}))$$

$$(\vec{H}(\vec{r}), \hat{\Theta} \vec{H}(\vec{r}))^* = \left(\frac{\omega^2}{c^2} \right)^* (\vec{H}(\vec{r}), \vec{H}(\vec{r}))^*$$



Autovalori reali

Proprietà generali $\vec{\nabla} \cdot [\vec{A} \times \vec{B}] = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B}$

$$\begin{aligned}
 & \frac{\omega^2}{c^2} (\vec{H}(\vec{r}), \vec{H}(\vec{r})) = (\vec{H}(\vec{r}), \hat{\Theta} \vec{H}(\vec{r})) = \\
 &= \int d^3 r \vec{H}^*(\vec{r}) \hat{\Theta} \vec{H}(\vec{r}) = \\
 &= \int d^3 r \vec{H}^*(\vec{r}) \cdot \vec{\nabla} \times \left(\frac{1}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \right) = \\
 &= \int d^3 r \frac{1}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \cdot \vec{\nabla} \times \vec{H}^*(\vec{r}) = \int d^3 r \frac{1}{\epsilon} |\vec{\nabla} \times \vec{H}(\vec{r})|^2
 \end{aligned}$$

→ Autovalori non negativi
 ω reale per ϵ positivo

Proprietà generali

$$\begin{aligned}\omega_1^2 \left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) &= c^2 \left(\vec{H}_2(\vec{r}), \hat{\Theta} \vec{H}_1(\vec{r}) \right) = \\ &= c^2 \left(\hat{\Theta} \vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) = \omega_2^2 \left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right)\end{aligned}$$

$$\left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r}) \right) = 0$$

Ortogonalità autovettori