Fermionic systems: just an idea...

Giacomo Roati (lab #45)

Introduction

All the particles in nature belong to two families: Bosons & Fermions

Satyendranath Bose (1894-1974)

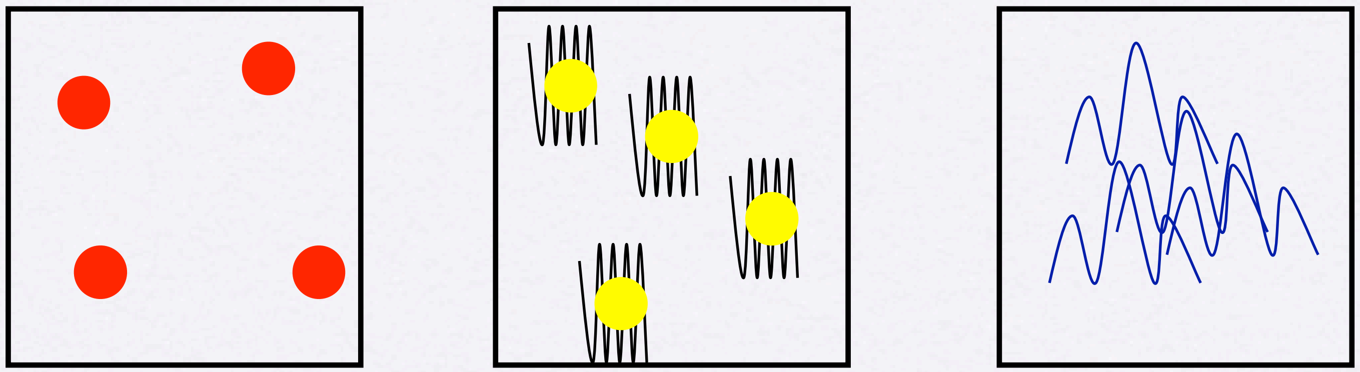
Enrico Fermi (1901-1954)

The spin of a boson is an multiple integer of $\hbar$

The spin of a fermion is a multiple integer of $\hbar/2$

De Broglie wavelength λ associated to each particle comparable with the mean inter-particle distance:

$\lambda \ll d$ $\lambda \gg d$



T = 300 K T @10 μ K T @ 100 nK

$$n\lambda^3 \approx 1 \rightarrow \begin{cases} \lambda = \sqrt{\dfrac{2\pi\hbar^2}{mk_BT}} \\ n = \dfrac{N}{V} \propto \dfrac{1}{\bar{d}^3} \end{cases}$$

Bosons can occupy macroscopically the ground state of the system, while fermions arrange themselves individually in each quantum state up to the Fermi energy E_F.

Introduction

Why ultracold?

$$n\lambda^3 \approx 1 \rightarrow \begin{cases} \lambda = \sqrt{\dfrac{2\pi\hbar^2}{mk_B T}} \\ n = \dfrac{N}{V} \propto \dfrac{1}{\bar{d}^3} \end{cases}$$

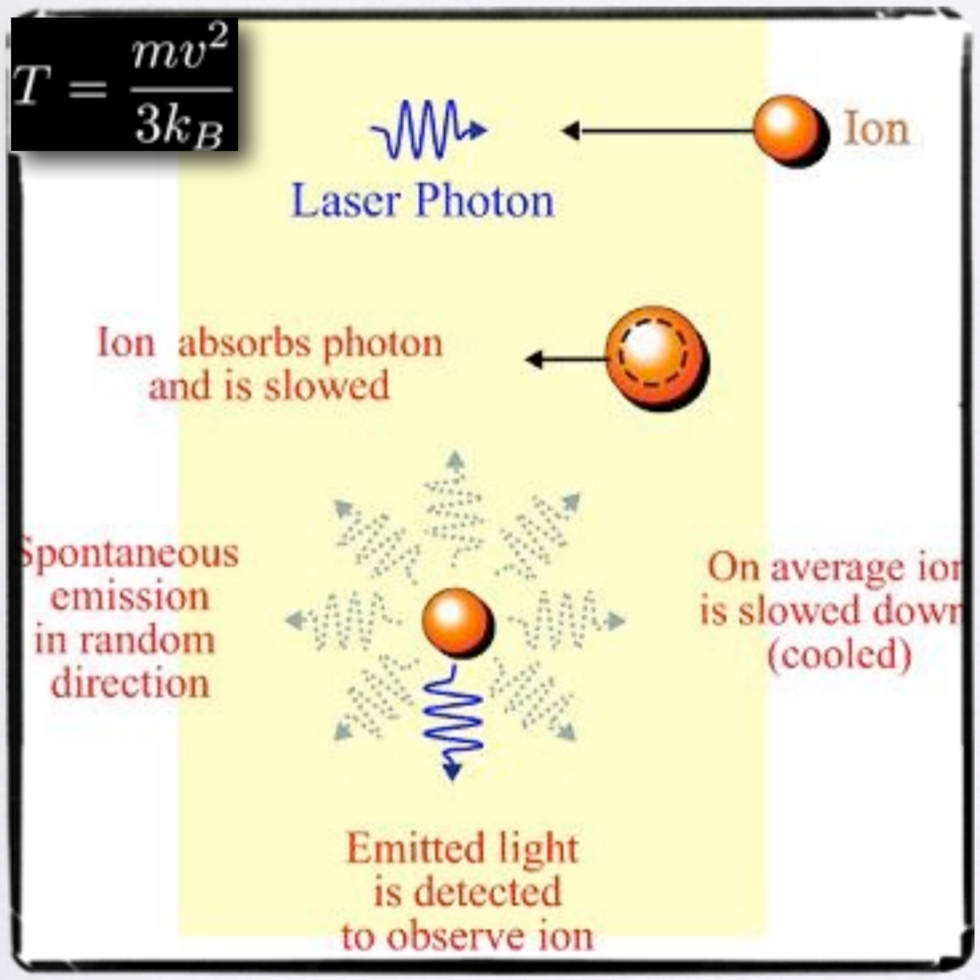
Relevant parameter is phase-space density: $\rho = n\lambda^3_{dB} > 1$

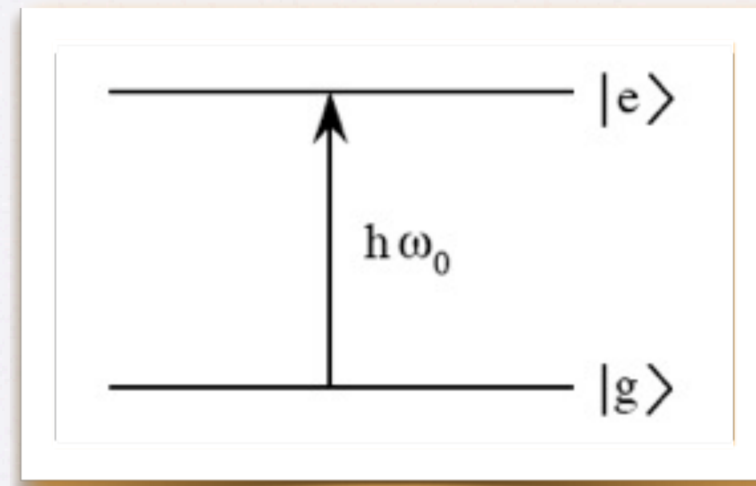
$T \approx 300$ K		$\rho \approx 10^{-20}$
$T \approx 10\ \mu$K	Laser cooling	$\rho \approx 10^{-6}$
$T \approx 100$ nK	Evaporative cooling	$\rho > 1$

$\boxed{\text{Quantum behaviour}}$

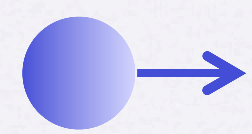
Atom-photon interactions: conservation of momenta

Competition between absorption (directional) and emission (isotropic): cooling





atoms

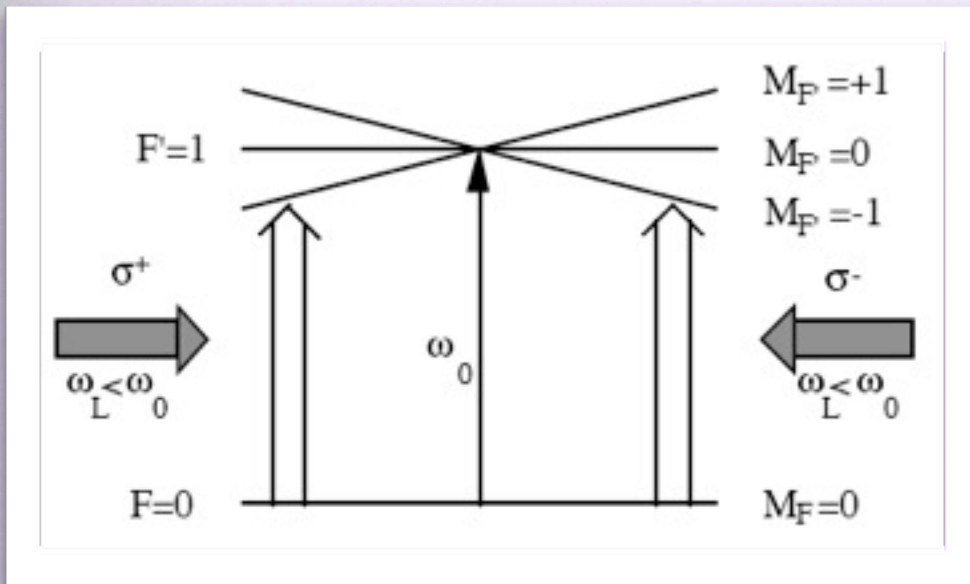


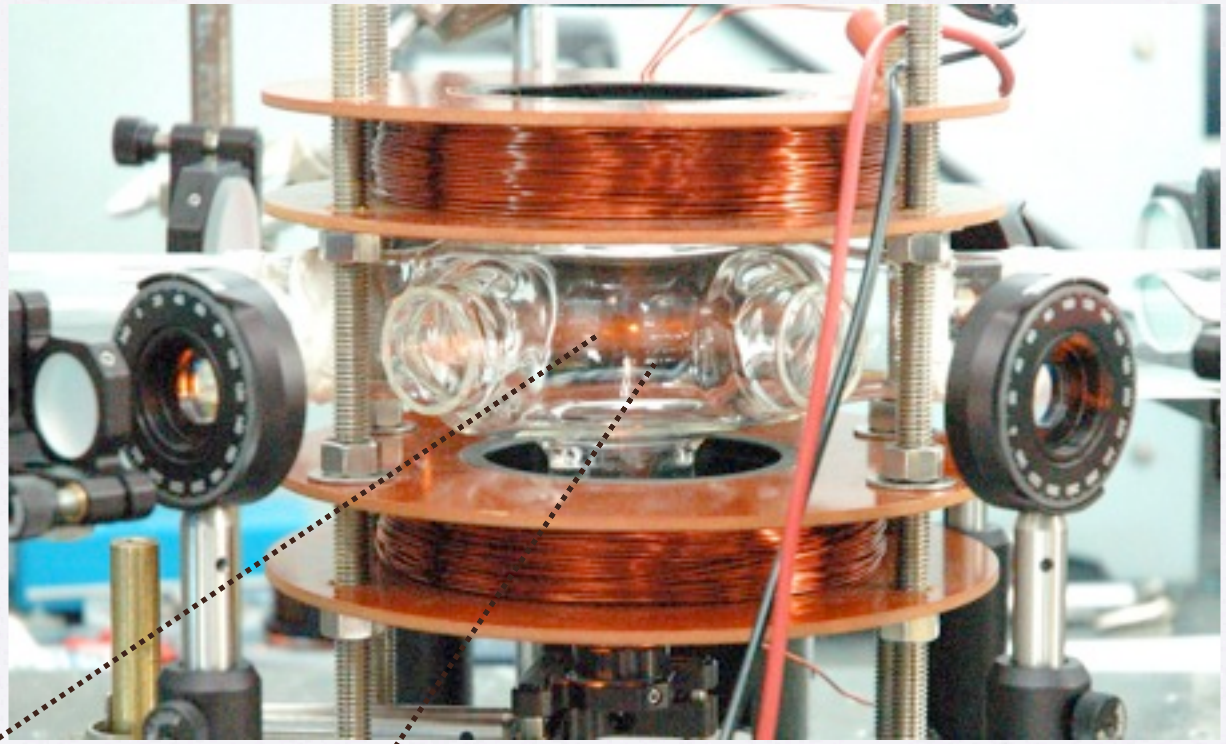
$$\vec{p} = m\vec{v}$$

photons

$$\vec{p} = \hbar\vec{k}$$

Many cycles (10^6 photons per second) : reduction of the kinetic energy (T) of the atoms



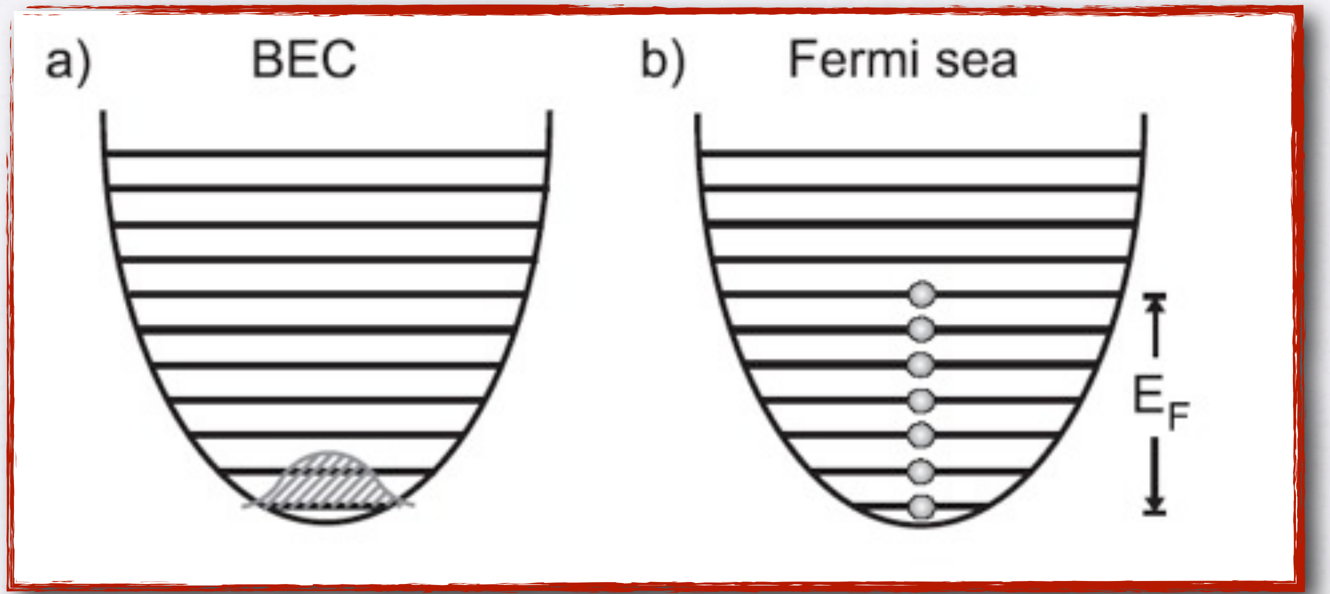


$$\vec{F} = -k\,\vec{z}$$

$$\vec{F} = -\gamma\;\vec{v} - k\,\vec{z}$$

viscous force

restoring force



- Statistical “attraction” between the particles

- Statistical “repulsion” between the particles

- Atomic gases
- Photons
- Phonons in crystal
- ^{4}He

- Atomic gases
- Electrons
- White Dwarf
- ^{3}He

Introduction

The many-body wavefunction of identical fermions must be anti-symmetric under the exchange of two particles

The many-body wavefunction of bosons is symmetric:

$$\Psi_{1,2}^{+/-} = (\phi_1 \chi_2 \pm \phi_2 \chi_1)/\sqrt{2} \qquad \phi,\chi \text{ single particle wavefunctions}$$

The (+/-) describe symmetric (bosons) and anti-symmetric (fermions). If:

$$\phi = \chi \Rightarrow \Psi_{1,2}^{-} = 0$$

Two fermions cannot occupy the same quantum state: Pauli exclusion principle

Summary

Cooling fermions to quantum degeneracy

a) Properties of trapped ultracold fermions

b) Scattering theory: interactions between ultracold fermions

Experimental achievement of Fermi Degeneracy (FD):

a) Evidence of FD

b) Comparison with Bose-Einstein Condensation (BEC)

Experiments

a) Mean-field interactions

a) Fermi gases in optical potentials

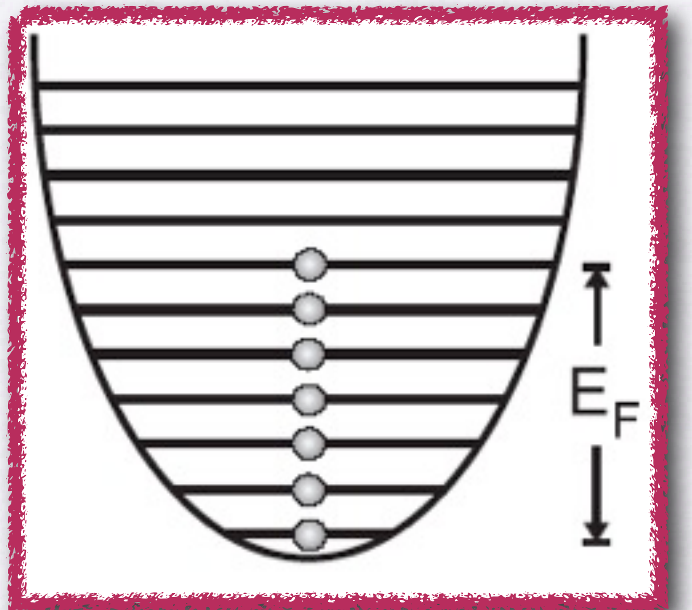
Consider N identical fermions trapped in a cylindrical harmonic potential:

$$V(\rho) = \frac{1}{2} m \omega_r^2 \rho^2$$

where we define $\rho^2=(x^2+y^2+\lambda z^2)$ with $\lambda=\omega_a/\omega_r$ is the anisotropy of the trap

The Pauli principle forbids multiple occupation of a single quantum state: fermions go to occupy one by one every state. The energy of the last occupied state is named Fermi energy E_F.

We define the Fermi temperature $T_F=E_F/k_B$



Fermi gases in harmonic potentials

All the physics here depends from the ratio T/T_F:

If $T/T_F \rangle 1$ the system is behaving "classically"

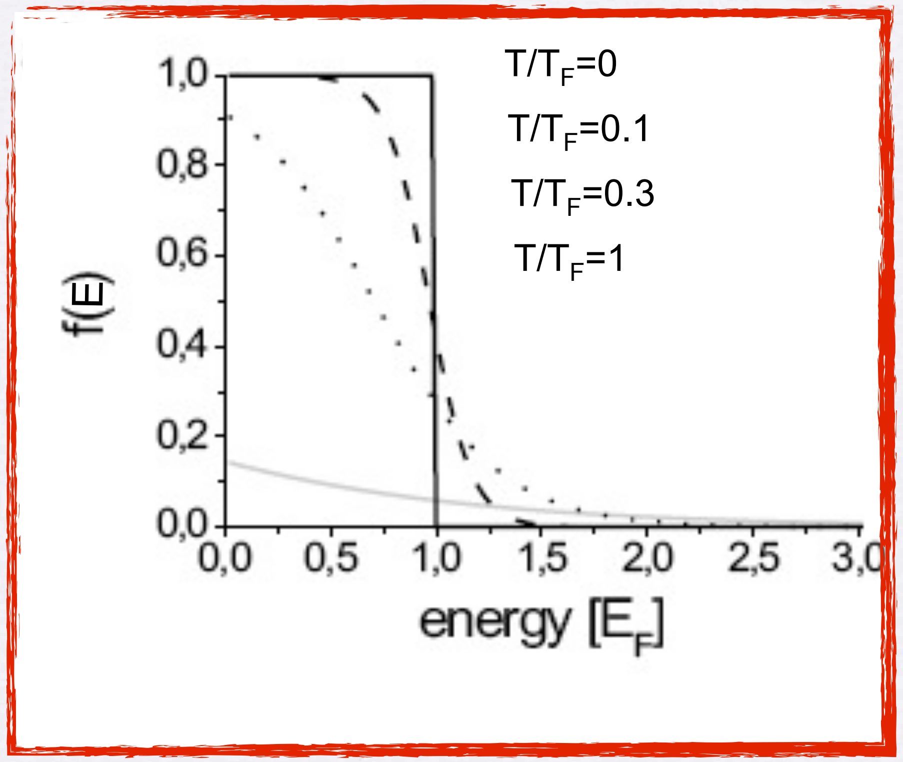
If $T \langle T_F$ the system is entering in the quantum degenerate regime

Consider for simplicity T=0. The Fermi-Dirac distribution is given by:

$$f_F(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}$$

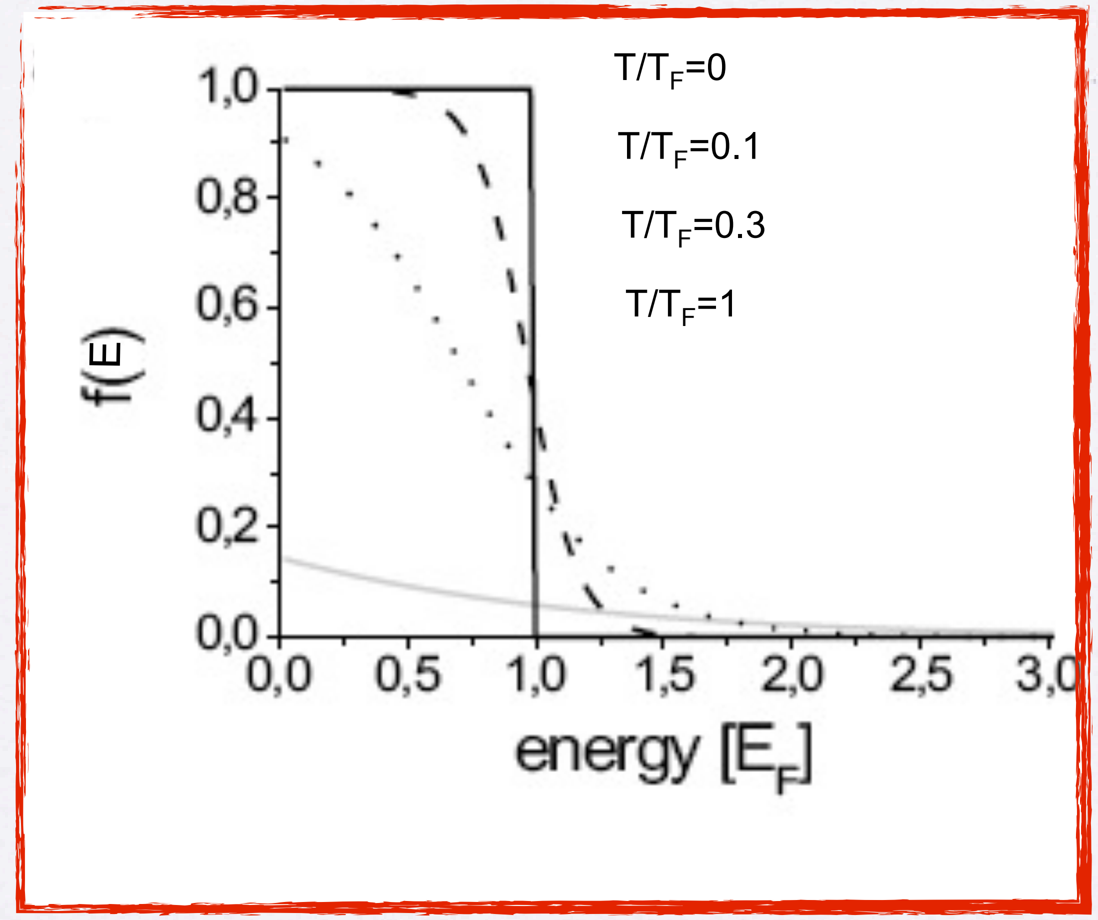
where $\beta = 1/k_BT$ and μ is the chemical potential. In particular $E_F = \mu$ for T=0.

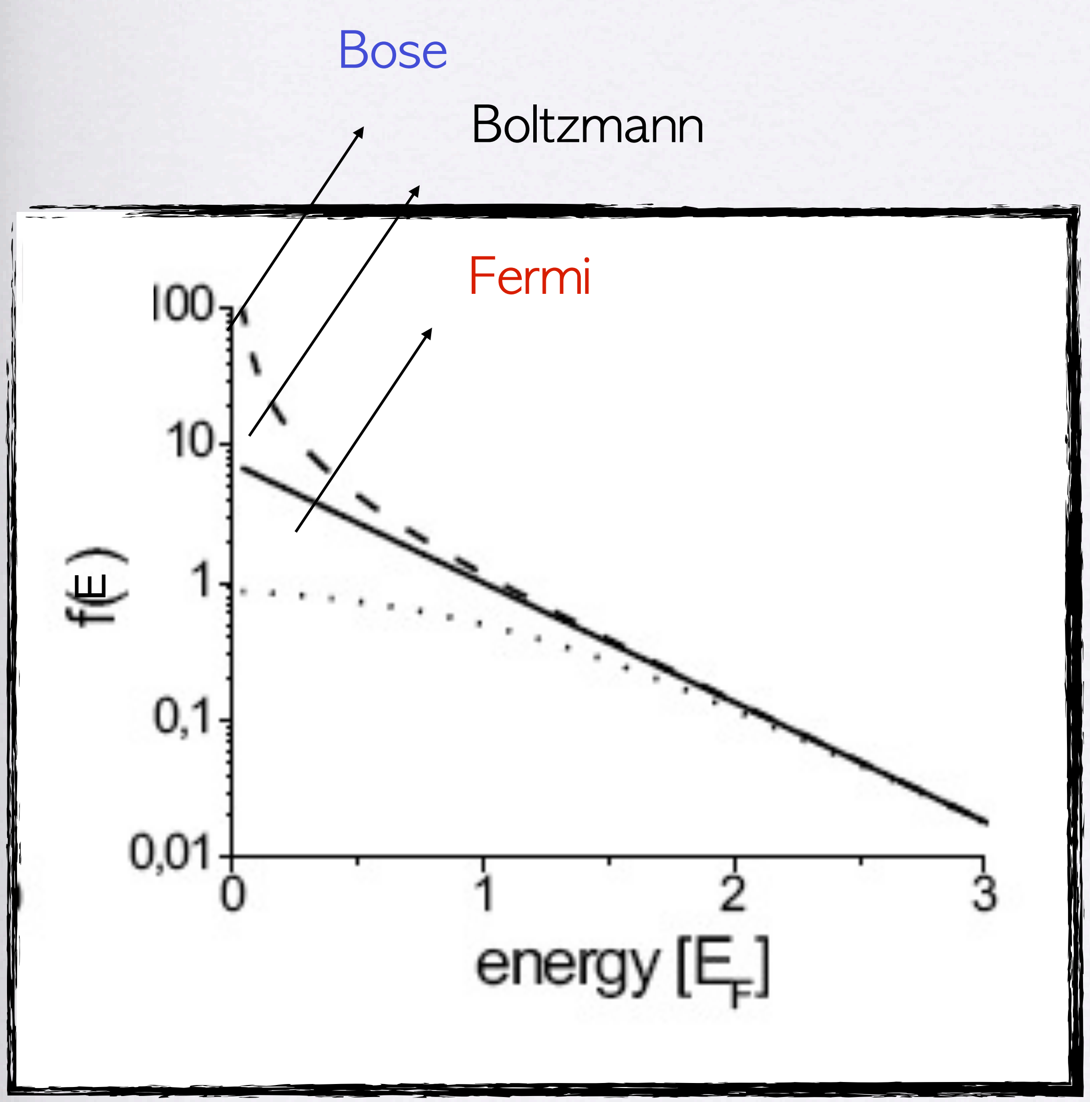
The Fermi-Dirac distribution never exceeds 1, reflecting the Pauli exclusion principle



Fermi gases in harmonic potentials

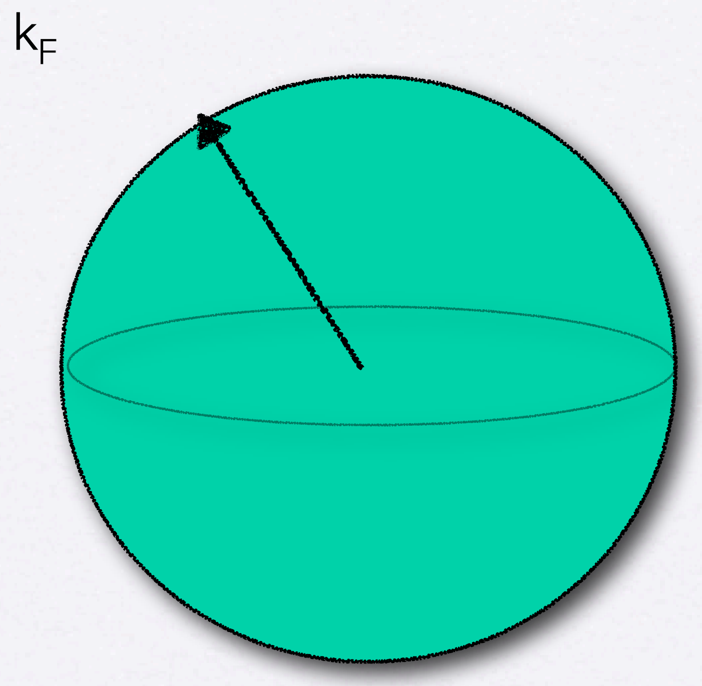
In case of $T/T_F \neq 0$ the step function is smeared out around E_F with a width of the order of $E_F T/T_F$.





If $T/T_F \gg 1$ the Fermi-Dirac is like the "classical" Boltzmann distribution:

$$f(\varepsilon) \approx e^{-\beta(\varepsilon-\mu)}$$



$$k_F = \frac{1}{\hbar}\sqrt{2mE_F}$$

In momentum space, the step function corresponds to a sphere of radius k_F: @ T=0 all the momentum states $\leq k_F$ are occupied!

-> consequences on the collisions between fermions (Pauli Blocking)

Another important quantity which can be defined is the Fermi radius of our trapped degenerate gas:

$$R_F = \sqrt{\frac{2E_F}{M\omega_r^2}}$$

If $\hbar\omega_r, \hbar\omega_a << k_B T$ we can define the density of state as:

$$g(\varepsilon) = \frac{\varepsilon^2}{2(\hbar\varpi)^3}$$

The total number of atoms of the system is given simply by:

$$N = \int d\varepsilon \frac{g(\varepsilon)}{e^{\beta(\varepsilon-\mu)}+1}$$

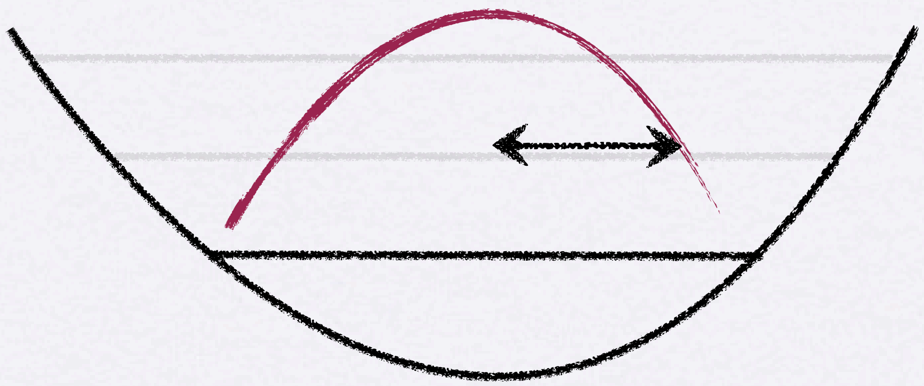
from which it is possible to obtain the explicit relation for the Fermi energy:

$$\boxed{E_F = \hbar\varpi\sqrt[3]{6N}}$$

Fermi gases in harmonic potentials

$$\boxed{E_F = \hbar\varpi\sqrt[3]{6N} \qquad R_F = \sqrt{\frac{2E_F}{M\omega_r^2}}}$$

It is worth to compare the size of the Fermi gas with the harmonic oscillator length of the ground state of our trap:



$$a_{ho} = \sqrt{\frac{\hbar}{m\omega}}$$

We directly get: $R_F \propto a_{ho}\sqrt[6]{N}$

$$R_F \propto a_{ho} \sqrt[6]{N}$$

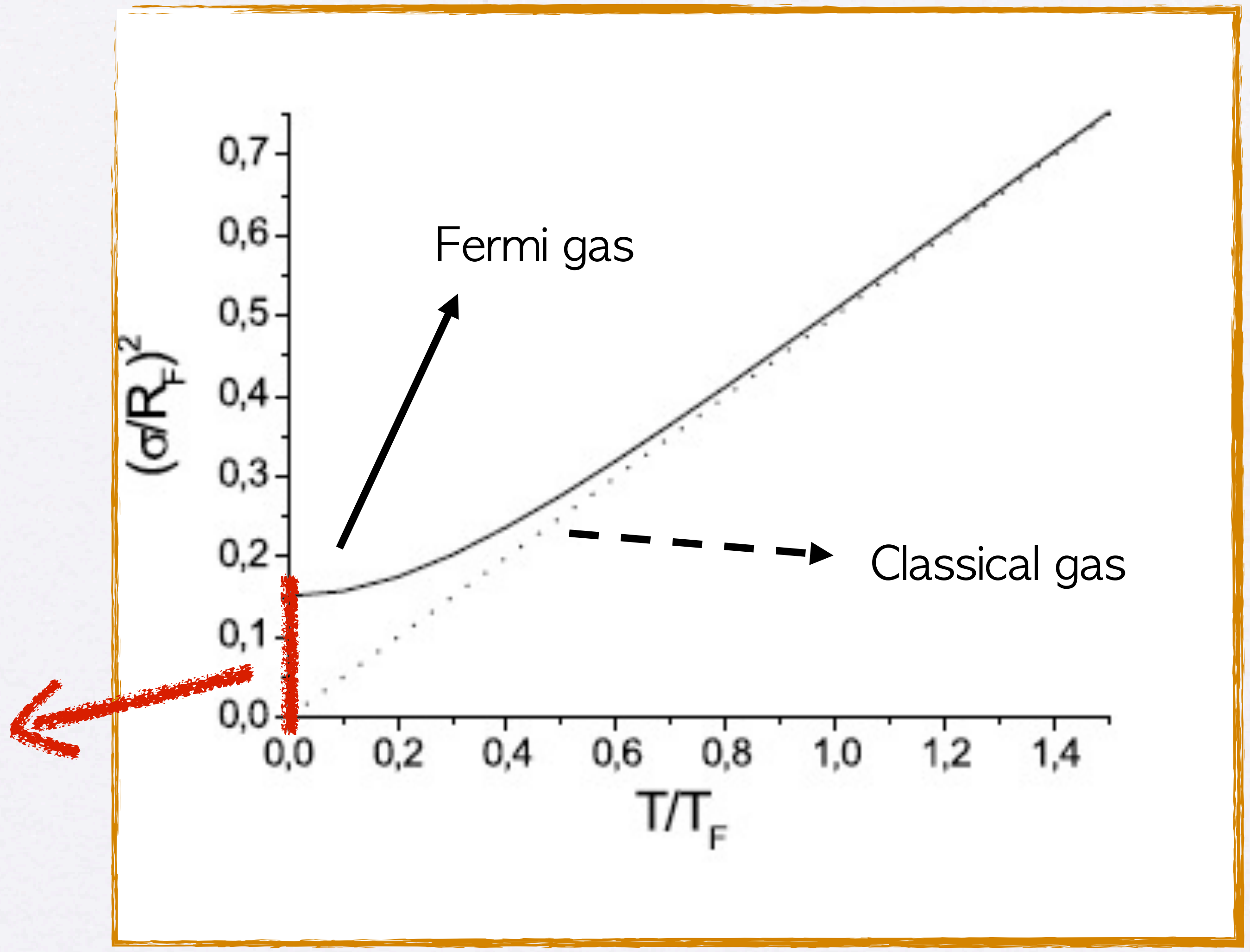
If $N \gg 1$ then the Fermi radius R_F largely exceeds a_{h0}. This is another consequence of the Pauli exclusion principle.

In fact the Pauli exclusion principle induce an "effective" repulsion between the fermions in the trap -> Fermi pressure.

This behavior differentiates the Fermi gas respect to the Bose gas:

- ☑ Interacting BEC: interactions + harmonic oscillator length (typically $U_{int} < E_F$)
- ☑ Ideal BEC: harmonic oscillator length

Fermi gases in harmonic potentials



Comparison with a classical gas: the size R_C -> 0 if T ->0, since $R^2_C \approx$ (T)

It is possible to calculate the momentum distribution of a degenerate trapped Fermi gas @ T=0

$$n(\vec{k}, T=0) = \frac{N}{k_F^3}\frac{8}{\pi^2}\left[1 - \frac{|\vec{k}|^2}{k_F^2}\right]^{3/2}$$

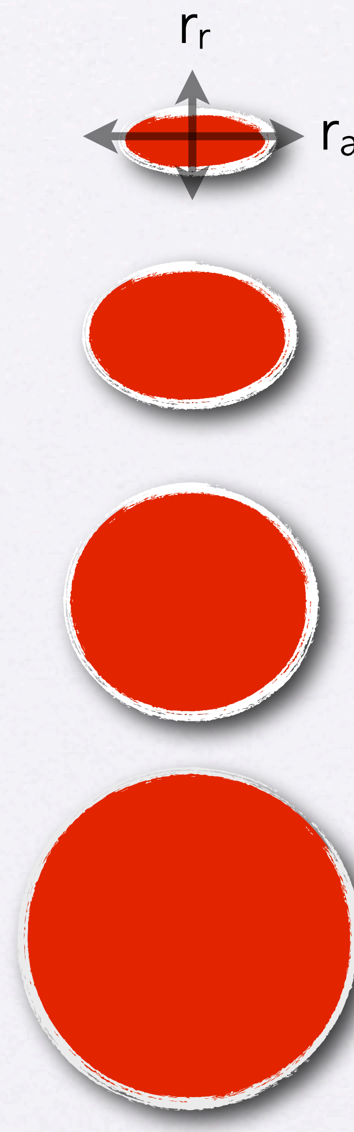
Note that is isotropic despite the anisotropy of the trapping potential, as the one of a "classical" gas:

$$f(\overrightarrow{r}, \overrightarrow{k}, T) = \frac{N\lambda}{(2\pi)^3\sigma_x^3\sigma_k^3}\exp\left(-\beta\frac{\hbar^2\overrightarrow{k}^2}{2m}\right)\exp\left(-\beta\frac{m\omega_{rad}^2}{2}\rho^2\right).$$

This represents a big difference with respect to a BEC!

In more detail we consider the evolution of the "aspect ratio" of a classical gas versus the expansion time:

$$\boxed{\Re = \sqrt{\frac{< r_a^2 >}{< r_r^2 >}} = \frac{1}{\lambda}\sqrt{\frac{1+\omega_a^2 t^2}{1+\omega_r^2 t^2}}} \longrightarrow 1 \quad \text{if } t \to \infty$$



In case of a degenerate Fermi gas, it is possible to obtain:

$$\boxed{\Re = \sqrt{\frac{< r_a^2 >}{< r_r^2 >}} = \frac{1}{\lambda}\sqrt{\frac{1+\omega_a^2 t^2}{1+\omega_r^2 t^2}}} \longrightarrow 1 \quad \text{if } t \to \infty$$

As we see we get the same result: a degenerate Fermi gas will show the same asymptotic behavior of a classical gas, independently from the initial anisotropy of the trapping potential !

Elastic collisions are the main and necessary tool for cooling atoms

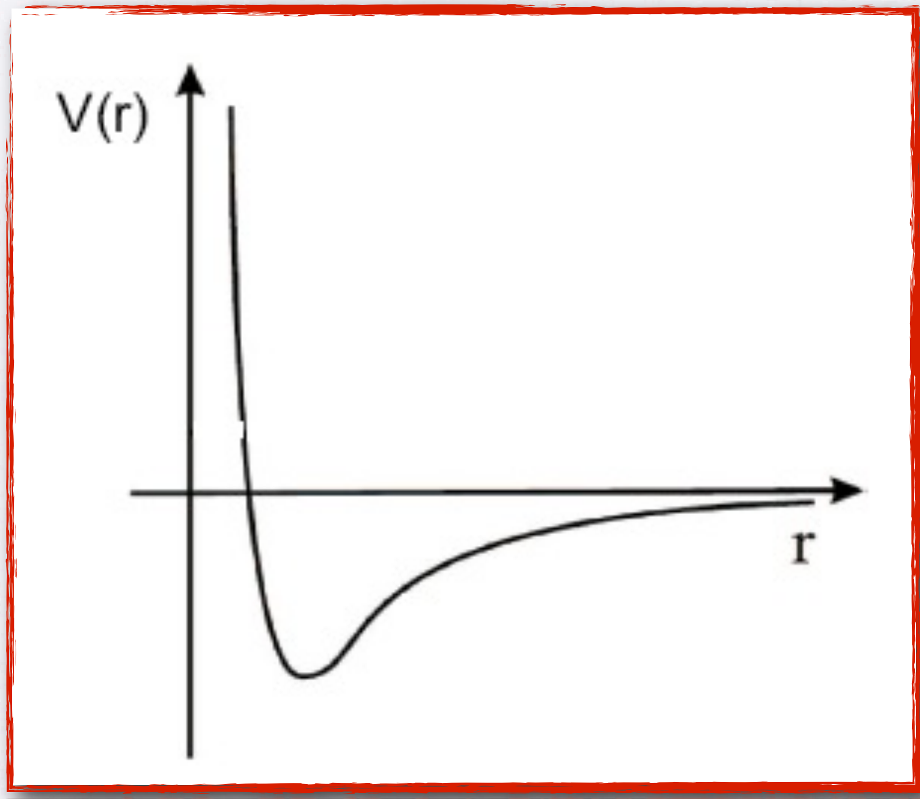
a) Evaporative cooling -> degenerate gas
b) Sympathetic cooling -> degenerate mixture

We briefly discuss how the Fermi statistics affect collisions between identical fermions.

The main result is that a degenerate gas of identical fermions is almost similar to a non-interacting ideal gas.

Scattering theory: basics

Dilute systems -> binary collisions!



The scattering problem of two colliding atoms can be easily reduced to the problem of one particle with reduced mass M/2 in a molecular potential between the two atoms V(r).

The solution of the Shrodinger equation associated:

$$\Psi_k(\overrightarrow{r}) = \underline{e^{i\overrightarrow{k}\overrightarrow{r}}} + f(k, \overrightarrow{n}, \overrightarrow{n}')\underline{\frac{e^{ikr}}{r}}$$

incoming plane wave

outgoing scattered spherical wave

Scattering theory: basics

The physical measurable quantities are related to the scattering amplitude $\boxed{f(k, \overrightarrow{n}, \overrightarrow{n}')}$

In fact we can define the differential
and the total cross sections as:

$$\frac{d\sigma}{d\Omega} = |f(k, \overrightarrow{n}, \overrightarrow{n}')|^2$$

$$\sigma(k, \overrightarrow{n}) = \int |f(k, \overrightarrow{n}, \overrightarrow{n}')|^2 \, d^2n'$$

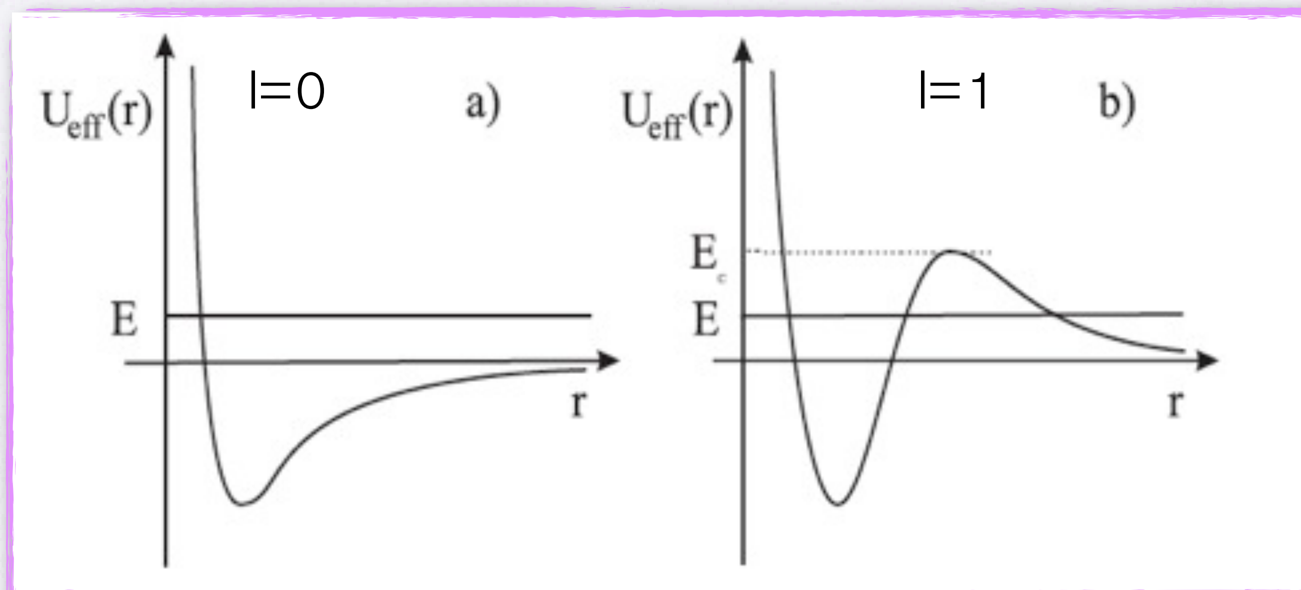
If we consider now the case: V(r) = V(r) -> spherical symmetry, it is possible to expand the wave-function in terms of angular momentum partial waves, named s,p,d,f..(L=0,1,2,3) $\hbar$

Scattering theory: basics

Plugging this expansion in the Shrodinger equation, we get an equation for the radial wave-function.

Beside the molecular potential V(r), a new centrifugal potential appears: $$\frac{\hbar^2 l(l+1)}{mr^2}$$

For $l>0$ this new term generates a centrifugal barrier:



If the energy of the collisional energy E is lower than the height of the barrier the collision is highly suppressed...

-> @ sufficiently low energy ($T<1$ mK) the collisions with $l>0$ are suppressed!

Scattering theory: basics

Coming back to the collisional cross section, after the expansion it reads:

$$\sigma(\mathrm{k}) = \sum_{\mathrm{l}=0}^{\infty} \sigma_l(\mathrm{k}), \ \sigma_l(k) = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l(k)$$

$\delta_l(k)$=phase shift associated to each partial wave l

So far, we have not considered the different statistics:

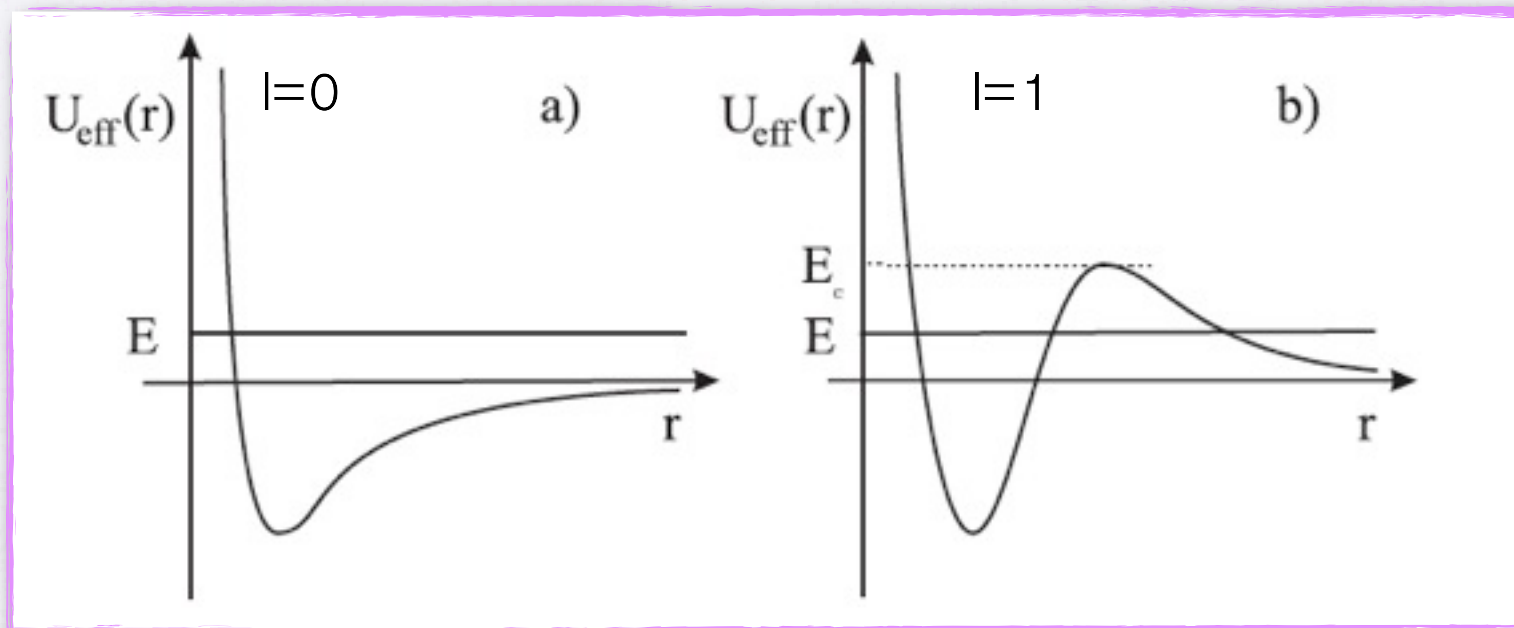
Symmetric wave-function -> bosons
Anti-symmetric wave-function -> fermions

$$\sigma(k) = \frac{8\pi}{k^2}\sum_{l\,\text{even}}(2l+1)\sin^2\delta_l(k) \qquad \text{bosons}$$

$$\sigma(k) = \frac{8\pi}{k^2}\sum_{l\,\text{odd}}(2l+1)\sin^2\delta_l(k) \qquad \text{fermions}$$

Scattering theory: basics

As we see for bosons the scattering cross section is a sum of all the even l partial waves l=0,2,4.. while for fermions only the odd ones l=1,3,5..



Only isotropic l=0 (s-wave scattering) @ low T is possible!

-> collisions between ultracold identical fermions are suppressed!

Scattering theory: basics

More in detail, it is possible to introduce the concept of scattering length a:

$$\boxed{\lim_{k \to 0} \sigma_{l=0}(k) = 4\pi a^2}$$

distinguishable particle (bosons)

while for indistinguishable particles (bosons):

$$\boxed{lim_{k\to 0}\sigma_{l=0}(k) = 8\pi a^2}$$

The expression used for defining the elastic cross-section is then:

$$\boxed{\sigma = 4\pi a^2}$$

The "geometric meaning" is clear: each atom is a scattering surface of area $4\pi a^2$. The scattering length a is the only parameter needed for describing the scattering properties of ultracold atoms.

In case of l〉0, it is possible to derive $\sigma_{l\neq 0}(k) \propto k^{4l}$

In other words, the contribution to the elastic scattering cross-section for l〉0 decreases as T^{2l}. We stress again that this result has dramatic consequences in the cooling of identical fermions.

Let's summarize the main results of this short discussion:

- ☑ The scattering @ this low T is only s-wave scattering.
- ☑ The collisional properties of ultracold atoms-〉 scattering length a.
- ☑ Identical fermions do not collide @ this low T.

Scattering theory: example

Fermionic ^{40}K

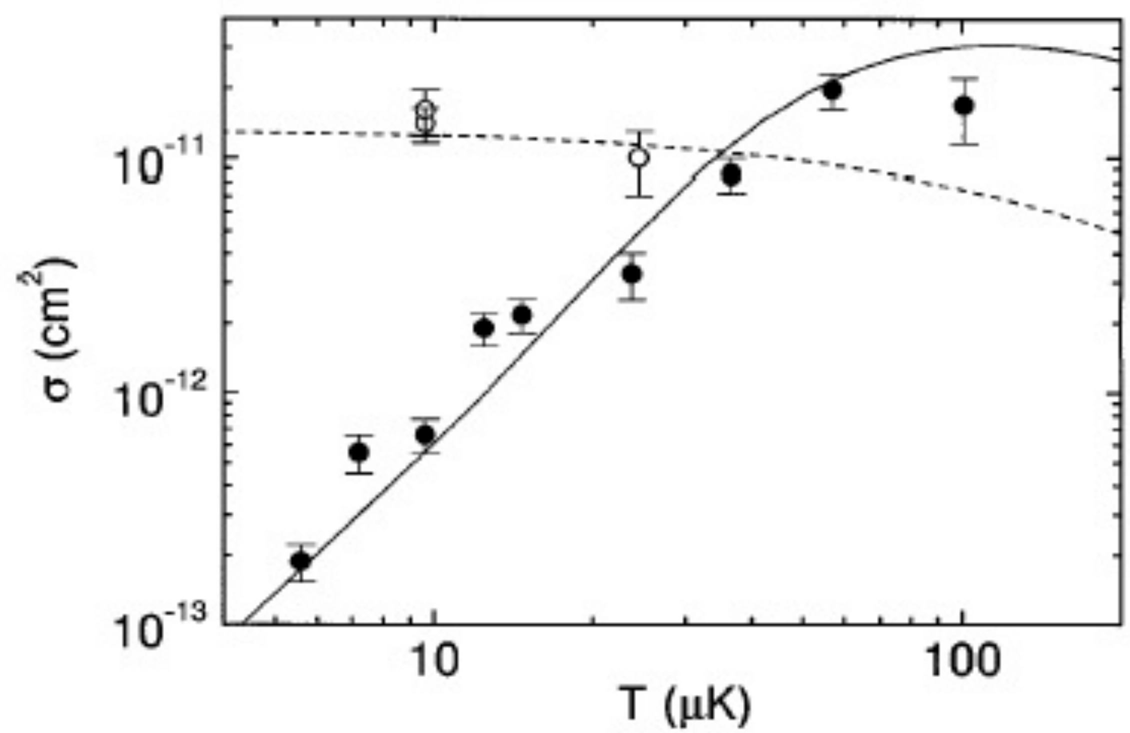
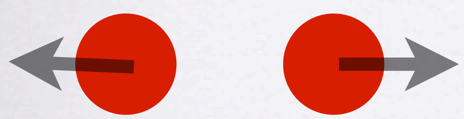


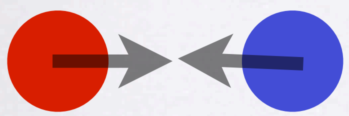
FIG. 2. Elastic cross sections vs temperature. The s-wave cross section (○), measured using a mixture of spin states, shows little temperature dependence. However, the p-wave cross section (●), measured using spin-polarized atoms, exhibits the expected threshold behavior and is seen to vary by over 2 orders of magnitude. The lines are a fit to the data, as described in the text, yielding $a_t = (157 \pm 20)a_0$.

B. DeMarco, J. L. Bohn, J. P. Burke, M. Holland, and D. S. Jin
Phys. Rev. Lett. 82, 4208–4211 (1999).

Consequences of the Fermi statistics on collisions of ultracold fermions.

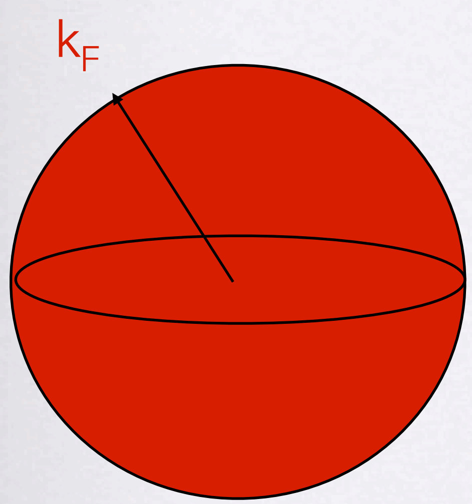


Identical fermions do not collide @ very low T ($T < 100\ \mu K$)

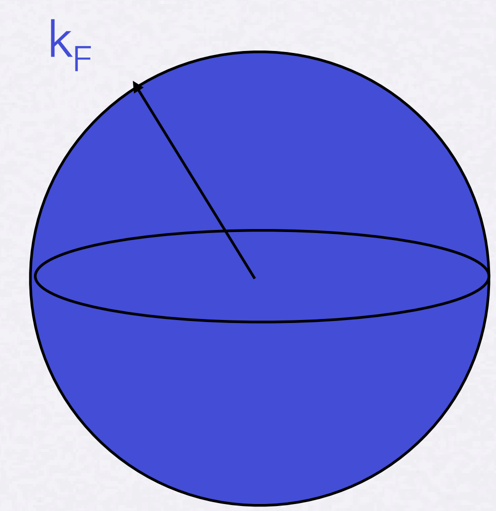


No-identical fermions collide (s-wave scattering!)

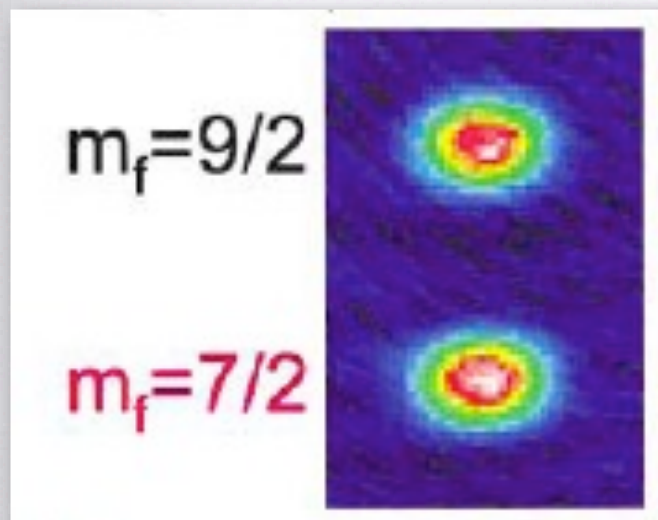
But..

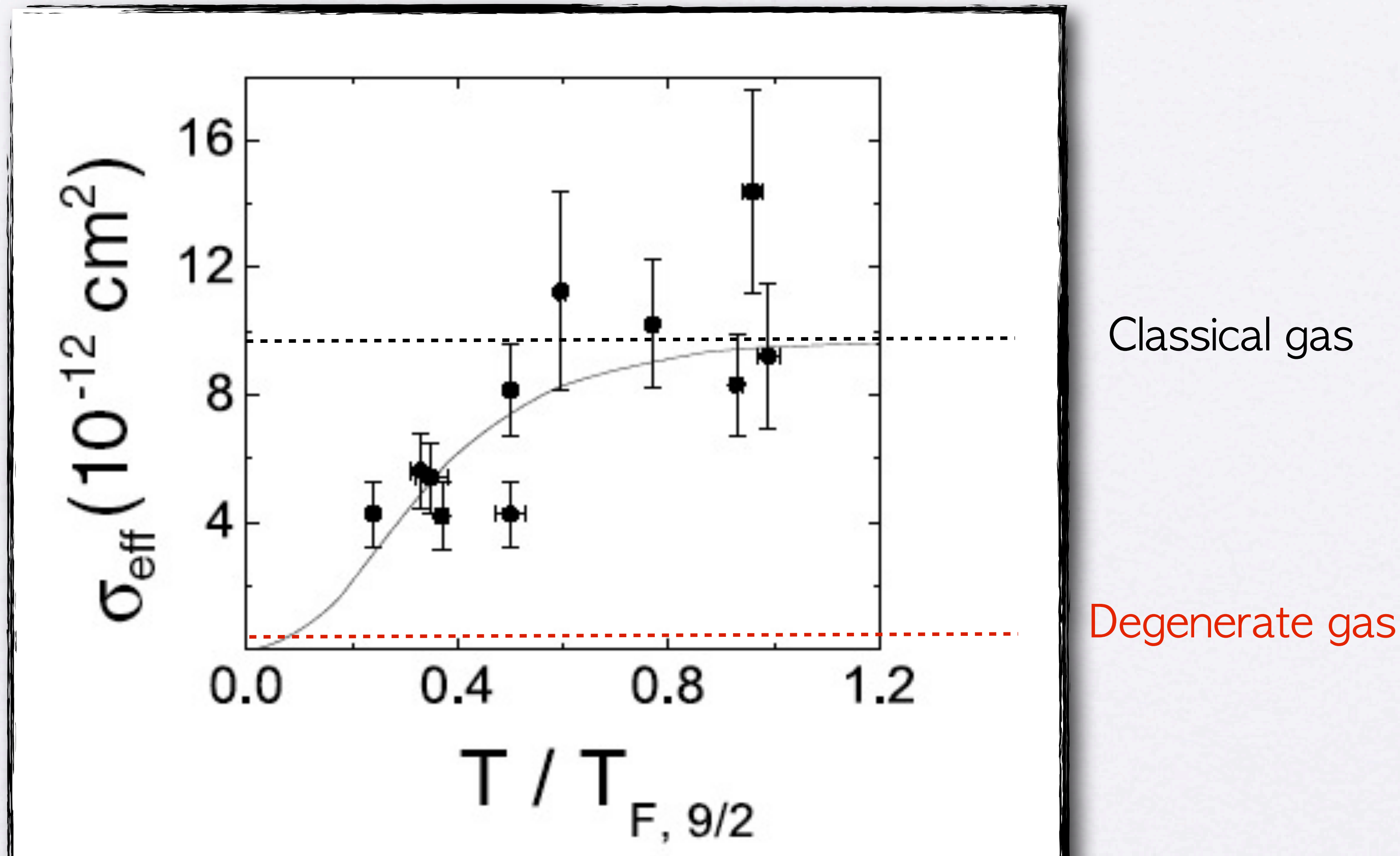


In the collisions between two degenerate Fermi gases every scattering process which produces fermions with $k \leq k_F$ is highly inhibited! In fact no final free momentum state is available



This inhibition of the collisions between non-identical fermions is known as Pauli Blocking of collisions. It is a direct consequence of Pauli exclusion principle.

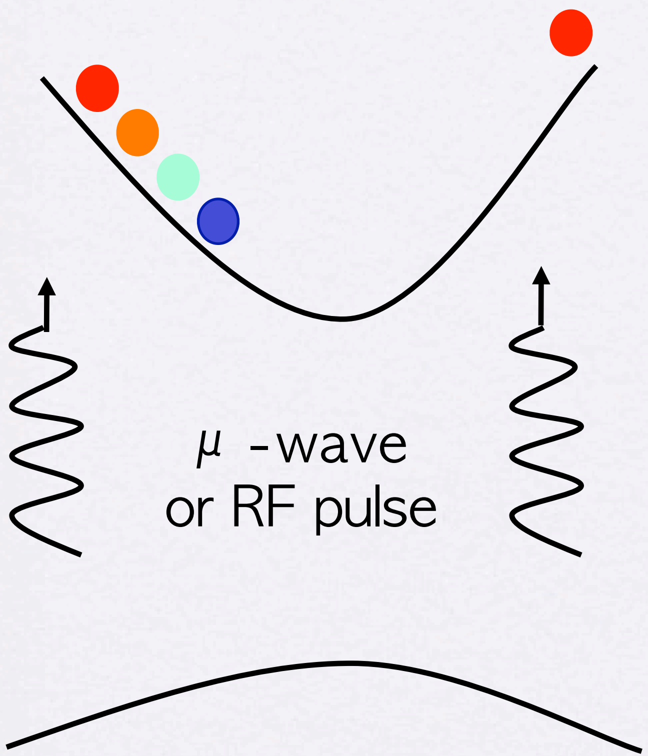




In the experiment the effect of Pauli blocking appears in a reduction of the scattering cross-section between a mixture of two different Fermi gases of ^{40}K.

B. DeMarco, S. B. Papp, and D. S. Jin*, Phys. Rev. Lett. 86, 5409–5412 (2001)

Cooling the atoms to degeneracy



untrapping
potential

removing only hottest atoms

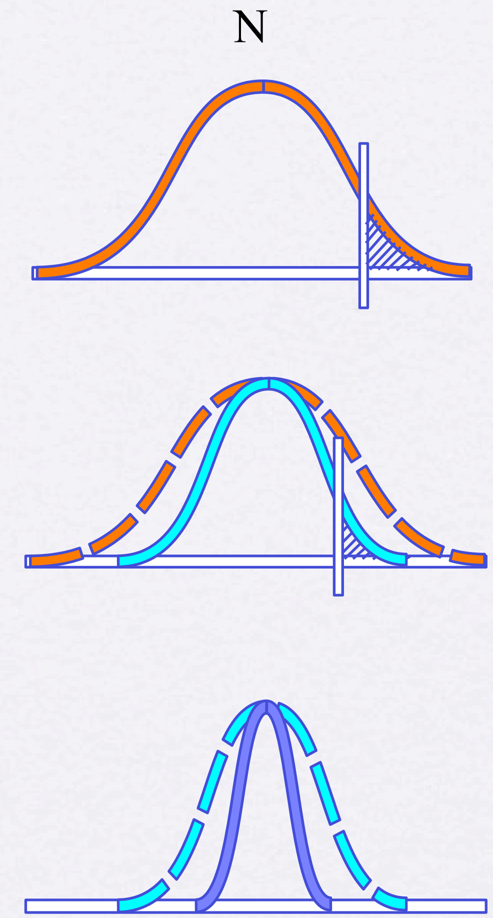
+

thermalization

(elastic collisions)

⬇

cooling and high density



Evaporative cooling exploits elastic collisions between atoms. The initial T of the atoms in the MOT is around 100 μ K.

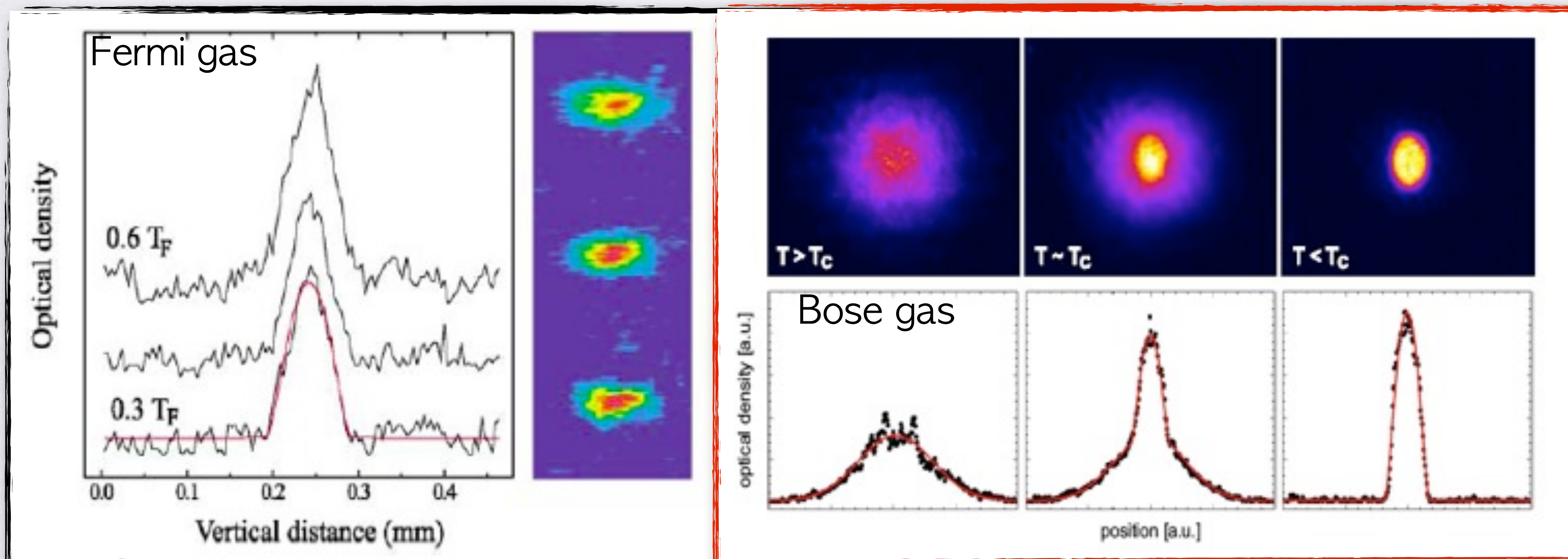
Since the only contribution to the elastic cross-section is given by the s-wave scattering, $\sigma = 4\pi a^2$

No evaporative cooling is possible in a sample of identical fermions

-> no degenerate regime is possible ???

BEC vs Fermi gases

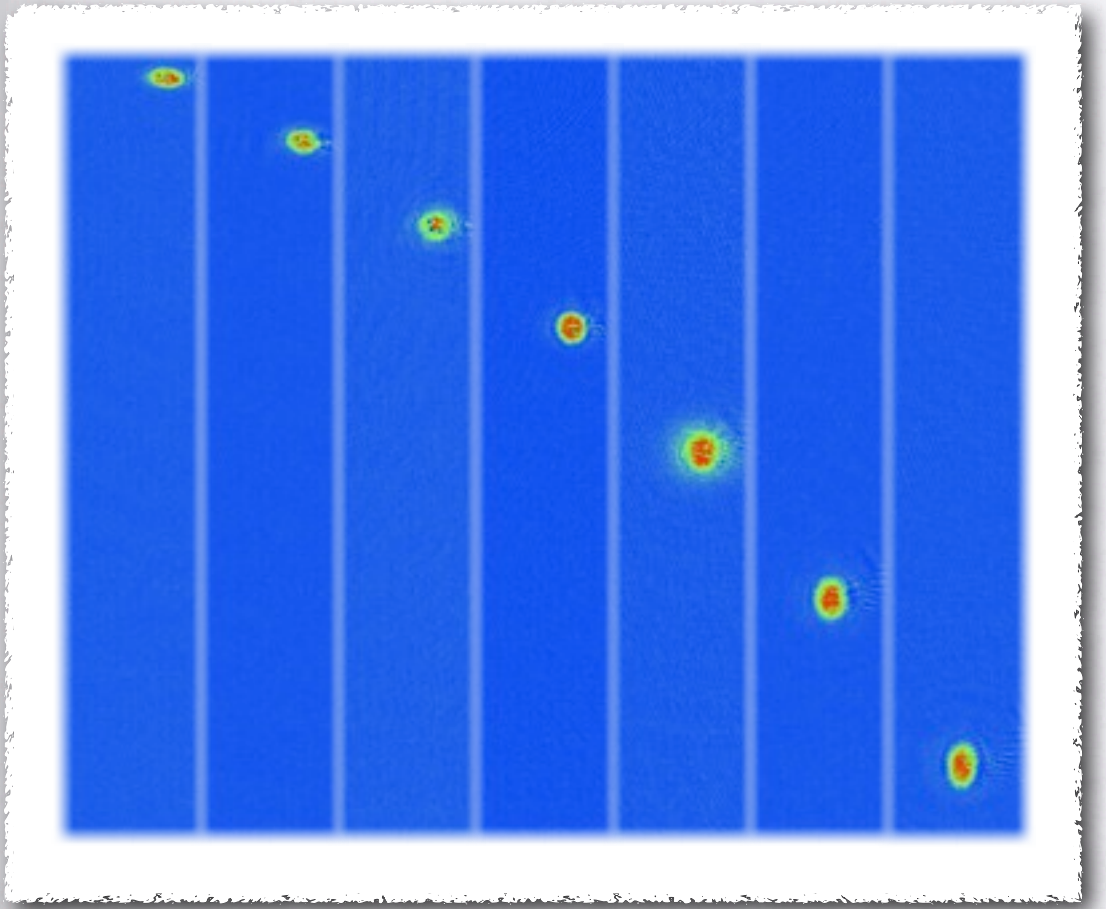
The transition between a classical gas of fermions and a degenerate Fermi gas is not a phase-transition… not spectacular !!



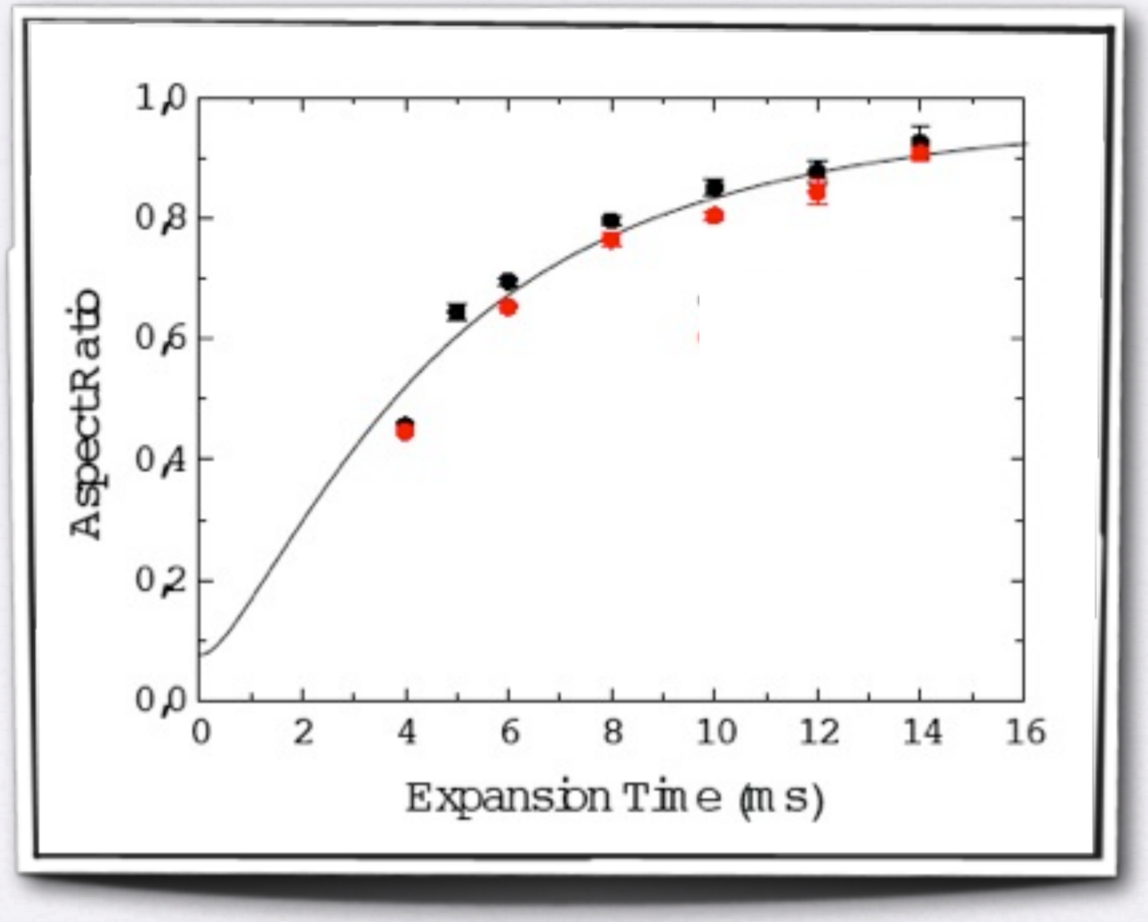
No dramatic change in the momentum distribution

BEC vs Fermi gases

The mean-field interactions is almost negligible in case of identical fermions: the released energy is $E_F \gg E_{mf}$.



Bose gas

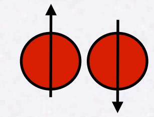


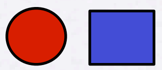
Fermi gas

The momentum distribution is isotropic: no inversion in the aspect ratio

Cooling the atoms to degeneracy

Two different strategies for overcoming the absence of s-wave scattering

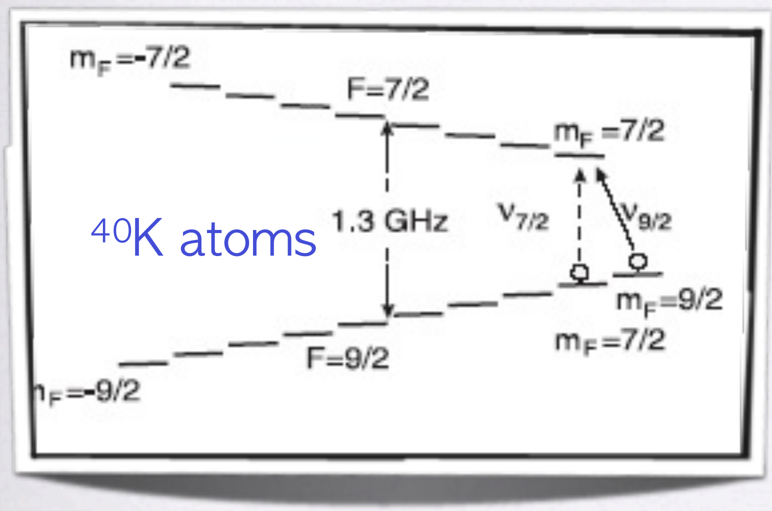
Mixture composed by fermions in 2 spin states  JILA,Duke,Innsbruck...

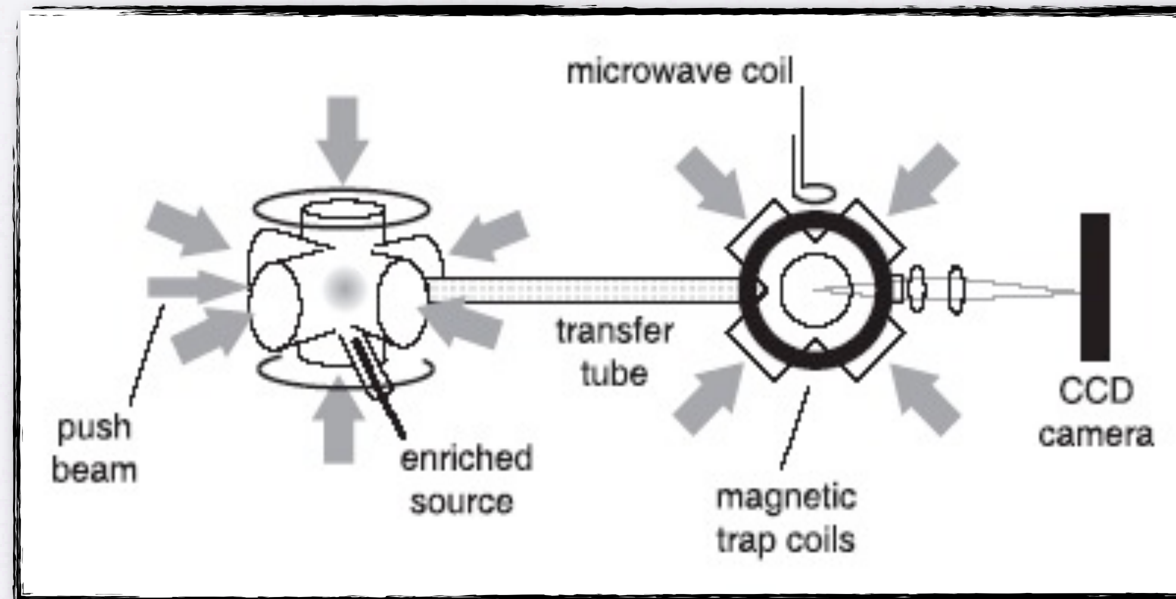
Mixture composed by fermions and bosons
-> sympathetic cooling  ENS, Rice, LENS, MIT
JILA, Zurich, Hamburg...

The efficiency of evaporative cooling is assured by elastic collisions between non-identical particles: achievement of degenerate regime.

Achieving Fermi degeneracy

Fermions in 2 spin states measurement of the thermodynamic properties of Fermi gas



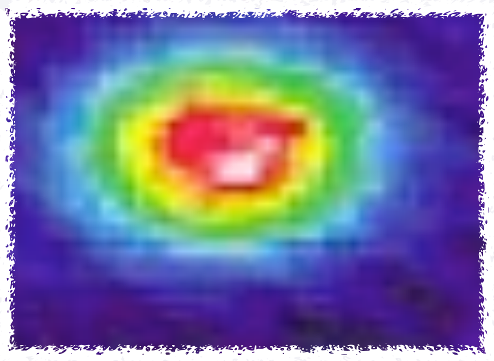


60%-40% mixture of potassium atoms in two different spin states

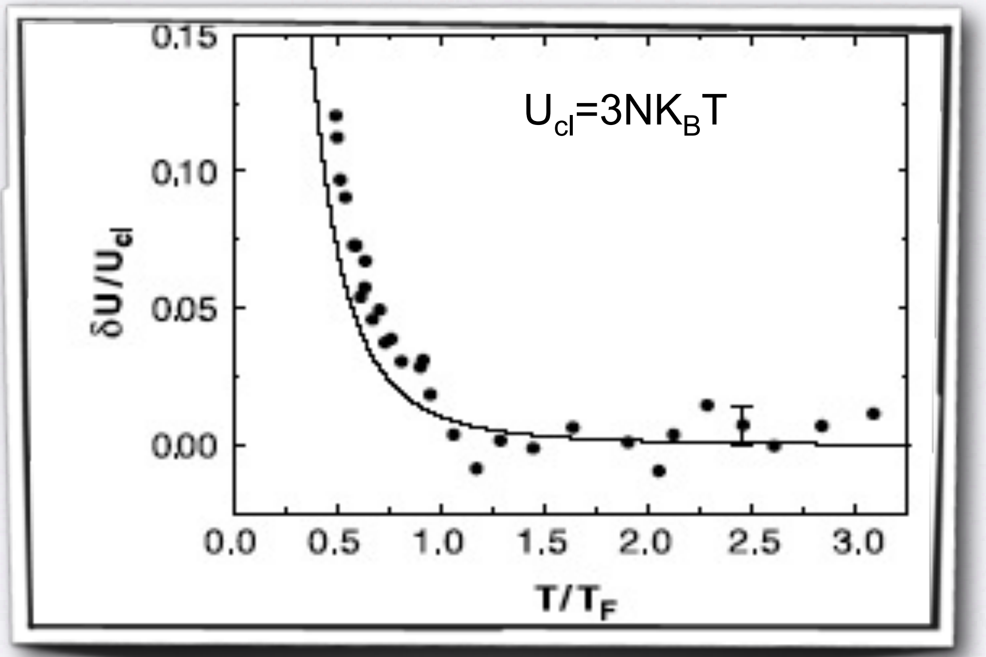
B. De Marco and D. S. Jin, Science 285, 1703 (1999)

Achieving Fermi degeneracy

After the evaporative cooling stage, removal of one of the two spin state from magnetic trap: single degenerate Fermi gas.



$N = 5 \times 10^5$, $T = 350$ nK, $T/T_F=0.5$



Analysis of the momentum distribution extracted from TOF signal:
TOF signal-〉 momentum distribution in the trap

Classical gas -〉 Gaussian fit
Fermi gas -〉 Thomas-Fermi profile

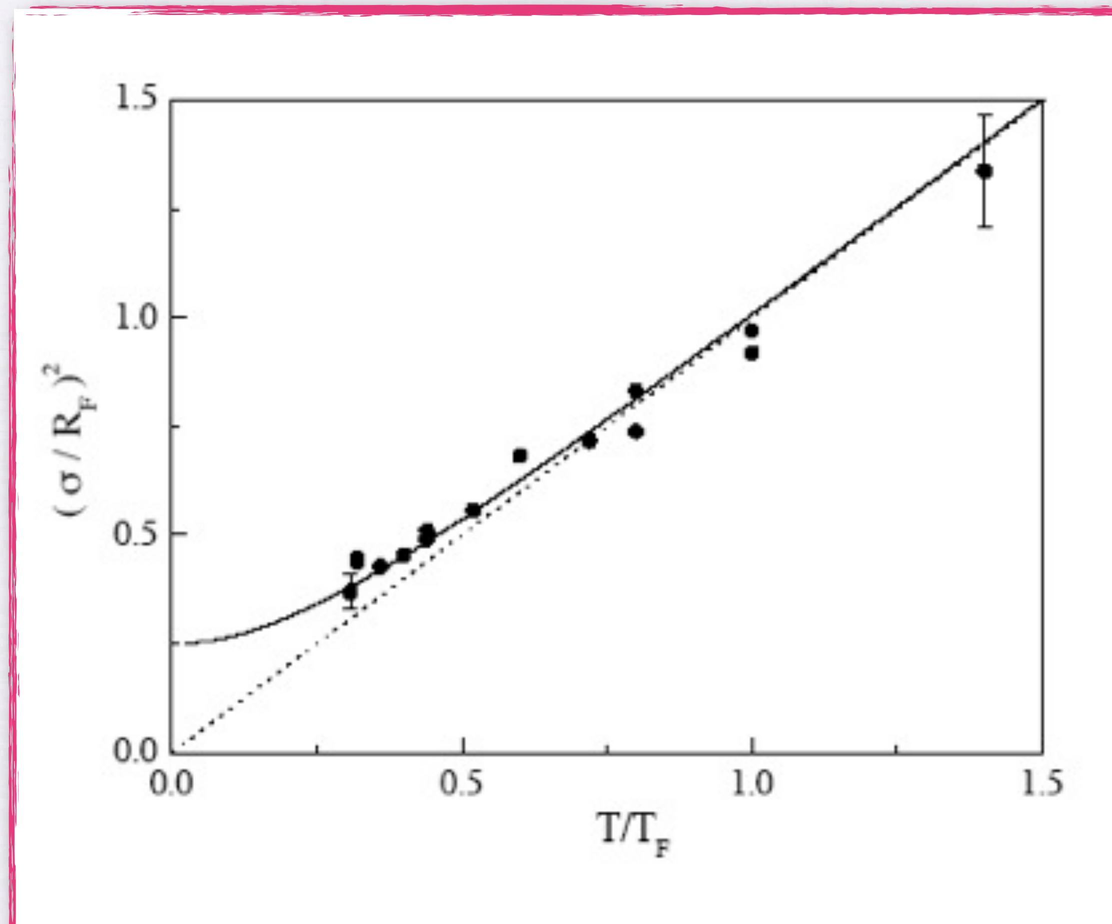
B. De Marco and D. S. Jin, Science 285, 1703 (1999)

Achieving Fermi degeneracy

In case of thermal distribution, the width of the TOF signal is directly giving the temperature of the atoms in the trap: decreasing the width means decreased T!

In case of BEC, T_C is determined by the wings of the thermal component!

In case of a Fermi gas, an accurate fit is needed. The size of the clouds saturates due to Fermi pressure.



Achieving Fermi degeneracy

Mixture composed by fermions and bosons

Two ways for detecting FD

1) Direct measurement on Fermi gas
2) Bosonic thermometer

G. Truscott, *et al.*, Science **291**, 2570 (2001).

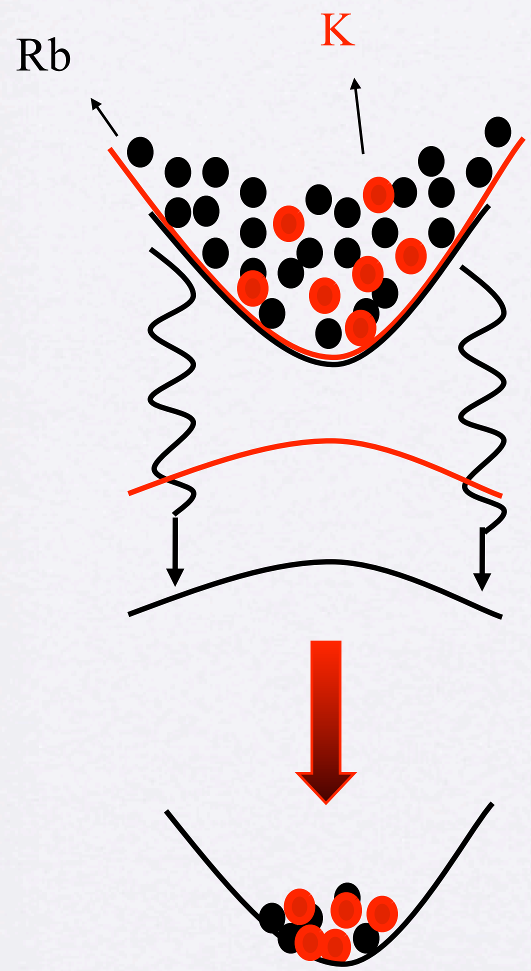
F. Schreck, *et al.*, Phys. Rev. Lett. **87**, 080403 (2001).

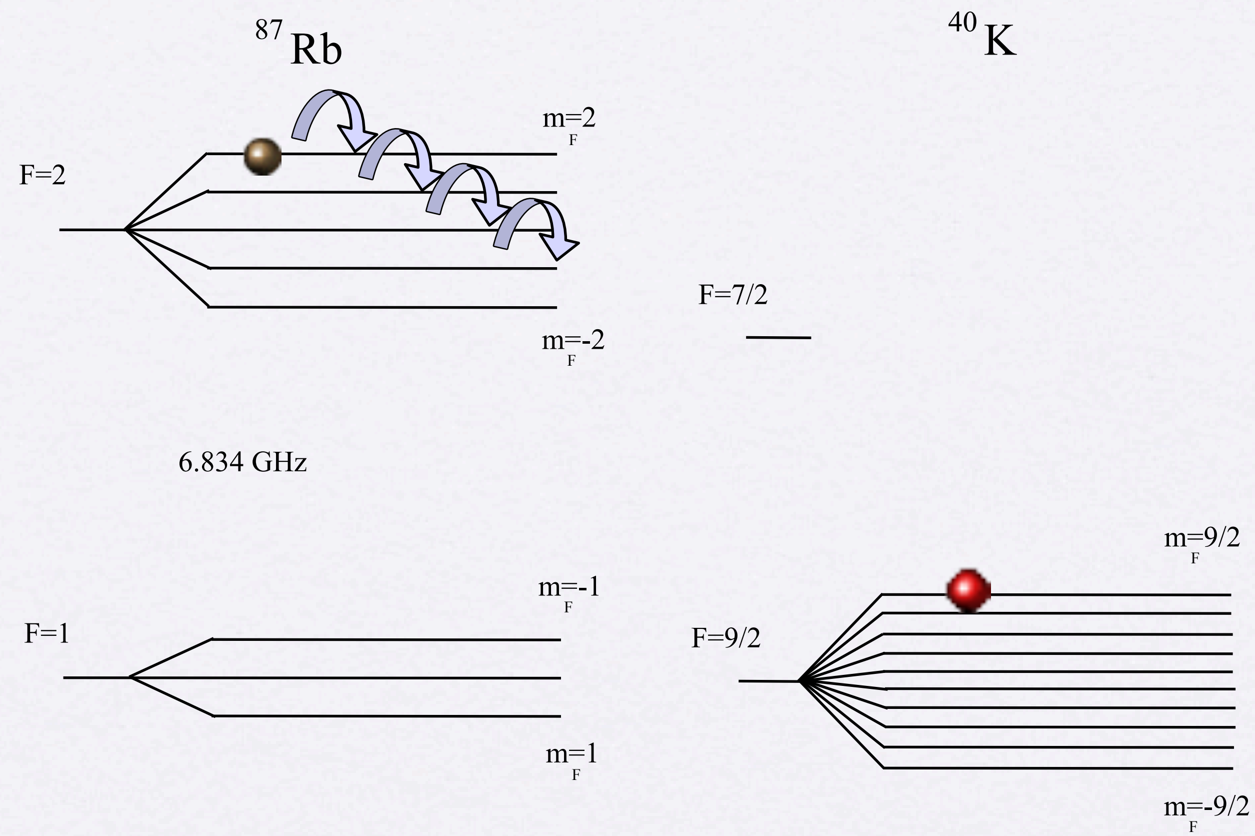
Z. Hadzibabic, *et al.*, Phys. Rev. Lett. **88**, 160401 (2002)

G. Roati, F. Riboli, G. Modugno, and M. Inguscio Phys. Rev. Lett **89**, 150403 (2002).

Achieving Fermi degeneracy

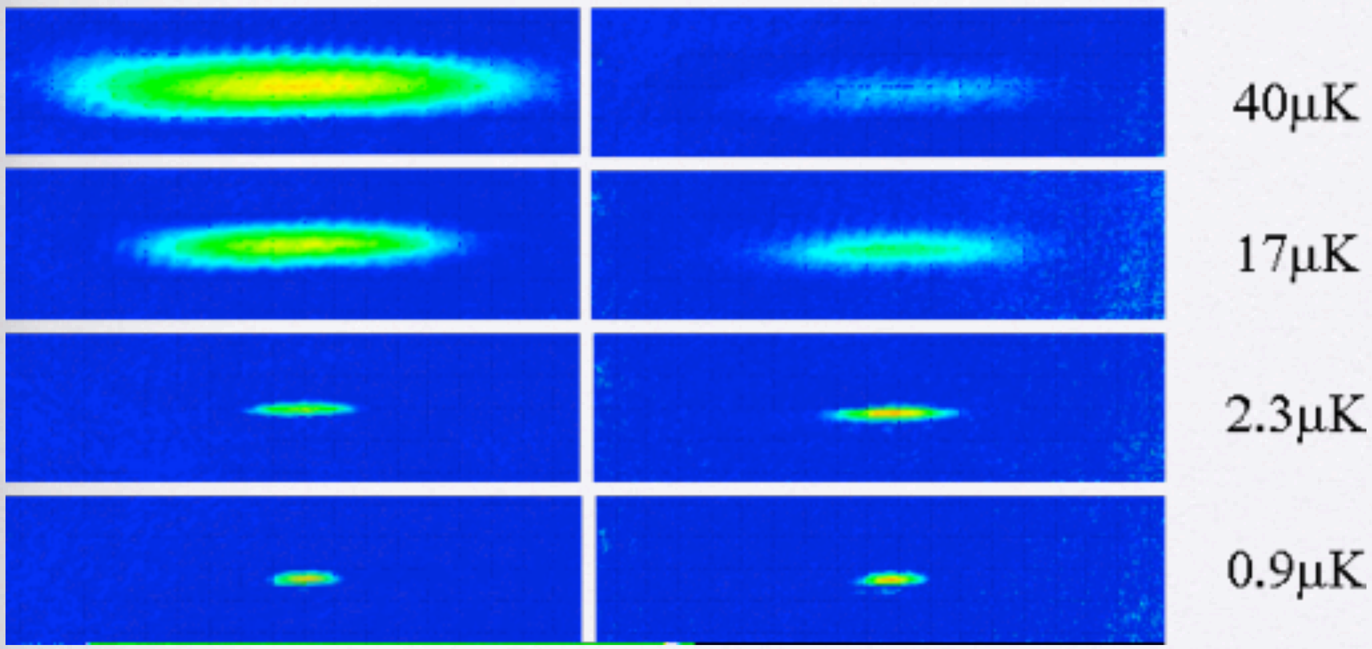
In this case the fermions are cooled to FD by means of elastic collisions with a cooler gas of bosons.
In detail, the evaporative cooling is performed selectively on the bosonic component: sympathetic cooling.





G. Modugno, G. Ferrari, G. Roati, R. J. Brecha, A. Simoni, M. Inguscio, Science, 294, 1320 (2001)

Achieving Fermi degeneracy



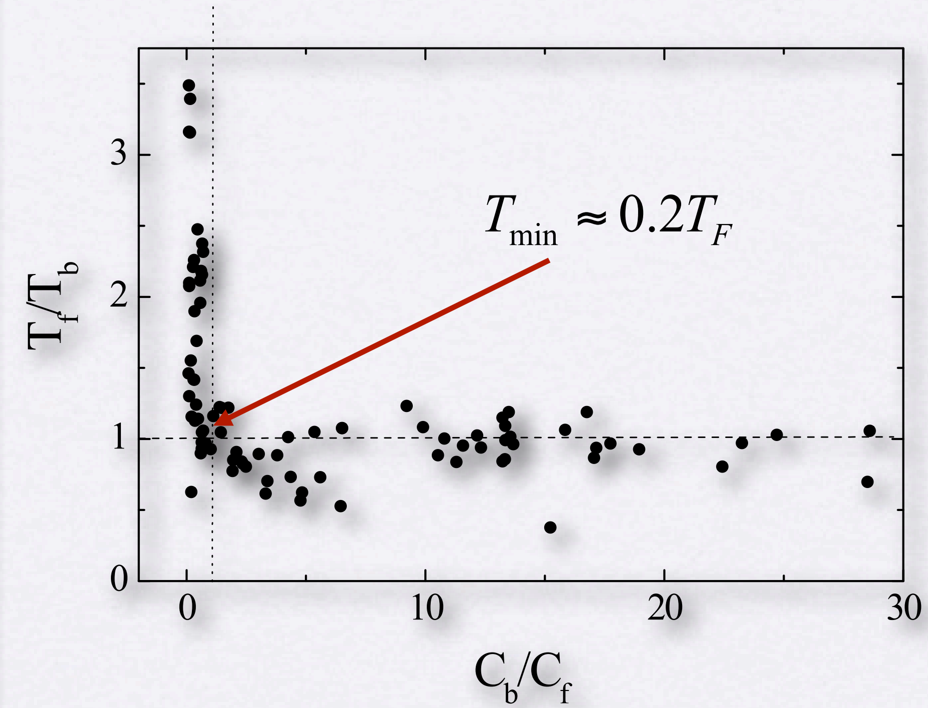
Efficiency criteria:

- many elastic collisions:

$$\Gamma_{coll} > \frac{100}{\tau_{evap}} \approx 10 \ \mathrm{s}^{-1}$$

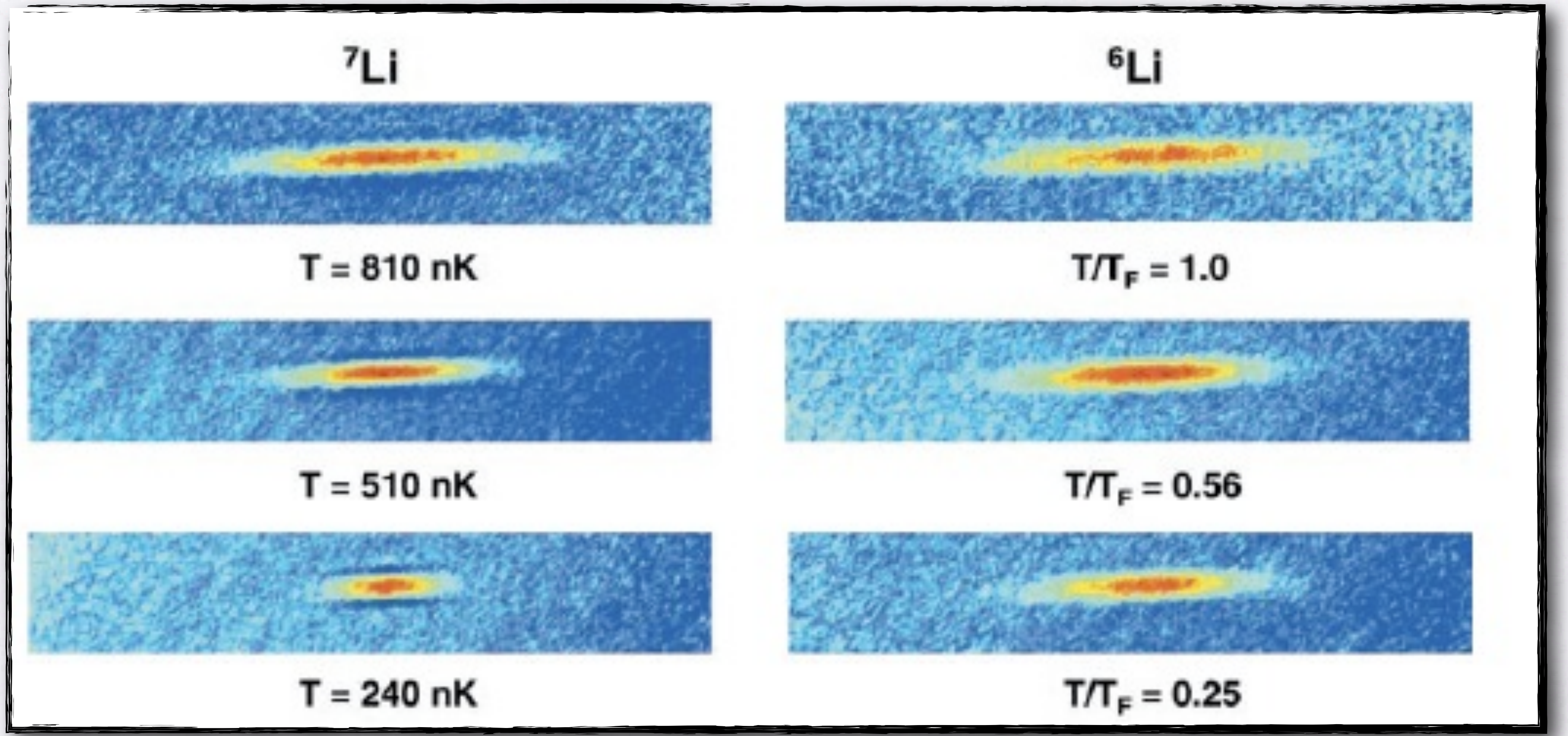
•proper ratio of thermal capacities:

$$C_b > C_f\,, \quad C = 3Nk_B \quad \Rightarrow \quad N_b > N_f$$



G.Roati, F. Riboli, G. Modugno, and M. Inguscio Phys. Rev. Lett. 89, 150403 (2002)

Achieving Fermi degeneracy



bosons : $n(r) = n_0(1-(r/R_B)^2)$, $R_B = \ell(15Na/\overline{\ell})^{1/5} \approx 25\mu\mathrm{m}$

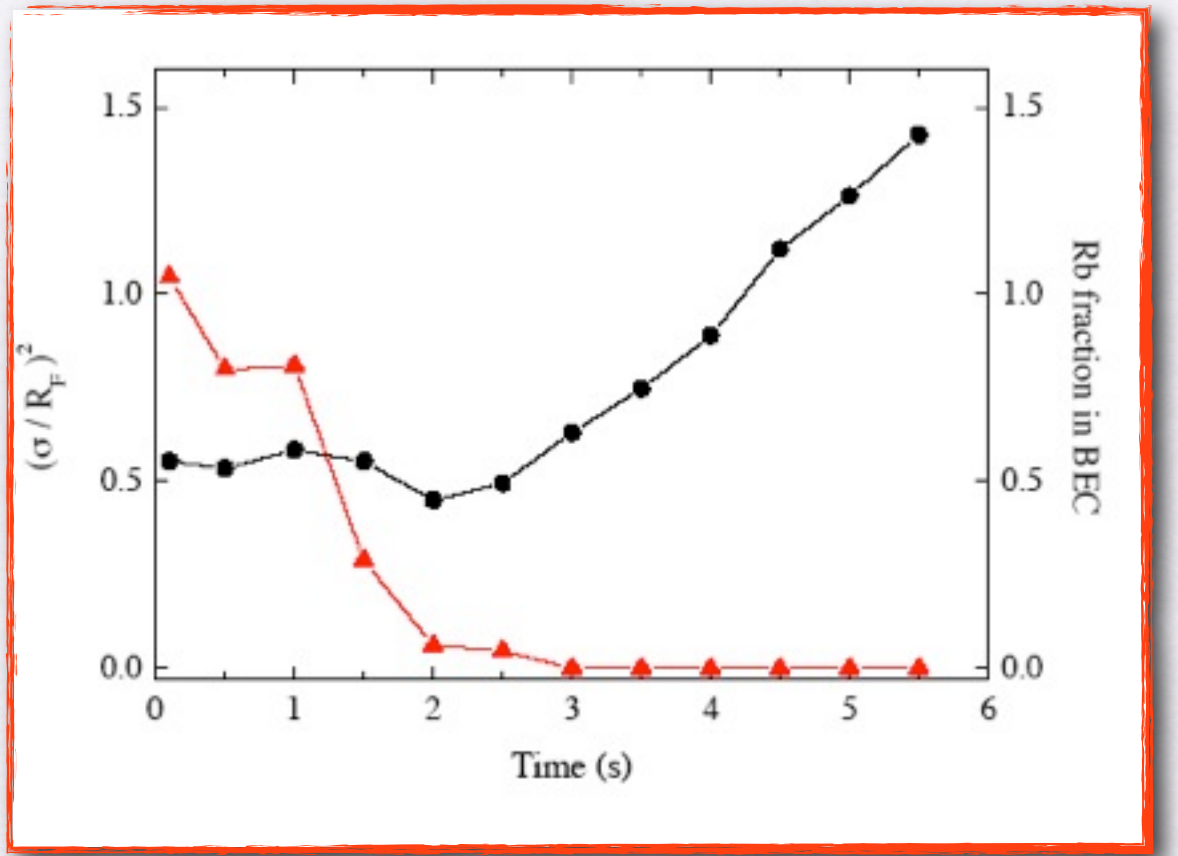
fermions: $n(r) = n_0(1-(r/R_F)^2)^{3/2}$, $R_F = \overline{\ell}(48N)^{1/6} \approx 50\mu\mathrm{m}$

A. G. Truscott et al., Science 291, 2569 (2001).

The idea is to measure the T of the boson cloud and to infer the same temperature to the fermions.

This method works if the thermal between the two clouds is present.

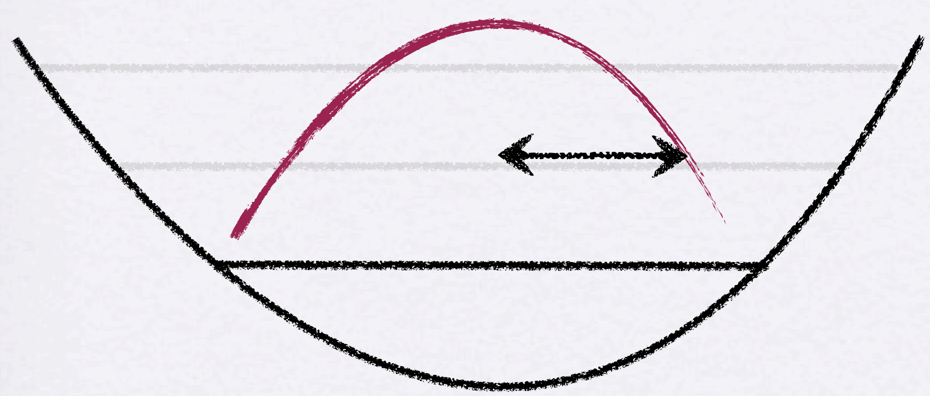
In case of degenerate clouds (a Fermi gas interacting with a BEC), this condition is not automatically accomplished! In case of mean-field repulsion, the two clouds separate (phase separation)
-> $T_{fermions} \neq T_{bosons}$



Summary

$$\boxed{\begin{aligned} E_F &= \hbar\varpi\sqrt[3]{6N} \\ R_F &= \sqrt{\frac{2E_F}{M\omega_r^2}} \end{aligned}}$$

It is worth to compare the size of the Fermi gas with the harmonic oscillator length of the ground state of our trap:



$$a_{ho} = \sqrt{\frac{\hbar}{m\omega}}$$

$$R_F \propto a_{ho}\sqrt[6]{N}$$

If N$\gg$ 1 then the Fermi radius R_F largely exceeds a_{h0}. This is another consequence of the Pauli exclusion principle.

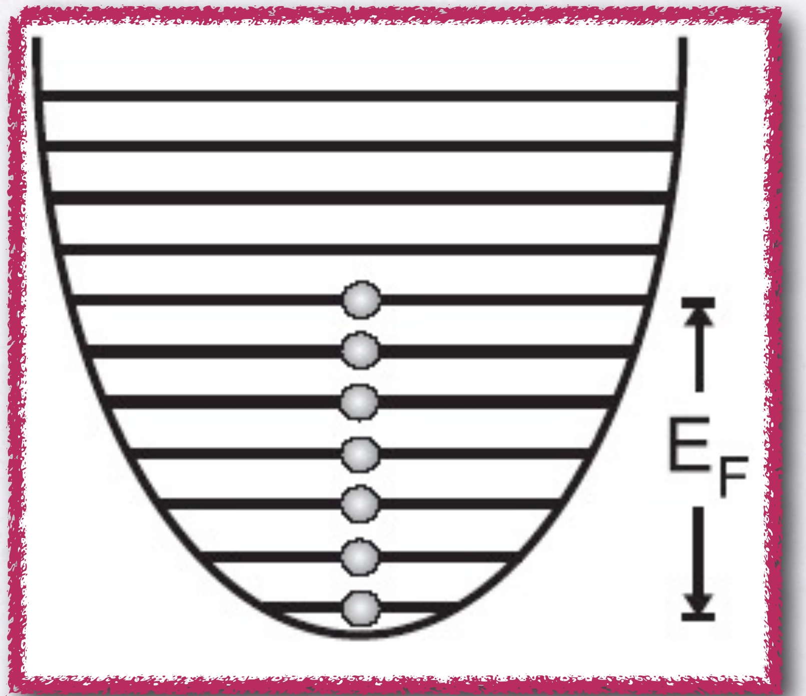
The Pauli exclusion principle induce an “effective” repulsion between the fermions in the trap -> Fermi pressure.

End lecture I

Summary

The Pauli principle forbids multiple occupation of a single quantum state: fermions go to occupy one by one every state. The energy of the last occupied state is named Fermi energy E_F.

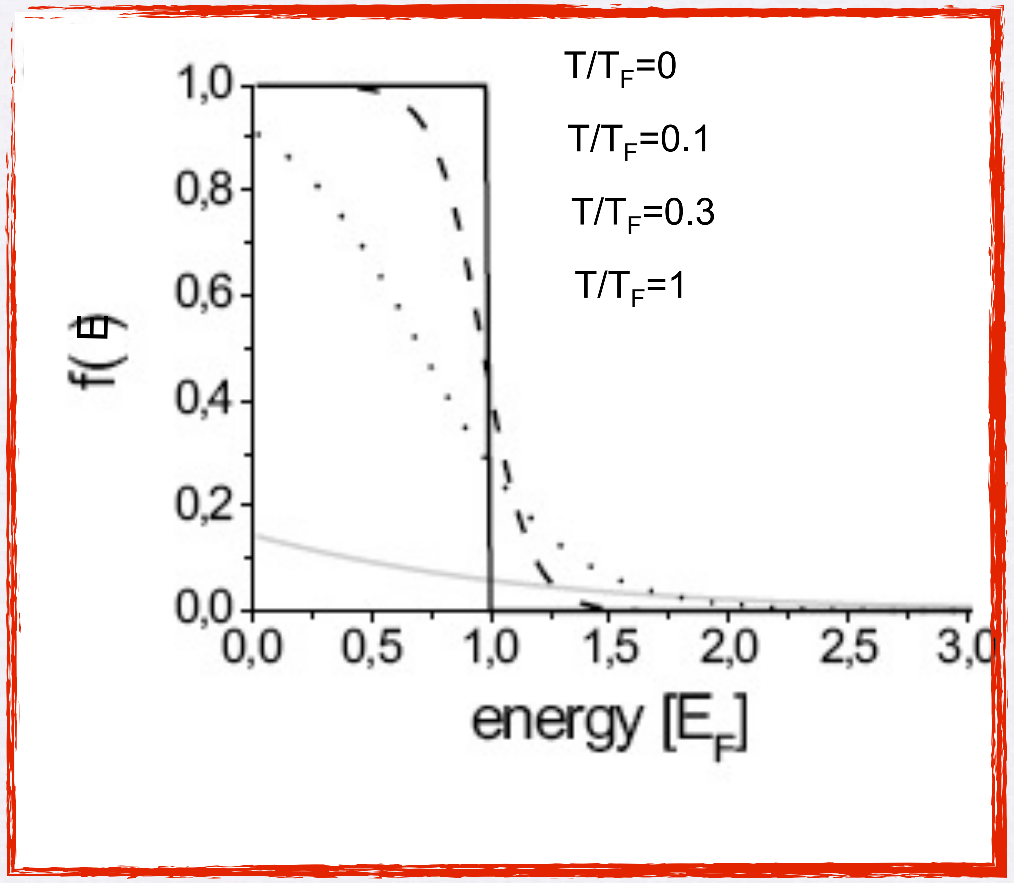
We define the Fermi temperature $T_F = E_F / k_B$



$$\boxed{f_F(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}}$$

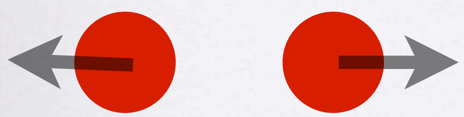
where $\beta = 1/k_B T$ and μ is the chemical potential. In particular $E_F = \mu$ for $T=0$.

The Fermi-Dirac distribution never exceeds 1, reflecting the Pauli exclusion principle

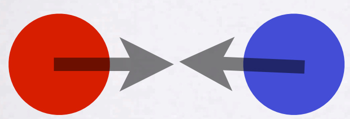


Summary

Consequences of the Fermi statistics on collisions of ultracold fermions.



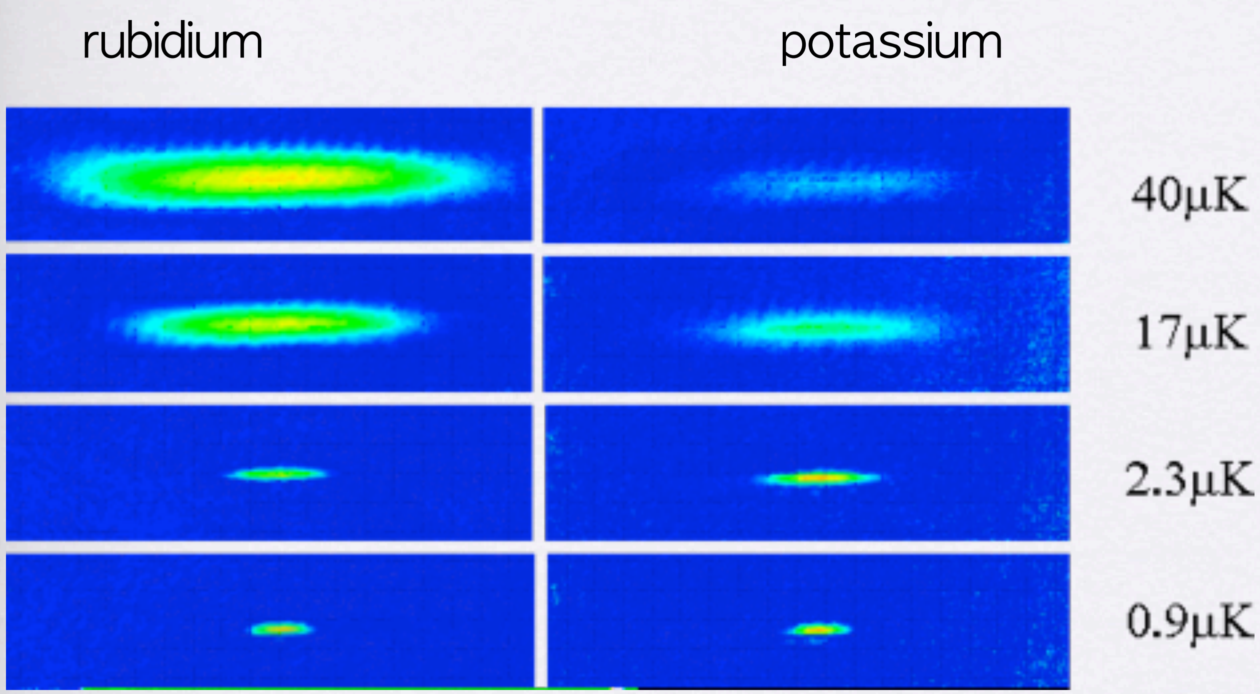
Identical fermions do not collide @ very low T (T〈 100 μ K)

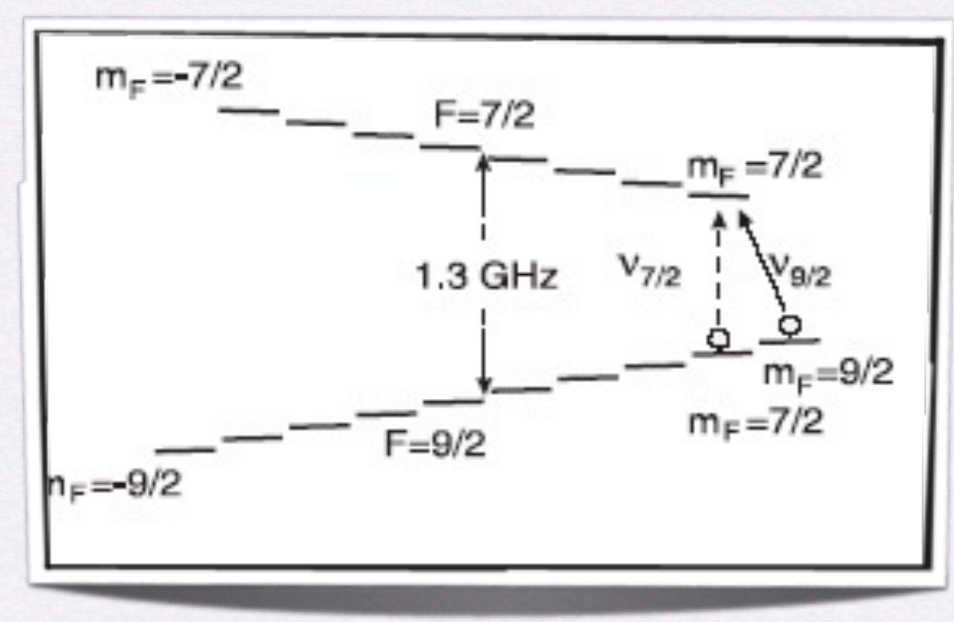


No-identical fermions collide (s-wave scattering!)

Mixture of fermions and bosons

Mixture of fermions in 2 spin states





What does it mean interaction between ultracold atoms?

We have seen that the key parameter for describing interactions @ this ultralow T is the scattering length a.

$$\boxed{\sigma = 4\pi a^2}$$

The scattering cross-section is depending from a^2. But what about the sign of the scattering length a?
Is the sign important when considering the interactions properties of ultracold atomic gases?

It depends:

a) If the system is not degenerate we do not care about the sign of a

b) If the system is degenerate the sign of a is very critical!

Mean-field interactions

Since these systems are very dilute, each atom in the sample sees the other ones by means of mean-field interactions.

This is the leading term in defining, for example, the stability of a Bose-Einstein condensate.

$$\boxed{i\hbar \frac{d}{dt}\Phi(\vec{r},t) = \left(-\frac{\hbar^2 \nabla^2}{2m} + V_{ext}(\vec{r}) + g\,|\Phi(\vec{r},t)|^2 \right) \Phi(\vec{r},t)} \qquad g = \frac{4\pi\hbar^2 a}{m}$$

Mean-field interaction depends only from the s-wave scattering length a, magnitude and sign!

1) a〉0 repulsive interactions (stable BEC, molecules formations)
2) a〈0 attractive interactions (instability, collapse..)

In case of a Fermi gas composed by identical particle $E_{mf}=0$. This is not true in case of two-spin states Fermi systems, or in case of Fermi-Bose systems.

In this last case, it is worth do define mean-field equations similar to GPE equation used for a single BEC.

$$n_F(x) = \frac{\sqrt{2m_F^3}}{3\pi^2}\left[\mu_F - U_F(x) - \frac{4\pi\hbar^2 a_{BF}}{2m_{BF}} n_B(x)\right]^{3/2} \qquad \text{fermions}$$

$$\left[-\frac{\hbar^2}{2m_B}\nabla^2 - \mu_B + U_B(x) + \frac{4\pi\hbar^2 a_{BF}}{2m_{BF}} n_F(x) + \frac{4\pi\hbar^2 a_B}{m_B}\Phi_B^2(x)\right]\Phi_B(x) = 0 \qquad \text{bosons}$$

In this case, the mean-field term is proportional to the interspecies scattering length a_{FB} and the stability of the mixture depends on the sign of a_{FB}.

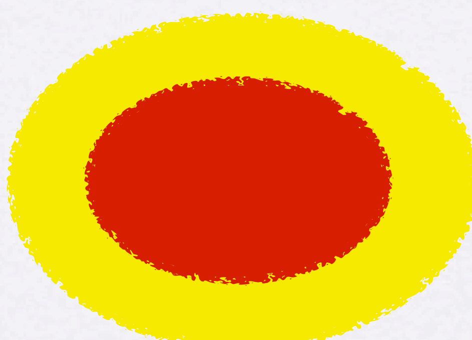
Mean-field interactions

The efficiency of evaporative-sympathetic cooling depends from the scattering cross section $\sigma = 4\pi a^2$: only the magnitude is involved!

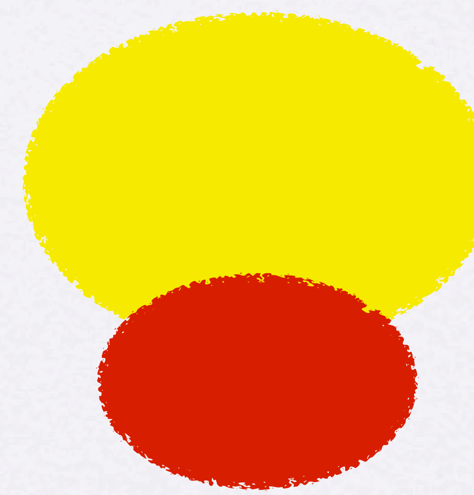
In case of sympathetic cooling, however, it is necessary to say that the sign of a_{ij} is important when the degenerate regime is reached.

If $E_{mf} \propto a_{ij} > k_B T$

$a_{ij} < 0$



$a_{ij} > 0$



If aij < 0 the sympathetic cooling is efficient also in the degenerate regime…
(limited only by Pauli blocking ,superfluidity)

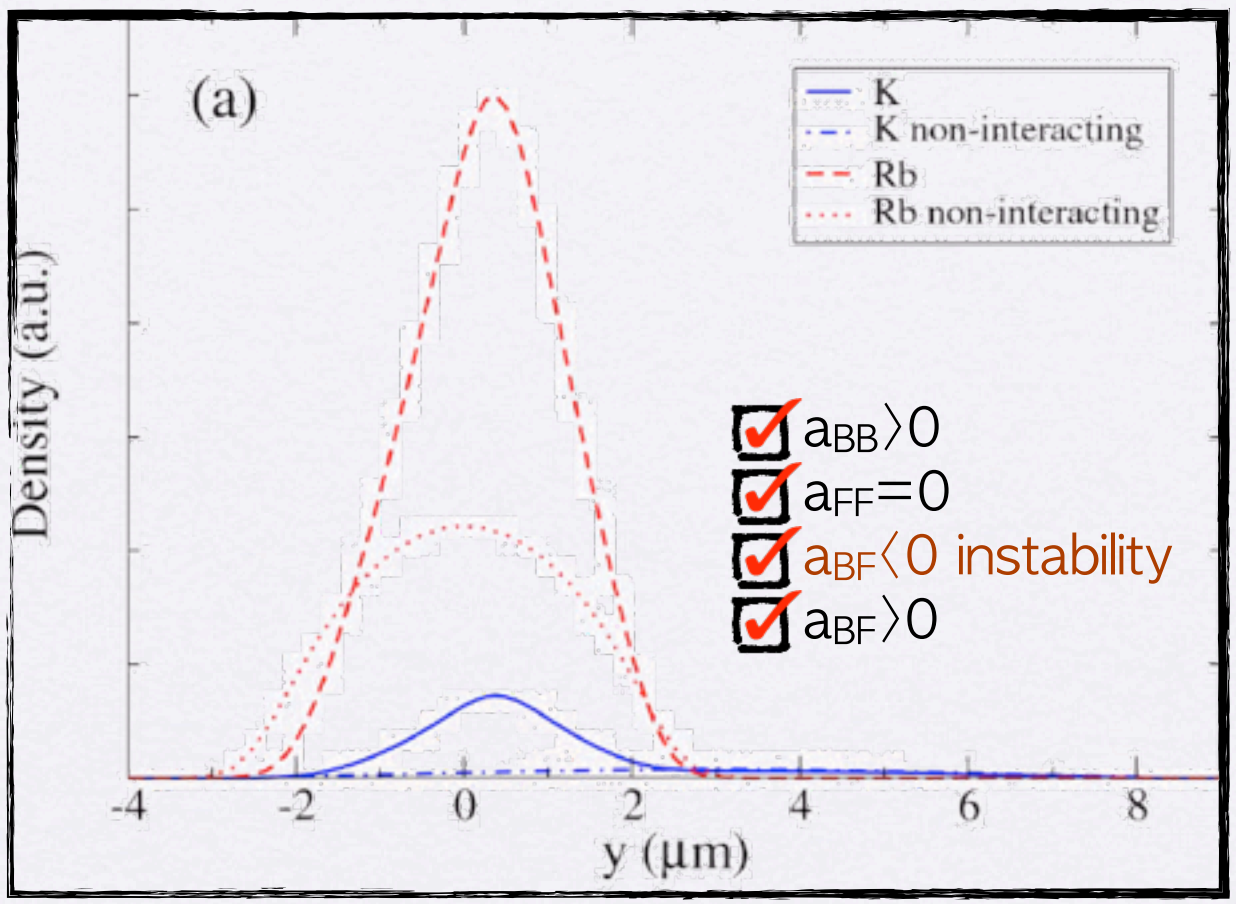
If aij > 0 the sympathetic cooling stops: the thermal contact is missing.

Molmer, Phys. Rev. Lett. 80, 1804,1998

Mean-field interactions

$$n_F(x) = \frac{\sqrt{2m_F^3}}{3\pi^2}\left[\mu_F - U_F(x) - \frac{4\pi\hbar^2 a_{BF}}{2m_{BF}}\, n_B(x)\right]^{3/2}$$

$$\left[-\frac{\hbar^2}{2m_B}\nabla^2 - \mu_B + U_B(x) + \frac{4\pi\hbar^2 a_{BF}}{2m_{BF}}\, n_F(x) + \frac{4\pi\hbar^2 a_B}{m_B}\Phi_B^2(x)\right]\Phi_B(x) = 0$$



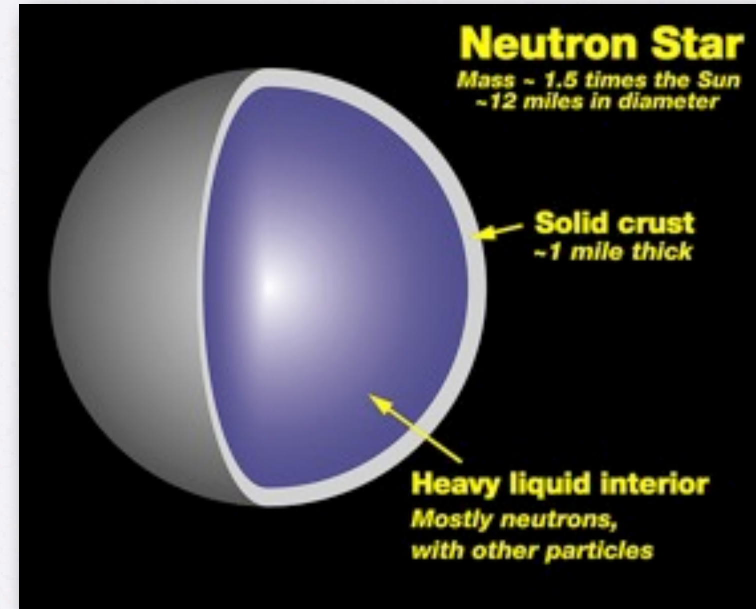
Mean-field Collapse of a Fermi system

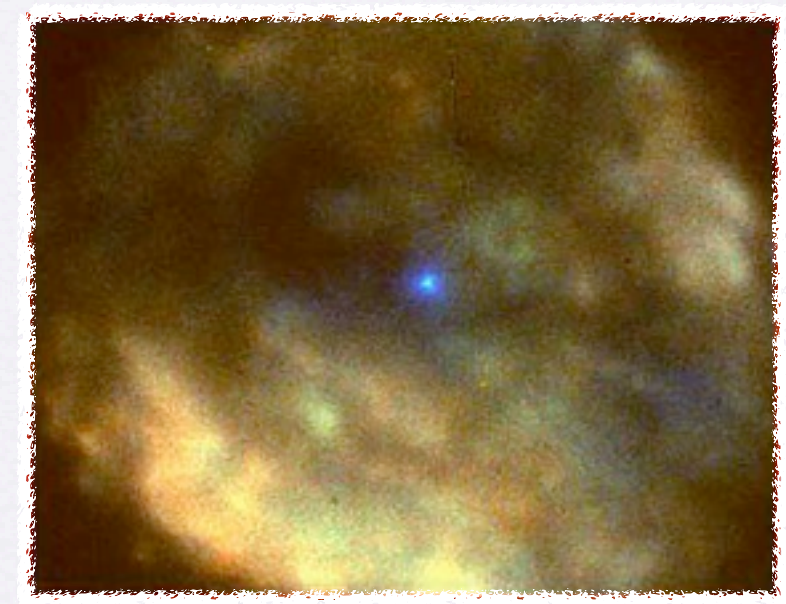
A neutron star is a type of remnant that can result from the gravitational collapse of a massive star during a supernova event. Such stars are composed almost entirely of neutrons, which are subatomic particles with close to zero electrical charge and roughly the same mass as protons. Neutron stars are very hot and are supported against further collapse because of the Pauli exclusion principle. This principle states that no two neutrons (or any other fermionic particle) can occupy the same quantum state simultaneously.

A typical neutron star has a mass between 1.35 and about 2.1 solar masses, with a corresponding radius of about 12 km. In contrast, the Sun's radius is about 60,000 times that. Neutron stars have overall densities predicted of 3.7×10^{17} (2.6×10^{14} times Solar density) to 5.9×10^{17} kg/m^3 (4.1×10^{14} times Solar density), which compares with the approximate density of an atomic nucleus of 3×10^{17} kg/m^3.The neutron star's density varies from below 1×10^9 kg/m^3 in the crust increasing with depth to above 6 or 8×10^{17} kg/m^3 deeper inside.

A neutron star is so dense that one teaspoon (5 millilitres) of its material would have a mass over 5×10^{12} kg. The resulting force of gravity is so strong that if an object were to fall from just one meter high it would only take one microsecond to hit the surface of the neutron star, and would do so at around 2000 kilometres per second, or 4.3 million miles per hour.

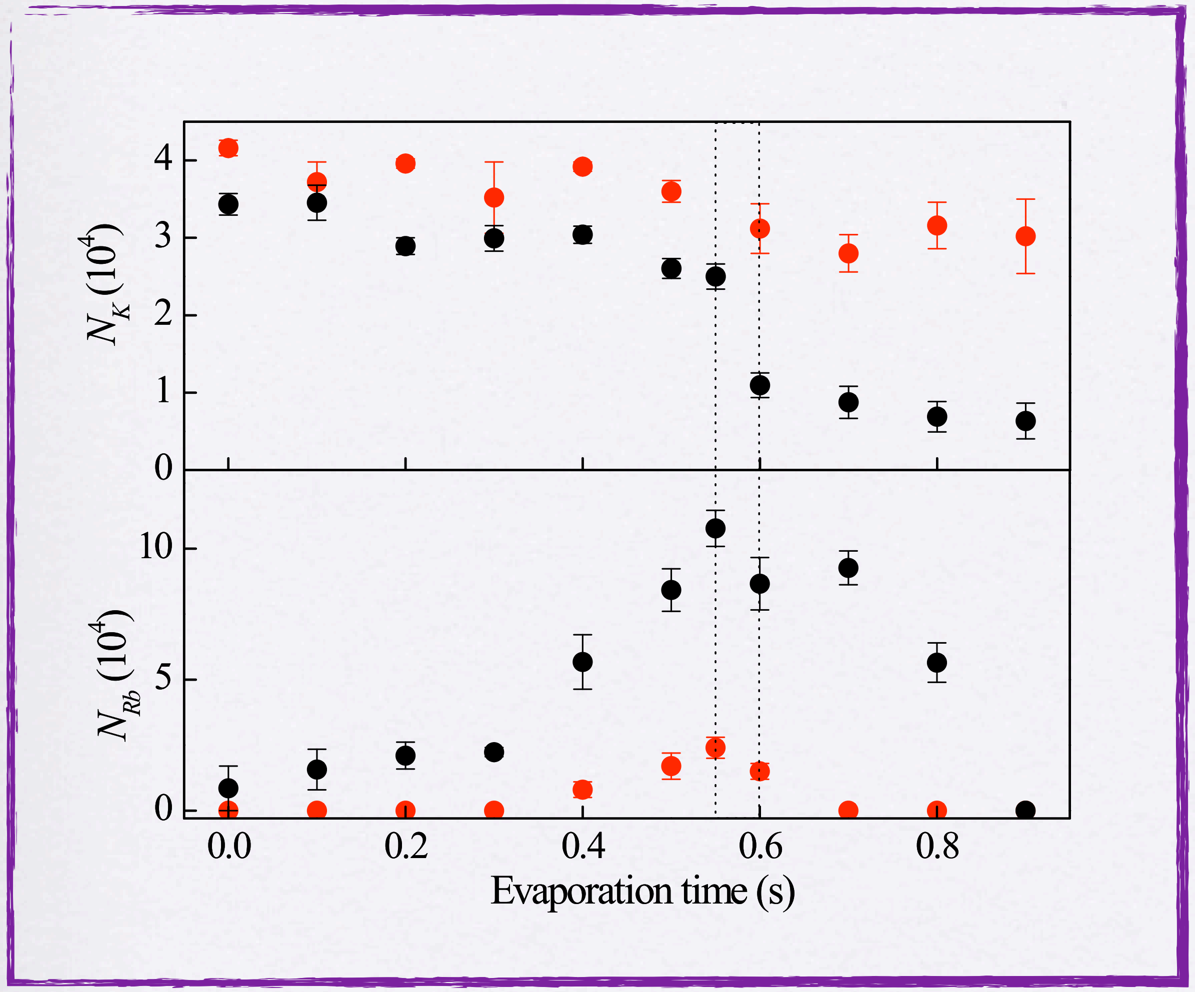
The temperature inside a newly formed neutron star is from around 10^{11} to 10^{12} Kelvin..



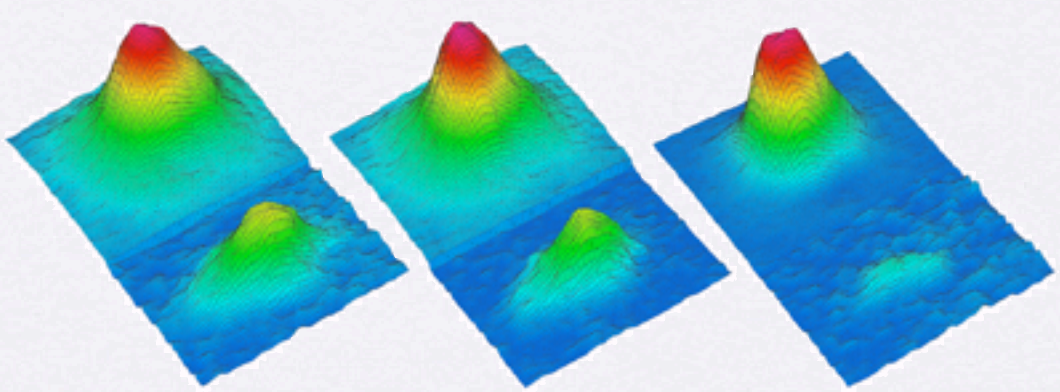


If M>3Ms Fermi pressure is not able to counteract the gravitational energy: collapse of the star & black hole formation

Mean-field Collapse of a Fermi system

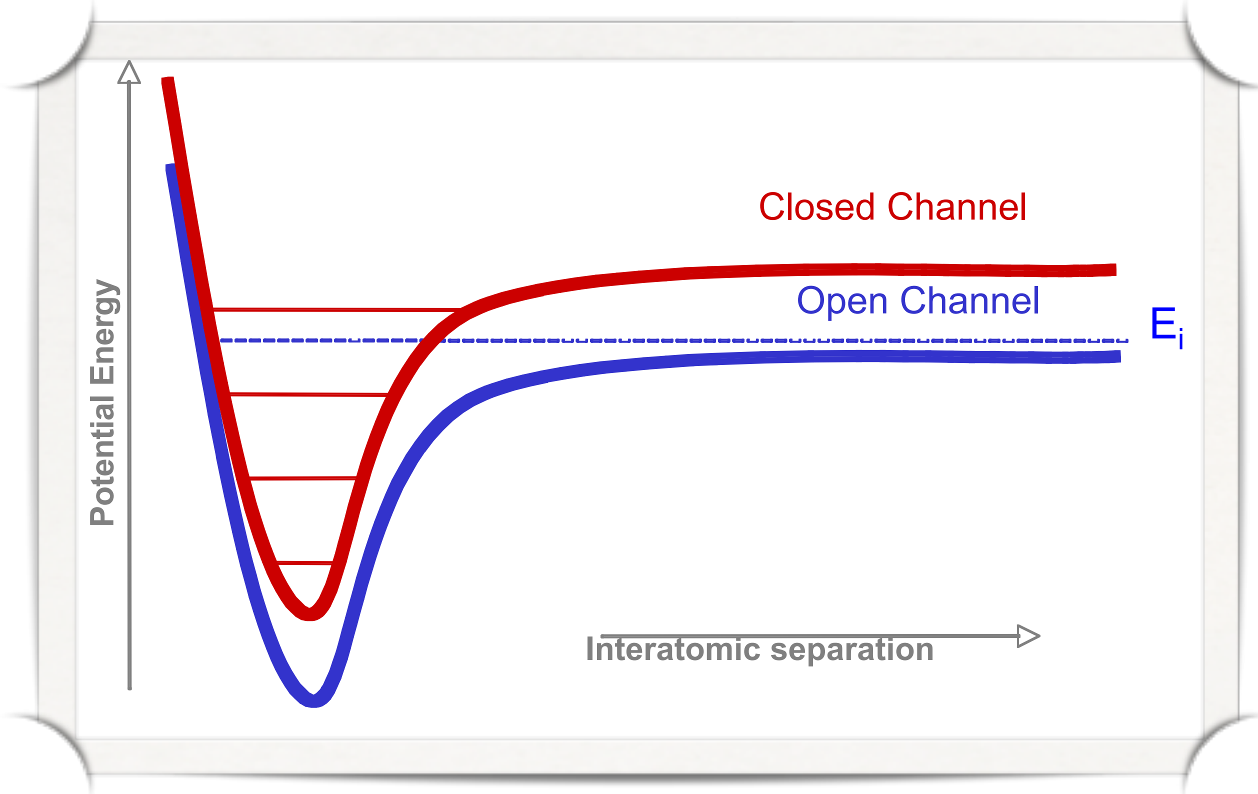


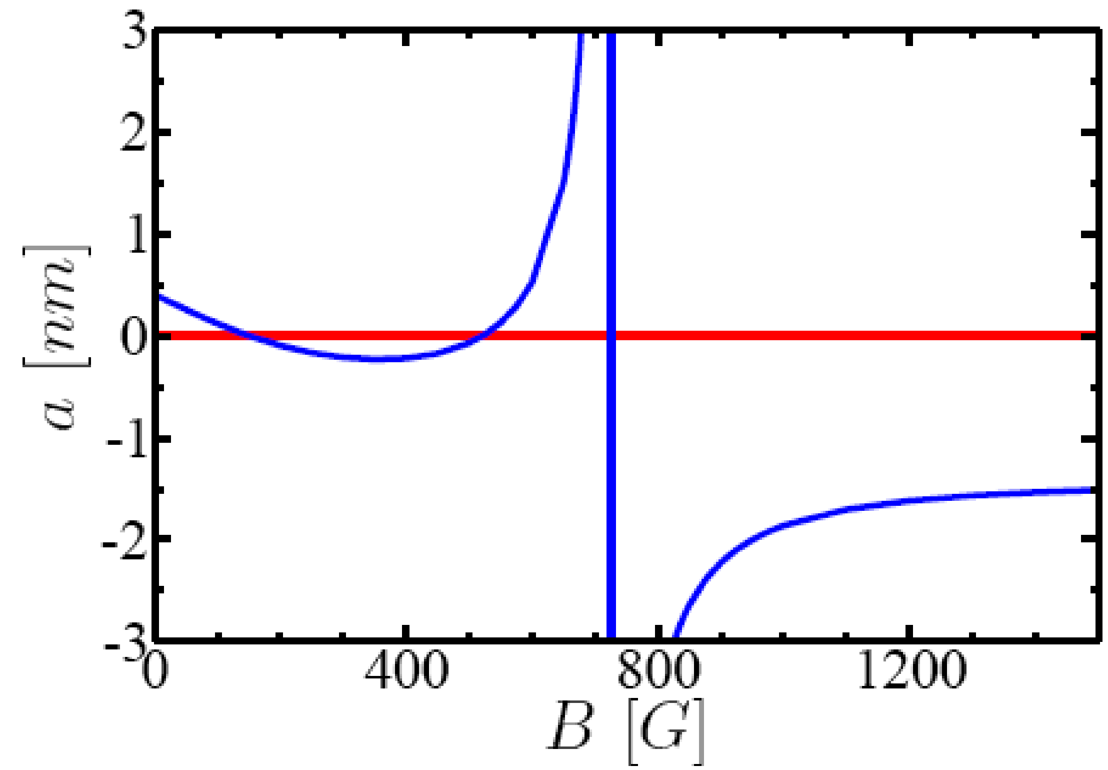
Fermions disappear from the trap as the evaporation increases BEC above a threshold



The attractive interaction beats Fermi pressure: analogy with neutron stars

Feshbach resonances



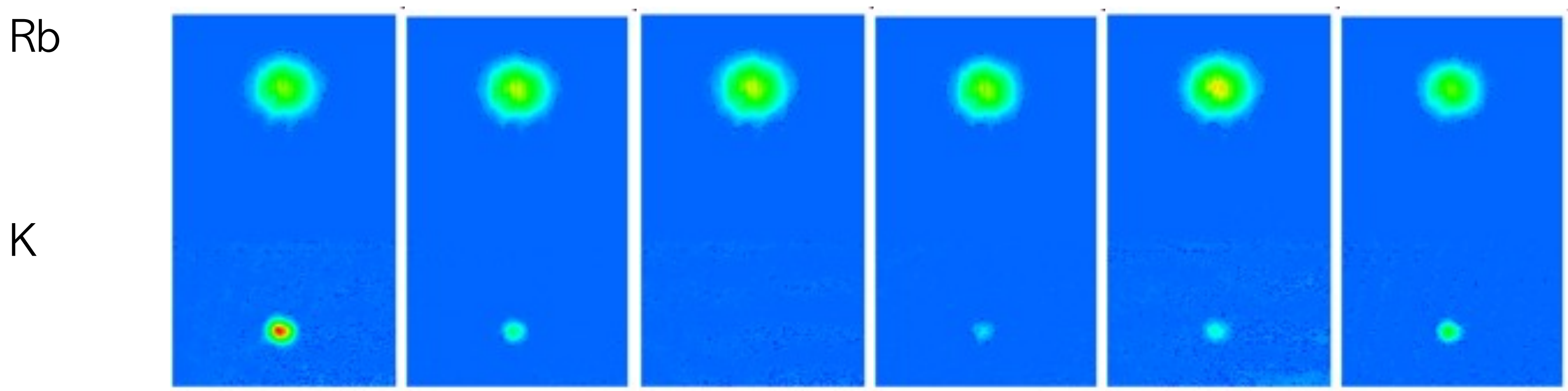


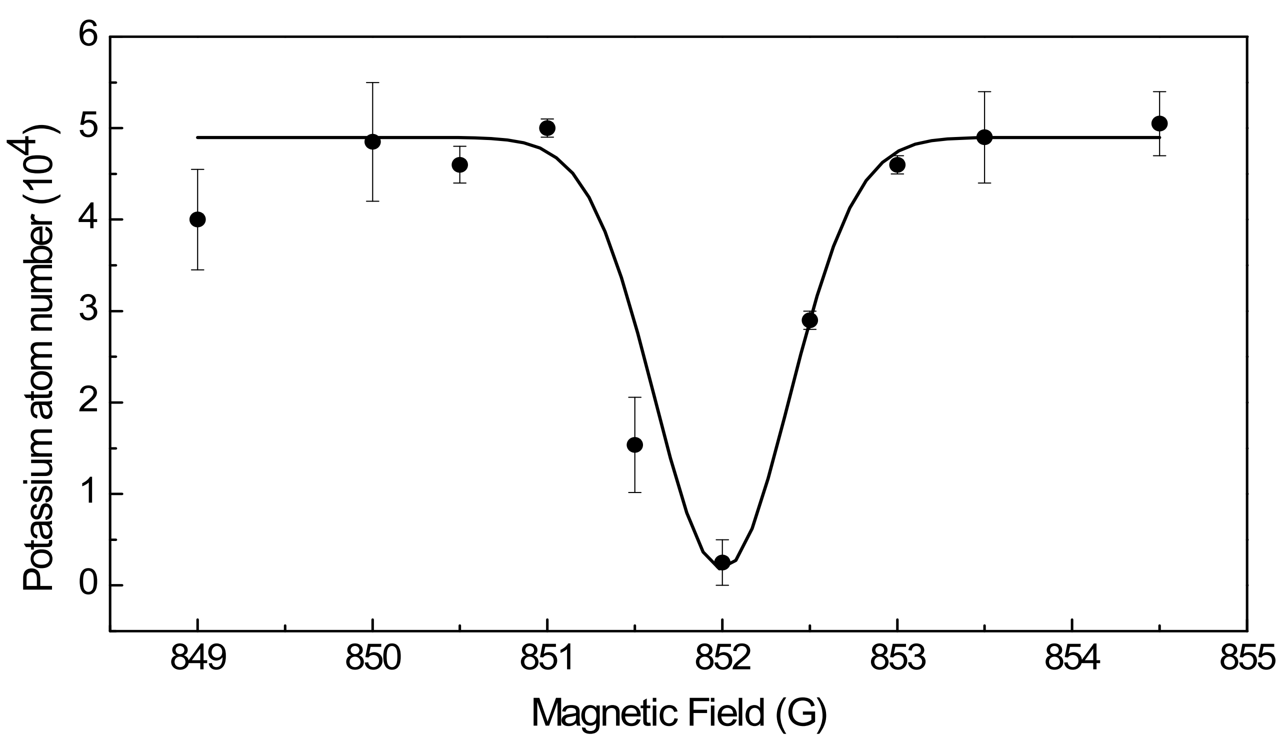
$$a(B) = a_{bg}\left(1 - \frac{\Delta}{B - B_{peak}}\right)$$

When a collisional channel of 2 colliding atoms has the same energy of a molecular state, we observe Feshbach resonance

Feshbach resonances

The first signature of Feshbach resonance are inelastic losses:



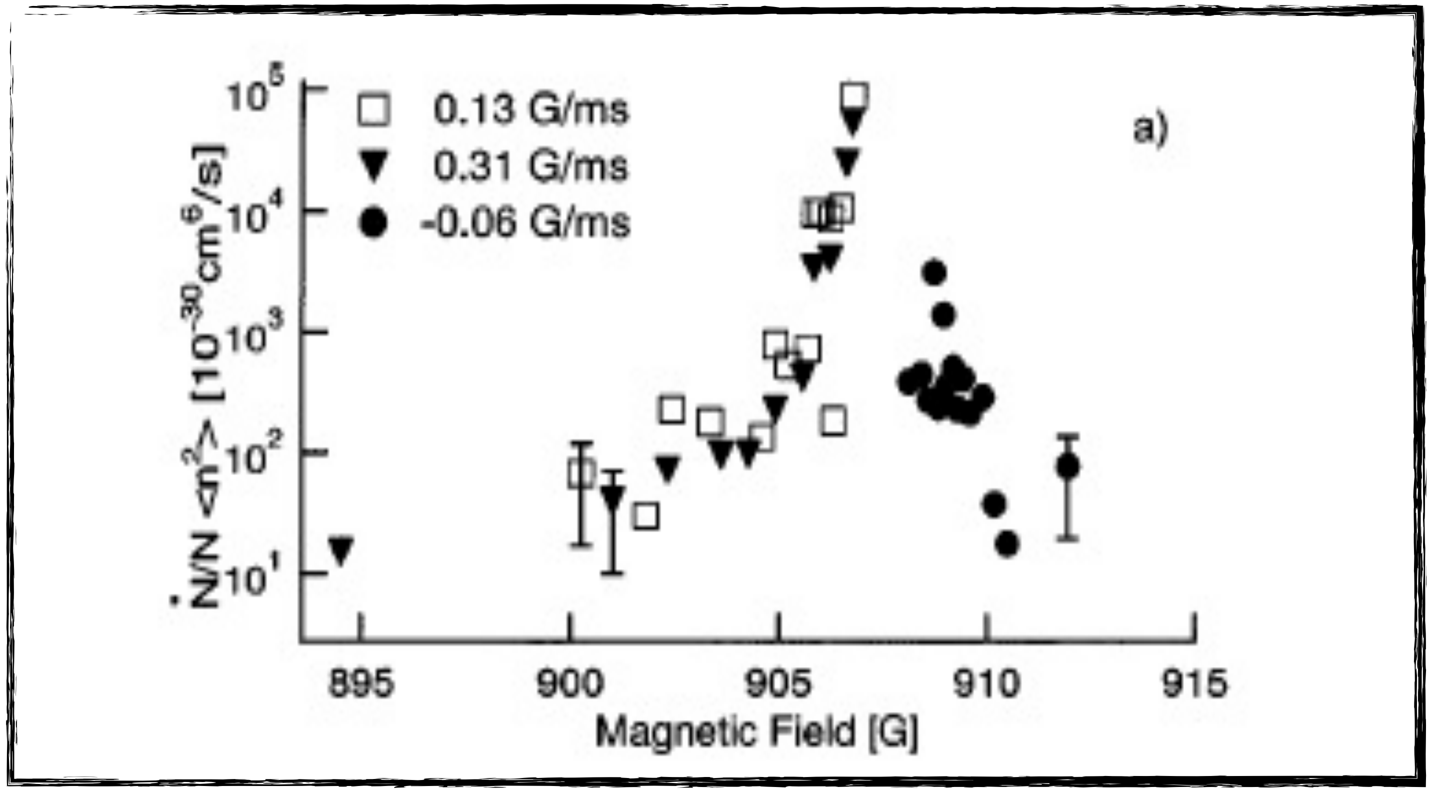


In fact these molecules are weakly bound:
collisions with atoms
decay and heating
molecules are not revealed by TOF

Feshbach resonances

The measured inelastic losses indicate the presence of a Feshbach resonance, but..
Playing with FR means to know the behavior of the elastic cross-section versus the magnetic field B (region when a>0).

The inelastic losses have a strong dependence on the scattering length @ Feshbach resonance:

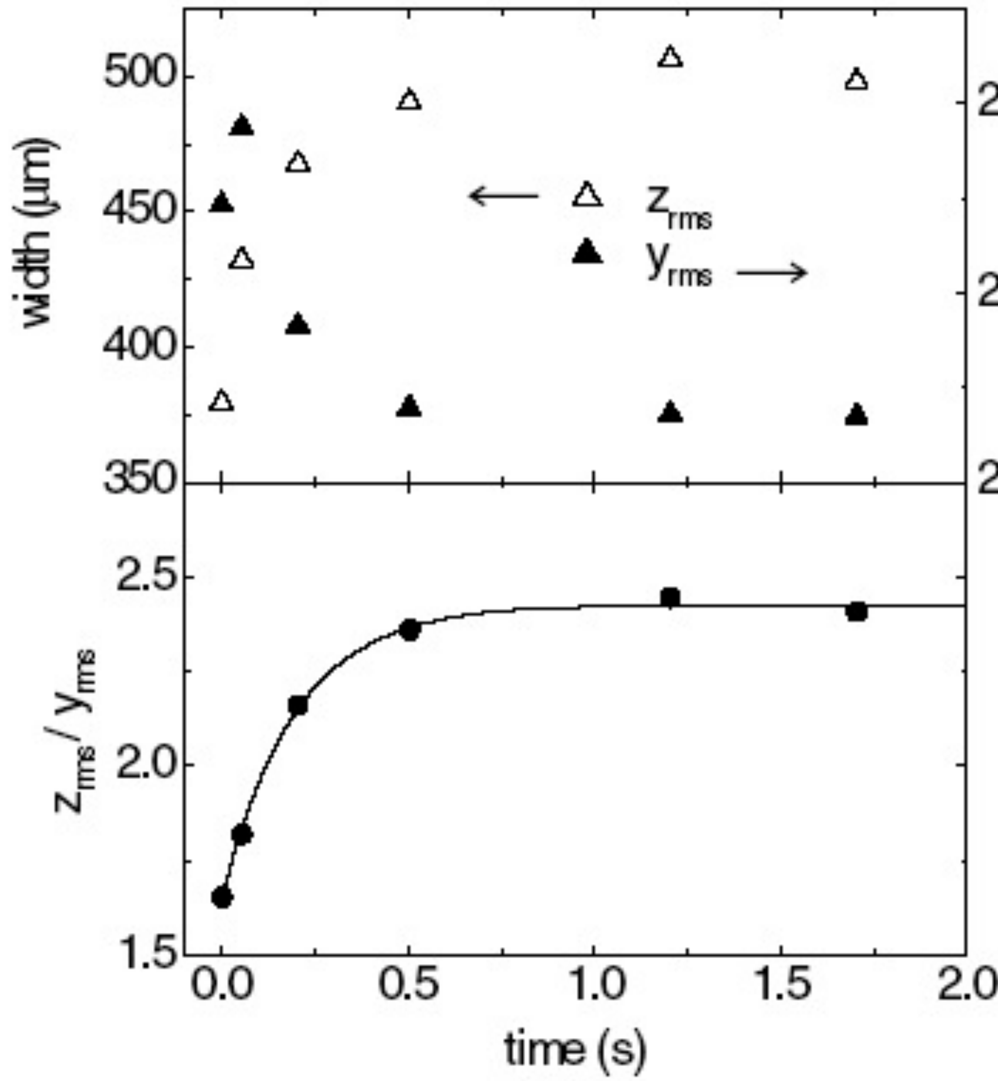


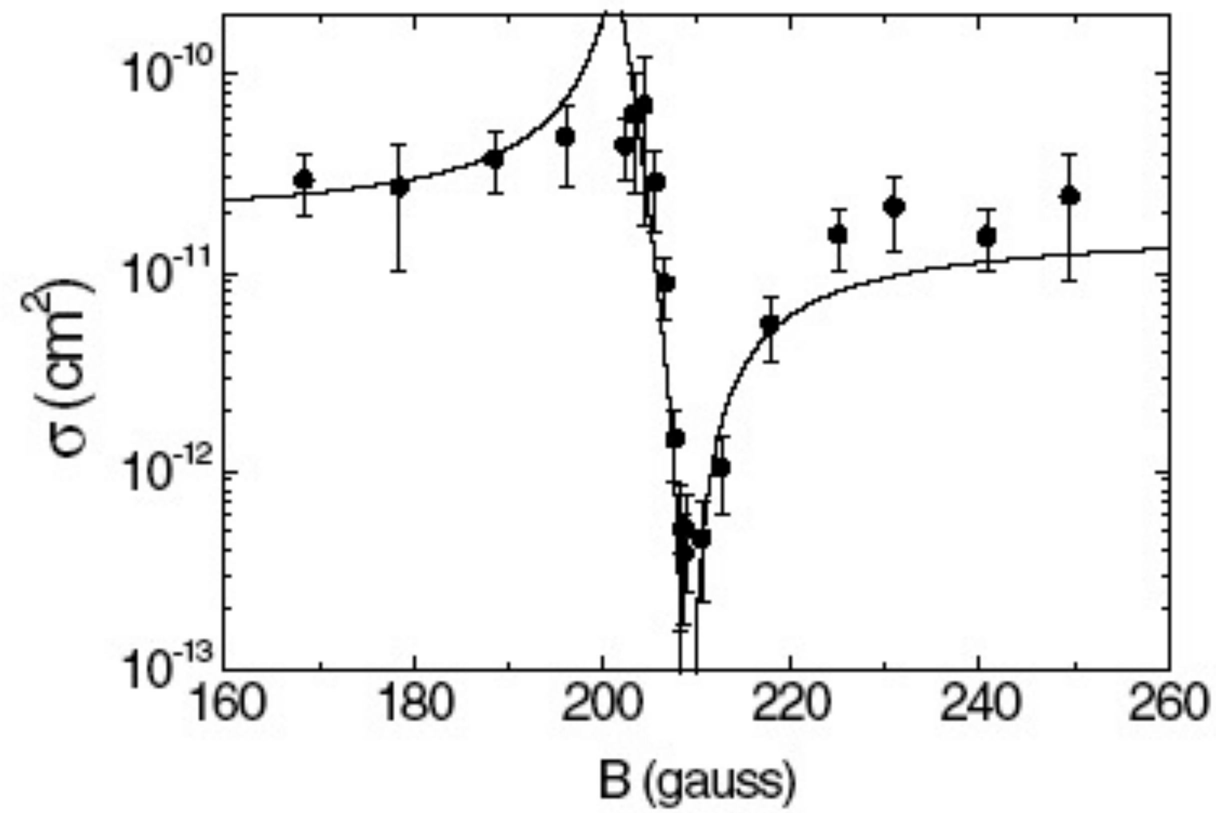
$$\boxed{K_3 \propto a^4}$$

The width of the resonance found by inelastic losses: the real width of the Feshbach resonance.

Feshbach resonances

To measure the dependence of the elastic cross-section versus the magnetic field is sufficient a cross-thermalization measurement versus the magnetic field:



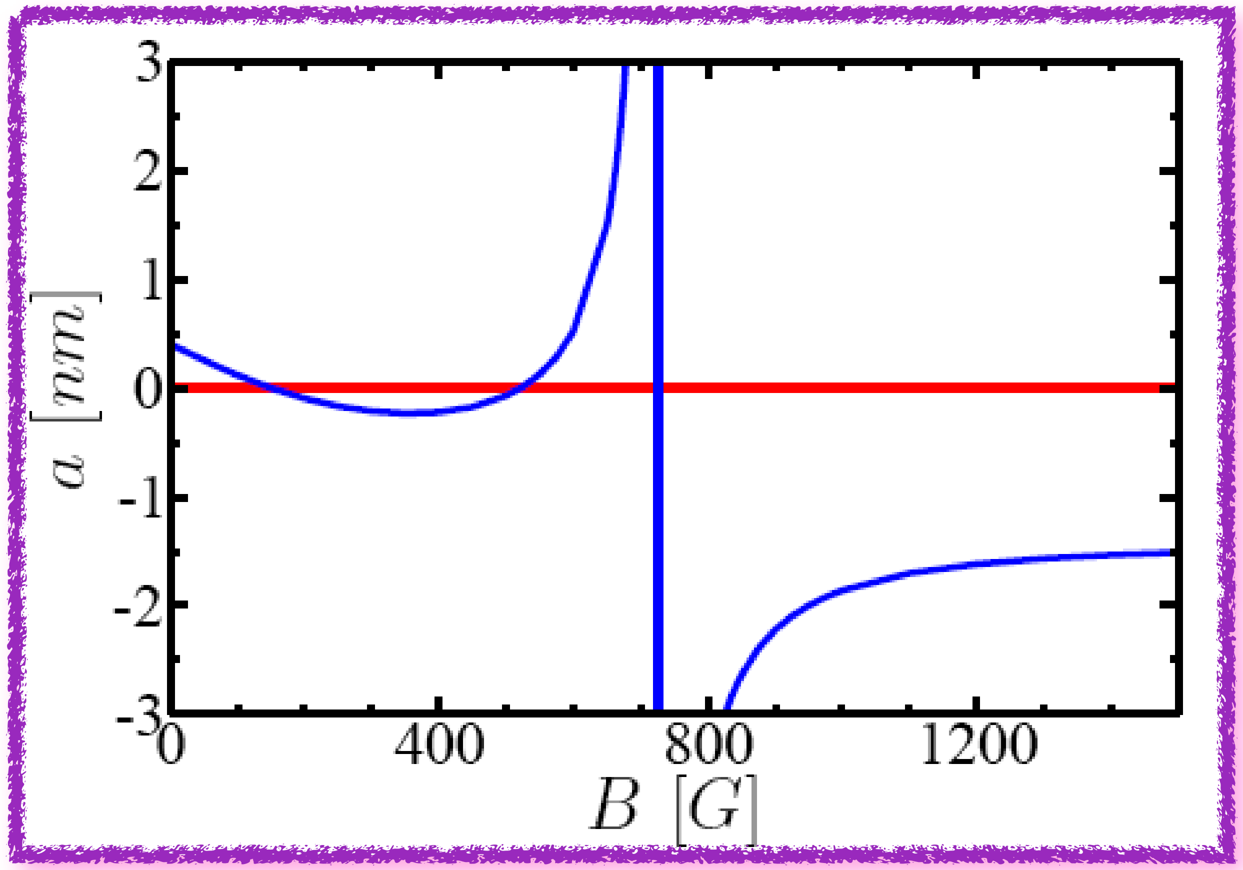


Exciting one degree of freedom and observing the consequent thermalization with the other degree by mean of elastic collisions:

$$\frac{1}{\tau} = \beta n \sigma v$$

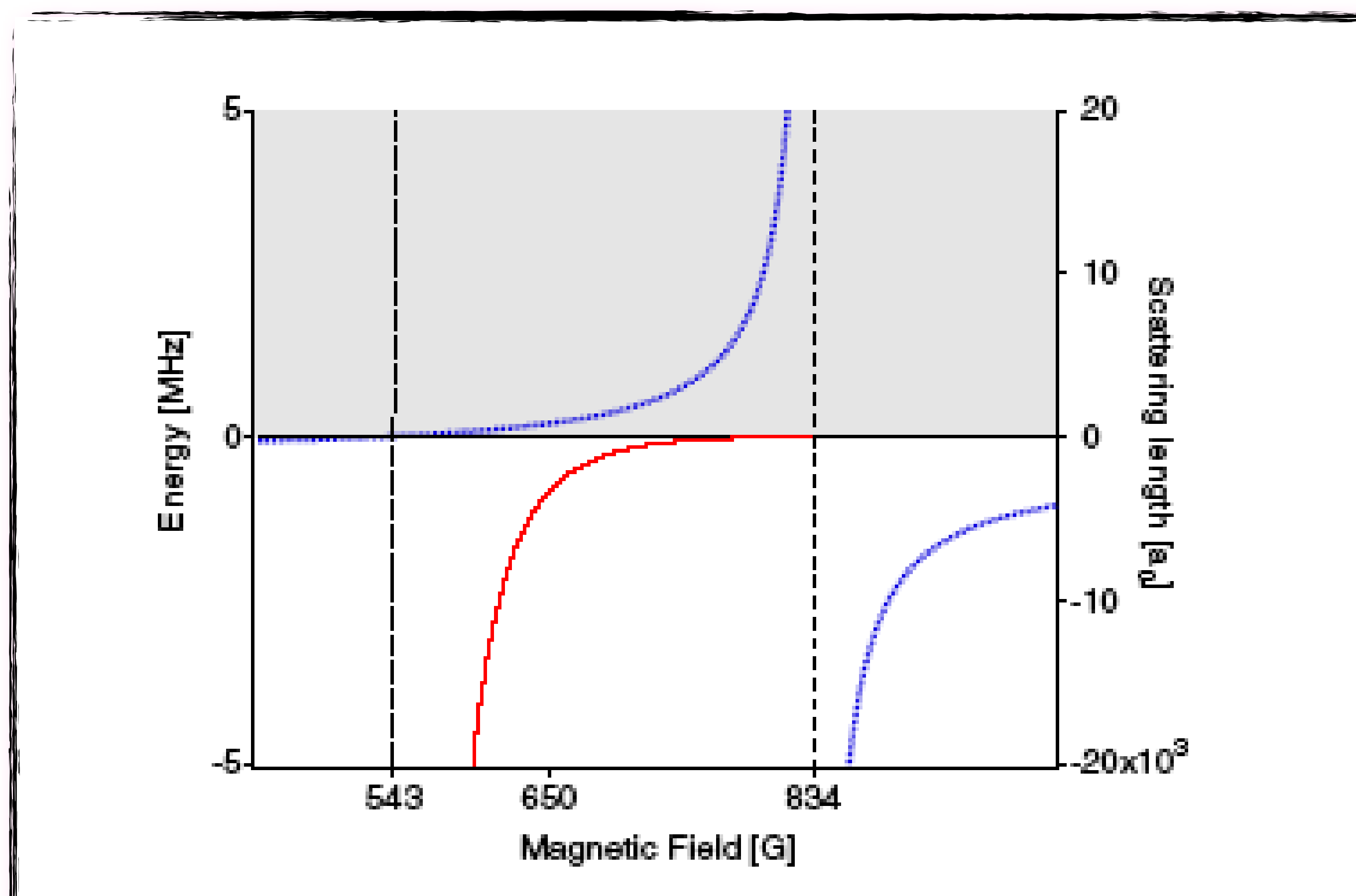
$$\sigma = 4\pi a^2(B)$$

Feshbach resonances



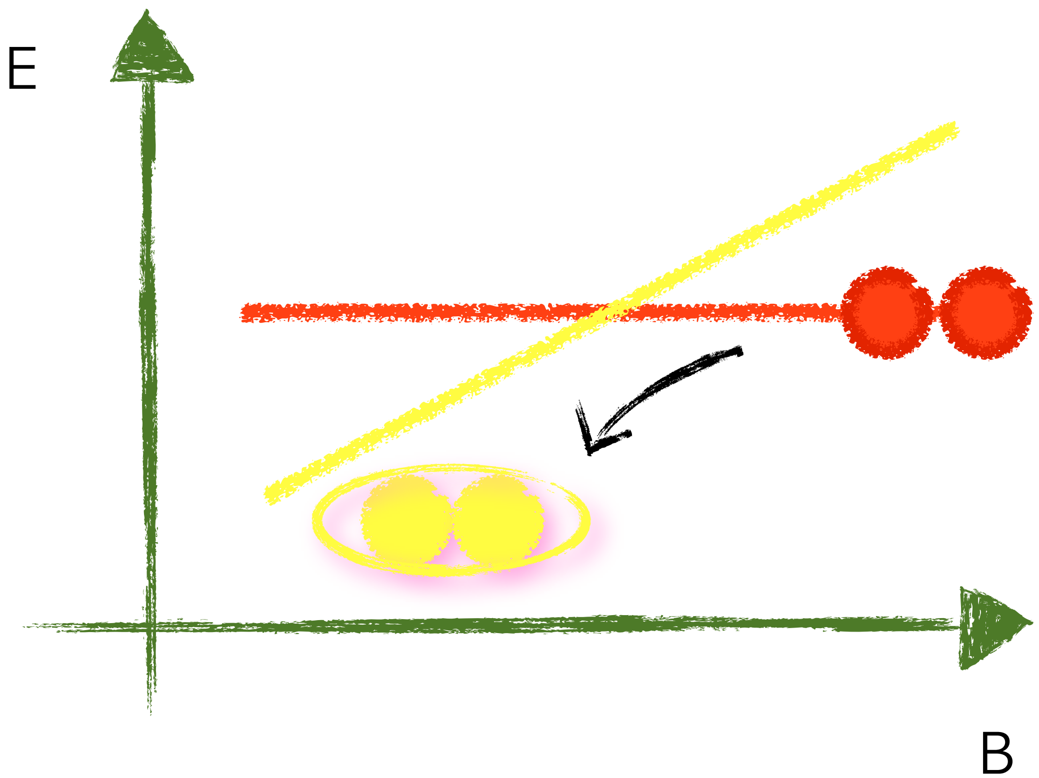
By tuning the magnetic field it is possible to change the value of a: strong interactions!!

By sweeping across the resonance it is possible to form molecules



Feshbach resonances

By adiabatic sweeping the magnetic field across a Feshbach resonance is possible to convert pair of atoms in molecules (a〉0).



$$E_{bind} = -\frac{\hbar^2}{ma^2} \sim 200 - 10KHz$$

$$\phi(r) \approx \frac{\exp(-r/a)}{r}$$

These molecules which are well all in the same internal state are highly vibrationally excited , very large and weakly bound

$d \approx a/2 \geq 1000\ \mathrm{a}_0$ (more than 10 times H_2 molecules)

Superfluidity is a phenomena shown in many different aspects of nature:

- ✓ Liquid Helium: neutral atoms (1908)
- ✓ Superconductors: charged particles (1911)
- ✓ Nuclei
- ✓ Neutron stars

Superfluidity and Superconductivity are quantum phenomena:

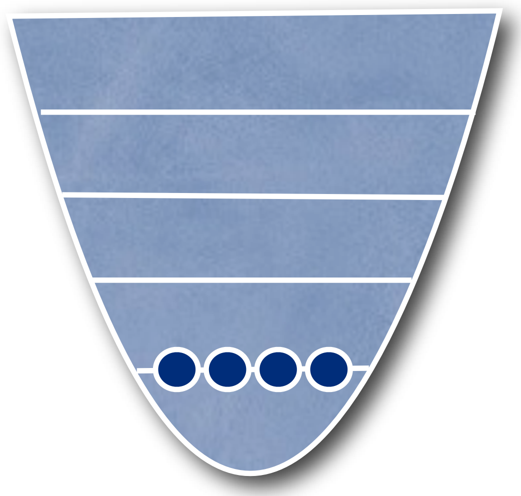
"..the density of the helium, which at first quickly drops with the temperature, reaches a maximum at 2.2 K approximately, and if one goes down further even drops again. Such an extreme could possibly be connected with the quantum theory"

Kamerlingh Onnes, Nobel lecture 1913

Interacting Fermi gases: superconductivity

Superfluidity:
condensed bosons (T_{BEC}) ?

Superconductor:
condensed fermions (T_F) ?

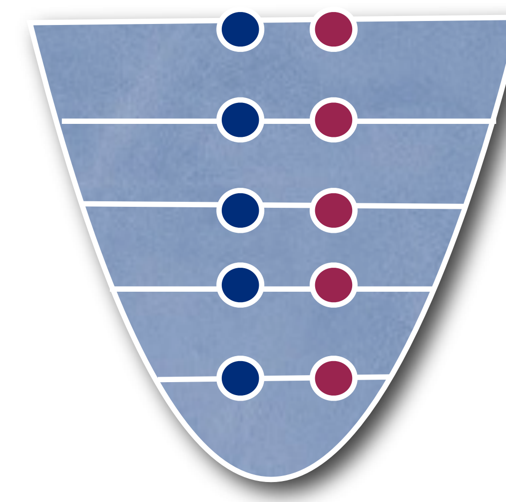


Degenerate regime:

$\lambda_B = h/\sqrt{(2\pi m k_B T)}$

$N_{atoms} = N_{states}\ (\rho)$

$N_{states} = \rho/\lambda^3_B$



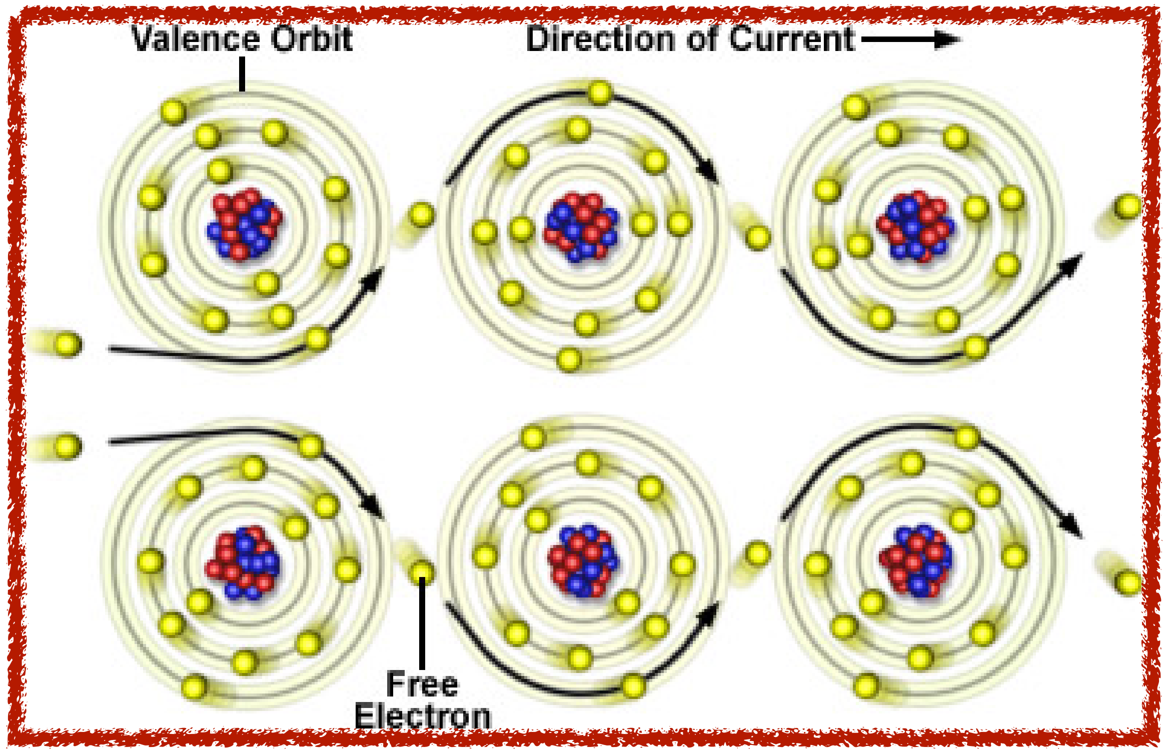
$T_{BEC} \approx 1/m\ (N/\rho)^{2/3}$
$m = m_{He}$
$T_{BEC} \approx 3K$

$T_F \approx 1/m\ (N/\rho)^{2/3}$
$m_{He}/m_e = 7000$
$T_F \approx 50000K$

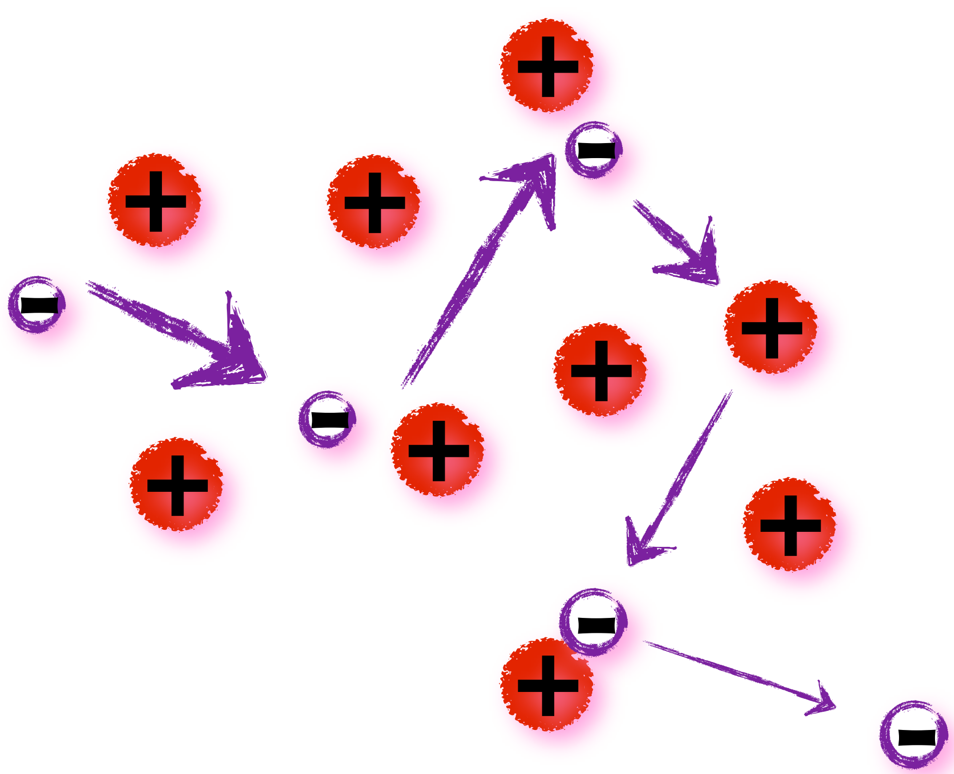
Problem: to find a mechanism for pairing fermions (electrons):

✦ Coulomb repulsion
✦ Critical temperature of the order of E_F/k_B (thousands K), in contrast with the experimental findings: $T(^3He) \approx mK$

Superconductivity

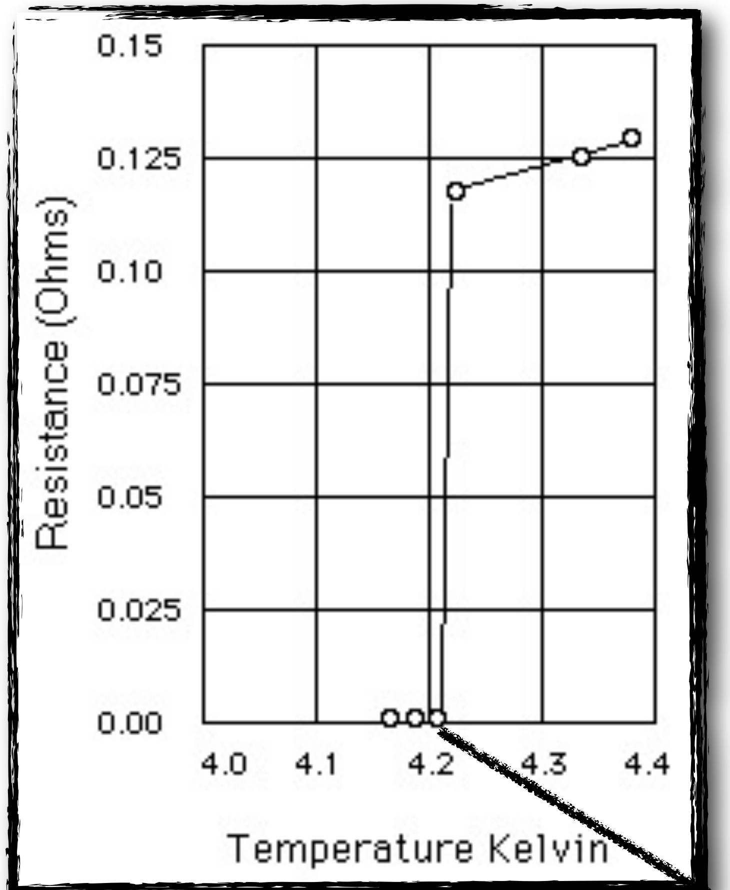


When current flows in an ordinary conductor some energy is lost. Electricity is conducted as outer energy level electrons migrate as individuals from one atom to another. The atoms in the metal form a vibrating lattice within the metal conductor; the warmer the metal the more it vibrates.



As the electrons begin moving through the maze, they collide with tiny impurities or imperfections in the lattice. When the electrons bump into these obstacles they fly off in all directions and lose energy in the form of heat.

Superconductivity



$10^{-5}\ \Omega$

The resistivity of mercury drops to almost zero @ T=4.21 K (Kamerlingh-Omnes, 1911): superconductivity!

Superconducting elements in the periodic system

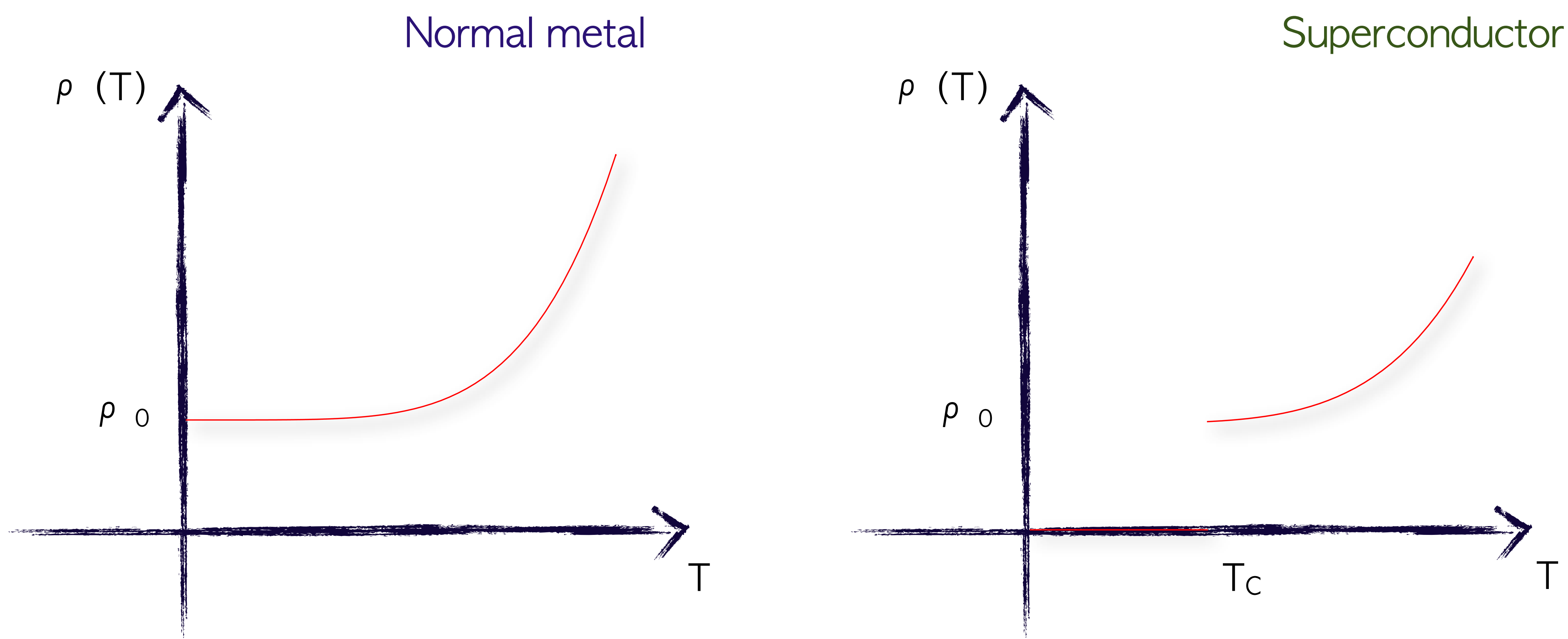
Transition temperatures in °K for the common crystal forms of the elements, after Matthias, Geballe, and Compton. Columns not shown contain no known superconductors in the common crystal forms.

B											C
Al 1.18											Si
Sc	Ti 0.39	V 5.03	Cr	Mn	Fe	Co	Ni	Cu	Zn 0.85	Ga 1.09	Ge
Y	Zr 0.55	Nb 9.1	Mo 0.92	Tc 11.2	Ru 0.49	Rh	Pd	Ag	Cd 0.52	In 3.41	Sn 3.72
La(β) 6	Hf 0.16	Ta 4.48	W 0.01	Re 1.7	Os 0.66	Ir 0.14	Pt	Au	Hg(α) 4.15	Tl 2.37	Pb 7.19
Ac											

Ce	Pr	Nd
Th 1.37	Pa 1.4	U

Superconductivity

I. Current flows without dissipation (analogy with superfluids ^{4}He)



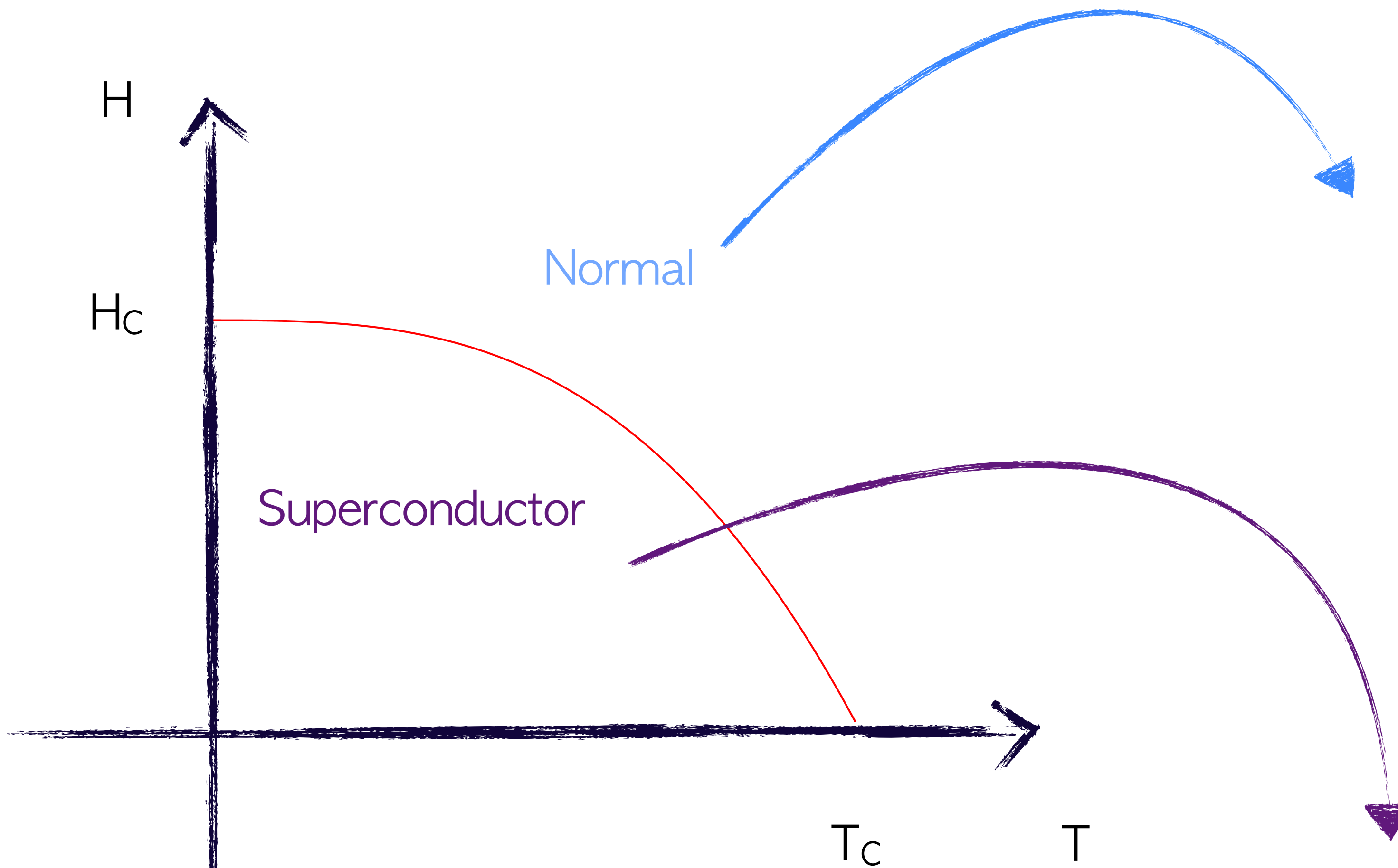
$$\boxed{\rho(T) = A + BT^5}$$

Impurities and
defect scattering

Phonons scattering

II. Perfect diamagnetism ($H_{in}=0$) : Meissner effect

Superconductivity



If the external magnetic field is too strong, the system prefers to come back to the normal state: too much energy needed for "expelling" the field

The external magnetic field cannot penetrate inside the superconductor. Screening currents, located close to surface (10^{-5} cm), do cancel the "internal" magnetic field.

Superconductivity

Conductivity, $\sigma \rightarrow \infty$ for a superconductor + $$\mathrm{E} = -\frac{d\phi_B}{dt}$$ = Meissner effect??

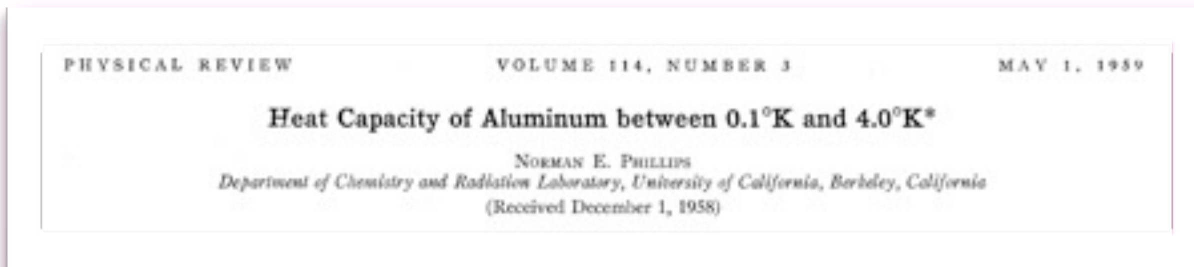
No: The Meissner effect cannot be explained only by the infinite conductivity of a superconductor! Infact the superconductor expels the magnetic field even if stationary!!

Note: Superconductors are not perfect conductors but perfect diamagnet

Superconductivity

III. Existence of an energy gap 2Δ around the Fermi energy: it is possible to add or extract an electron only paying an energy 2Δ

Superconductivity


PHYSICAL REVIEW VOLUME 114, NUMBER 3 MAY 1, 1959

Heat Capacity of Aluminum between 0.1°K and 4.0°K*

NORMAN E. PHILLIPS
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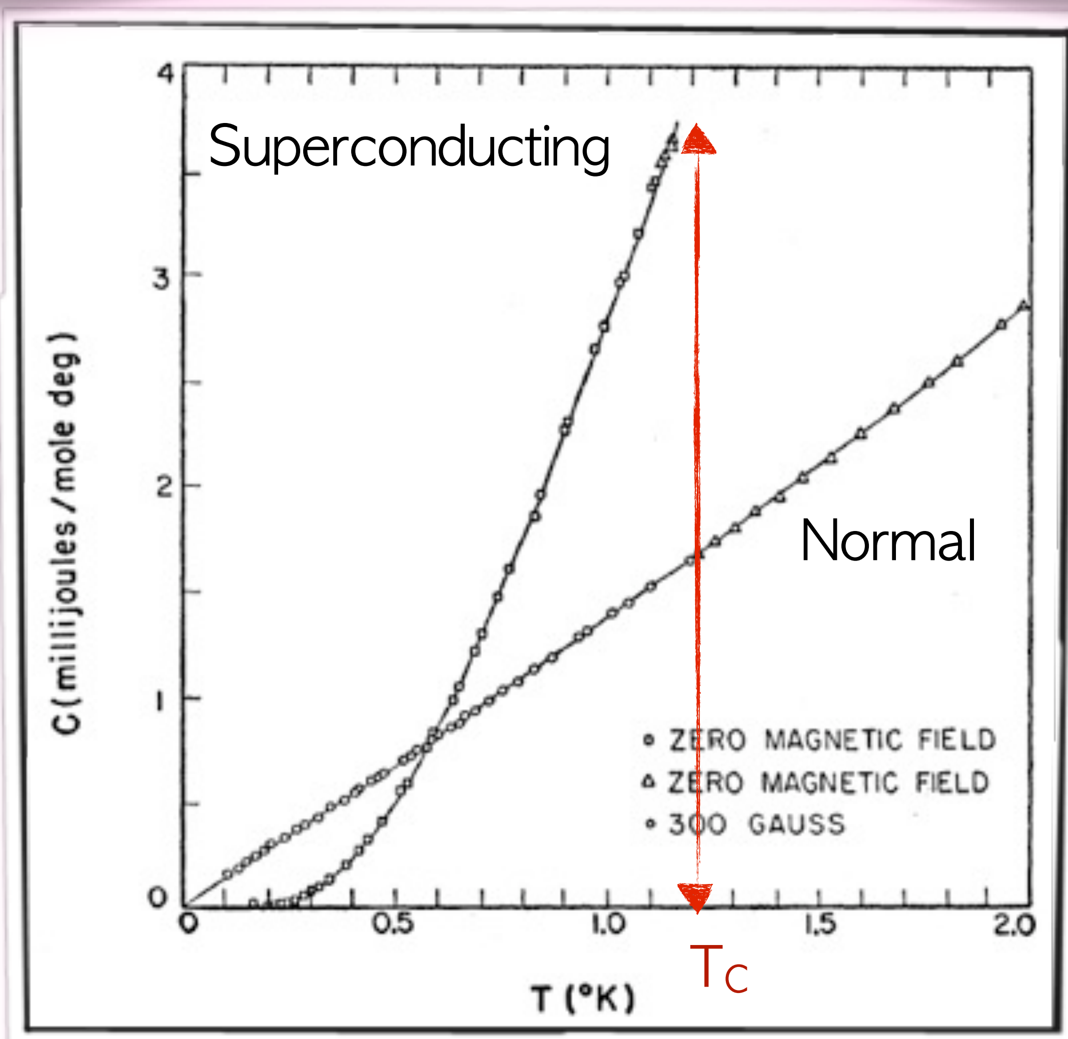


$$C_s^{Norm}(T) = AT + BT^3$$

Electronic excitations

Phonons (lattice vibrations)

$$C_s^{Super}(T \rightarrow 0) \propto e^{-\frac{\Delta}{k_B T}}$$

Superconductivity

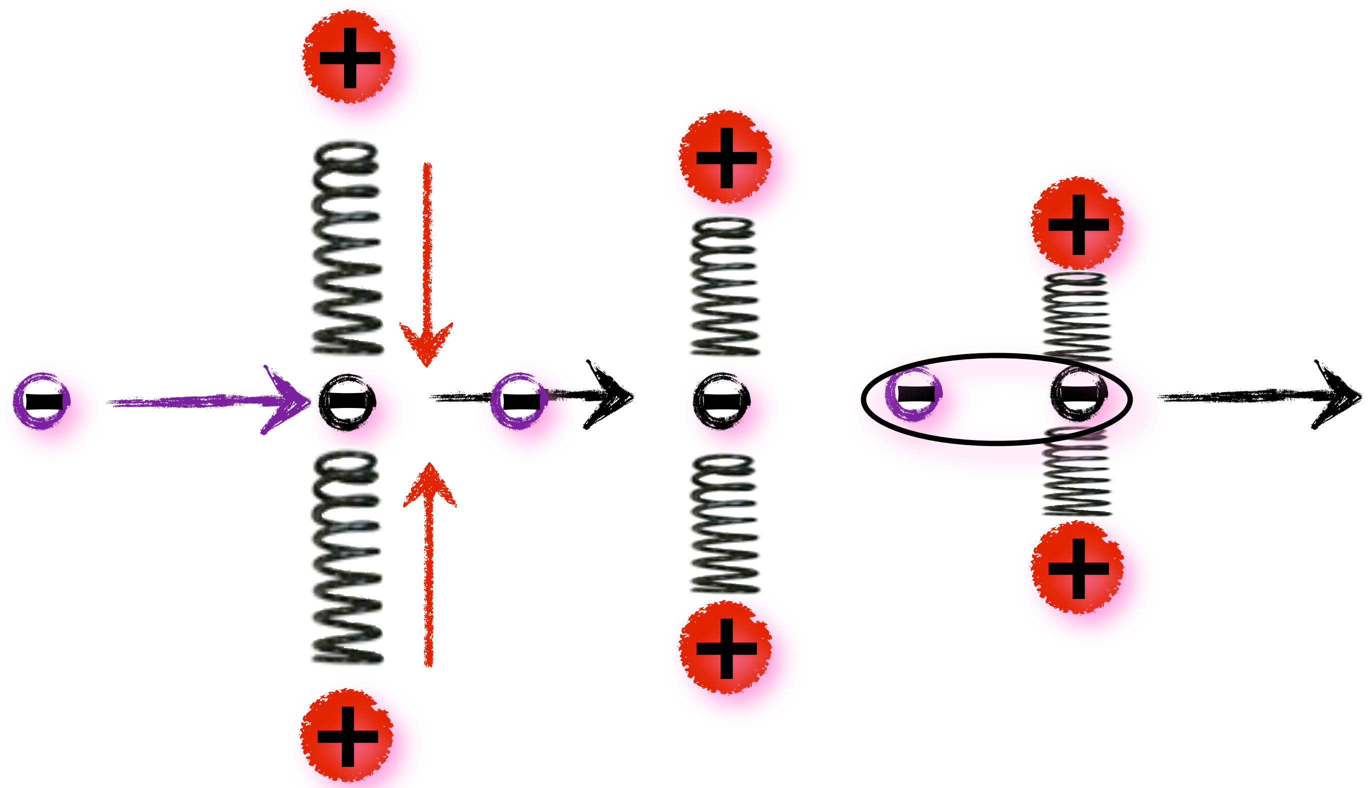
Counterintuitive physics:

Opposites attract:

☑ North pole, south poles of magnets
☑ Positive and negative charges

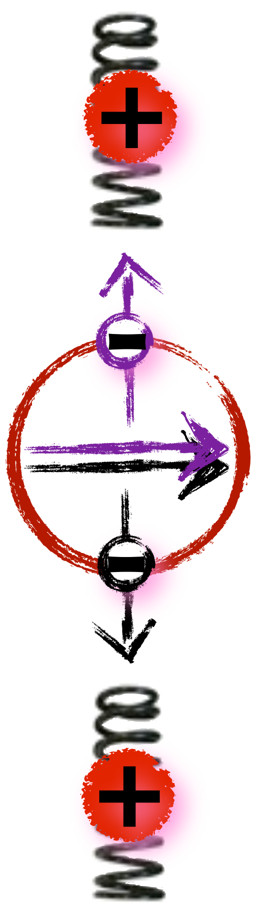
☆ In case of superconductors, two electrons do not repel, but attract, forming Cooper pairs (bosons).

The understanding of superconductivity was advanced in 1957 by three American physicists-John Bardeen, Leon Cooper, and John Schrieffer (BCS theory).



Naive picture: when one electron passes by positively charged ions in the lattice of the superconductor, the lattice distorts. Before the electron passes by and before the lattice springs back to its normal position, a second electron is drawn into the trough. It is through this process that two electrons, which should repel one another, link up. The electron pairs are coherent (are bosons!!!!) with one another as they pass through the conductor in unison.

Quantum picture: In the interaction process between an electron and the lattice there is an exchange of phonons, which are the elementary excitation of the lattice. This exchange of phonons wins the “electrons” natural repulsion. When one of the electrons that make up a Cooper pair passes close to an ion in the crystal lattice, the attraction between the negative electron and the positive ion cause a vibration to pass from ion to ion until the other electron of the pair absorbs the vibration. The net effect is that the electron has emitted a phonon and the other electron has absorbed the phonon. It is this exchange that keeps the Cooper pairs together.



Superconductivity

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Resonance Superfluidity in a Quantum Degenerate Fermi Gas

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We consider the superfluid phase transition that arises when a Feshbach resonance pairing occurs in a dilute Fermi gas. We apply our theory to consider a specific resonance in potassium (^{40}K), and find that for achievable experimental conditions, the transition to a superfluid phase is possible at the high critical temperature of about $0.5T_F$. Observation of superfluidity in this regime would provide the opportunity to experimentally study the crossover from the superfluid phase of weakly coupled fermions to the Bose-Einstein condensation of strongly bound composite bosons.

$$\frac{T_c}{T_F} \sim \exp\left(-\frac{\pi}{2|a|k_F}\right)$$

Superconductivity

^{6}Li

^{40}K

^{87}Sr

^{171}Yb, ^{173}Yb

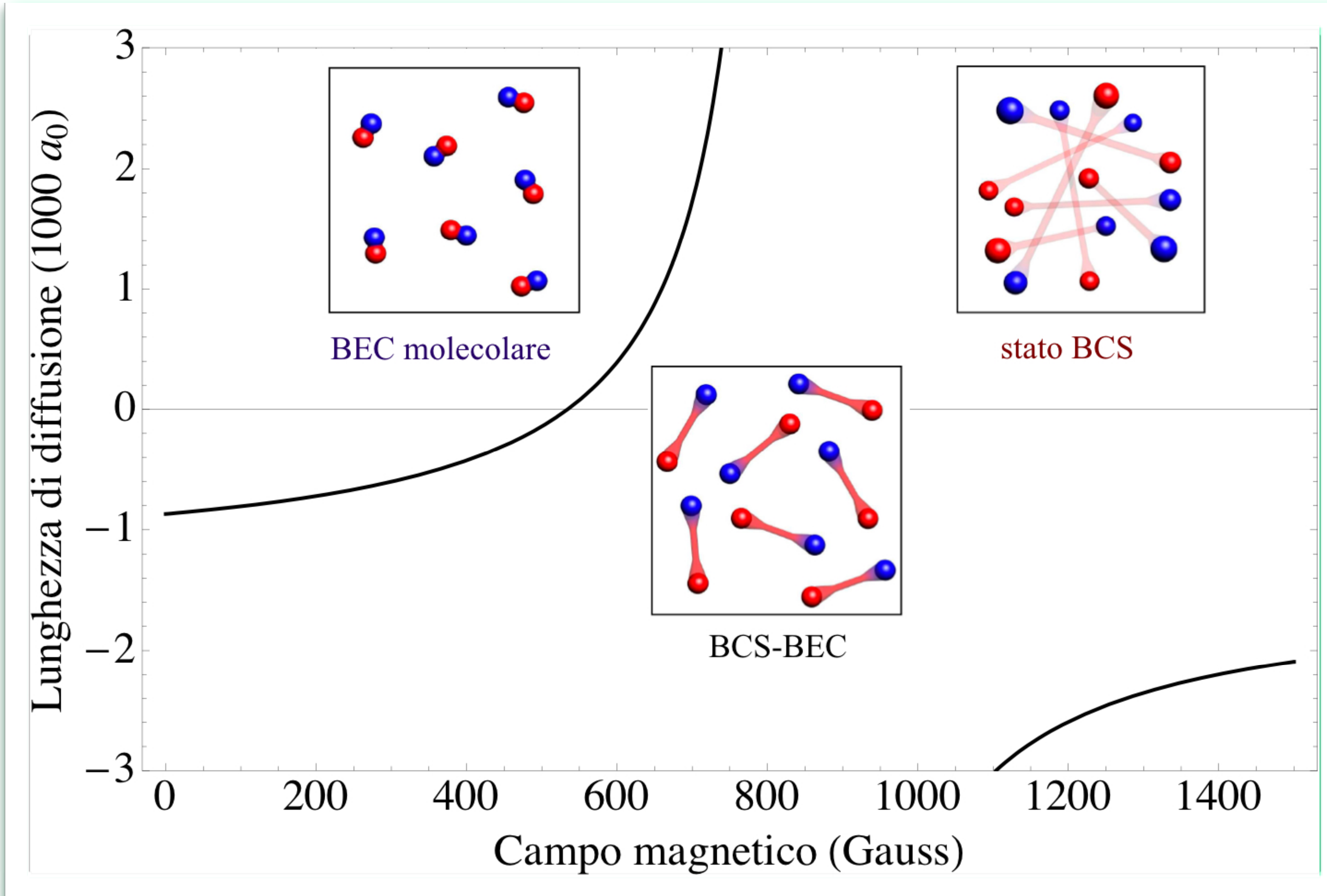
Typical scattering length of alkali atoms are of the order of the Van der Waals range $r_0 \approx 50\text{-}100\ a_0$.

Ultracold atoms are dilute systems: the mean inter-particle distance is of the order of $n^{-1/3} \approx 10^4\ a0$.

$k_F|a| \approx 10^{-2}$ and $T_c/T_F \approx e^{-\pi/2k_F|a|}$ -> BCS phase not observable!!!

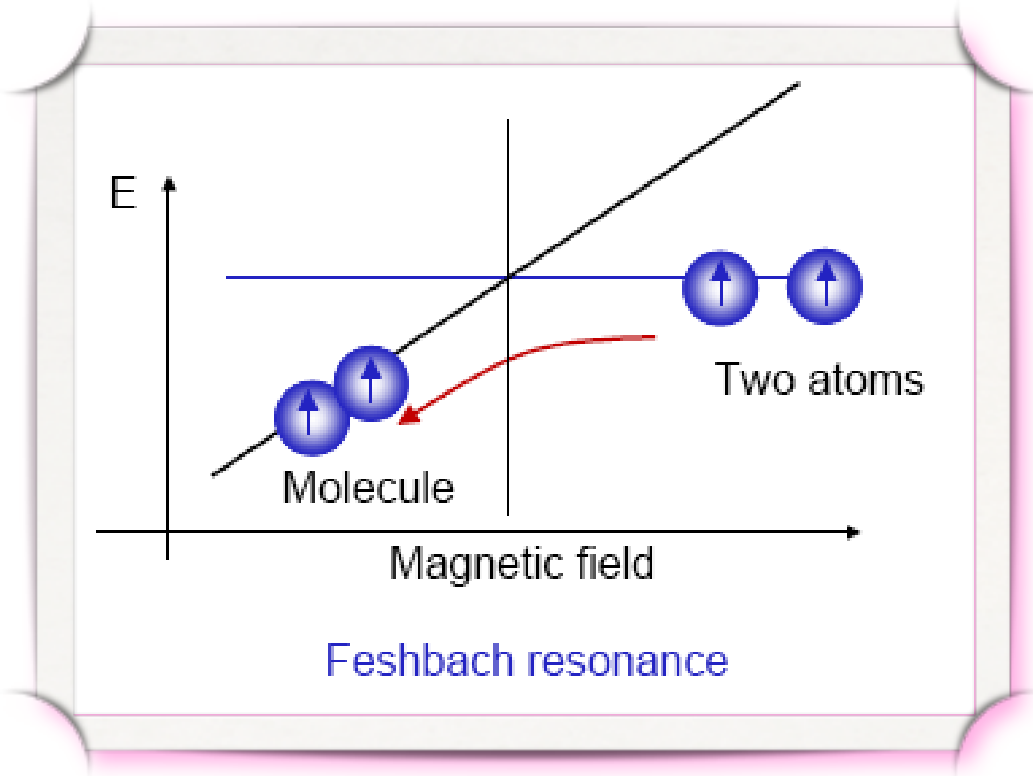
We need to increase the interactions a between the atoms!!

Feshbach resonance



The physics: by changing the interactions between the fermions from positive to negative (from repulsion to attraction) we are able to form pairs of fermions. The phonon-mediated electron-electron interactions that we have discussed in case of ordinary crystal, is here replaced by a direct collisional interaction between the fermions

Ultracold molecules formation

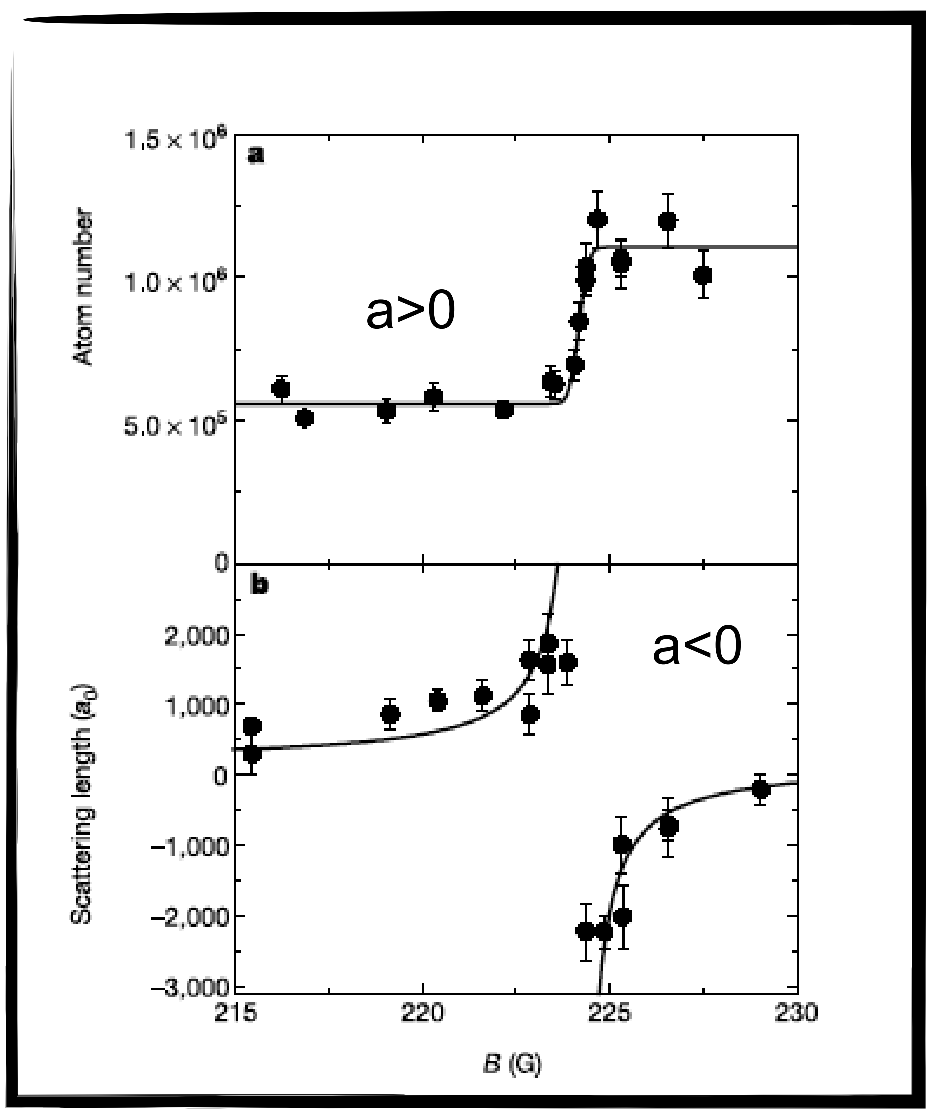


These molecules (in our case bosons) which are well all in the same internal state, are highly vibrationally excited , very large and weakly bound:

✷ $\psi\ (r) \approx 1/r\ e^{-r/a}$

✷ $d \approx a/2 \rangle 1000\ a_0$

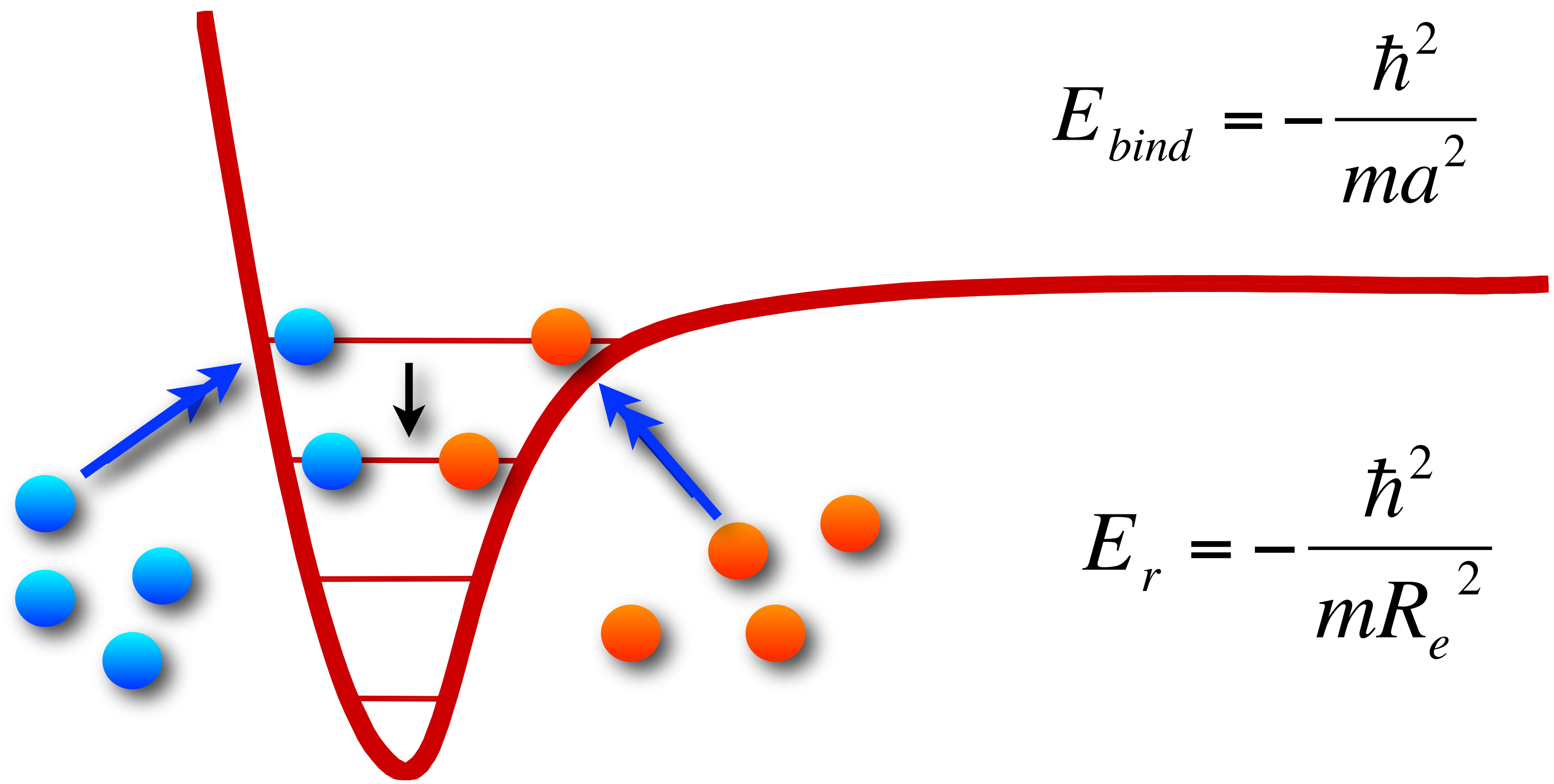
✷ $E_{bind} \approx -\hbar^2/(2ma^2)$



Feshbach molecules are typically invisible to the resonant imaging light used for the atoms:

- the first evidence of the molecules formation is the disappearing of the atoms after the sweep across the Feshbach resonance!

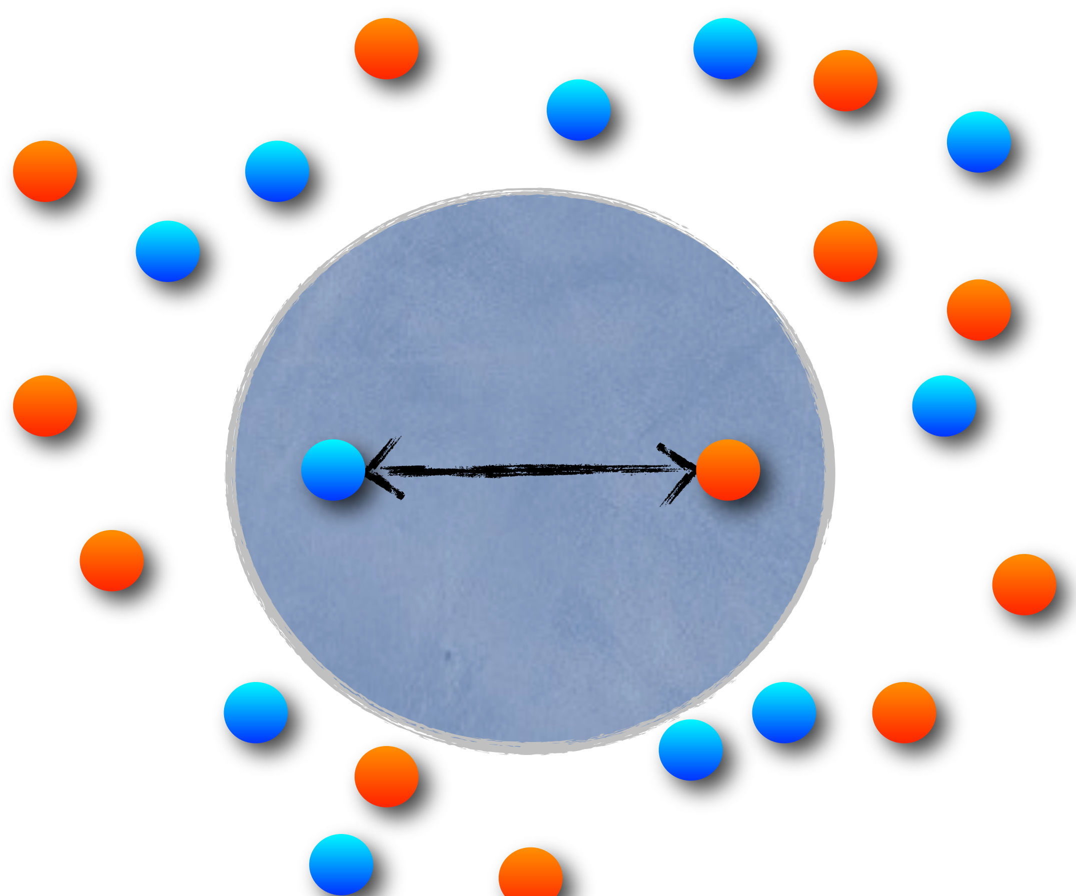
Ultracold molecules



$$E_{bind} = -\frac{\hbar^2}{ma^2}$$

$$E_r = -\frac{\hbar^2}{mR_e^{\,2}}$$

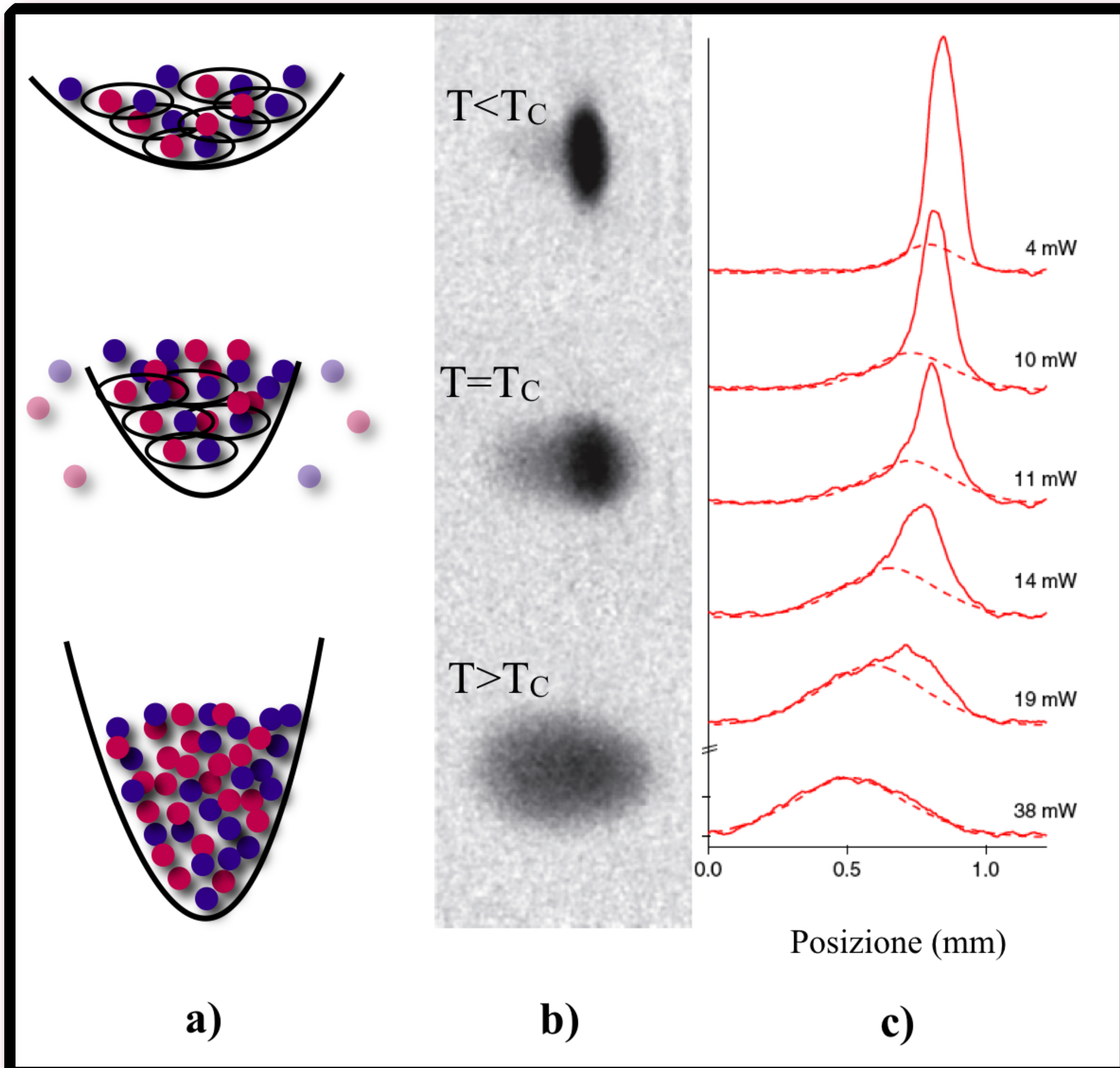
The collisions with the non-associate atoms are the possible limiting factor!!



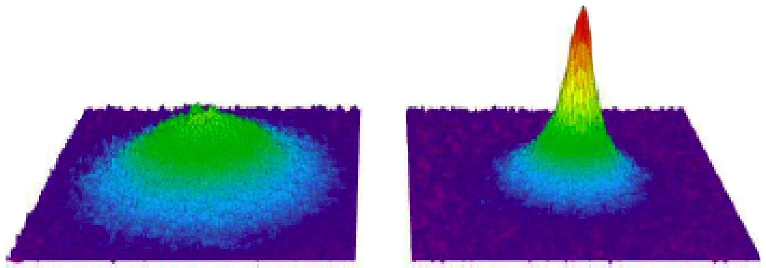
Close to resonance these molecules show an "atomic" behavior: the fermionic nature of the single atom assures the stability of these bosonic molecules. Fermi statistics does not allow the collisions between identical fermions:

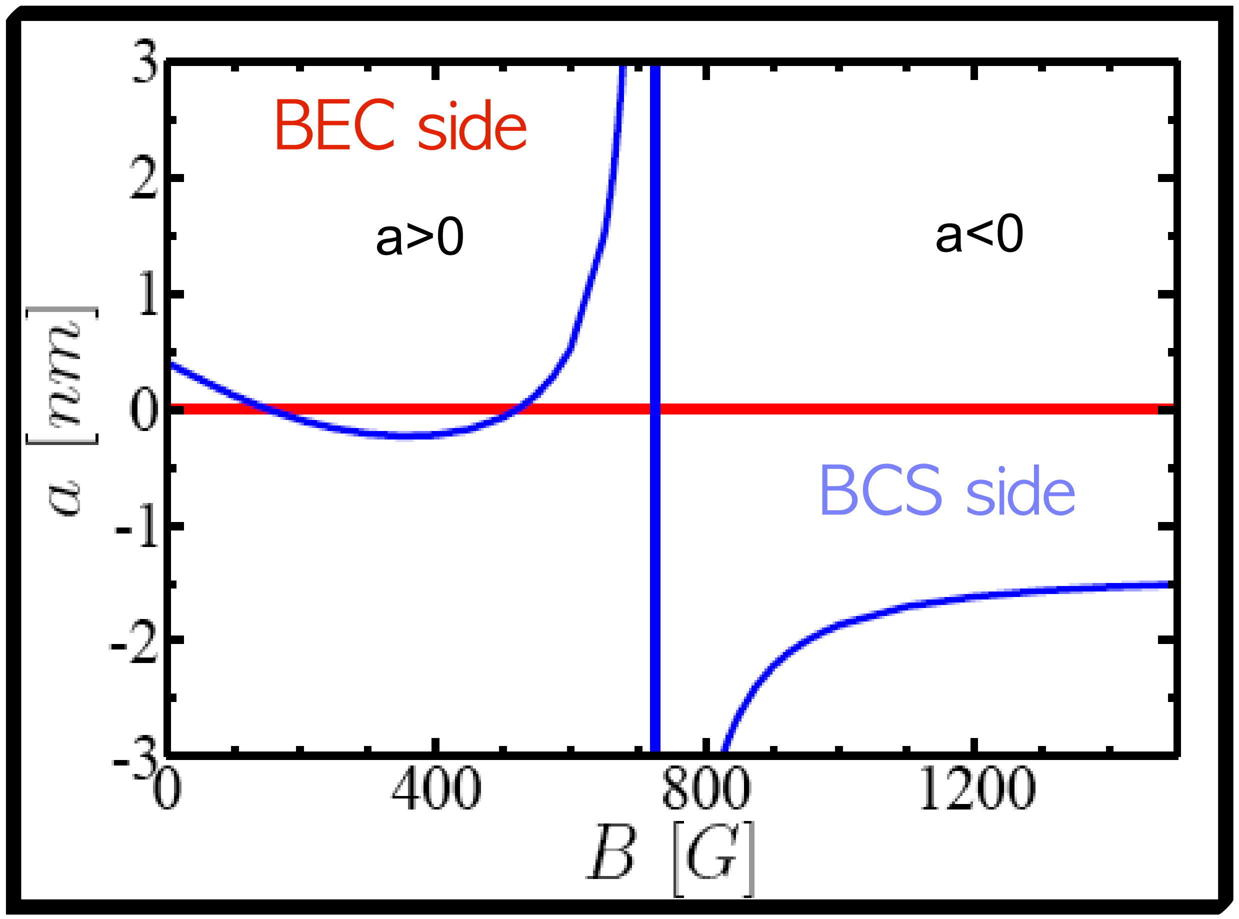
Lifetimes of the order of seconds!!
Ideal situation for BEC!!!

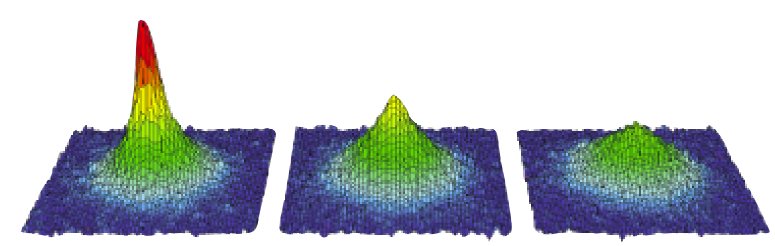
Ultracold molecules



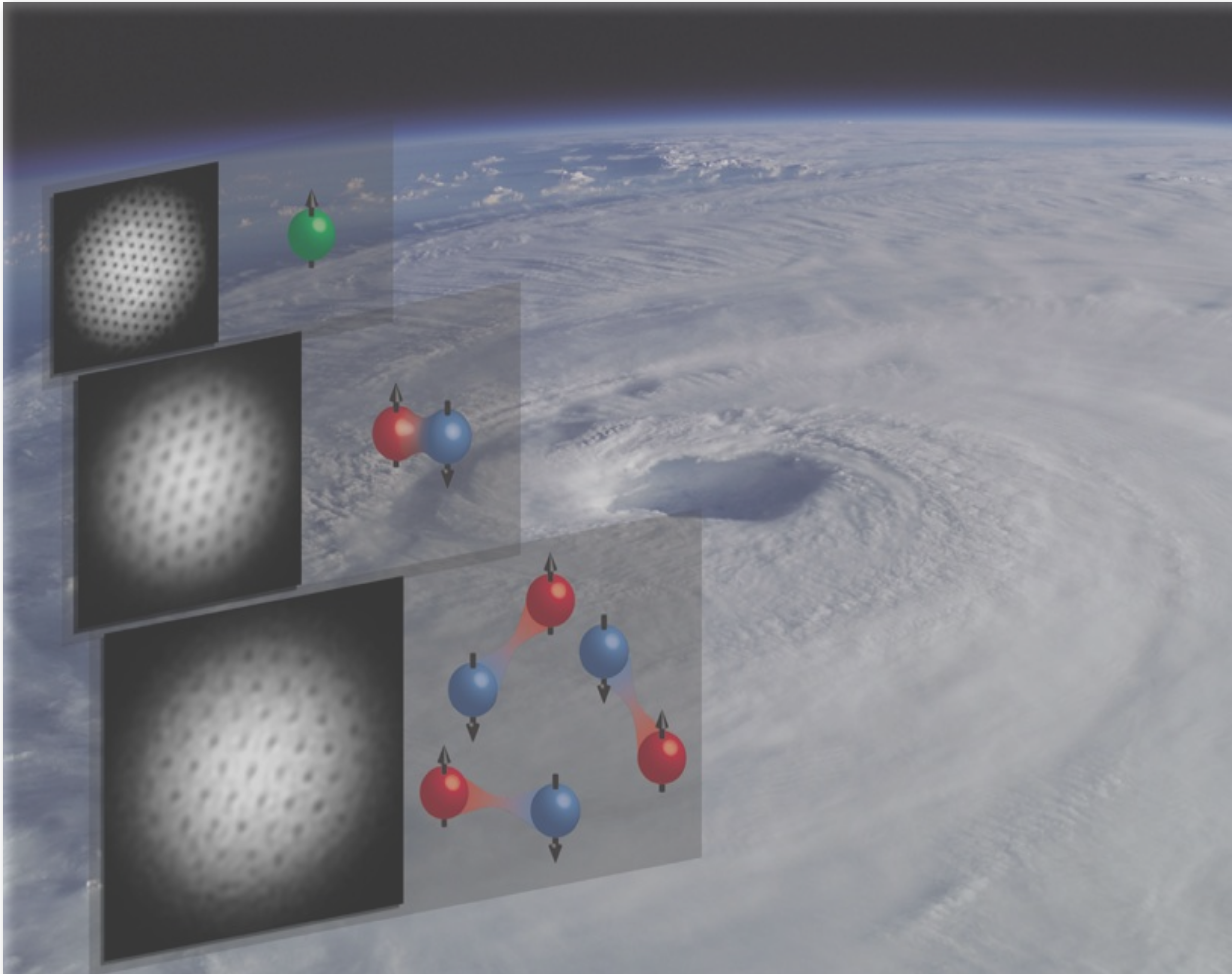
BEC-BCS







Vortices: test of superfluidity



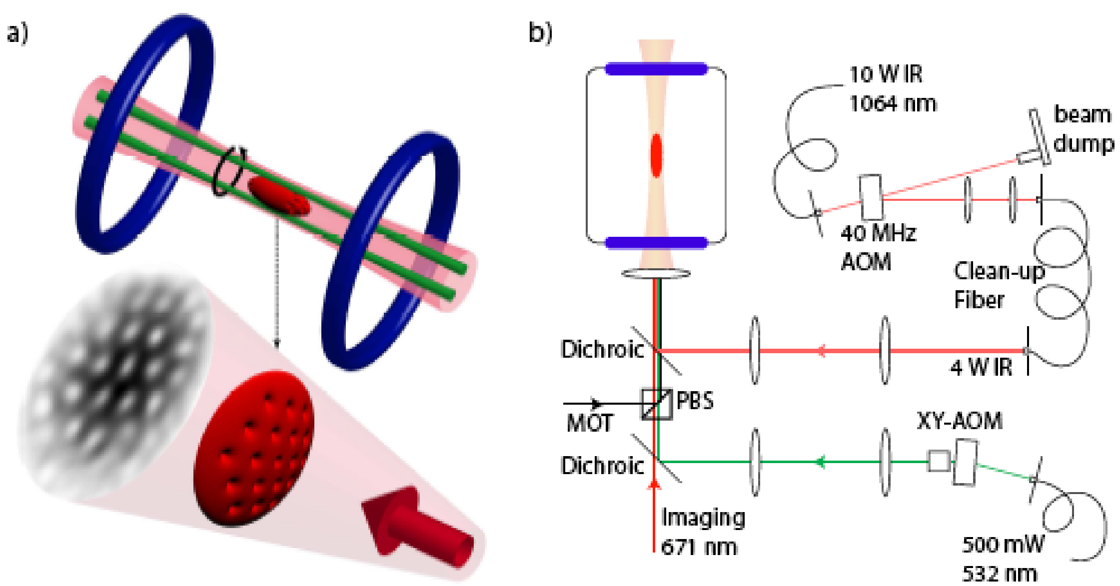


Figure 6-3: Experimental setup for the observation of vortices in a Fermi gas. a) Sketch of the geometry. The atomic cloud (in red) is trapped in a weakly focussed optical dipole trap (pink). The coils (blue) provide the high magnetic offset field to access the Feshbach resonance as well as the axial confinement (additional curvature coils not shown). Two blue-detuned laser beams (green) rotate symmetrically around the cloud. An absorption image of the expanded cloud shows the vortices. b) Optical setup for the vortex experiment. The laser beam forming the dipole trap is spatially filtered using an optical fiber tolerating high laser power. The stirring beam (green) passes through two crossed AOMs that deflect it in the transverse (XY) plane. These beams are overlapped with the imaging light by dichroic mirrors. The light for the magneto-optical trap (MOT) is overlapped on a polarizing beam splitter cube (PBS).

The idea is to first create vortices on the BCS side by stirring the cloud and then map the system in the more robust BEC side just after the release from the trap!

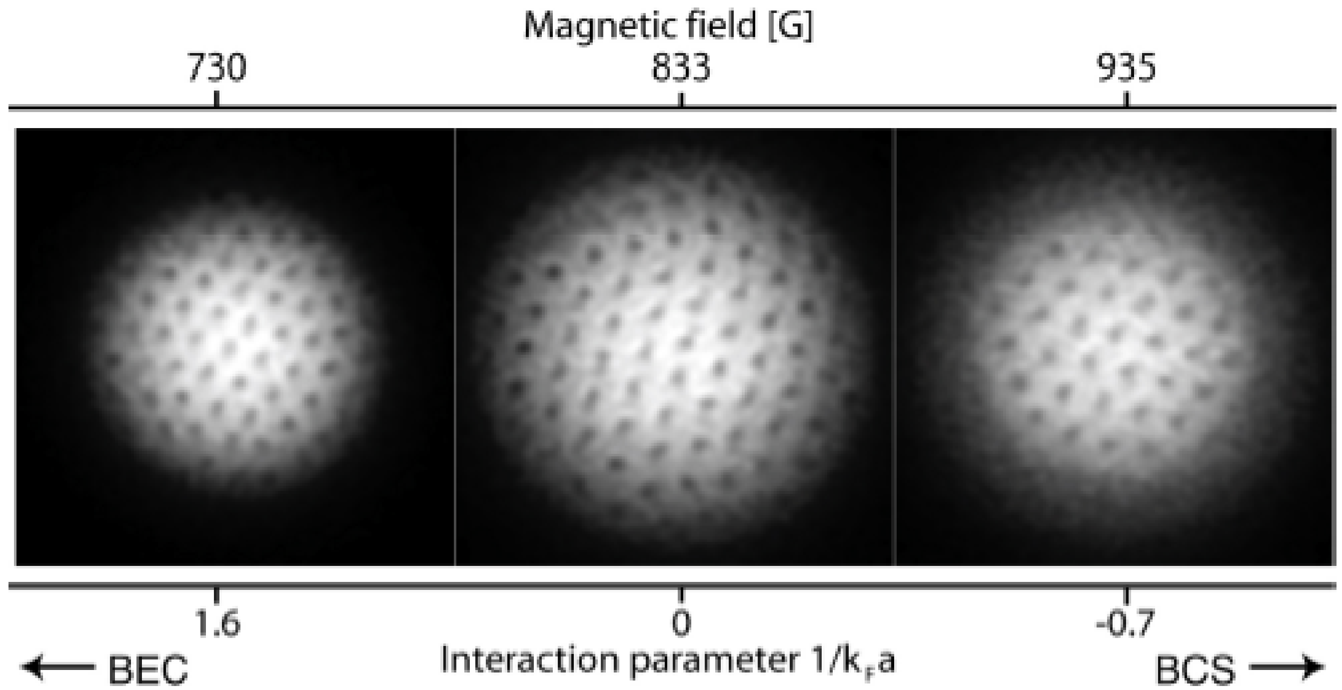


Figure 6-9: Observation of vortices in a strongly interacting Fermi gas. This establishes superfluidity and phase coherence in gases of fermions.

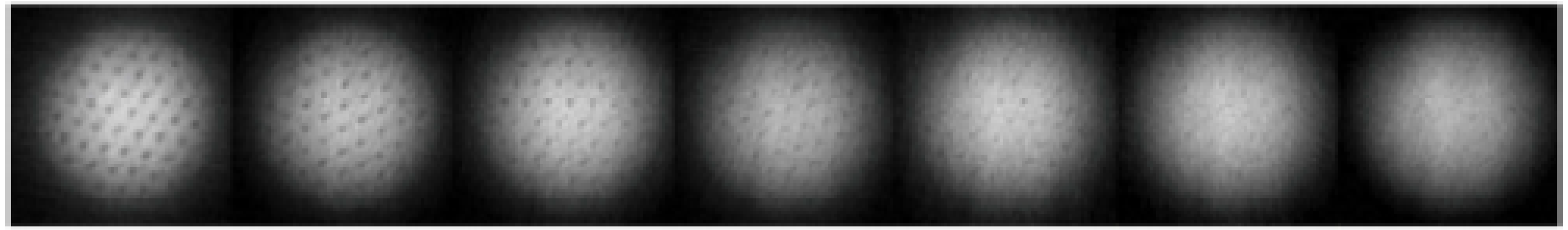


Figure 6-14: Loss of vortex contrast on resonance, at $B = 834$ G. Shown are absorption images at fixed total time-of-flight, but for different expansion times on resonance (2, 2.5, 3, 3.5, 4,5, and 6 ms) before the magnetic field ramp to the BEC-side for further expansion. The vortex contrast decreases uniformly across the cloud from 15% (for 2 ms resonant expansion) to about 3% (for 5 ms). The field of view of each image is 1.2 mm $\times$ 1.2 mm.

The Cooper pairs are really fragile, so the BCS state. By mapping the BCS state at different times the vortices are destroyed. This is the indication of the pair breaking.

... "we observe superfluid flow in an expanded Fermi gas down to densities of about 10^{11} cm^{-3}.
At these densities, the average distance between two atoms is 2 μ m!

The average distance between neutrons in a neutron star is a few fm, corresponding to densities of 10^{38} cm^{-3}.

This nicely illustrates how general the phenomenon of fermionic superfluidity is."

Prof. Martin Zwierlein, MIT, Cambridge, USA