CORRELAZIONE

CROSS - CORRELAZIONE

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t+\tau)\, d\tau \quad (x * h)$$

DATA LE FUNZIONI x E h OCCORRE <u>SPOSTARE</u> TEMPORALMENTE h, <u>MOLTIPLICARE</u> x PER h SPOSTATO E INFINE <u>INTEGRARE</u>.

AUTO - CORRELAZIONE

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, x(t+\tau)\, d\tau \quad (x * x)$$

$$\int_{-\infty}^{\infty} x(\tau)\, h(t+\tau)\, d\tau$$

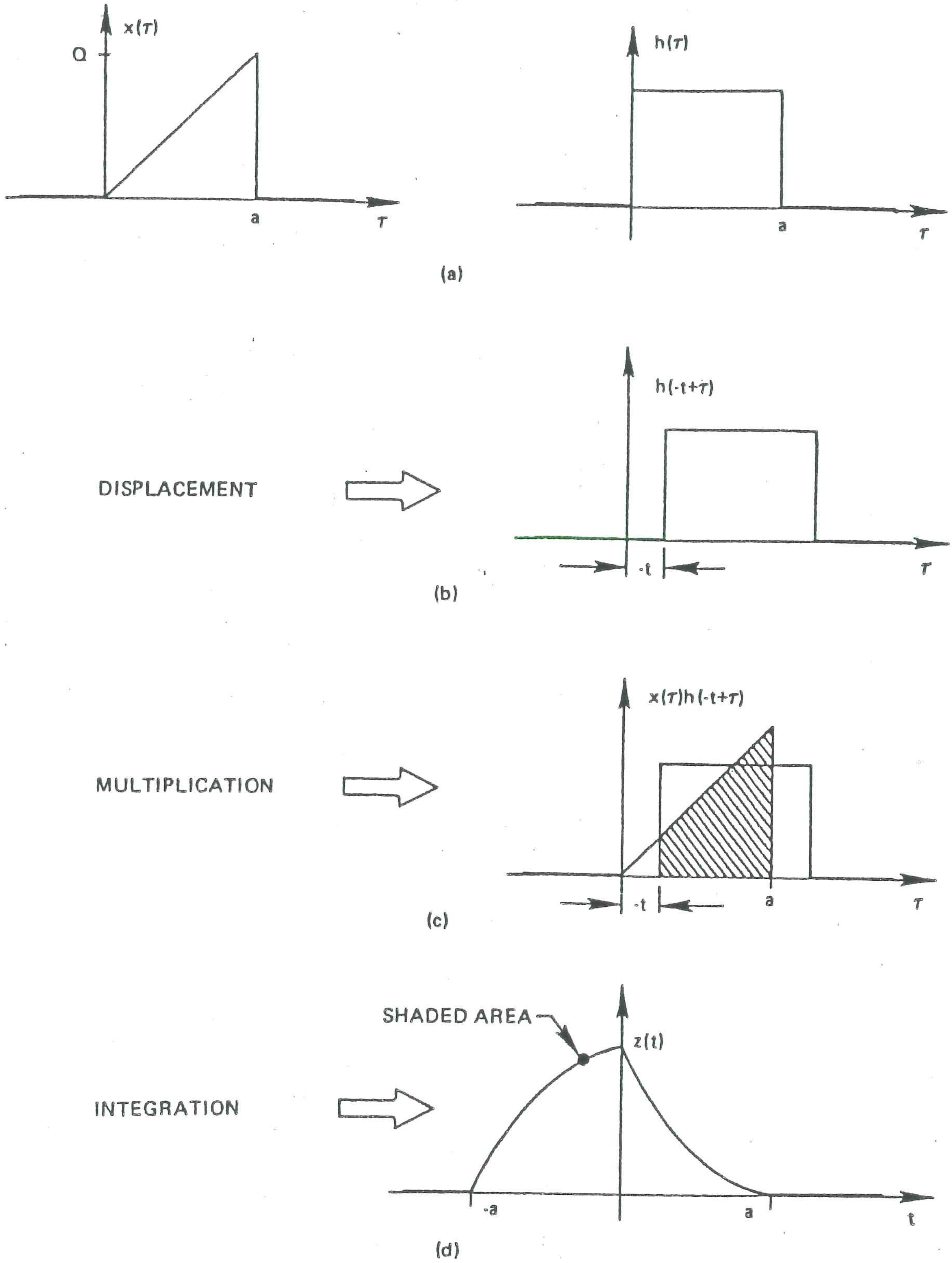

Figure 4-12. Correlation procedure: displacement, multiplication, and integration.

The autocorrelation function

The self-convolution of a function $f(x)$ is given by

$$f * f = \int_{-\infty}^{\infty} f(u)f(x - u)\, du.$$

Suppose, however, that prior to multiplication and integration we do not reverse one of the two component factors; then we have the integral

$$\int_{-\infty}^{\infty} f(u)f(u - x)\, du,$$

which may be denoted by $f \star f$. A single value of $f \star f$ is represented by

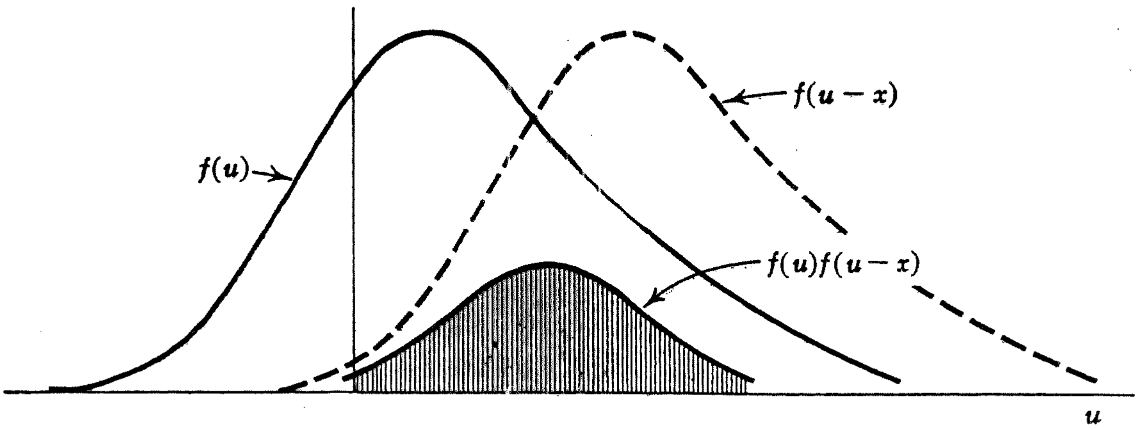

Fig. 3.11 The autocorrelation function represented by an area (shown shaded).

the shaded area in Fig. 3.11. A moment's thought will show that if the function f is to be displaced relative to itself by an amount x (without reversal), then the integral of the product will be the same whether x is positive or negative. In other words, if $f(x)$ is a real function, then $f \star f$ is an even function, a fact which is not true in general of the convolution integral. It follows that

$$f \star f = \int_{-\infty}^{\infty} f(u)f(u - x)\, du = \int_{-\infty}^{\infty} f(u)f(u + x)\, du,$$

which, of course, is deducible from the previous expression by substitution of $w = u - x$.

Appendix

Prove that the autocorrelation of the real function $f(x)$ is a maximum at the origin, that is,

$$\int_{-\infty}^{\infty} f(u)f(u+x)\,du \leqslant \int_{-\infty}^{\infty} [f(u)]^2\,du.$$

Let ϵ be a real number. Then

$$\int_{-\infty}^{\infty} [f(u) + \epsilon f(u+x)]^2\,du > 0$$

and
$$\int_{-\infty}^{\infty} [f(u)]^2\,du + 2\epsilon \int_{-\infty}^{\infty} f(u)f(u+x)\,du + \epsilon^2 \int_{-\infty}^{\infty} [f(u+x)]^2\,du > 0;$$

that is,
$$a\epsilon^2 + b\epsilon + c > 0,$$

where
$$a = c = \int_{-\infty}^{\infty} [f(u)]^2\,du$$
$$b = 2\int_{-\infty}^{\infty} f(u)f(u+x)\,du.$$

Now, if the quadratic expression in ϵ may not be zero, that is, if it has no real root, then

$$b^2 - 4ac \leqslant 0.$$

Hence in this case $b/2 \leqslant a$, or

$$\frac{\int_{-\infty}^{\infty} f(u)f(u+x)\,du}{\int_{-\infty}^{\infty} [f(u)]^2\,du} \leqslant 1.$$

The equality is achieved at $x = 0$; consequently the autocorrelation function can nowhere exceed its value at the origin.