Caratterizzazione ottica

6.1c Calculation of the Absorption Coefficient

The total probability W_T of photon absorption is calculated by summing Eq. (6.22) over all the initial states of the solid (valence band states) and all possible final states (conduction band states):

$$W_T = \sum_{\substack{\mathbf{k}_{i,v} \\ \mathbf{k}_{f,c}}} \frac{2\pi}{\hbar} \frac{e^2 A^2}{4m^2}\, p^2_{\mathbf{k}_i}\, \delta(\mathbf{k}_i - \mathbf{k}_f)\, \delta(E_f - E_i - \hbar\omega) \tag{6.30}$$

with

$$E_f - E_c = \frac{\hbar^2 k^2_{f,c}}{2m_e}, \tag{6.31}$$

$$E_i - E_v = -\frac{\hbar^2 k^2_{i,v}}{2m_h} \tag{6.32}$$

for a "standard" band structure (Fig. 6.2).

Summing over $\mathbf{k}_i$ we get

$$W_T = \frac{2\pi}{\hbar} \frac{e^2 A^2}{4m^2} \sum_{\mathbf{k}_f} p^2_{\mathbf{k}_f}\, \delta\left[\frac{\hbar^2 k_f^2}{2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right) + E_g - \hbar\omega\right]. \tag{6.33}$$

We can transform the discrete summation into an integral by using the density of states in $\mathbf{k}$ space. For a sample of volume $s dx$ this density is $s dx/4\pi^3$ so that

$$W_T = \frac{2\pi}{\hbar} \frac{e^2 A^2}{4m^2} \frac{s\, dx}{4\pi^3} \int d^3\mathbf{k}_f\, p^2_{\mathbf{k}_f}\, \delta\left[\frac{\hbar^2 k_f^2}{2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right) + E_g - \hbar\omega\right]. \tag{6.34}$$

The periodic functions u_v and u_c vary slowly with $\mathbf{k}$ (see for example the $\mathbf{k} \cdot \mathbf{p}$ method, Appendix 2.4), so we can regard $p^2_{\mathbf{k}_f}$ as independent of k and write

$$p^2_{\mathbf{k}_f} = p^2. \tag{6.35}$$

We define the reduced mass m_r by

$$m_r^{-1} = m_e^{-1} + m_h^{-1}. \tag{6.36}$$

We set $x = (\hbar^2/2m_r)\mathbf{k}_f^2 + E_g - \hbar\omega$. It remains to integrate

$$\frac{e^2 A^2 p^2}{8\pi^2 m^2 \hbar} s \, dx \cdot 2\pi \left(\frac{2m_r}{\hbar^2}\right)^{3/2} \int (\hbar\omega - E_g + x)^{1/2} \, \delta(x) \, dx. \tag{6.37}$$

We obtain

$$W_T = \frac{s \, dx}{4\pi} \frac{e^2 A^2}{\hbar^4 m^2} (2\, m_r)^{3/2} p^2 (\hbar\omega - E_g)^{1/2}. \tag{6.38}$$

The average incident photon flux is $\overline{\Pi}/\hbar\omega$ per unit area, and by definition the fraction of incident photons absorbed over a depth dx is

$$W_T = \alpha \frac{\overline{\Pi}}{\hbar\omega} s \, dx. \tag{6.39}$$

Substituting Eqs. (6.7) and (6.38) in Eq. (6.39) we get

$$\alpha = \frac{(2m_r)^{3/2}}{2\pi\epsilon_0\, cn} \frac{e^2 p^2}{m^2} \frac{1}{\hbar^3 \omega} (\hbar\omega - E_g)^{1/2}. \tag{6.40}$$

This establishes Eq. (2.106) for the case of direct transitions. This result was obtained by J. Bardeen, F.J. Blatt, and L.H. Hall (Atlantic City Photoconductivity Conference, 1954).

Absorption varies from one semiconductor to another through the values m_r, p and the refractive index n. An average order of magnitude is

$$\alpha(\mathrm{cm}^{-1}) = 4.10^4 [(h\nu - E_g)(\mathrm{eV})]^{1/2}. \tag{6.41}$$

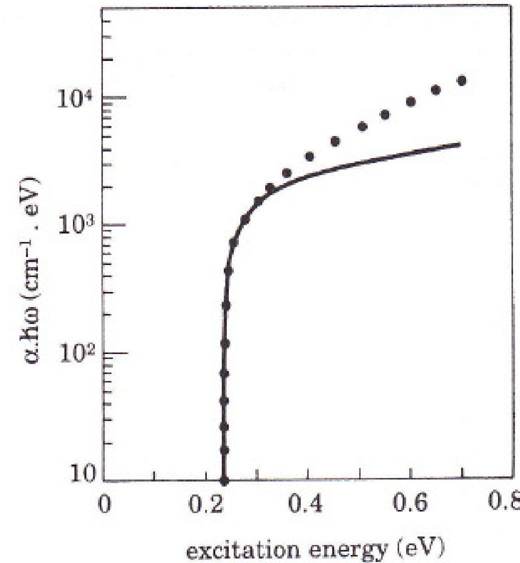
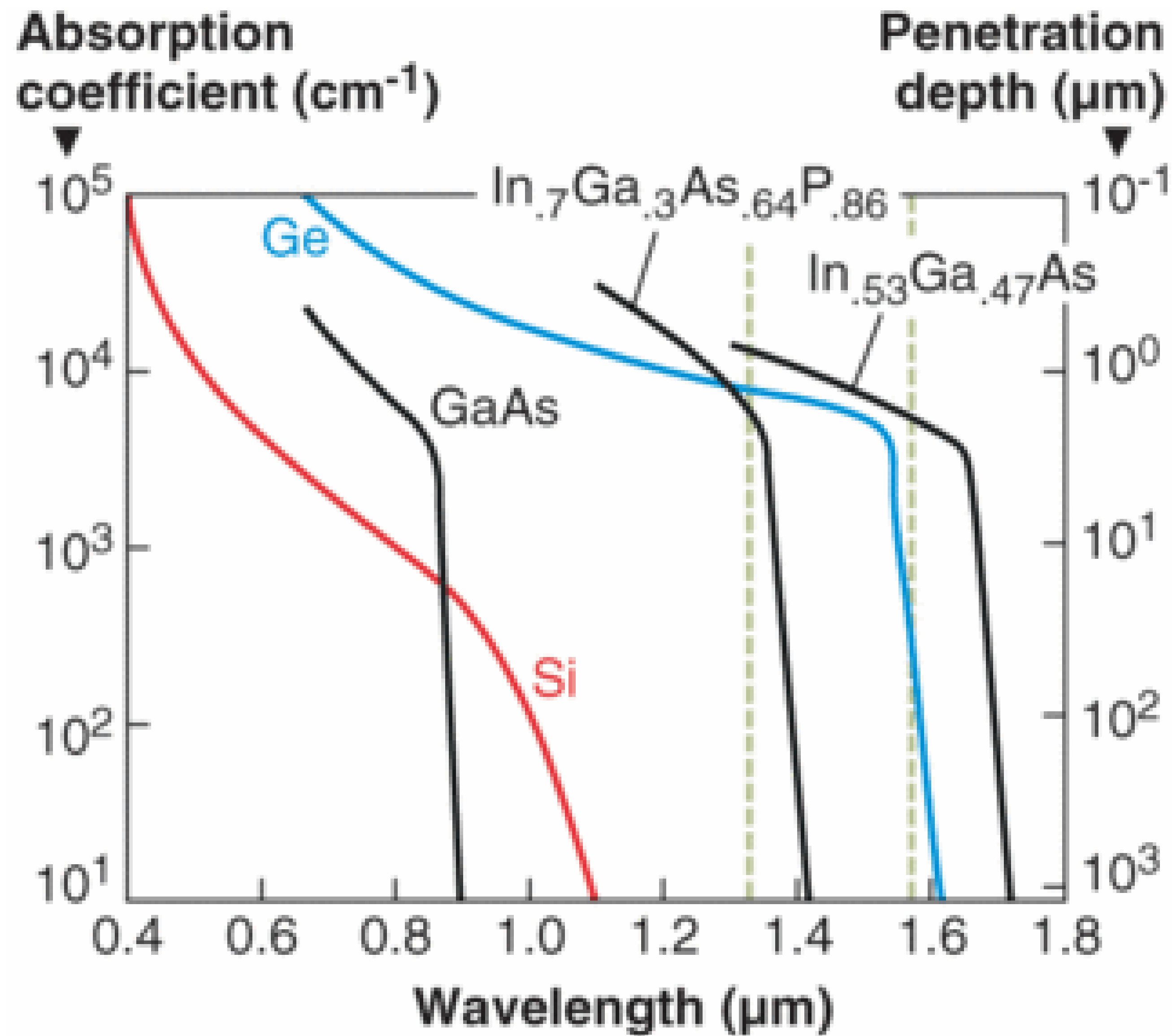
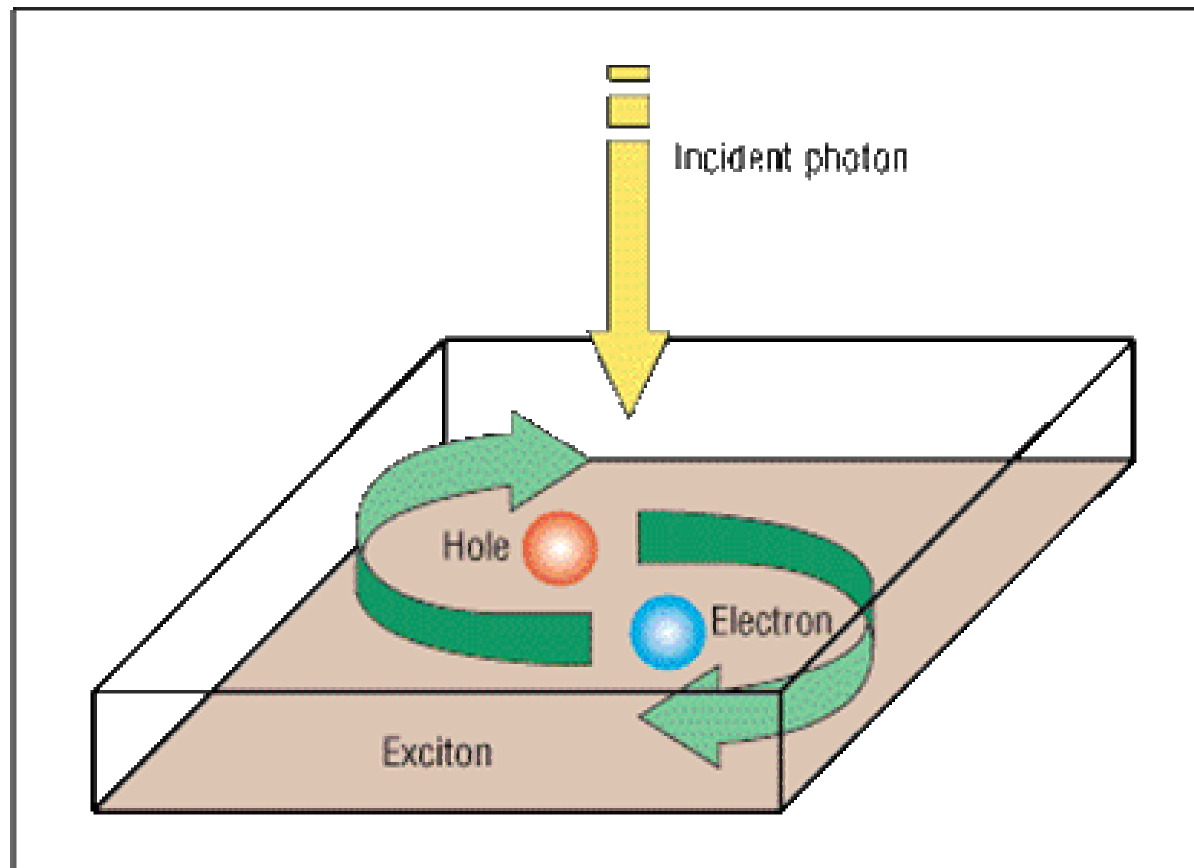


Fig. 6.3. Absorption threshold of InSb at $T = 5$ K. The measurements (points) are compared with the predictions of Eq. (6.40). The deviation at high energy is suppressed by taking account of the correct density of states, which is no longer parabolic at large **k**, and of the variation of the matrix element $p_{\mathbf{k}_f}$ with $\mathbf{k}_f$.

Assorbimento

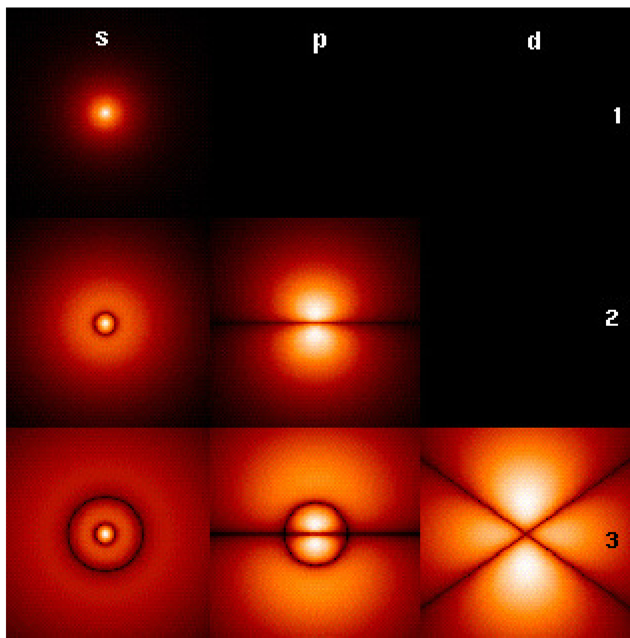


Eccitone

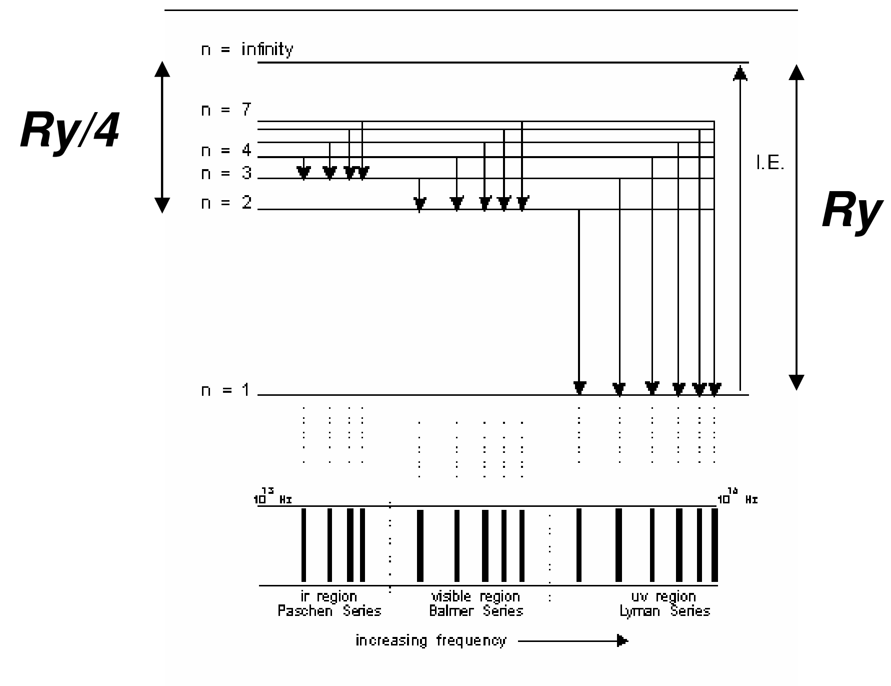


Atomo di idrogeno

$$E_{\text{total}} = \frac{-m_e e^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n^2}$$



Ry/4



Ry

$$Ry = \frac{m_e e^4}{8\varepsilon_o^2 h^2} \longrightarrow Ry^* = \frac{m_e^* e^4}{8(\varepsilon_o \varepsilon^*)^2 h^2} = Ry \frac{m_e^*}{m_e (\varepsilon^*)^2}$$

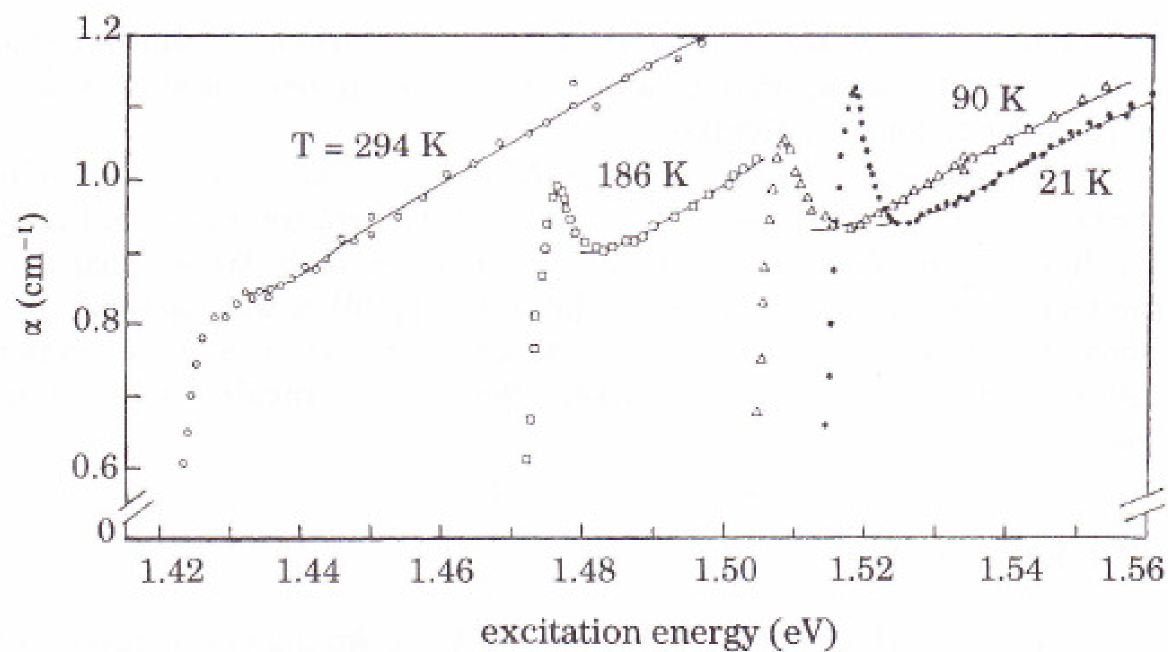


Fig. 6.4. Absorption in GaAs near threshold, at various temperatures. As the temperature increases the band gap decreases, basically because of the thermal expansion of the lattice (Appendix 2.3). The peak is caused by absorption leading to the creation of excitons and is more prominent at low temperature: if kT is of the order of the exciton binding energy, broadening of the peak will become significant.

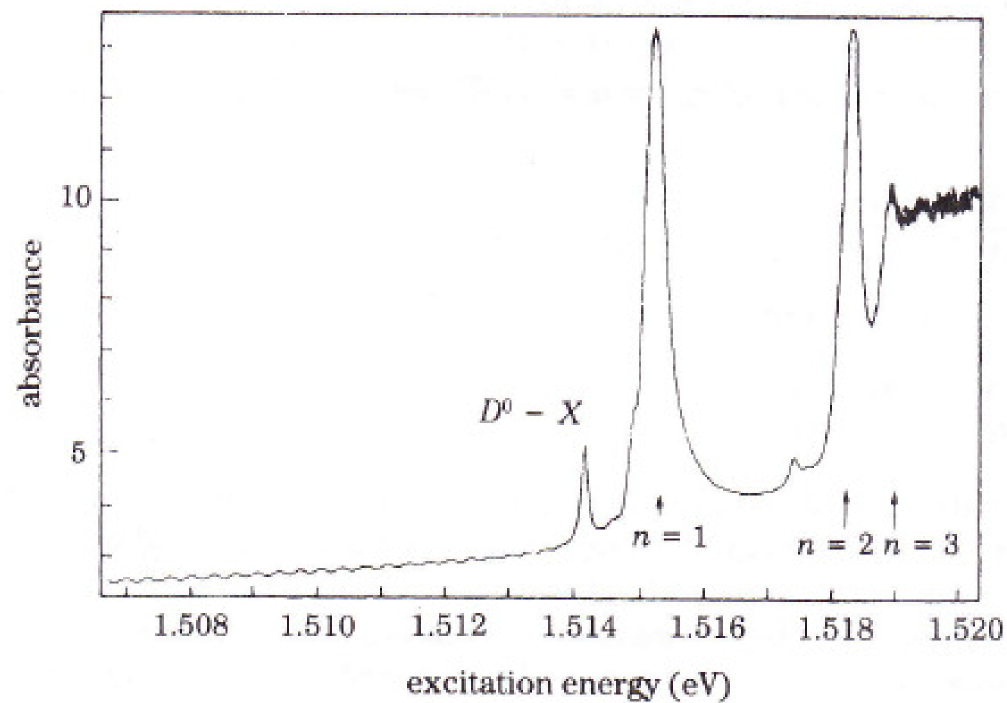
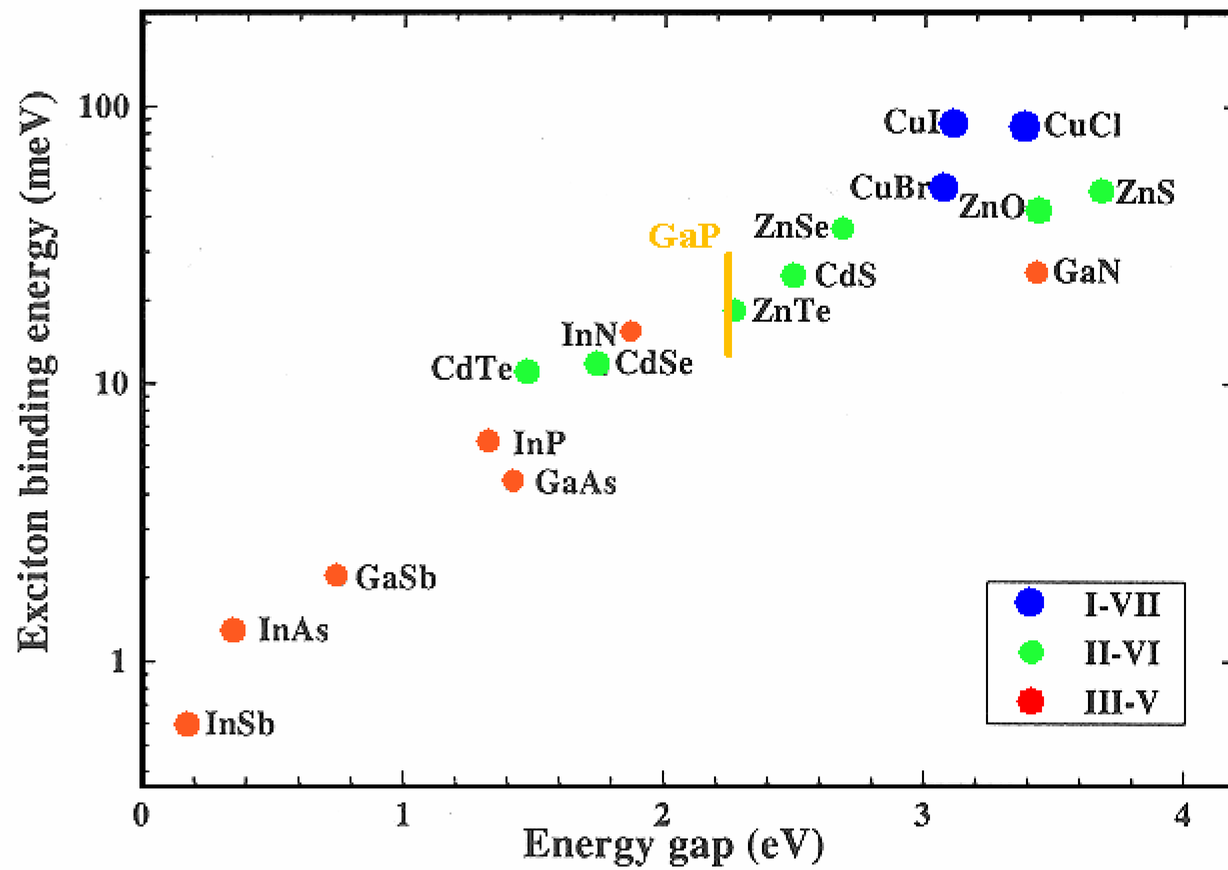
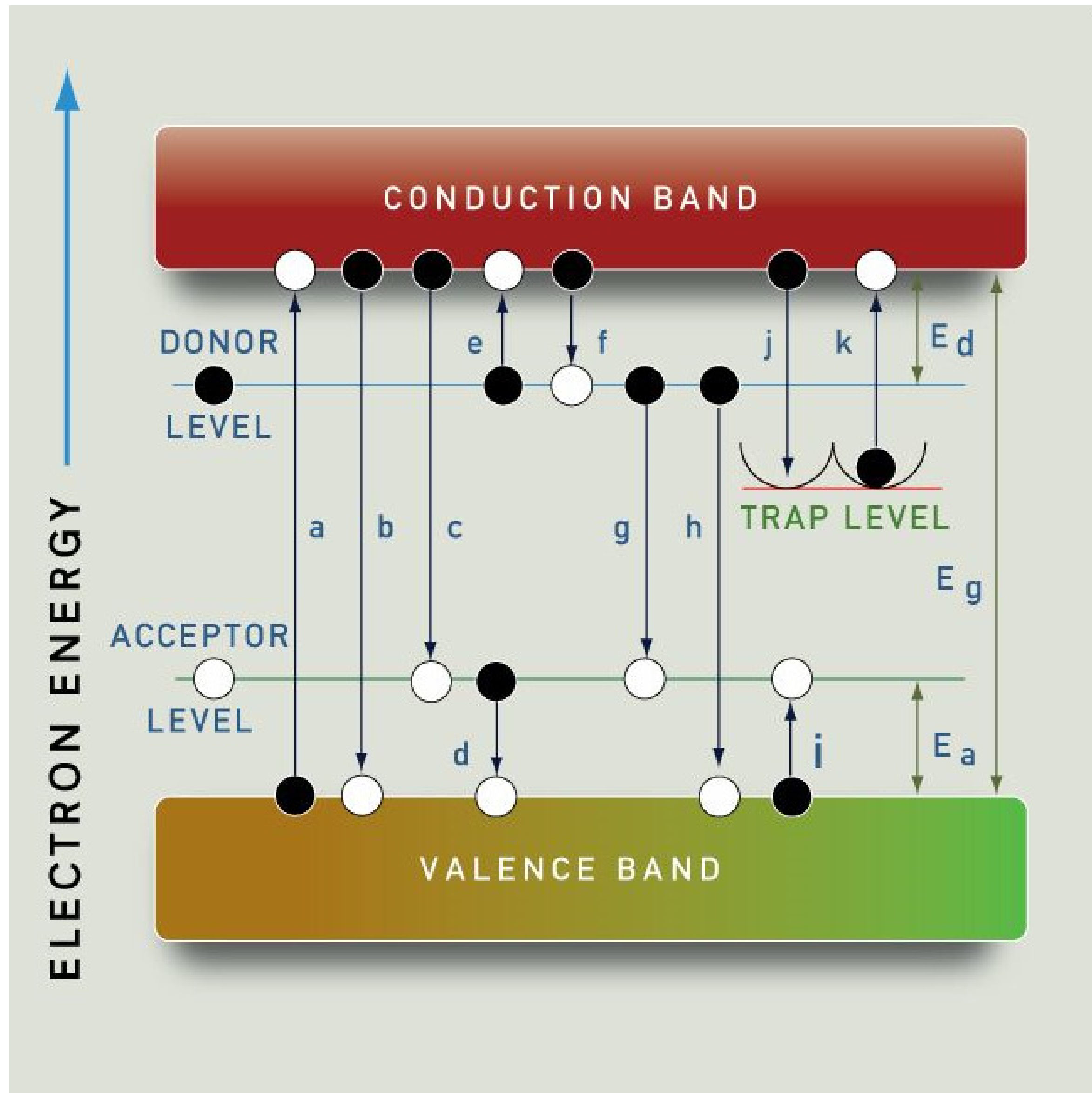


Fig. 6.5. Absorption measured at $T = 6$ K of a sample of pure GaAs containing residual donors. The absorbance is proportional to the absorption coefficient α. Note the peaks corresponding to the levels $n = 1, 2$, and 3 of the exciton, as well as a peak associated with an exciton bound to a neutral donor $(D^0 - X)$.


Exciton binding energy (meV)
Energy gap (eV)
100
10
1
0
1
2
3
4
CuI
CuCl
CuBr
ZnO
ZnS
GaP
ZnSe
CdS
GaN
ZnTe
InN
CdTe
CdSe
InP
GaAs
GaSb
InAs
InSb
I-VII
II-VI
III-V

Transizioni elettroniche



Fotoluminescenza

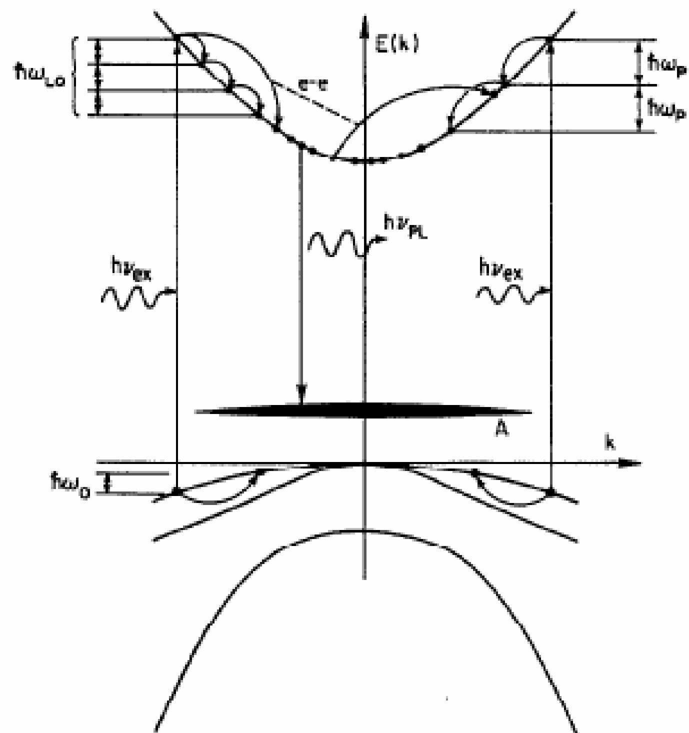


FIG. 11. A schematic illustration of hot electron thermalization and radiative recombination with holes bound to acceptors in GaAs. Incident light at the energy $\hbar\nu_{ex}$ excites an electron-hole pair and luminescence is emitted at an energy $\hbar\nu_{PL}$. Several scattering processes including LO phonon emission ($\hbar\omega_{LO}$), plasmon emission ($\hbar\omega_P$), and electron-electron scattering (indicated with the dotted line labeled e-e) are shown. The wide line marked A is the acceptor level. The figure is reprinted from Ref. 87.

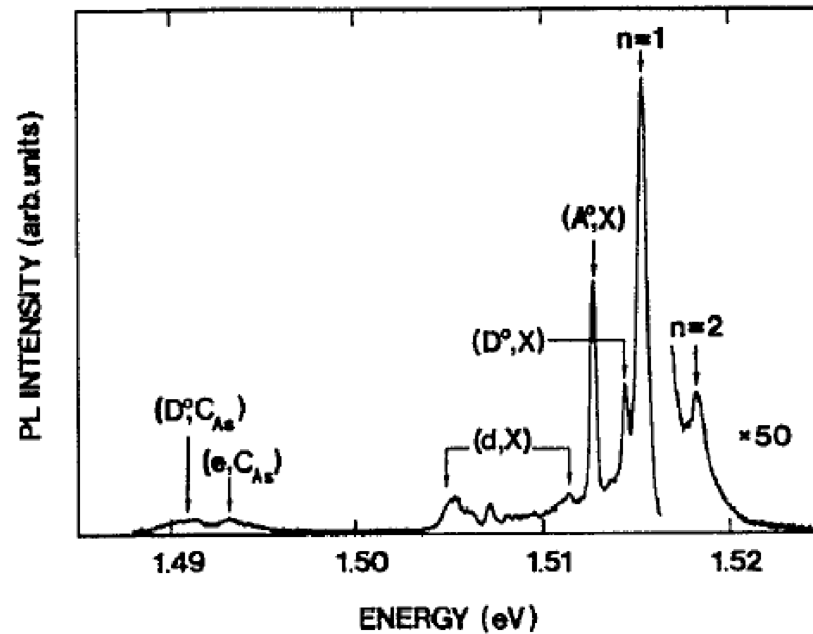


FIG. 30. The photoluminescence spectrum measured at $T=2$ K of a high quality GaAs MBE sample. The exciting power density is $\simeq$ 100 mW/cm^2. The different labels stand for: $n=1$ and $n=2$ fundamental and excited state of the free exciton, (D^0,X) exciton bound to a neutral donor, (A^0,X) exciton bound to a neutral acceptor, (d,X) defect exciton, (e,C_{As}) electron to a C acceptor transition, and (D^0,C_{As}) donor to C acceptor transition. The splitting of the (A^0,X) recombination, which is of the order of the spectral band pass ($\simeq$ 0.3 meV), is not resolved. The figure is reprinted from Ref. 61.