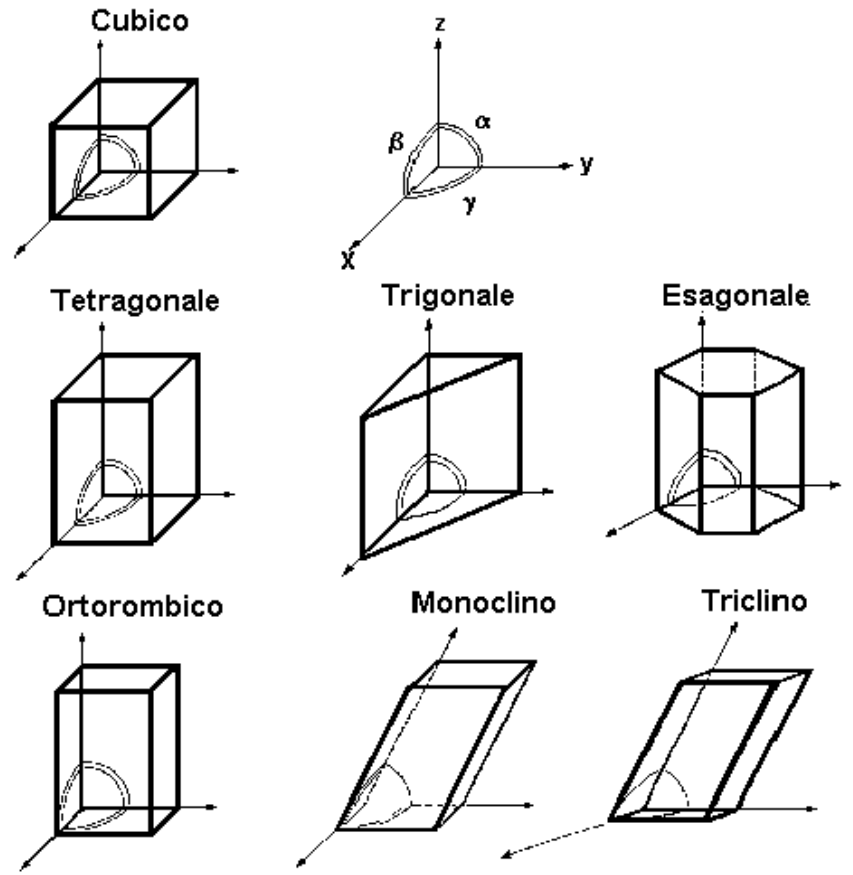


Propagazione in mezzi anisotropi



Equazioni Maxwell semplificate nella materia

$$\vec{\nabla} \cdot \vec{D} = 0 \quad \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

Dielettrici anisotropi

$$\vec{D} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \vec{E} \quad \varepsilon_{ij} = \varepsilon_{ji}$$

Assi principali

$$\vec{D} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \vec{E} \quad D_i = \varepsilon_i E_i$$

I° Equazione Maxwell

$$\begin{aligned}\vec{\nabla} \cdot \vec{D} &= \sum_i \frac{\partial D_i}{\partial x_i} = \sum_i \frac{\partial \varepsilon_i E_i}{\partial x_i} = \\ &= \sum_i \varepsilon_i \frac{\partial E_i}{\partial x_i} = 0 \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{E} \neq 0\end{aligned}$$

Se polarizzazione lungo asse principale $\vec{\nabla} \cdot \vec{E} = 0$

Equazioni onde

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Onde piane

$$\vec{k} \times (\vec{k} \times \vec{E}) = -\omega^2 \mu \varepsilon \vec{E}$$

$$\mu \varepsilon = \frac{n^2}{c^2}$$

Equazioni Onde mezzi anisotropi

$$\vec{k} \times (\vec{k} \times \vec{E}) = (\vec{k} \cdot \vec{E})\vec{k} - k^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E}$$

$$\begin{pmatrix} \omega^2 \mu \varepsilon_x - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_y k_x & \omega^2 \mu \varepsilon_y - k_x^2 - k_z^2 & k_y k_z \\ k_x k_z & k_y k_z & \omega^2 \mu \varepsilon_z - k_x^2 - k_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \mathbf{0}$$

Materiali uniassici dielettrici ($\mu = \mu_0$)

$$\varepsilon_x = \varepsilon_y = \varepsilon_0 n_o^2 \quad \varepsilon_z = \varepsilon_0 n_e^2$$

Equazioni Onde mezzi uniassici ($k_x=0$)

$$\begin{pmatrix} \frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 & 0 & 0 \\ 0 & \frac{\omega^2 n_o^2}{c^2} - k_z^2 & k_y k_z \\ 0 & k_y k_z & \frac{\omega^2 n_e^2}{c^2} - k_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \mathbf{0}$$

$$\left(\frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 \right) \left[\left(\frac{\omega^2 n_o^2}{c^2} - k_z^2 \right) \left(\frac{\omega^2 n_e^2}{c^2} - k_y^2 \right) - k_y^2 k_z^2 \right] = 0$$

Equazioni Onde mezzi uniassici ($k_x=0$)

Onda ordinaria

$$\left(\frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 \right) = 0 \quad \vec{E} = (E_x, 0, 0)$$

Onda straordinaria

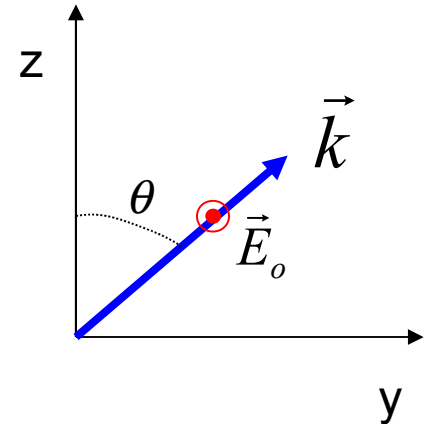
$$\left[\left(\frac{\omega^2 n_o^2}{c^2} - k_z^2 \right) \left(\frac{\omega^2 n_e^2}{c^2} - k_y^2 \right) - k_y^2 k_z^2 \right] = 0 \quad \vec{E} = (0, E_y, E_z)$$

Equazioni Onde mezzi uniassici ($k_x=0$)

Onda ordinaria (TE)

$$k^2 = \frac{\omega^2 n_o^2}{c^2}$$

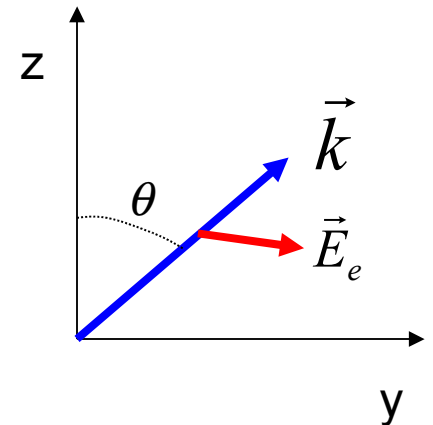
$$\vec{E}_o = (E_x, 0, 0)$$



Onda straordinaria (TM)

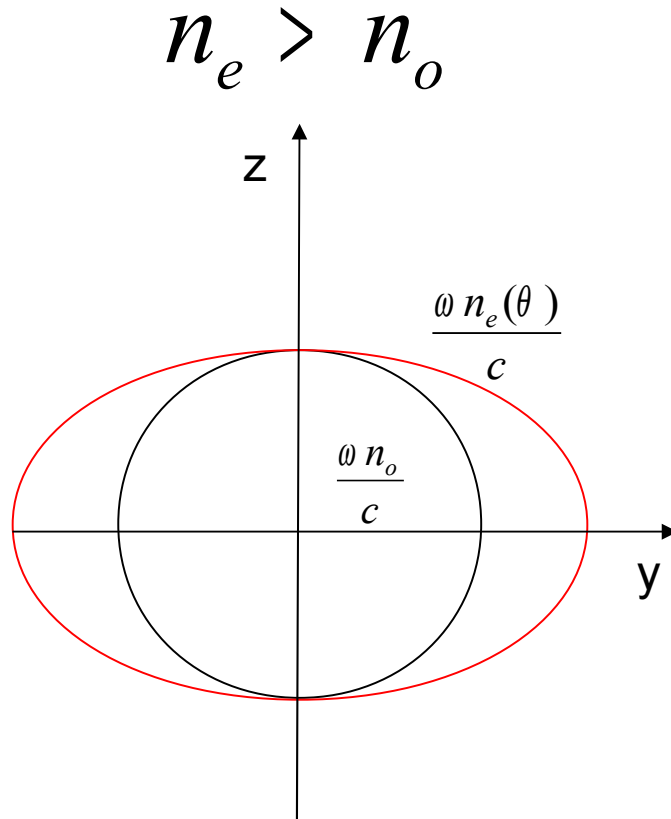
$$k^2 = \frac{\omega^2 n_e(\theta)^2}{c^2}$$

$$\vec{E}_e = (0, E_y, E_z)$$

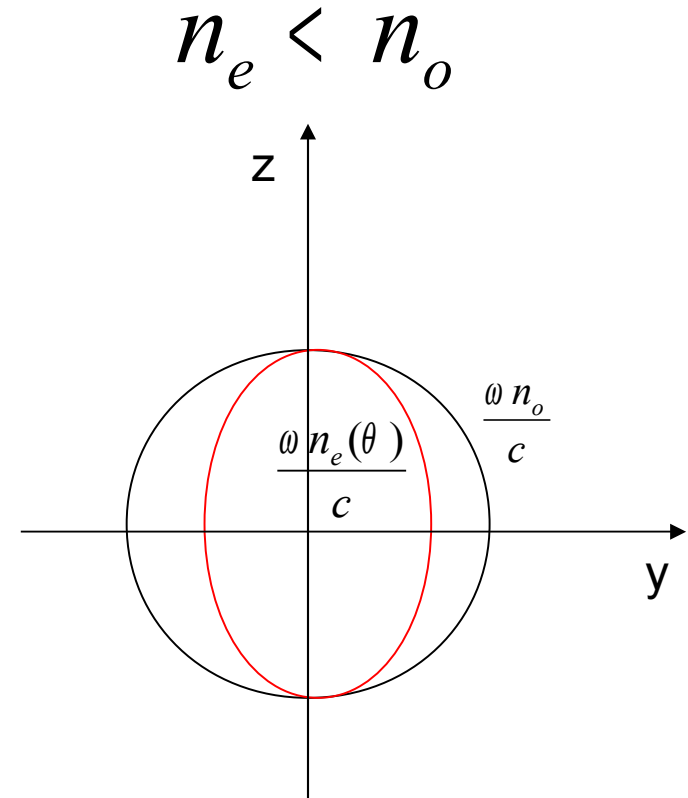


$$\frac{1}{n_e(\theta)^2} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}$$

Superfici isofrequenza



Ghiaccio	1.309	1.310
Quarzo	1.544	1.553
ZnS	2.354	2.358



Tormalina	1.638	1.618
Calcite	1.658	1.486
KDP	1.507	1.467

Equazioni Maxwell in onde piane

$$\vec{k} \cdot \vec{D} = 0 \quad \vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \cdot \vec{B} = 0 \quad \vec{k} \times \vec{H} = -\omega \vec{D}$$

$$\vec{D} = \varepsilon \vec{E} \quad \vec{H} = \frac{\vec{B}}{\mu_0}$$

Onda ordinaria

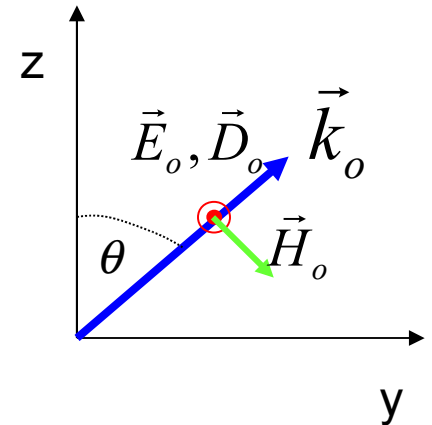
$$\vec{k}_o = \frac{\omega n_o}{c} (0, \sin \theta, \cos \theta)$$

$$\vec{D}_o = D(1, 0, 0)$$

$$\vec{E}_o = \frac{D}{\epsilon_x} (1, 0, 0) \quad \vec{H}_o = \frac{c}{n_o} D(0, \cos \theta, -\sin \theta)$$

$$\vec{k} \cdot \vec{D} = 0 \quad \vec{D} = \epsilon \vec{E}$$

$$\vec{k} \times \vec{H} = -\omega \vec{D}$$



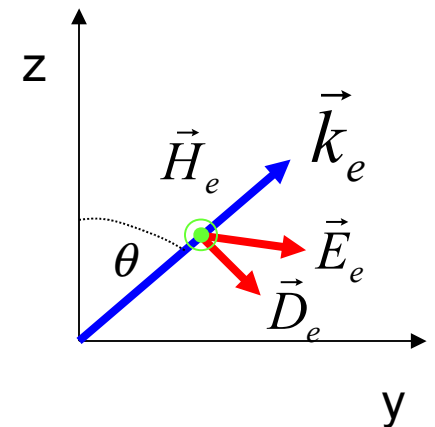
Onda straordinaria

$$\vec{k}_e = \frac{\omega n_e(\theta)}{c} (0, \sin \theta, \cos \theta)$$

$$\vec{D}_e = D(0, \cos \theta, -\sin \theta)$$

$$\vec{E}_e = D \left(0, \frac{\cos \theta}{\epsilon_x}, -\frac{\sin \theta}{\epsilon_z} \right)$$

$$\vec{H}_e = \frac{c}{n_e(\theta)} D(1, 0, 0)$$

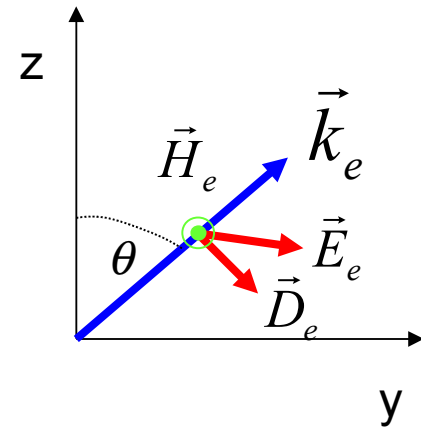


Onda straordinaria:
Componente longitudinale

$$\vec{k}_e = \frac{\omega n_e(\theta)}{c} (0, \sin \theta, \cos \theta)$$

$$\vec{D}_e = D(0, \cos \theta, -\sin \theta)$$

$$\vec{E}_e = D \left(0, \frac{\cos \theta}{\epsilon_x}, -\frac{\sin \theta}{\epsilon_z} \right) \quad \vec{H}_o = \frac{c}{n_e(\theta)} D(1, 0, 0)$$



$$\vec{E}_{e, \text{long}} = \frac{\vec{k}_e \cdot \vec{E}_e}{|\vec{k}_e|} = D \left(\frac{\cos \theta \sin \theta}{\epsilon_x} - \frac{\sin \theta \cos \theta}{\epsilon_z} \right) =$$

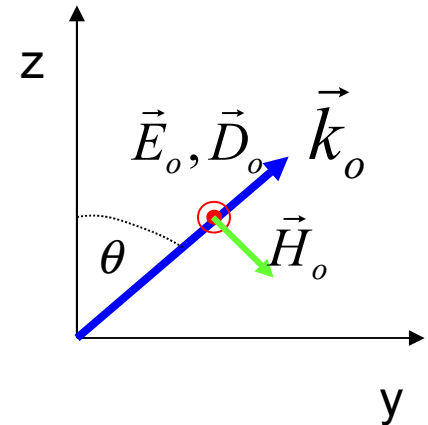
$$= \frac{D \cos \theta \sin \theta}{\epsilon_0} \frac{n_e^2 - n_o^2}{n_o^2 n_e^2} = \frac{D \cos \theta \sin \theta}{\epsilon_0} \frac{(n_e - n_o)(n_e + n_o)}{n_o^2 n_e^2}$$

Onda ordinaria

$$\vec{k}_o = \frac{\omega n_o}{c} (0, \sin \theta, \cos \theta)$$

$$\vec{E}_o = \frac{D}{\epsilon_x} (1, 0, 0) \quad \vec{H}_o = \frac{c}{n_o} D (0, \cos \theta, -\sin \theta)$$

$$\vec{S} = \frac{c}{\epsilon_x n_o} |D|^2 (0, \sin \theta, \cos \theta) \quad // \vec{k}$$

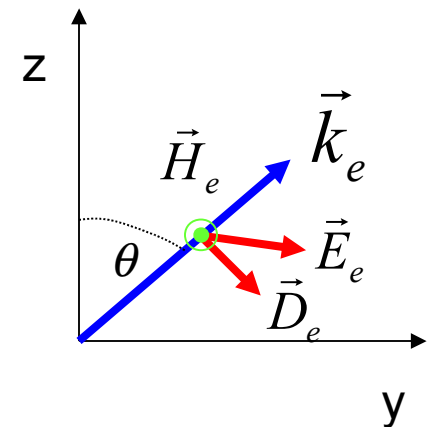


Onda straordinaria

$$\vec{k}_e = \frac{\omega n_e(\theta)}{c} (0, \sin \theta, \cos \theta)$$

$$\vec{E}_e = D \left(0, \frac{\cos \theta}{\epsilon_x}, -\frac{\sin \theta}{\epsilon_z} \right) \quad \vec{H}_e = -\frac{c}{n_e(\theta)} D (1, 0, 0)$$

$$\vec{S} = \frac{c}{n_e(\theta)} |D|^2 \left(0, \frac{\sin \theta}{\epsilon_x}, \frac{\cos \theta}{\epsilon_z} \right) \quad \nparallel \vec{k}$$



Velocità di gruppo

$$\omega^2 = \frac{k^2 c^2}{n_e(\theta)^2} = k^2 c^2 \left(\frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2} \right) = c^2 \left(\frac{k_z^2}{n_o^2} + \frac{k_y^2}{n_e^2} \right)$$

$$\omega = c \sqrt{\frac{k_z^2}{n_o^2} + \frac{k_y^2}{n_e^2}}$$

$$\vec{v}_g = \vec{\nabla}_k \omega = \frac{c^2}{\omega} \left(0, \frac{k_y}{n_e^2}, \frac{k_z}{n_o^2} \right) = n_e(\theta) c \epsilon_0 \left(0, \frac{\sin \theta}{\epsilon_z}, \frac{\cos \theta}{\epsilon_x} \right)$$

$$\vec{S} = \frac{c}{n_e(\theta)} |D|^2 \left(0, \frac{\sin \theta}{\epsilon_z}, \frac{\cos \theta}{\epsilon_x} \right) = \frac{1}{\epsilon_0 n_e^2(\theta)} |D|^2 \vec{v}_g$$

Velocità energia

$$\vec{S} \equiv U_{em} \vec{v}_e$$

$$\vec{D}_e = D(0, \cos \theta, -\sin \theta)$$

$$\vec{E}_e = D \left(0, \frac{\cos \theta}{\epsilon_x}, -\frac{\sin \theta}{\epsilon_z} \right) \quad \vec{H}_e = \frac{c}{n_e(\theta)} D(1, 0, 0)$$

$$U_{em} = \vec{E}_e \cdot \vec{D}_e^* = |D|^2 \left(\frac{\cos^2 \theta}{\epsilon_x} + \frac{\sin^2 \theta}{\epsilon_z} \right) =$$

$$= \frac{|D|^2}{\epsilon_0} \left(\frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2} \right) = \frac{|D|^2}{\epsilon_0 n_e^2(\theta)}$$

$$\vec{S} = \frac{1}{\epsilon_0 n_e(\theta)} |D|^2 \vec{v}_g = U_{em} \vec{v}_g$$

$$\vec{v}_e = \vec{v}_g$$

$$\vec{k} \times \vec{E} = \omega \mu \vec{H} \quad \vec{k} \times \vec{H} = -\omega \epsilon \vec{E}$$

$$\begin{cases} \delta \vec{k} \times \vec{E} + \vec{k} \times \delta \vec{E} = \delta \omega \mu \vec{H} + \omega \mu \delta \vec{H} \\ \delta \vec{k} \times \vec{H} + \vec{k} \times \delta \vec{H} = -\delta \omega \epsilon \vec{E} - \omega \epsilon \delta \vec{E} \end{cases}$$

$$\begin{cases} \vec{H} \cdot (\delta \vec{k} \times \vec{E} + \vec{k} \times \delta \vec{E}) = \vec{H} \cdot (\delta \omega \mu \vec{H} + \omega \mu \delta \vec{H}) \\ \vec{E} \cdot (\delta \vec{k} \times \vec{H} + \vec{k} \times \delta \vec{H}) = -\vec{E} \cdot (\delta \omega \epsilon \vec{E} + \omega \epsilon \delta \vec{E}) \end{cases}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

$$\begin{cases} \delta \vec{k} \cdot (\vec{E} \times \vec{H}) + \vec{k} \cdot (\delta \vec{E} \times \vec{H}) = \delta \omega (\vec{H} \cdot \mu \vec{H}) + \omega (\vec{H} \cdot \mu \delta \vec{H}) \\ \delta \vec{k} \cdot (\vec{E} \times \vec{H}) + \vec{k} \cdot (\delta \vec{H} \times \vec{E}) = -\delta \omega (\vec{E} \cdot \epsilon \vec{E}) - \omega (\vec{E} \cdot \epsilon \delta \vec{E}) \end{cases}$$

$$\vec{E} \cdot \epsilon \delta \vec{E} = \delta \vec{E} \cdot \epsilon \vec{E} \quad \vec{H} \cdot \mu \delta \vec{H} = \delta \vec{H} \cdot \mu \vec{H}$$

$$\begin{aligned} 2\delta \vec{k} \cdot (\vec{E} \times \vec{H}) + \vec{k} \cdot (\delta \vec{E} \times \vec{H}) - \vec{k} \cdot (\delta \vec{H} \times \vec{E}) = \\ \delta \omega (\vec{H} \cdot \mu \vec{H} + \vec{E} \cdot \epsilon \vec{E}) + \omega (\delta \vec{H} \cdot \mu \vec{H} + \delta \vec{E} \cdot \epsilon \vec{E}) \end{aligned}$$

$$\begin{aligned} 2\delta \vec{k} \cdot (\vec{E} \times \vec{H}) - \delta \omega (\vec{H} \cdot \mu \vec{H} + \vec{E} \cdot \epsilon \vec{E}) = \\ -\vec{k} \cdot (\delta \vec{E} \times \vec{H}) + \vec{k} \cdot (\delta \vec{H} \times \vec{E}) + \omega (\delta \vec{H} \cdot \mu \vec{H} + \delta \vec{E} \cdot \epsilon \vec{E}) \end{aligned}$$

Velocità energia=Velocità gruppo

3

$$2\delta\vec{k} \cdot (\vec{E} \times \vec{H}) - \delta\omega (\vec{H} \cdot \mu \vec{H} + \vec{E} \cdot \epsilon \vec{E}) = \\ \delta\vec{E} \cdot (\vec{k} \times \vec{H}) - \delta\vec{H} \cdot (\vec{k} \times \vec{E}) + \omega (\delta\vec{H} \cdot \mu \vec{H} + \delta\vec{E} \cdot \epsilon \vec{E})$$

$$2\delta\vec{k} \cdot (\vec{E} \times \vec{H}) - \delta\omega (\vec{H} \cdot \mu \vec{H} + \vec{E} \cdot \epsilon \vec{E}) = \\ \delta\vec{E} \cdot (\vec{k} \times \vec{H} + \omega \epsilon \vec{E}) - \delta\vec{H} \cdot (\vec{k} \times \vec{E} - \omega \mu \vec{H}) = 0$$

$$\delta\vec{k} \cdot (\vec{E} \times \vec{H}) = \delta\omega \frac{1}{2} (\vec{H} \cdot \mu \vec{H} + \vec{E} \cdot \epsilon \vec{E})$$

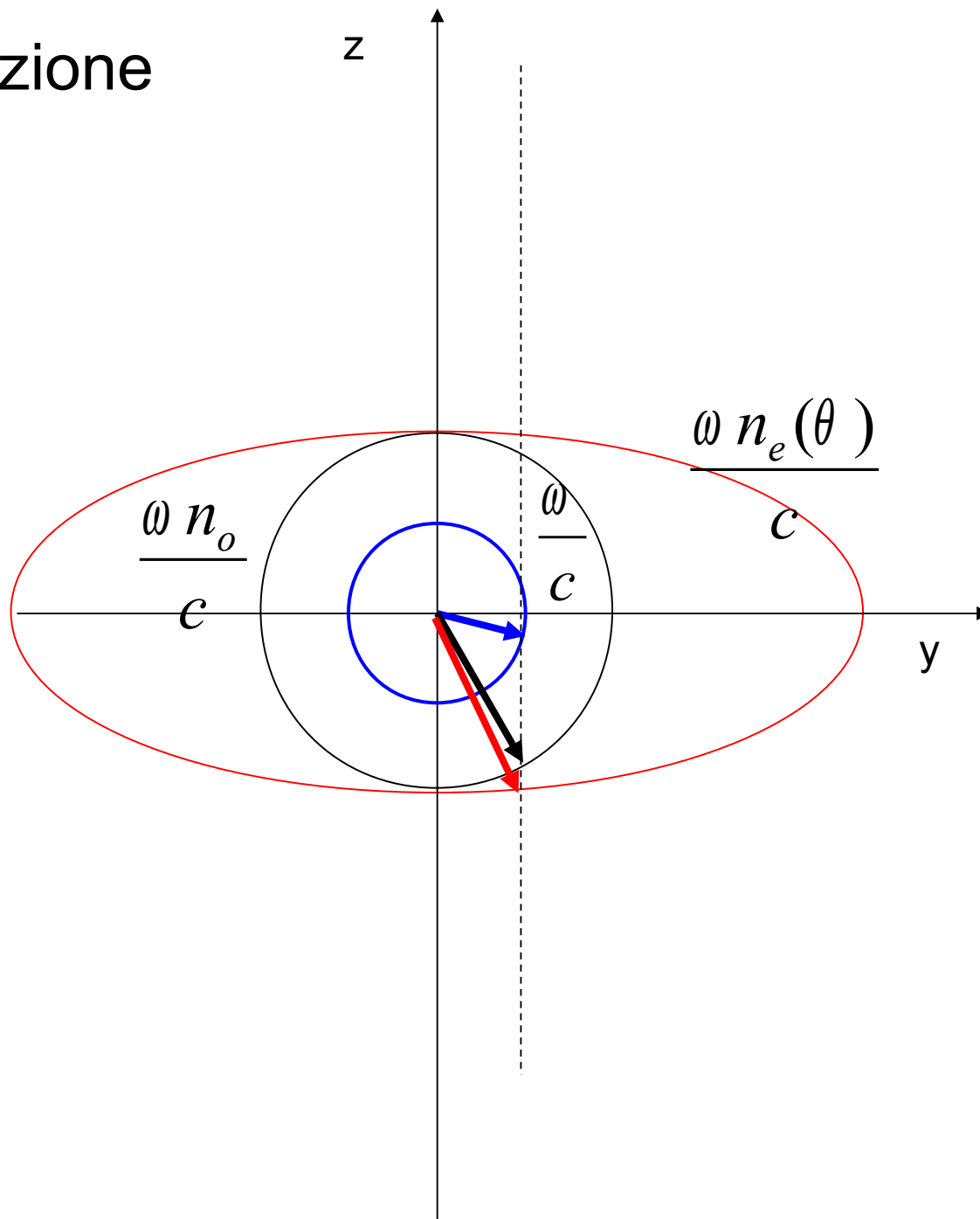
$$\delta\omega = \delta\vec{k} \cdot \frac{\vec{S}}{U_{el}} \quad \delta\omega = \delta\vec{k} \cdot \vec{\nabla}_k \omega(\vec{k}) \quad \therefore$$

Doppia rifrazione

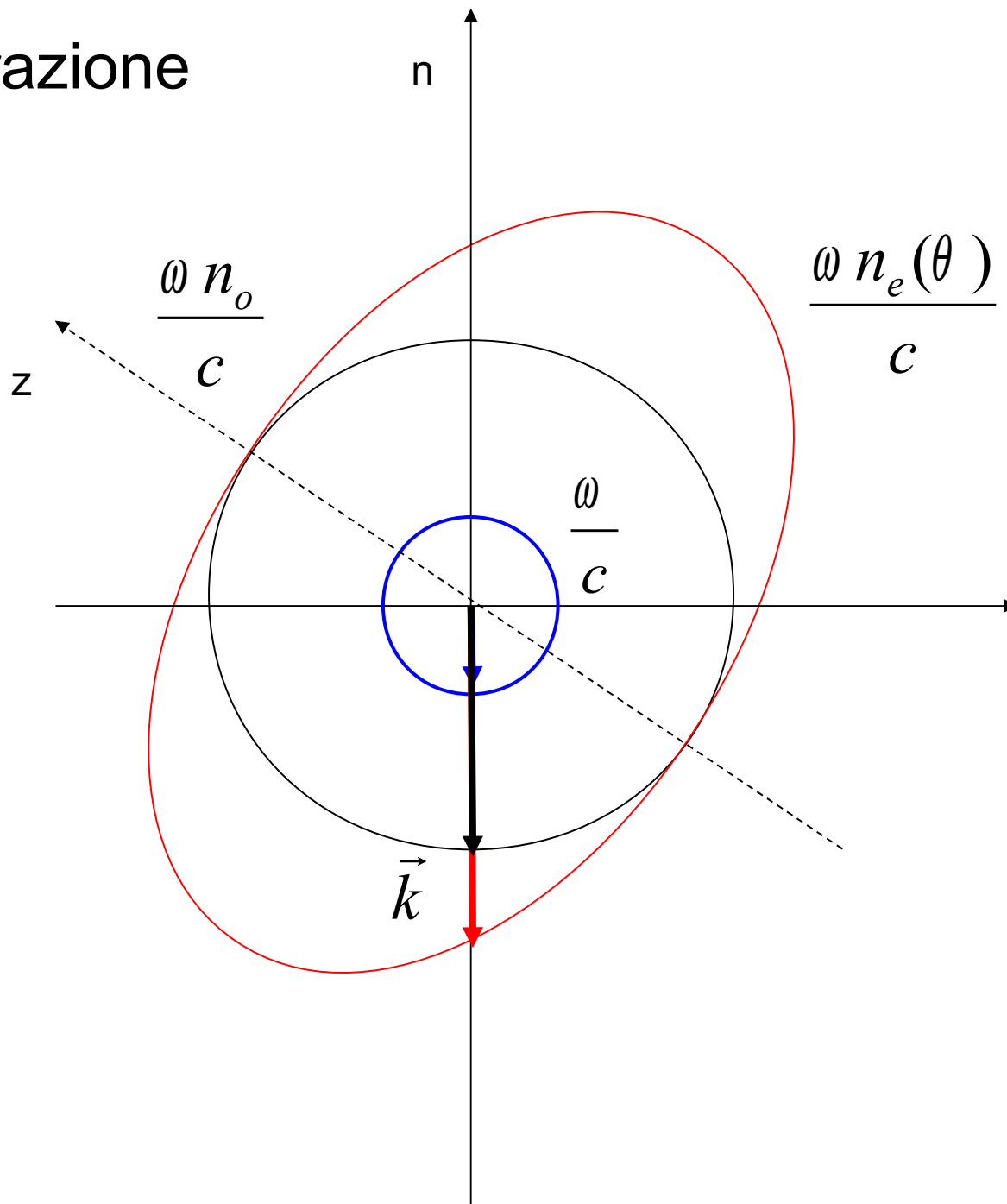


NOTA: effetto anche ad incidenza normale

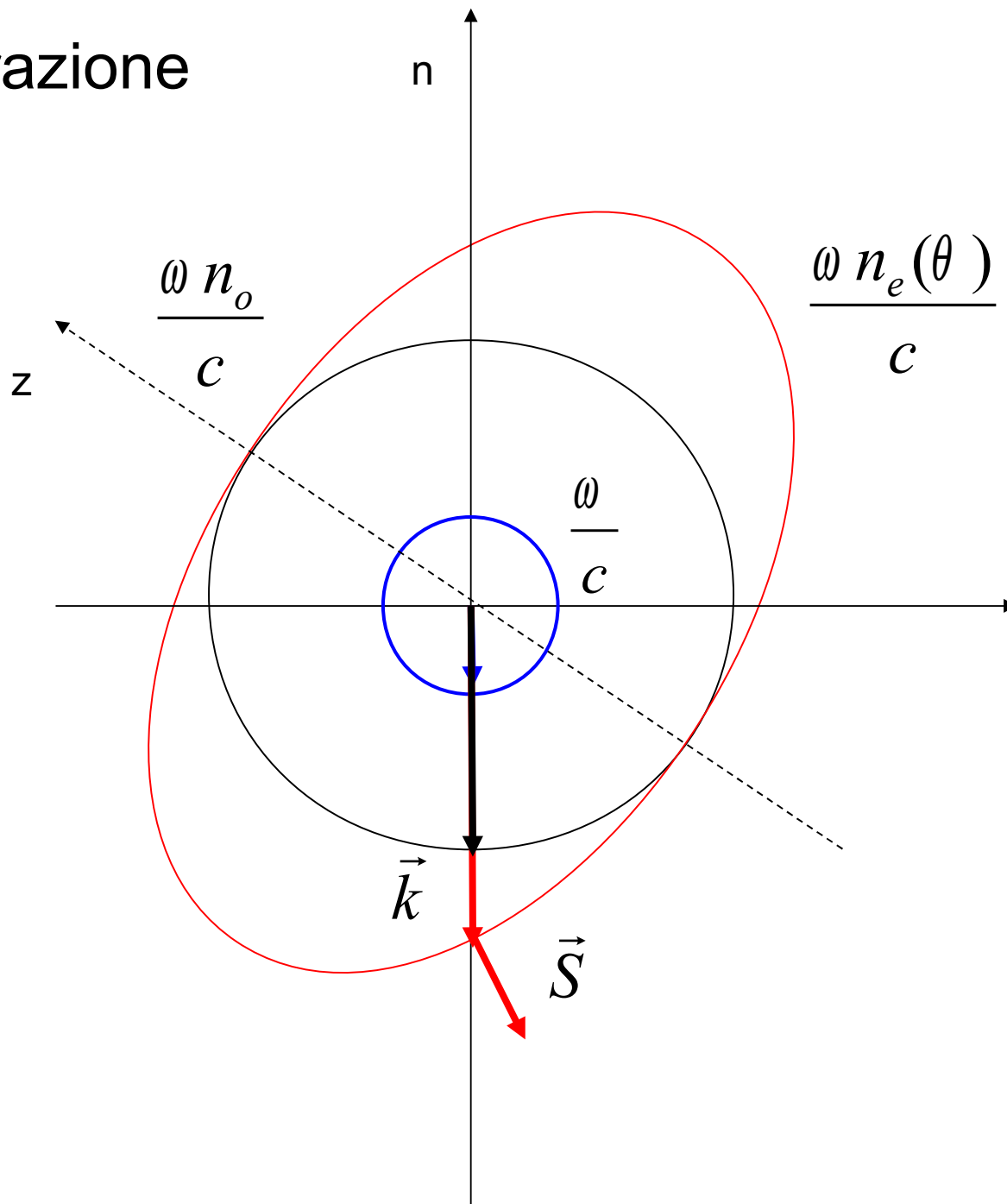
Doppia rifrazione



Doppia rifrazione

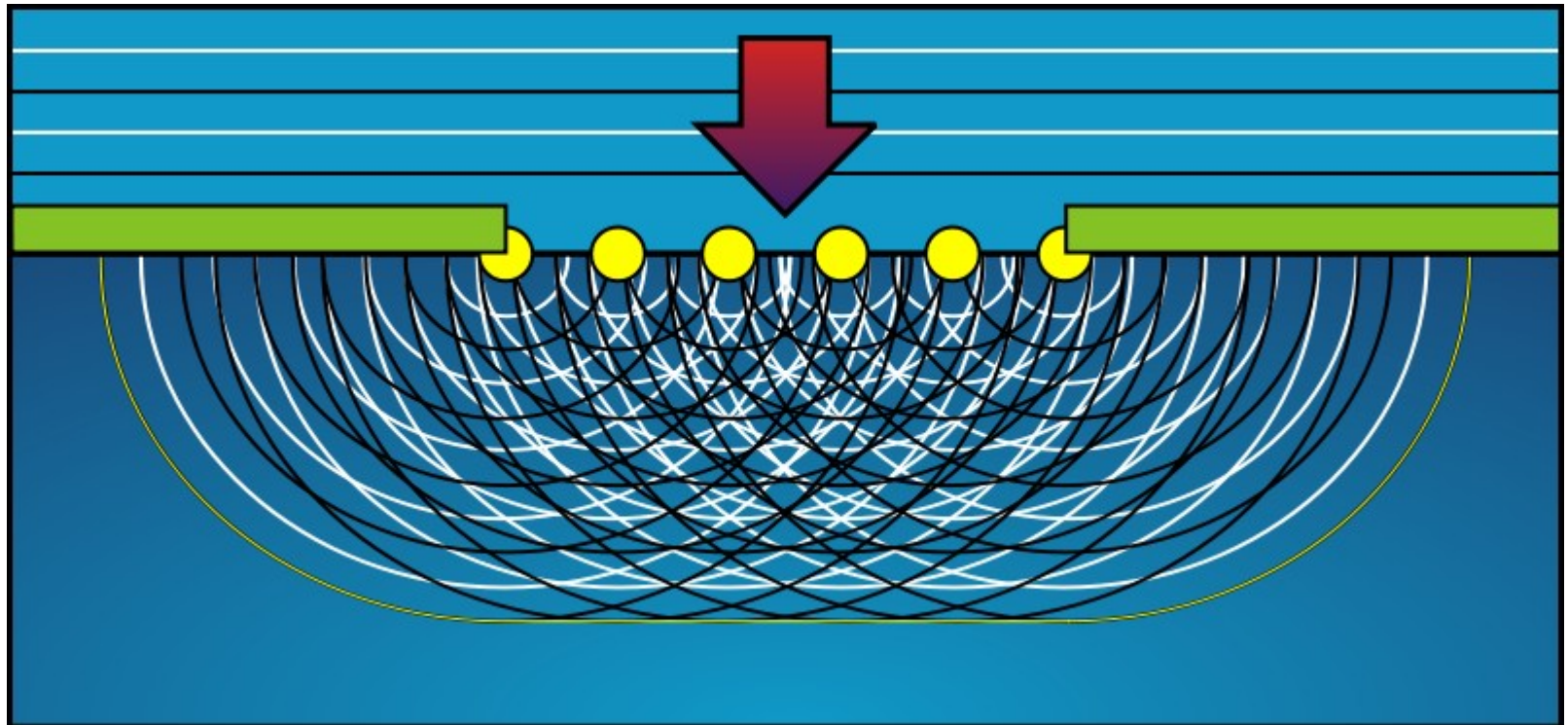


Doppia rifrazione



Principio di Huygens

$$\frac{u_0}{i\lambda} \int_{\Sigma} \frac{e^{ikr}}{r} f(\theta) d\Sigma$$



Principio di Huygens

componenti onda incidente

straordinaria

ordinaria

mostra raggio

FIL@poliTO

Principio di Huygens

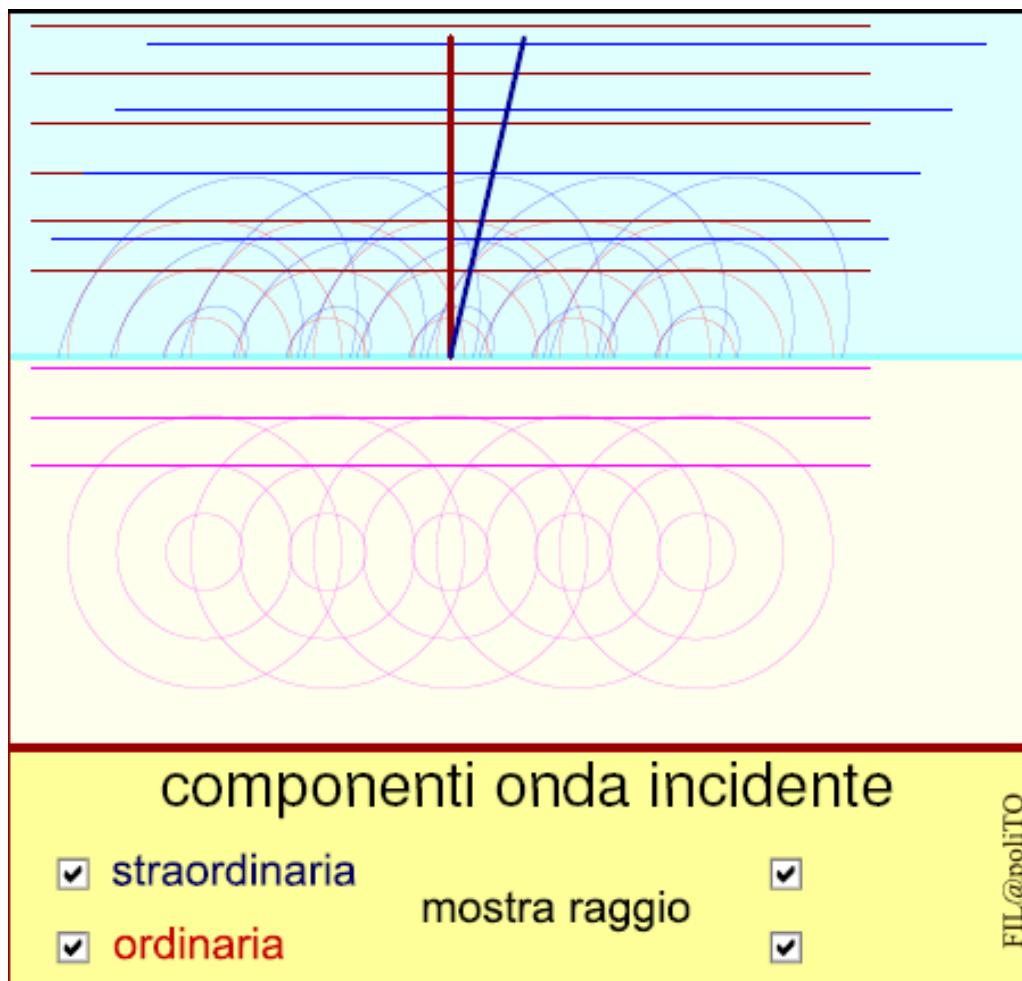
componenti onda incidente

straordinaria mostra raggio

ordinaria

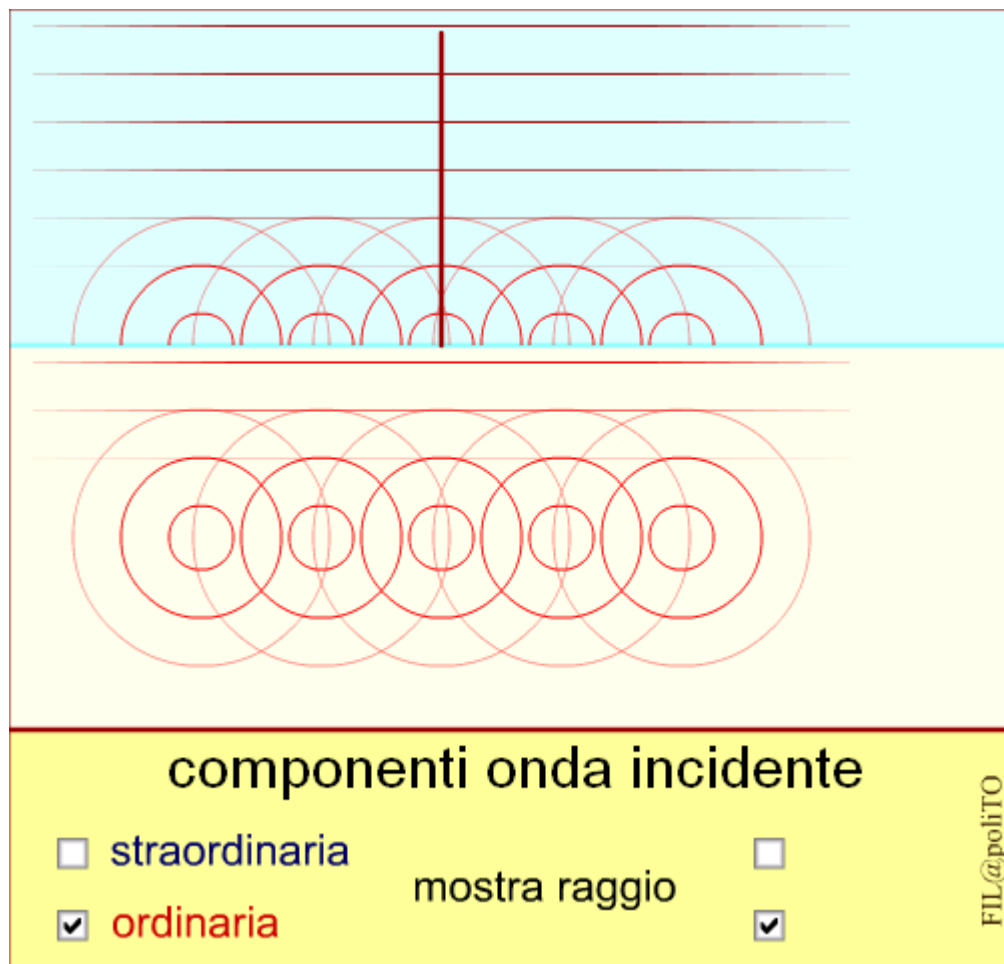
FIL@poliTO

Principio di Huygens



Per vedere immagine in movimento: andare sul sito, copiare tutto, tenere l'immagine su ppt e non spingere il PC. L'animazione resta solo sul ppt aperto!!!!
http://www2.polito.it/ricerca/qdbf/fil/indicegenerale/ottica/ottica_fisica/birifrangenza.htm

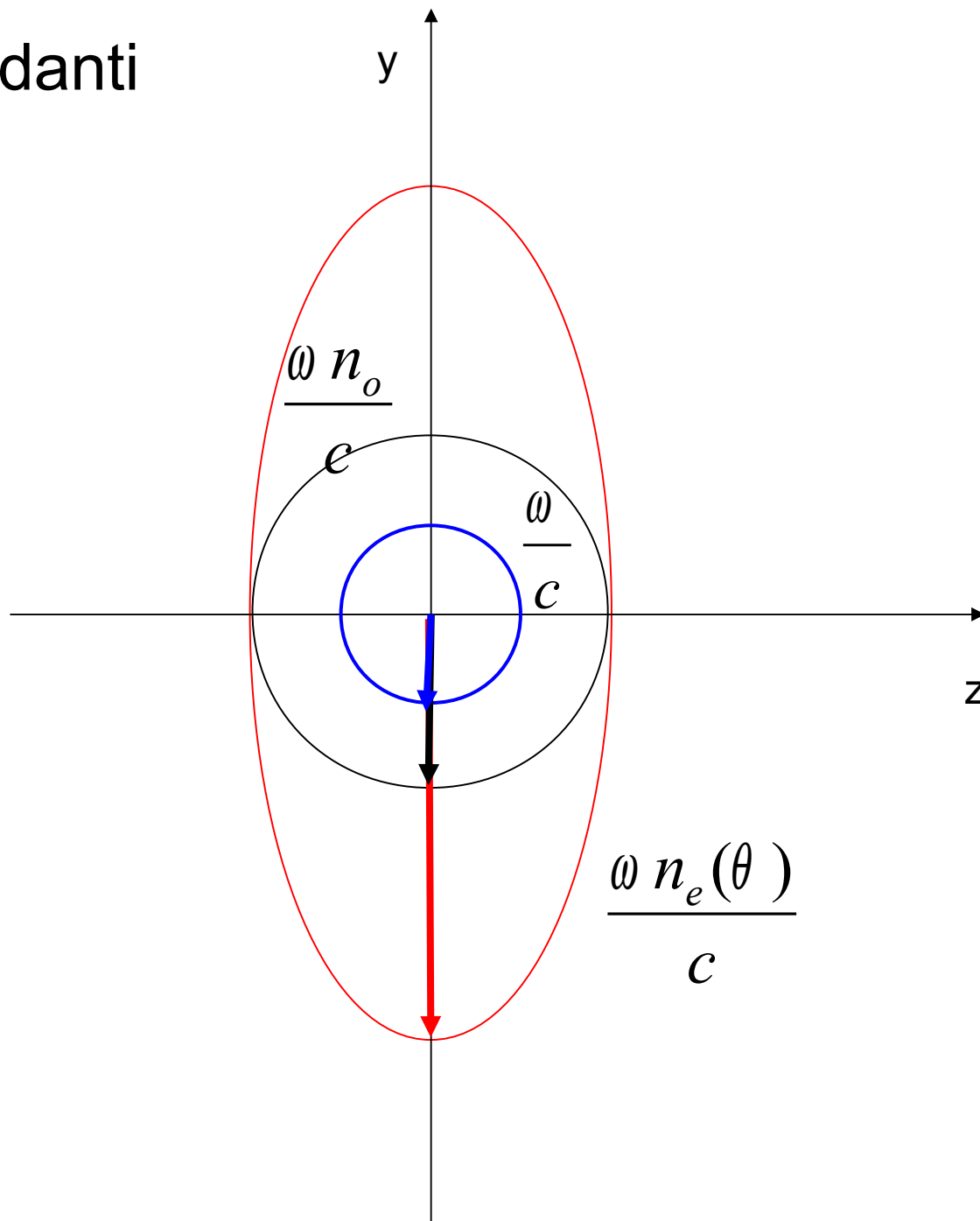
Principio di Huygens



Lamine ritardanti

$$\vec{k}_e = \frac{\omega n_e}{c} \hat{e}_y$$

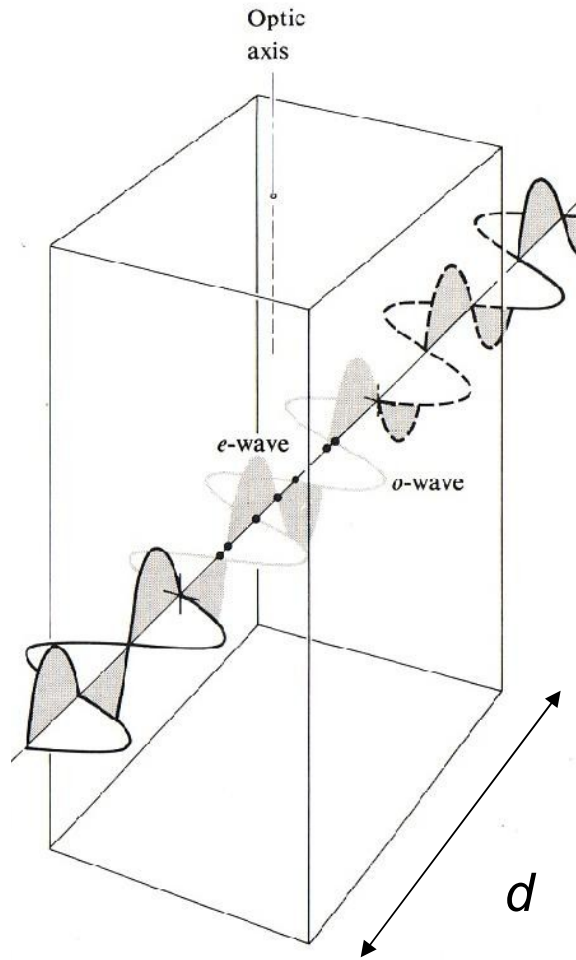
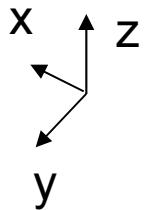
$$\vec{k}_o = \frac{\omega n_o}{c} \hat{e}_y$$



Lamine ritardanti

$$\vec{k}_e = \frac{\omega n_e}{c} \hat{e}_y$$

$$\vec{k}_o = \frac{\omega n_o}{c} \hat{e}_y$$



$$\varphi_e = \frac{\omega n_e}{c} d = \frac{2\pi n_e}{\lambda} d$$

$$\varphi_o = \frac{\omega n_o}{c} d = \frac{2\pi n_o}{\lambda} d$$

$$E_{in} = E_0 (\hat{e}_x + \hat{e}_z) e^{j(ky - \omega t)}$$

$$E_{out} = E_0 (\hat{e}_x e^{j\varphi_o} + \hat{e}_z e^{j\varphi_e}) e^{j(ky - \omega t)} =$$

$$= E_0 (\hat{e}_x + \hat{e}_z e^{j(\varphi_e - \varphi_o)}) e^{j\varphi_o} e^{j(ky - \omega t)}$$

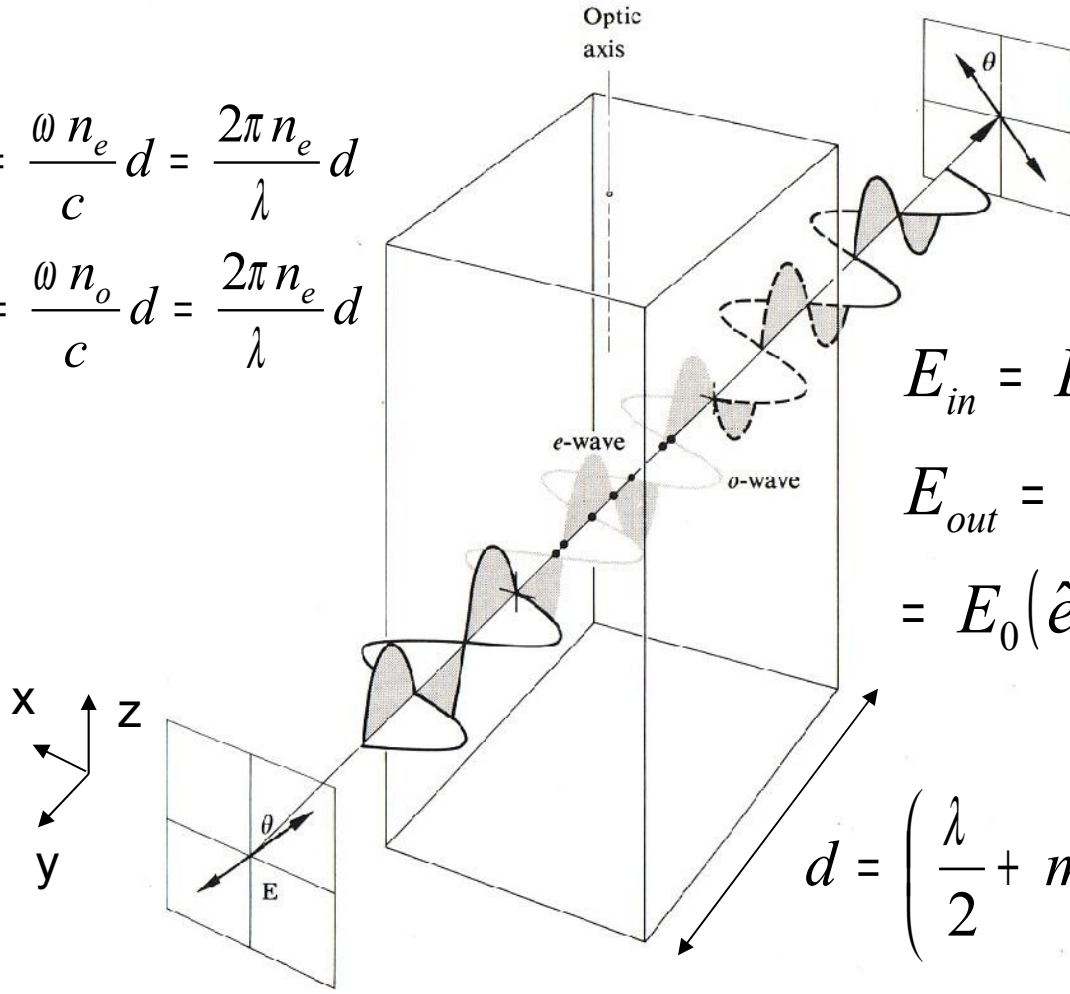
$$\varphi_e - \varphi_o = \frac{2\pi}{\lambda} d (n_e - n_o)$$

Lamine $\lambda/2$

$$\varphi_e - \varphi_o = \pi (2m + 1)$$

$$\varphi_e = \frac{\omega n_e}{c} d = \frac{2\pi n_e}{\lambda} d$$

$$\varphi_o = \frac{\omega n_o}{c} d = \frac{2\pi n_o}{\lambda} d$$



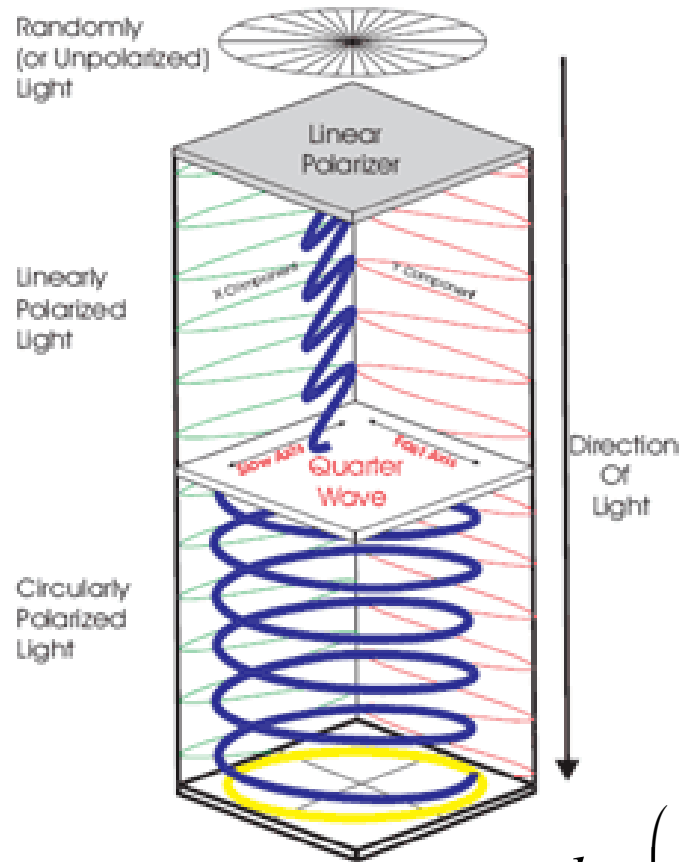
$$E_{in} = E_0 (\hat{e}_x + \hat{e}_z) e^{j(ky - \omega t)}$$

$$E_{out} = E_0 (\hat{e}_x + \hat{e}_z e^{j\pi}) e^{j\varphi_o} e^{j(ky - \omega t)} = E_0 (\hat{e}_x - \hat{e}_z) e^{j\varphi_o} e^{j(ky - \omega t)}$$

$$d = \left(\frac{\lambda}{2} + m\lambda \right) \frac{1}{n_e - n_o}$$

Lamine $\lambda/4$

$$\varphi_e - \varphi_o = \pi \left(2m + \frac{1}{2} \right)$$



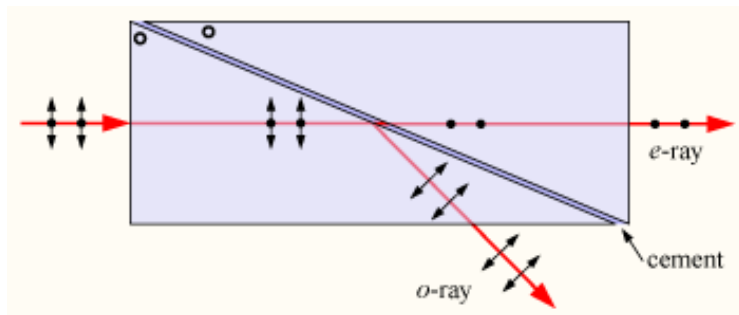
$$E_{in} = E_0 (\hat{e}_x + \hat{e}_z) e^{j(ky - \omega t)}$$

$$E_{out} = E_0 \left(\hat{e}_x + \hat{e}_z e^{j\frac{\pi}{2}} \right) e^{j\varphi_o} e^{j(ky - \omega t)} =$$

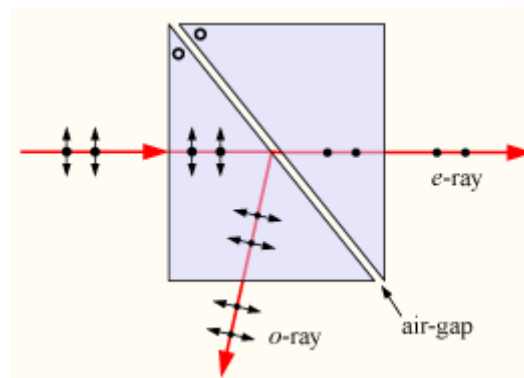
$$= E_0 (\hat{e}_x + j\hat{e}_z) e^{j\varphi_o} e^{j(ky - \omega t)}$$

$$d = \left(\frac{\lambda}{4} + m\lambda \right) \frac{1}{n_e - n_o}$$

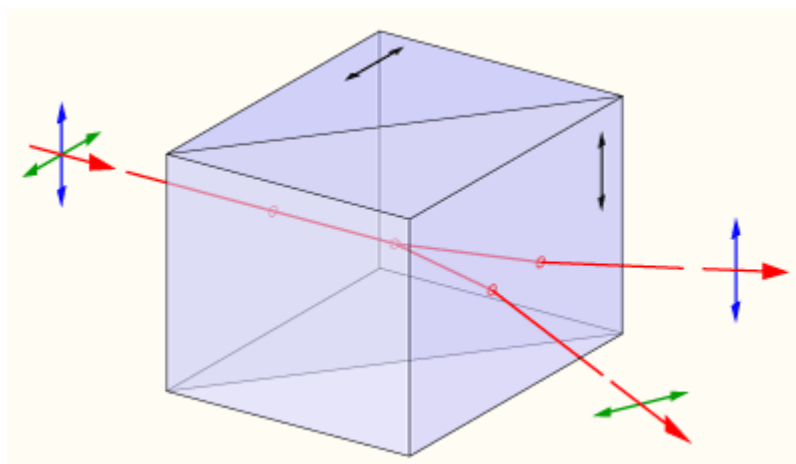
Polarizer prisms



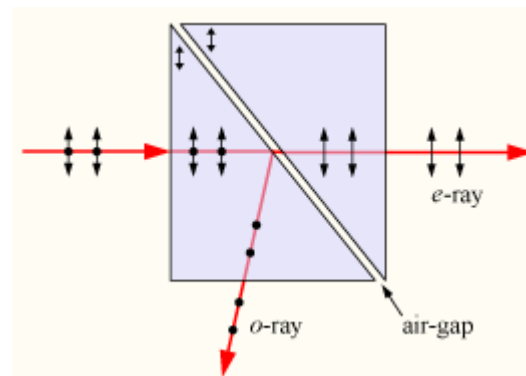
Glan Thompson



Glan Foucault



Wollaston



Glan Taylor