Condizioni al contorno
Relazioni di Fresnel

Equazioni Maxwell nella materia

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

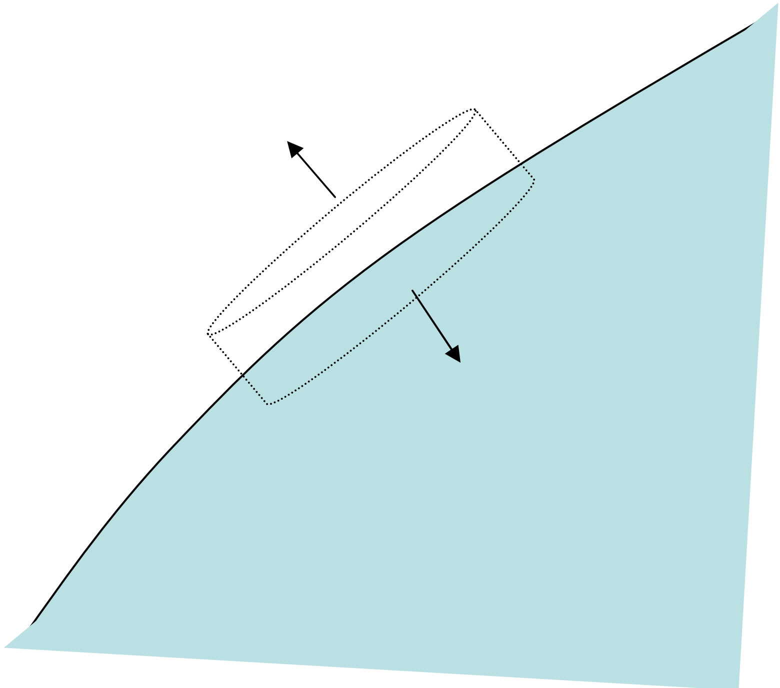
$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

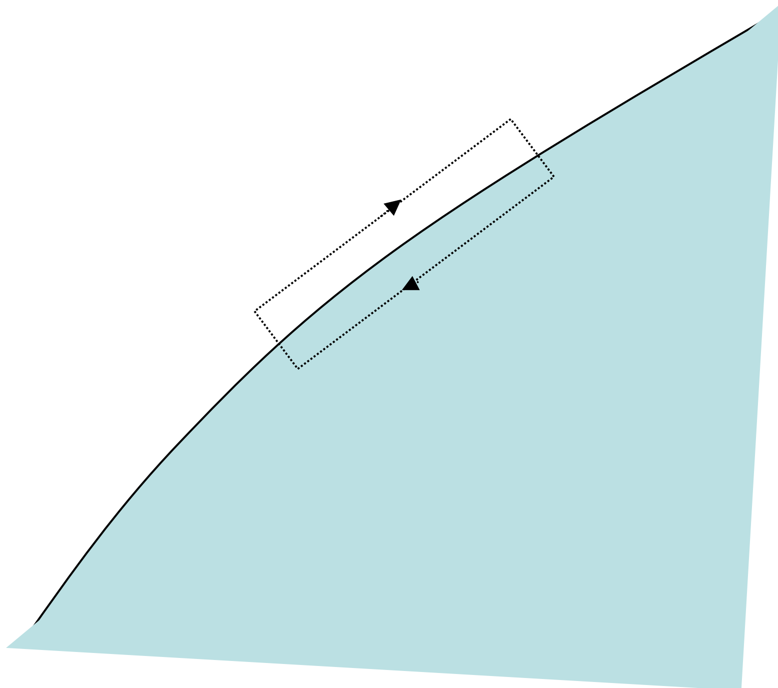
$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu_o \mu_r}$$

Boundary Conditions

$$E_{1,t} - E_{2,t} = 0 \qquad D_{1,n} - D_{2,n} = \sigma$$

$$H_{1,t} - H_{2,t} = J \qquad B_{1,n} - B_{2,n} = 0$$

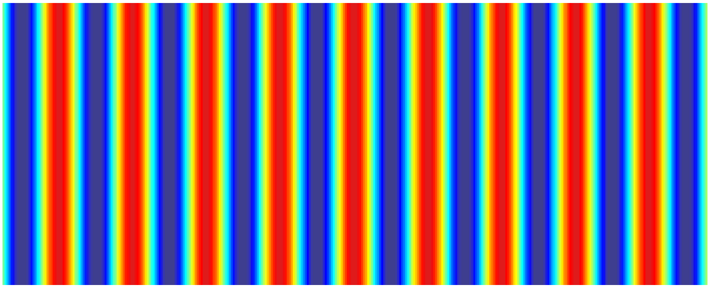


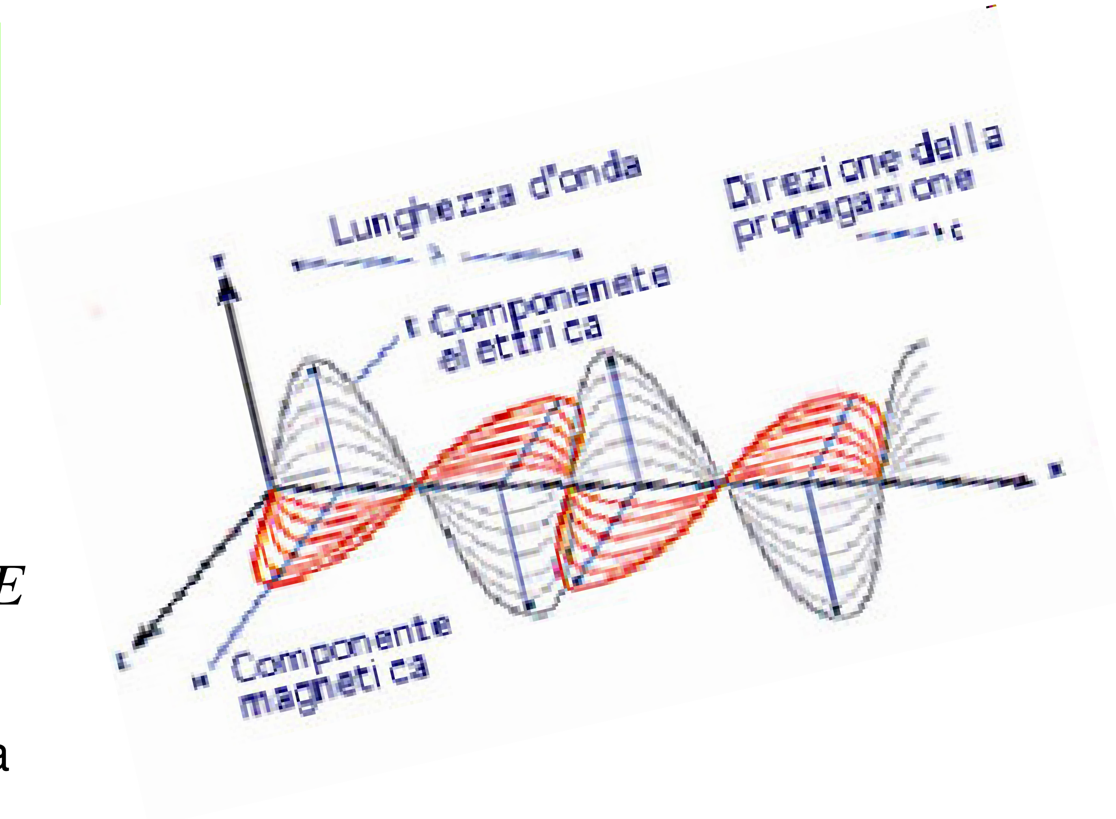


Plane wave

$$\vec{E} = \vec{E}_o e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{k}\cdot\vec{E} = 0 \qquad \vec{k}\times\vec{H} = \omega\varepsilon_o\varepsilon_r\vec{E} \qquad n = \sqrt{\varepsilon_r\mu_r}$$



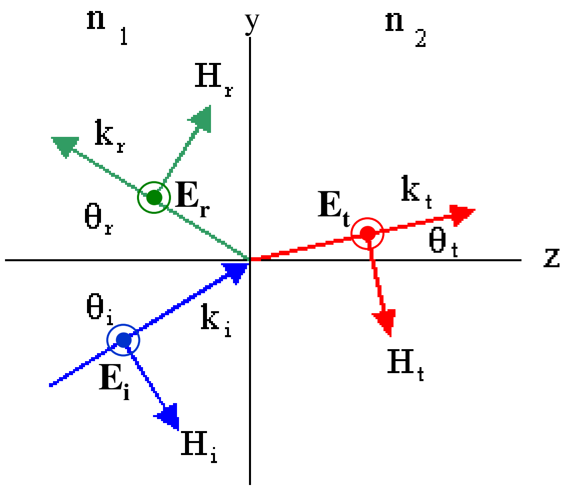


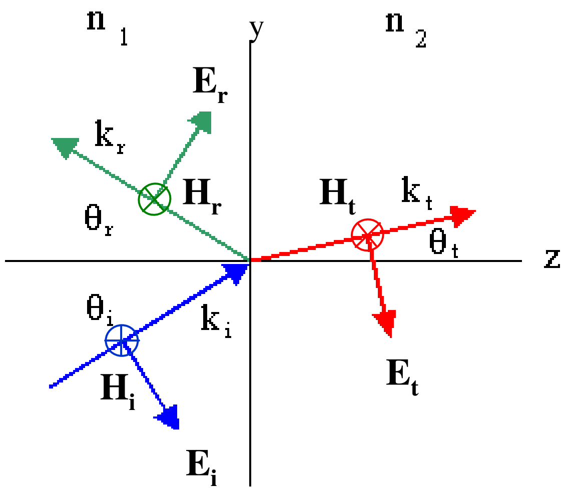
$(\vec{k},\vec{H},\vec{E})$ *right hand triplet*

$$H = \frac{c}{n}\varepsilon_o\varepsilon_r E = \sqrt{\frac{\varepsilon_o\varepsilon_r}{\mu_o\mu_r}}E = \eta_o\eta_r E$$

impedenza

Riflessione e rifrazione

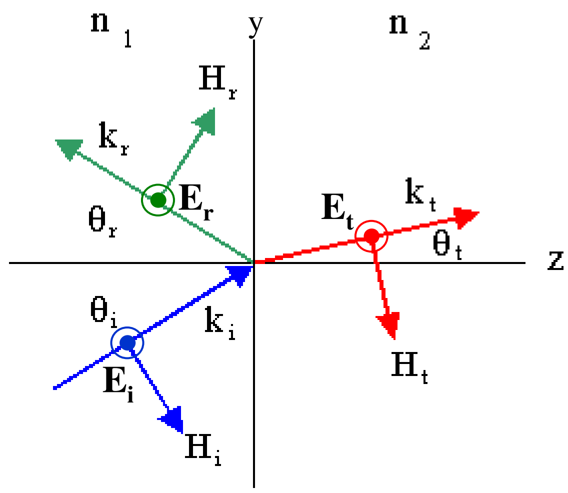




Onda s (senkrecht=perpendicolare)
Polarizzazione TE

Onda p (parallel)
Polarizzazione TM

Onda TE



$$\vec{E}_i(\vec{r},t) = E_i e^{j(\vec{k}_i\cdot\vec{r}-\omega_i t)}\hat{e}_x \qquad \vec{H}_i(\vec{r},t) = H_i e^{j(\vec{k}_i\cdot\vec{r}-\omega_i t)}(\hat{e}_z\cos\theta_i - \hat{e}_y\sin\theta_i)$$

$$\vec{E}_r(\vec{r},t) = E_r e^{j(\vec{k}_r\cdot\vec{r}-\omega_r t+\varphi_r)}\hat{e}_x \qquad \vec{H}_r(\vec{r},t) = H_r e^{j(\vec{k}_r\cdot\vec{r}-\omega_r t+\varphi_r)}(\hat{e}_z\cos\theta_r + \hat{e}_y\sin\theta_r)$$

$$\vec{E}_t(\vec{r},t) = E_t e^{j(\vec{k}_t\cdot\vec{r}-\omega_t t+\varphi_t)}\hat{e}_x \qquad \vec{H}_t(\vec{r},t) = H_t e^{j(\vec{k}_t\cdot\vec{r}-\omega_t t+\varphi_t)}(\hat{e}_z\cos\theta_t - \hat{e}_y\sin\theta_t)$$

$$H_i = \eta_o\eta_1 E_i \qquad H_r = \eta_o\eta_1 E_r \qquad H_t = \eta_o\eta_2 E_t$$

Onda TE e TM, conservazione fase all'interfaccia

$$\vec{k}_i \cdot \vec{r} - \omega_i t = \vec{k}_r \cdot \vec{r} - \omega_r t + \varphi_r = \vec{k}_t \cdot \vec{r} - \omega_t t + \varphi_t$$

$$\varphi_r = \varphi_t = 0$$

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

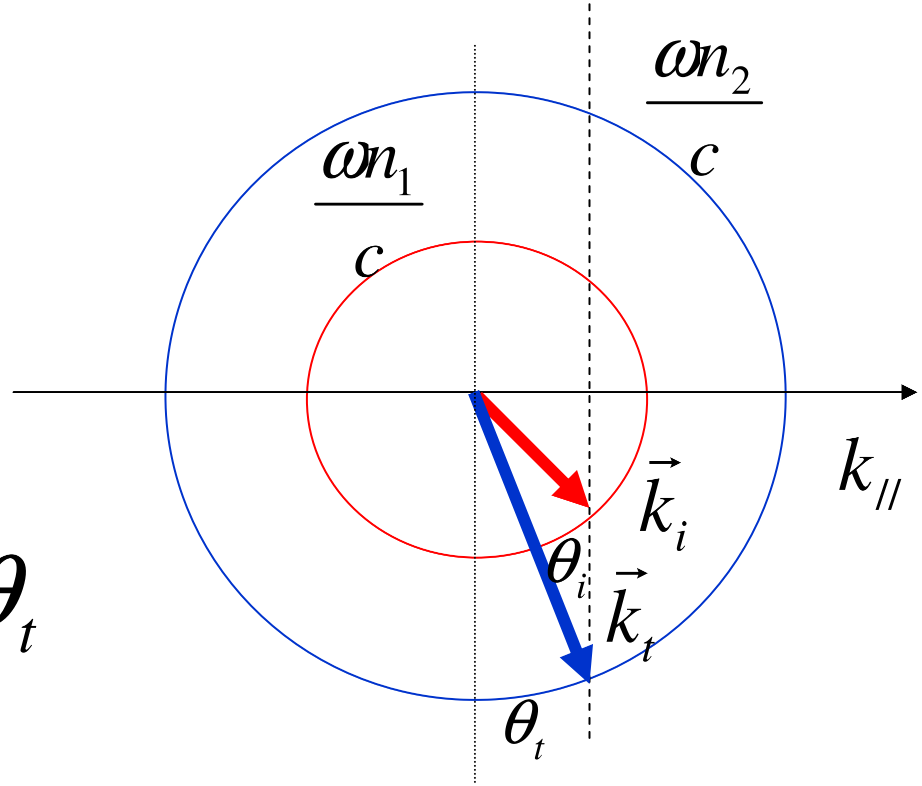
$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//} \qquad \begin{cases} \theta_i = \theta_r \\ n_1 \sin\theta_i = n_2 \sin\theta_t \end{cases}$$

Metodo grafico per rifrazione

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//}$$

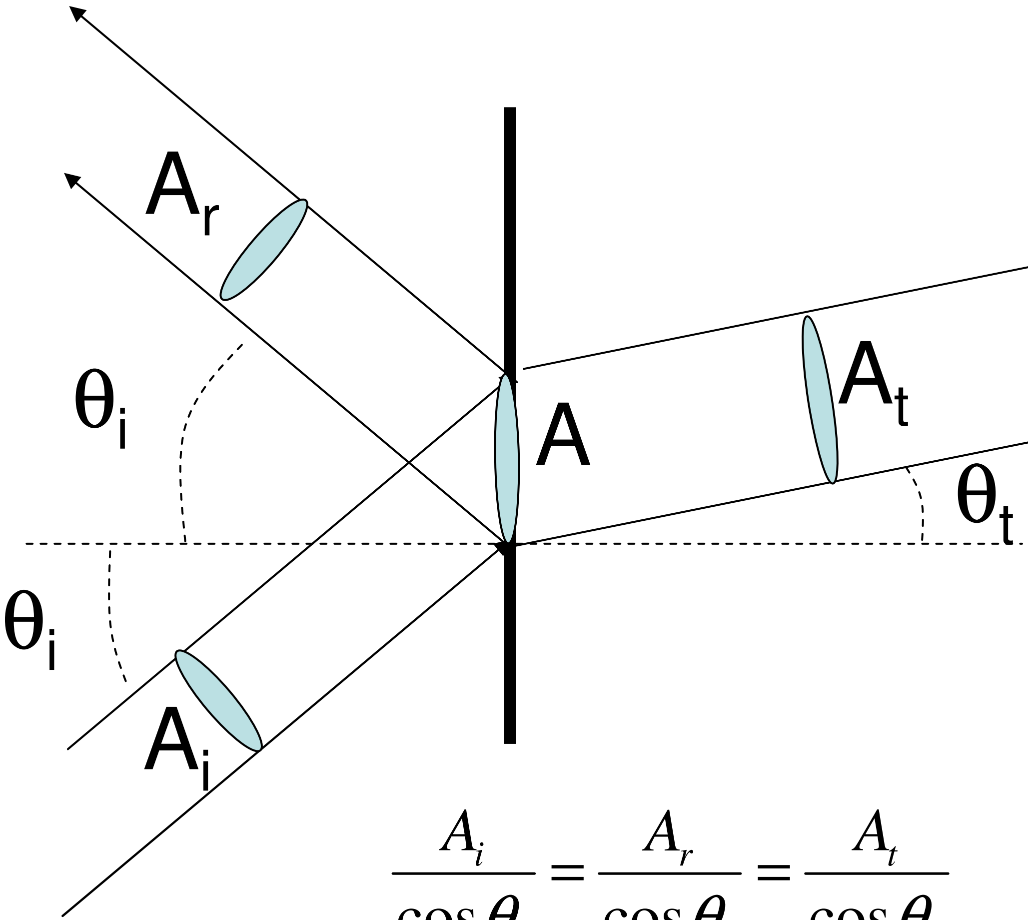
$$n_1 \sin\theta_i = n_2 \sin\theta_t$$



$$\left|\vec{S}\right| = \left|\vec{E}\times\vec{H}^*\right| = \left|\vec{E}\times\left(\frac{\vec{k}\times\vec{E}^*}{\mu\omega}\right)\right|$$

$$= \eta_o\eta_r\left|\vec{E}\right|^2 = nc\varepsilon_o\left|\vec{E}\right|^2$$

Velocità energia=v_g



Conservazione energia

$$\frac{A_i}{\cos\theta_i} = \frac{A_r}{\cos\theta_i} = \frac{A_t}{\cos\theta_t}$$

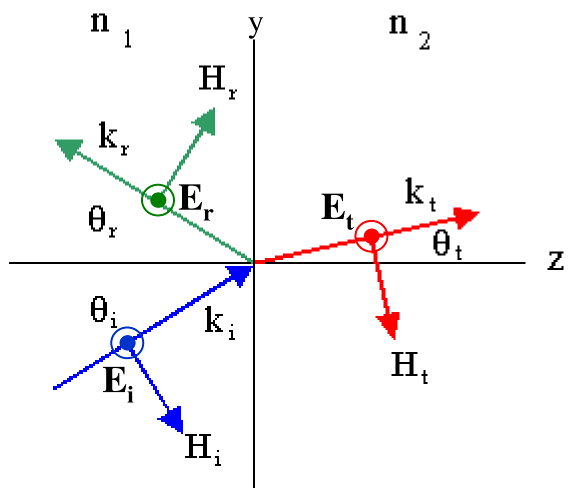
$$\left|\vec{S}_i\right|A_i = \left|\vec{S}_r\right|A_r + \left|\vec{S}_t\right|A_t$$

$$\eta_1\left|\vec{E}_i\right|^2 A_i = \eta_1\left|\vec{E}_r\right|^2 A_r + \eta_2\left|\vec{E}_t\right|^2 A_t$$

$$1 = |r|^2 + |t|^2\frac{\eta_2}{\eta_1}\frac{\cos\theta_i}{\cos\theta_t} = |r|^2 + |t|^2\frac{n_2}{n_1}\frac{\cos\theta_i}{\cos\theta_t}$$

Dielettrico non magnetico

Onda TE

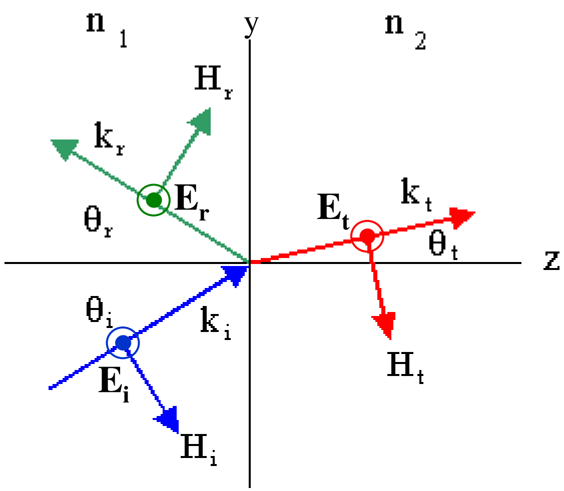


$$\vec{k}_i \cdot \vec{r} - \omega_i t = \vec{k}_r \cdot \vec{r} - \omega_r t + \varphi_r = \vec{k}_t \cdot \vec{r} - \omega_t t + \varphi_t$$

$$\begin{cases} E_i + E_r = E_t \\ H_i \cos\theta_i - H_r \cos\theta_r = H_t \cos\theta_t \end{cases}$$

$$\begin{cases} E_i + E_r = E_t \\ \eta_1 E_i \cos\theta_i - \eta_1 E_r \cos\theta_r = \eta_2 E_t \cos\theta_t \end{cases}$$

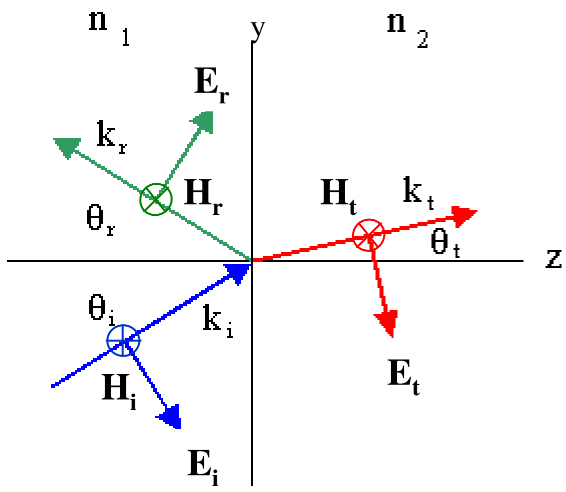
Onda TE



$$\begin{cases} E_i + E_r = E_t \\ \eta_1 E_i \cos\theta_i - \eta_1 E_r \cos\theta_r = \eta_2 E_t \cos\theta_t \end{cases}$$

$$\begin{cases} E_r = \dfrac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \end{cases} \longrightarrow \eta_i = \sqrt{\frac{\varepsilon_i}{\mu_i}} = \frac{n_i}{\mu_i} = \frac{\varepsilon_i}{n_i}$$

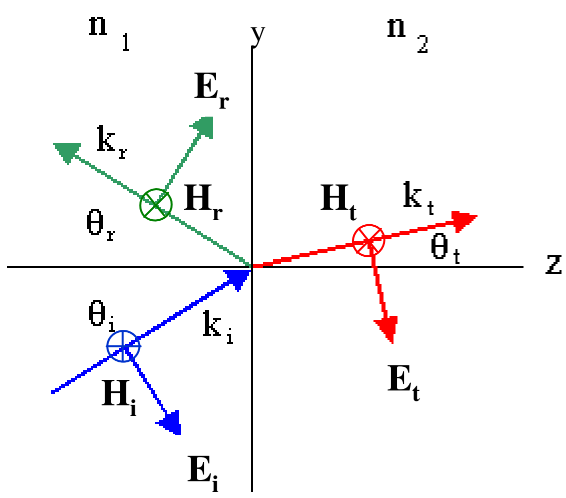
Onda TM



$$\begin{cases} \eta_1 E_i + \eta_1 E_r = \eta_2 E_t \\ E_i \cos\theta_i - E_r \cos\theta_i = E_t \cos\theta_t \end{cases}$$

$$\begin{cases} E_r = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \end{cases} \Longrightarrow \eta_i = \sqrt{\frac{\varepsilon_i}{\mu_i}} = \frac{n_i}{\mu_i} = \frac{\varepsilon_i}{n_i}$$

Onda TM

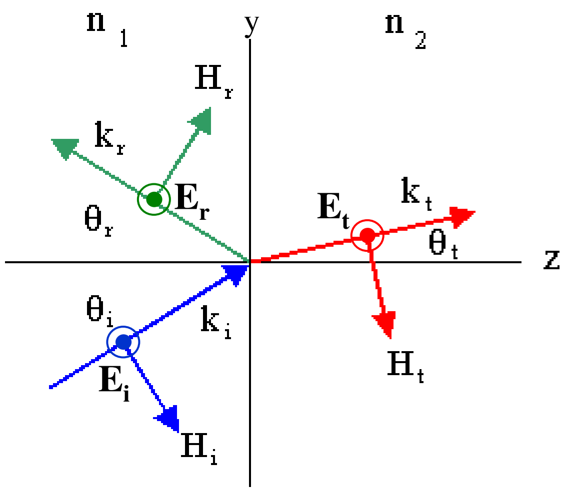


$$\vec{k}_i \cdot \vec{r} - \omega_i t = \vec{k}_r \cdot \vec{r} - \omega_r t + \varphi_r = \vec{k}_t \cdot \vec{r} - \omega_t t + \varphi_t$$

$$\begin{cases} H_i + H_r = H_t \\ E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t \end{cases}$$

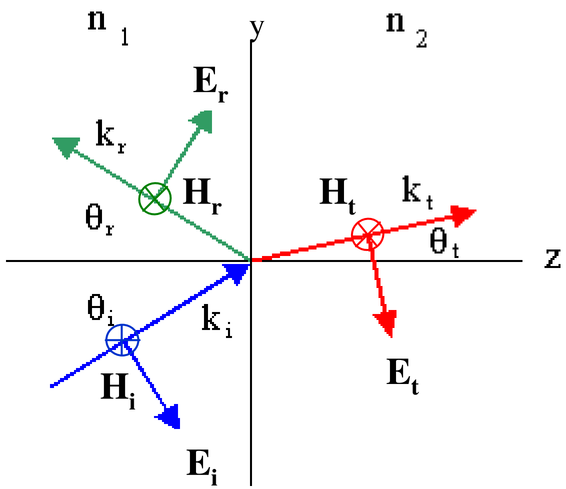
$$\begin{cases} \eta_1 E_i + \eta_1 E_r = \eta_2 E_t \\ E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t \end{cases}$$

TE wave



$$\begin{cases} E_r = \dfrac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \end{cases}$$

TM wave



$$\begin{cases} E_r = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \end{cases}$$

Normal incidence

TE = TM

$$\begin{cases} E_r = \dfrac{\eta_1 - \eta_2}{\eta_1 + \eta_2} E_i \\ E_t = \dfrac{2\eta_1}{\eta_1 + \eta_2} E_i \end{cases}$$

No reflection if same impedance (not same n!!)

Pure dielectric μ_i=1

$$n_i = \sqrt{\varepsilon_i \mu_i} = \sqrt{\varepsilon_i} \qquad \eta_i = \sqrt{\frac{\varepsilon_i}{\mu_i}} = n_i$$

TE
$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases} \Longrightarrow \begin{cases} E_r = \dfrac{\sin(\theta_t - \theta_i)}{\sin(\theta_i + \theta_t)} E_i \\ E_t = \dfrac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)} E_i \end{cases}$$

TM
$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases} \Longrightarrow \begin{cases} E_r = \dfrac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} E_i \\ E_t = \dfrac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} E_i \end{cases}$$

$$\frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}=\frac{\sin(\theta_i-\theta_t)\cos(\theta_i+\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}=$$

$$=\frac{(\sin\theta_i\cos\theta_t-\cos\theta_i\sin\theta_t)(\cos\theta_i\cos\theta_t-\sin\theta_i\sin\theta_t)}{(\sin\theta_i\cos\theta_t+\cos\theta_i\sin\theta_t)(\cos\theta_i\cos\theta_t+\sin\theta_i\sin\theta_t)}=$$

$$=\frac{(\sin\theta_i\cos\theta_i\cos^2\theta_t-\sin^2\theta_i\sin\theta_t\cos\theta_t-\cos^2\theta_i\sin\theta_t\cos\theta_t+\cos\theta_i\sin\theta_i\sin^2\theta_t)}{(\sin\theta_i\cos\theta_i\cos^2\theta_t+\sin^2\theta_i\sin\theta_t\cos\theta_t+\cos^2\theta_i\sin\theta_t\cos\theta_t+\cos\theta_i\sin\theta_i\sin^2\theta_t)}=$$

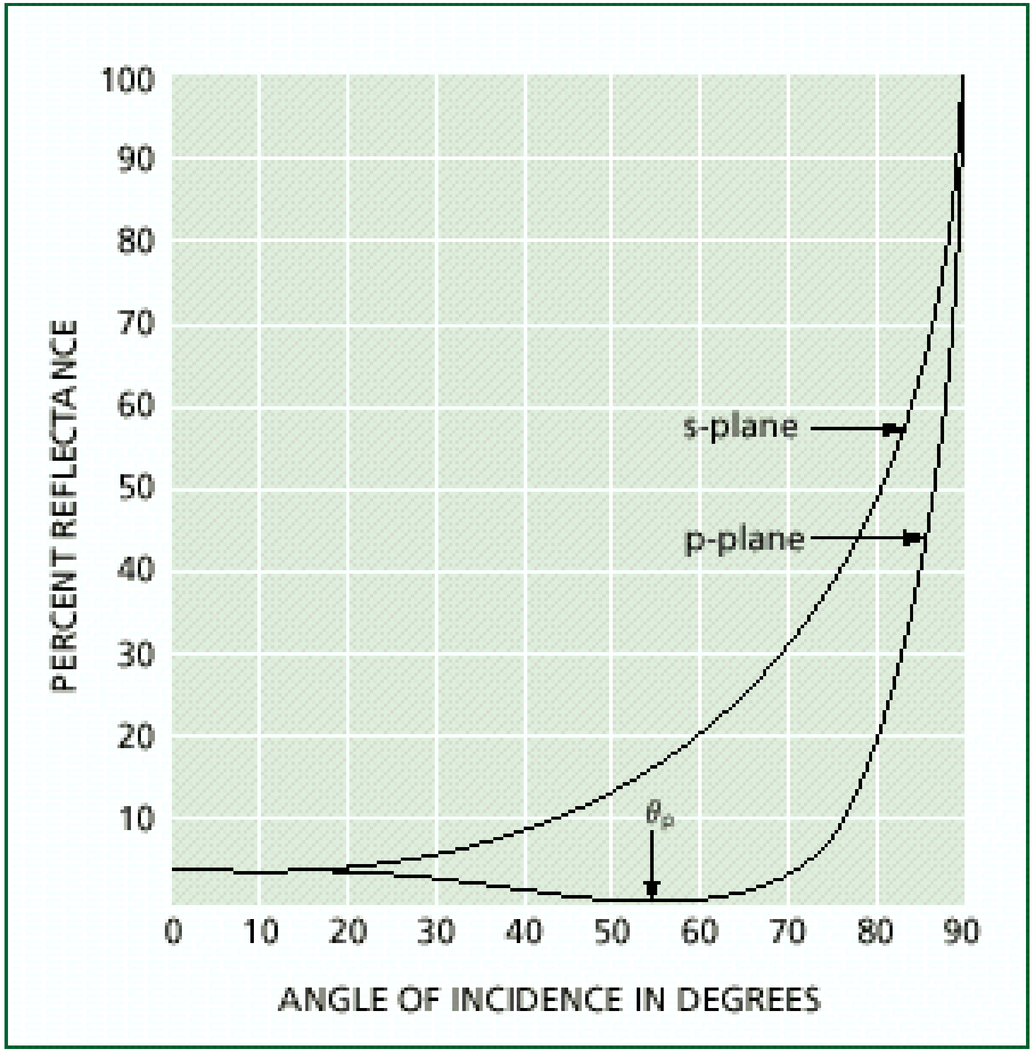
$$=\frac{(\sin\theta_i\cos\theta_i-\sin\theta_t\cos\theta_t)}{(\sin\theta_i\cos\theta_i+\sin\theta_t\cos\theta_t)}$$

$$\frac{(\sin\theta_i\cos\theta_t)}{(\sin\theta_i\cos\theta_i+\sin\theta_t\cos\theta_t)}=\frac{(\sin\theta_i\cos\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

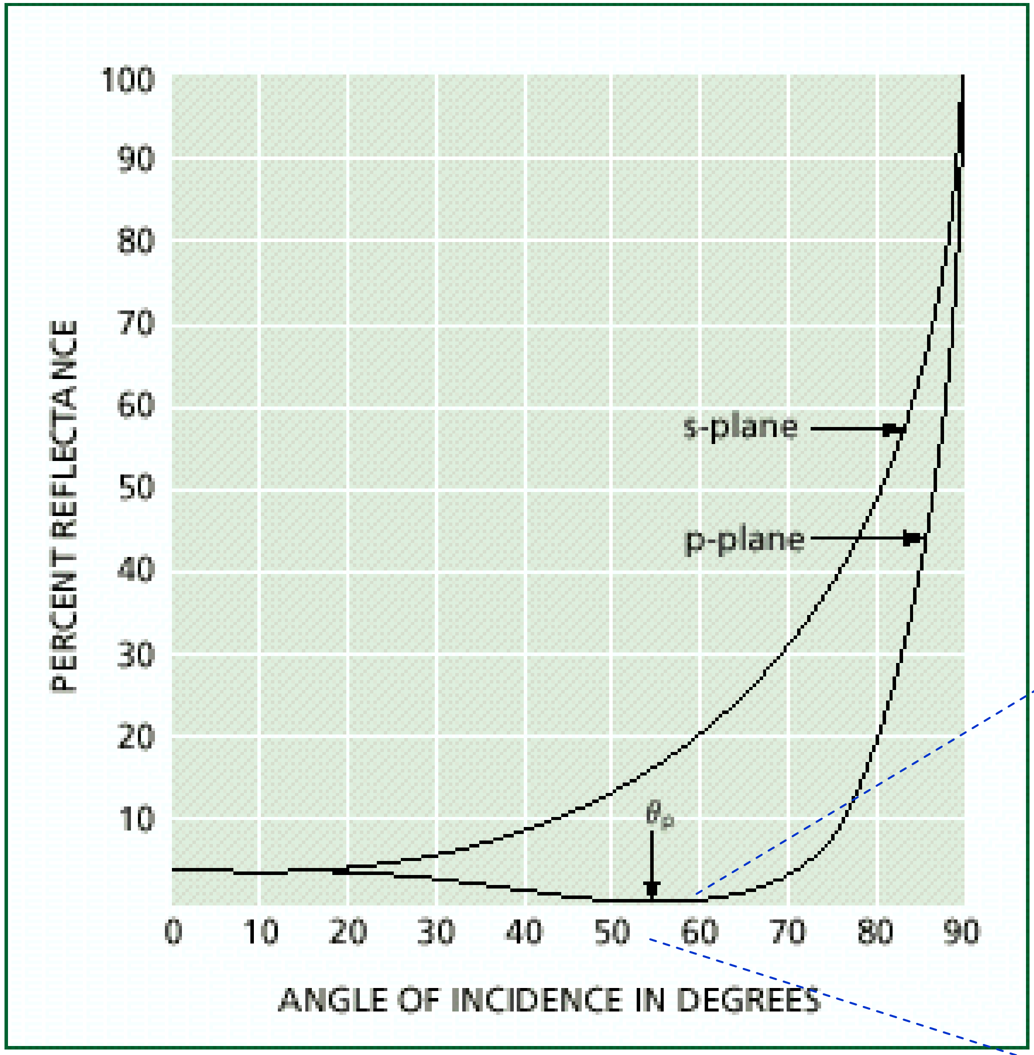
Aria-Vetro

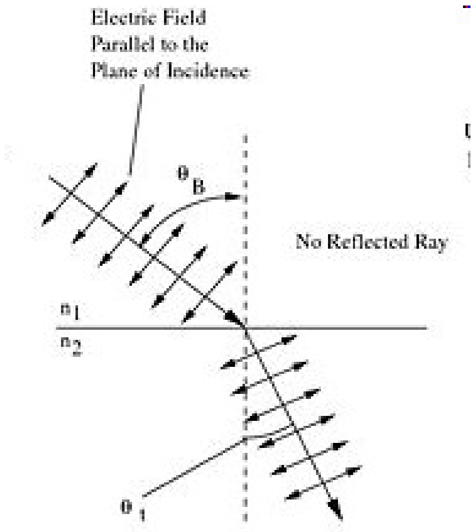
Onda s=Onda TE

Onda p=Onda TM

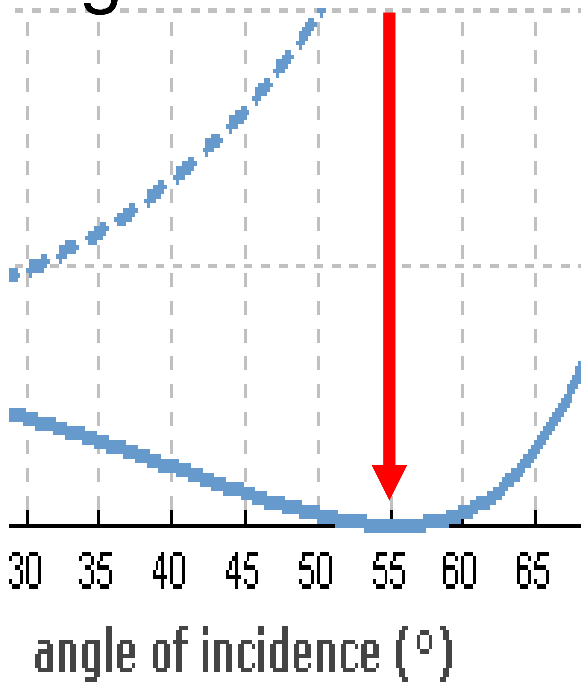


Aria-Vetro

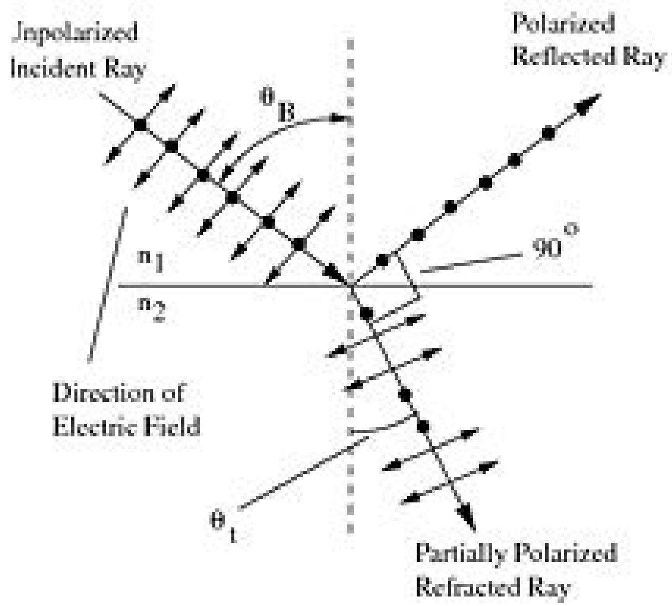




Angolo di Brewster



Angolo di Brewster (onda TM no riflessione)



$$E_r = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} E_i$$

$$\theta_i + \theta_t = \frac{\pi}{2}$$

Luce riflessa è polarizzata

Luce riflessa è polarizzata

Foto senza filtro polarizzatore

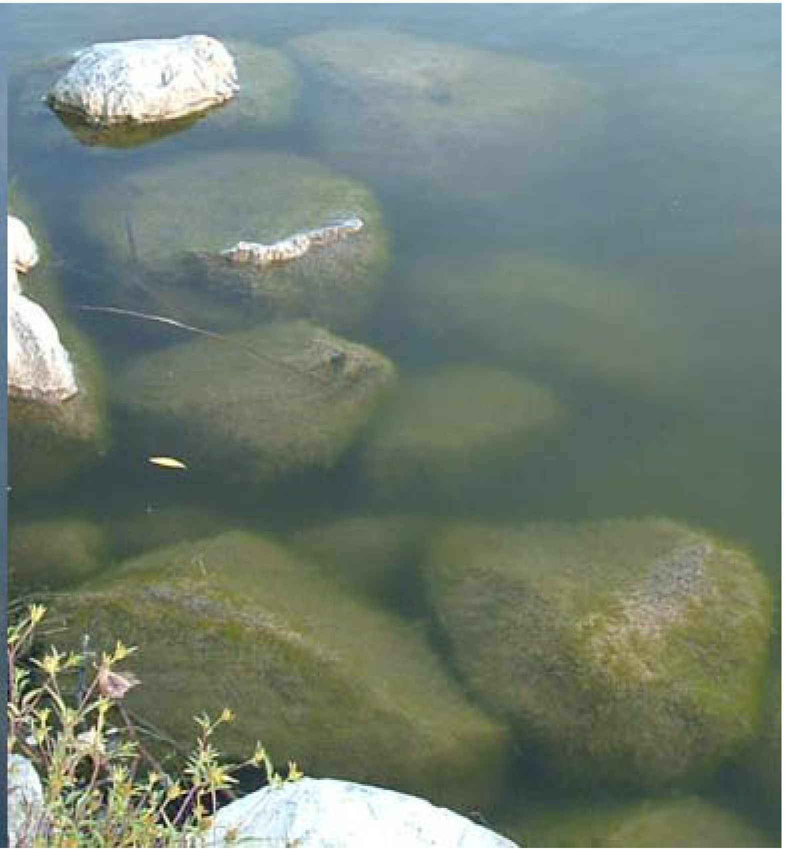
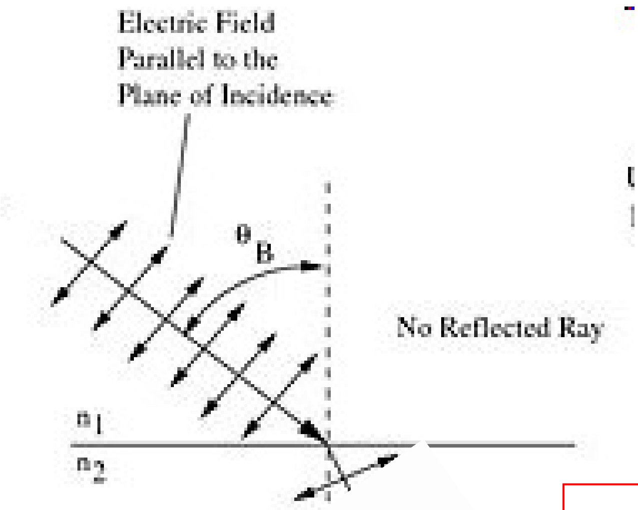
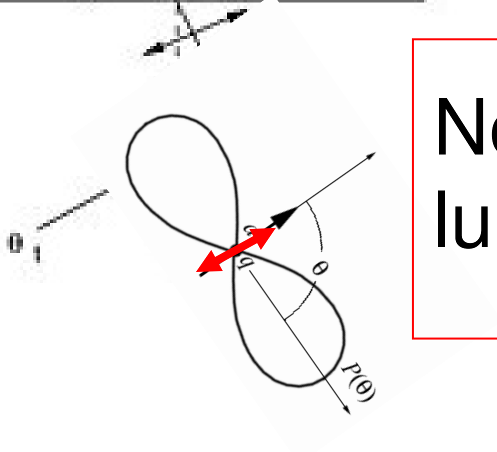


Foto senza filtro polarizzatore
che taglia la luce riflessa

Angolo Brewster e teorema estinzione



$$\vec{S} = \frac{p^2 \omega^4 \sin^2 \theta}{32\pi^2 \varepsilon_o c^3 r^2} \hat{r}$$



Non c'è emissione
lungo l'asse del dipolo

TE wave

$$\begin{cases} E_r = \dfrac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \end{cases}$$

TM wave

$$\begin{cases} E_r = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \end{cases}$$

Pure dielectric μ_i=1 η_i=n_i

$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases}$$

$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases}$$

TE wave

$$\begin{cases} E_r = \dfrac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \end{cases}$$

TM wave

$$\begin{cases} E_r = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \\ E_t = \dfrac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} E_i \end{cases}$$

Pure dielectric μ_i=1 η_i=n_i

$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases}$$

$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases}$$

Pure magnetic ε_i=1 η_i=1/n_i

$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2n_2 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases}$$

$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2n_2 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases}$$

Pure magnetic $\varepsilon_i=1$

$$n_i = \sqrt{\varepsilon_i \mu_i} = \sqrt{\mu_i} \qquad \eta_i = \sqrt{\frac{\varepsilon_i}{\mu_i}} = \frac{1}{n_i}$$

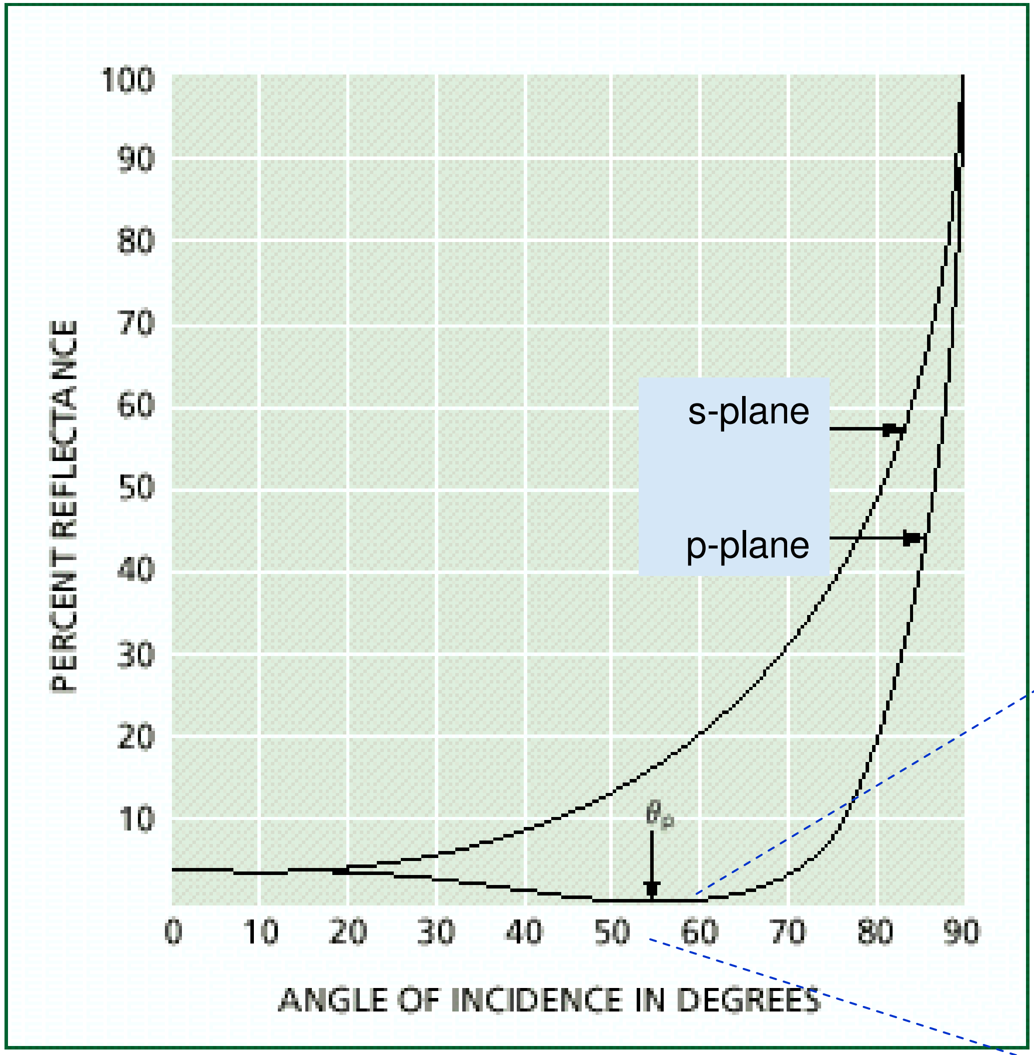
TM

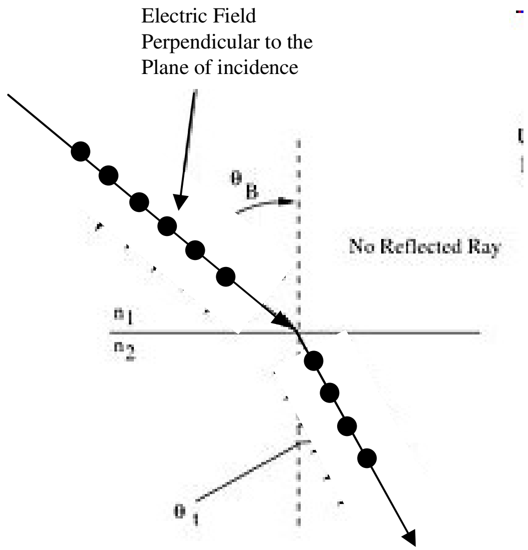
$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2n_2 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases} \longrightarrow \begin{cases} E_r = \dfrac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} E_i \\ E_t = \dfrac{2\cos\theta_i \sin\theta_i}{\sin(\theta_i + \theta_t)} E_i \end{cases}$$

TE

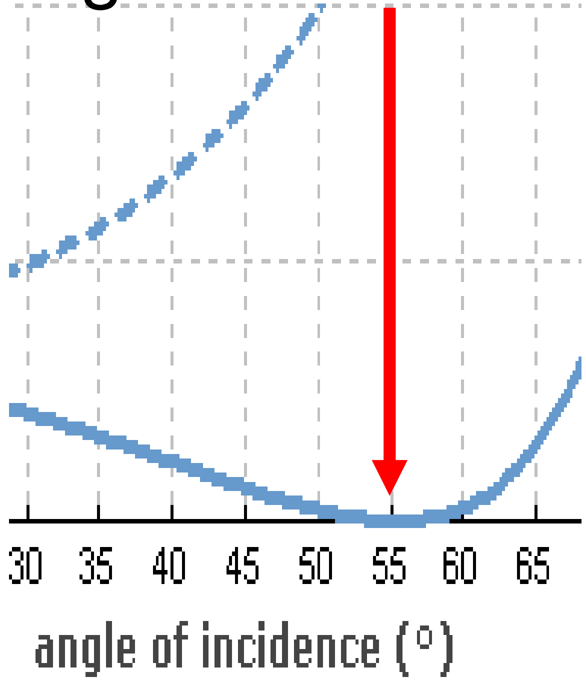
$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2n_2 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases} \longrightarrow \begin{cases} E_r = \dfrac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} E_i \\ E_t = \dfrac{2\cos\theta_i \sin\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} E_i \end{cases}$$

Aria-*Vetro magnetico* $\varepsilon_r=1; \mu_r=(1.5)^2$

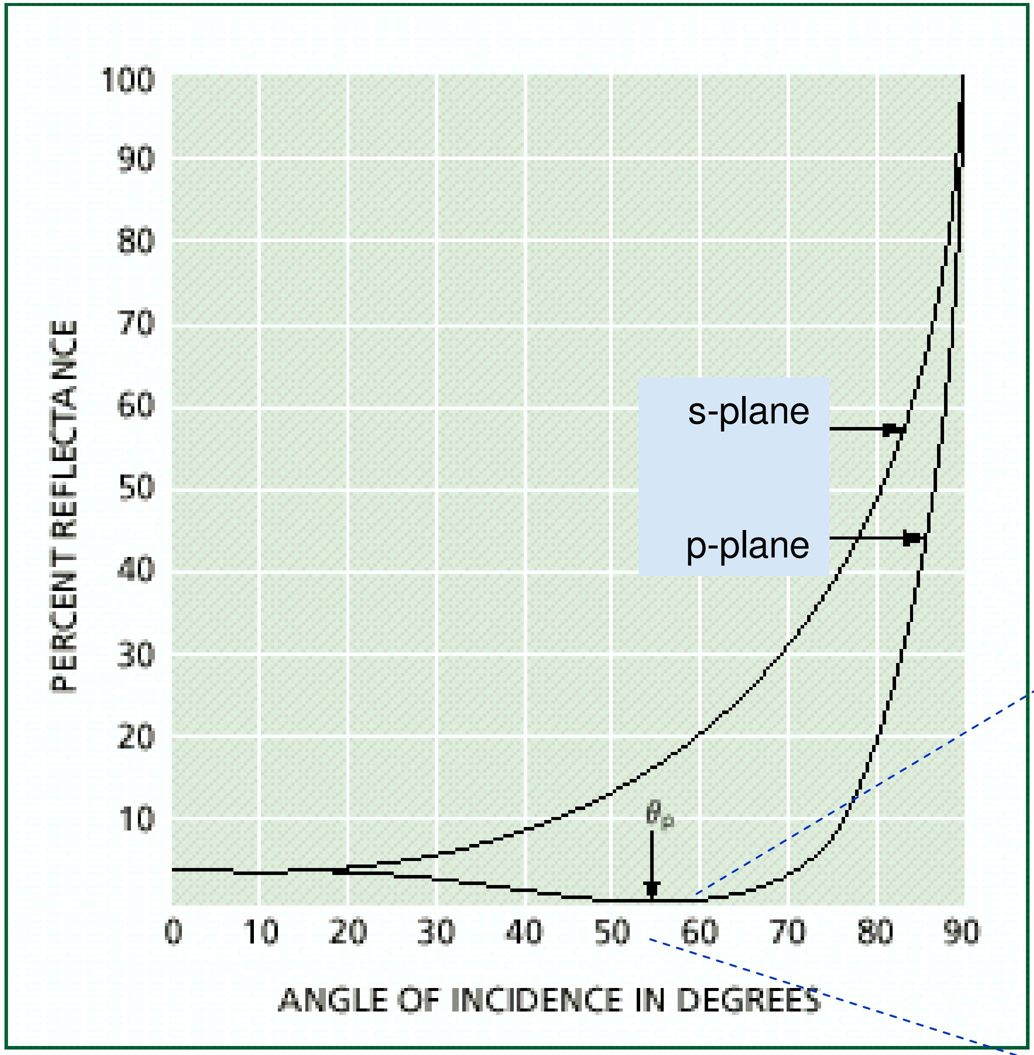


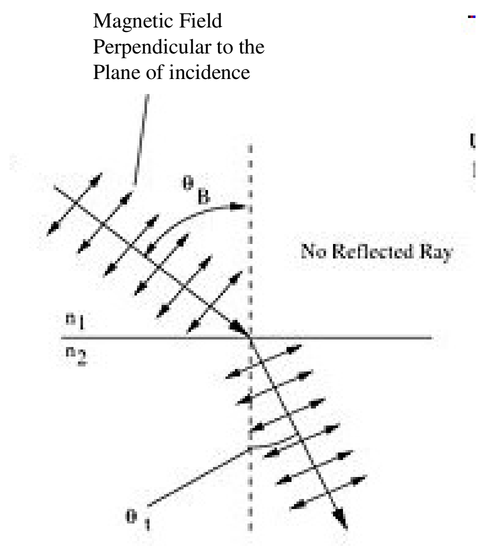


Angolo di Brewster

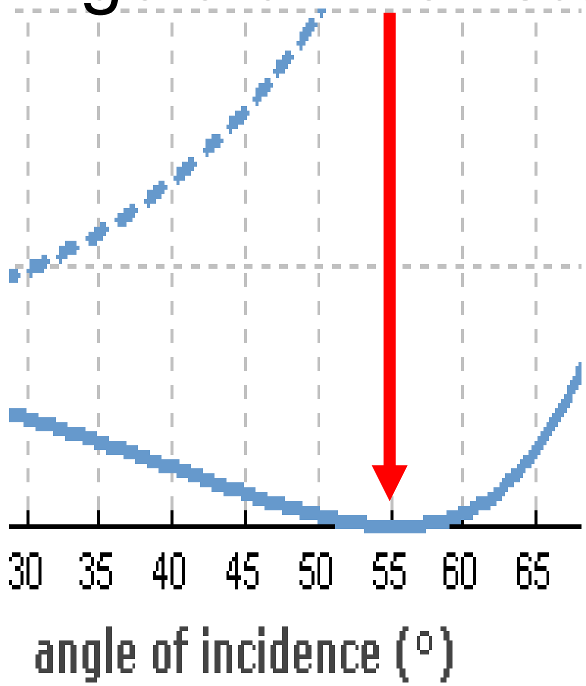


Aria-*Vetro magnetico ε_r=1; μ_r=(1.5)^2*

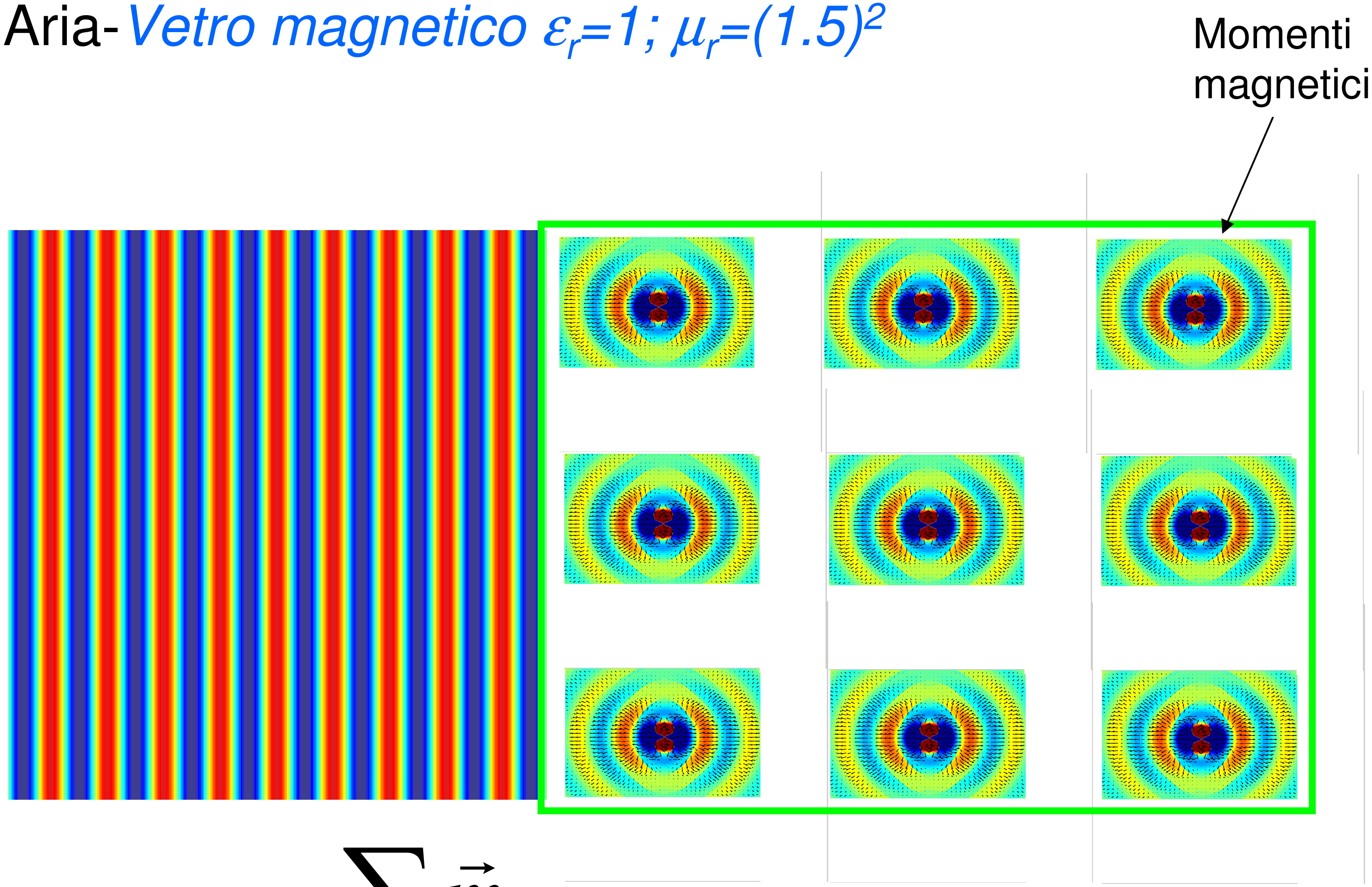




Angolo di Brewster



Aria-*Vetro magnetico* $\varepsilon_r=1;\ \mu_r=(1.5)^2$



$$\vec{M} = \frac{\sum_i \vec{m}_i}{\Delta V}$$

$$\vec{m}_i = \alpha \vec{B}_i$$

Left handed material

SOVIET PHYSICS USPEKHI VOLUME 10, NUMBER 4 JANUARY-FEBRUARY 1968

538.30

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P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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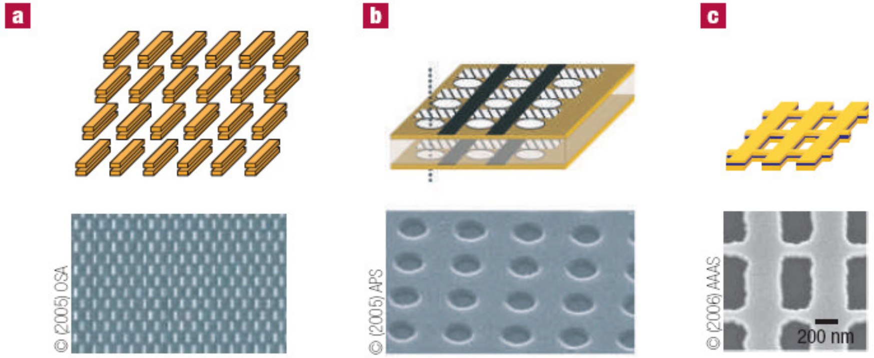
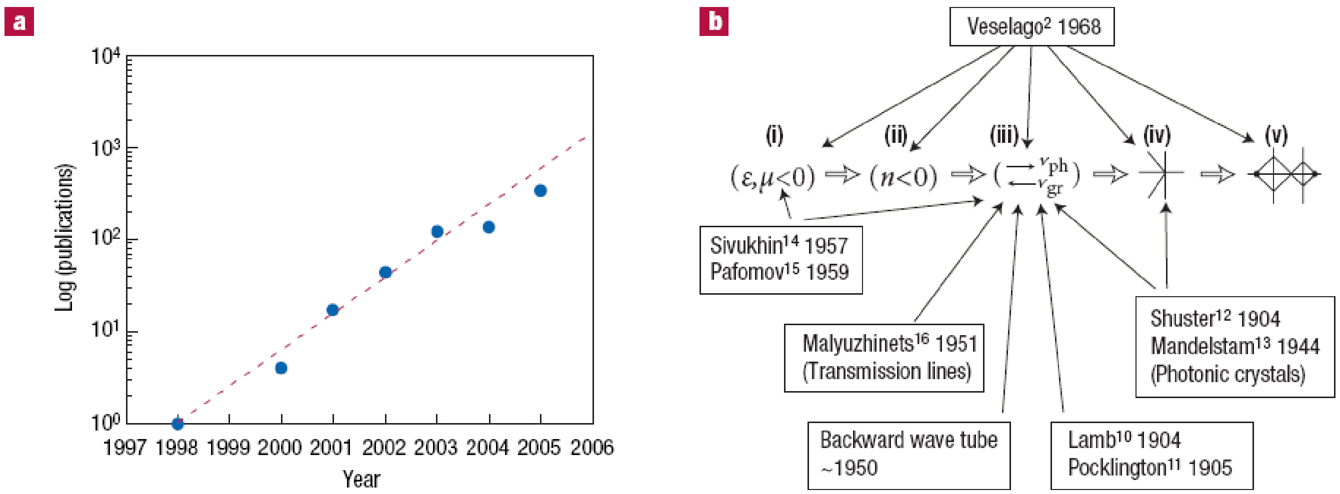




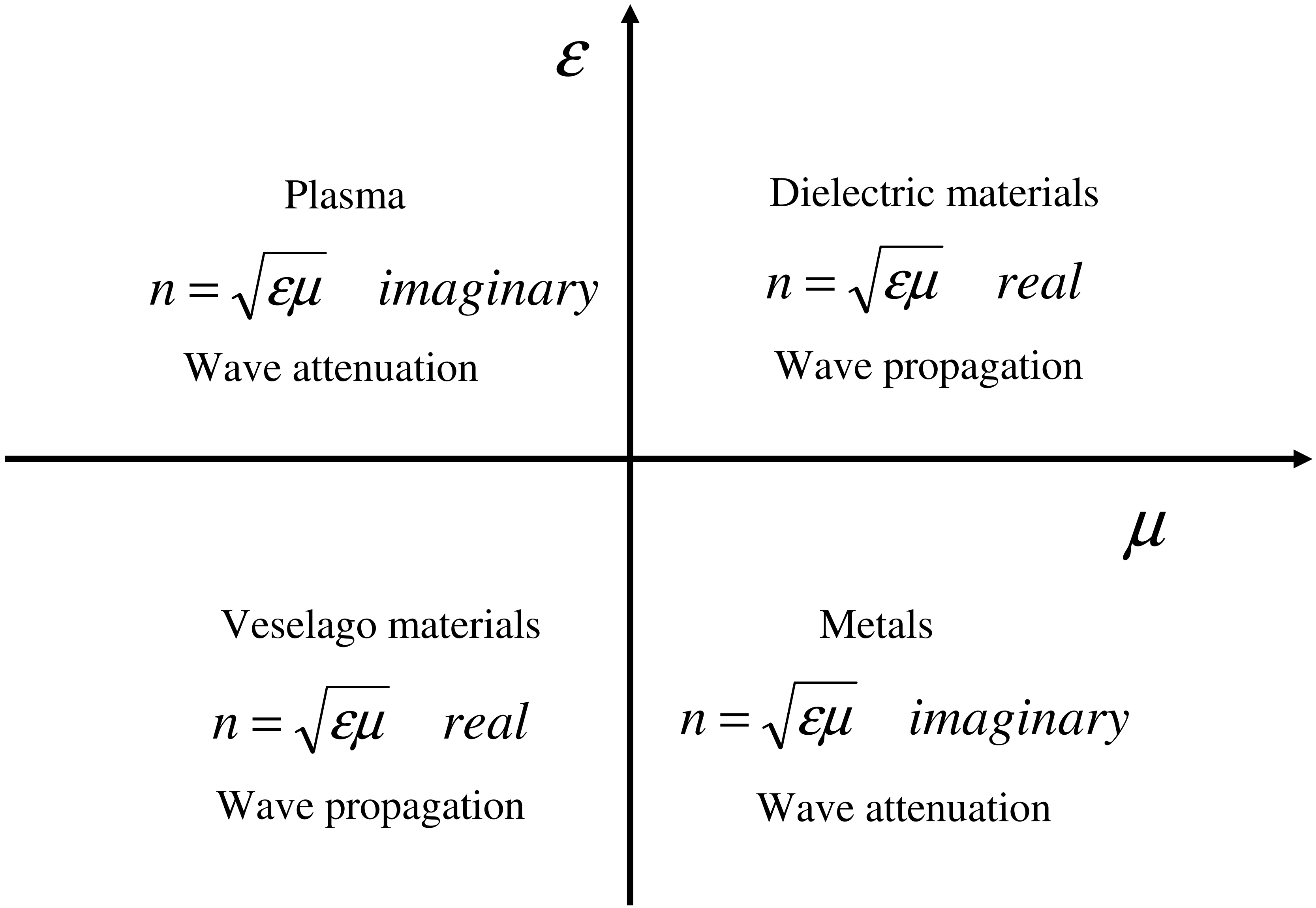
Waves equations

$$\nabla^2 \vec{E} = \mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2} = -k^2 \vec{E}$$

$$k^2 = \frac{\omega^2 n^2}{c^2} \qquad n = \sqrt{\boxed{\varepsilon\mu}}$$

$$\vec{k} \times \vec{E} = \omega \mu_o \boxed{\mu} \vec{H} \qquad \vec{k} \times \vec{H} = \omega \varepsilon_o \boxed{\varepsilon} \vec{E}$$

$$E_r = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} E_i \qquad \eta = \sqrt{\boxed{\frac{\varepsilon}{\mu}}}$$

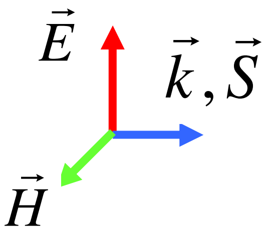

ε
Plasma
$n = \sqrt{\varepsilon\mu}$ imaginary
Wave attenuation
Dielectric materials
$n = \sqrt{\varepsilon\mu}$ real
Wave propagation
μ
Veselago materials
$n = \sqrt{\varepsilon\mu}$ real
Wave propagation
Metals
$n = \sqrt{\varepsilon\mu}$ imaginary
Wave attenuation

Dielectric materials | Veselago materials

$$\varepsilon>0 \qquad \mu>0$$ | $$\varepsilon<0 \qquad \mu<0$$

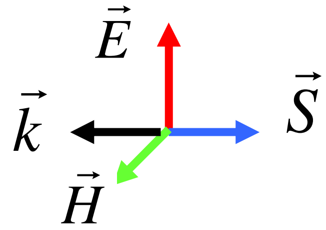
$$n=\sqrt{\varepsilon\mu} \quad \eta=\sqrt{\frac{\varepsilon}{\mu}} \quad real$$ | $$n=\sqrt{\varepsilon\mu} \quad \eta=\sqrt{\frac{\varepsilon}{\mu}} \quad real$$

Right handed materials



$$\vec{k}\times\vec{E}=\omega\mu_o\mu\vec{H}$$

Left handed materials



$$\vec{S}=\frac{S}{k}\vec{k}$$

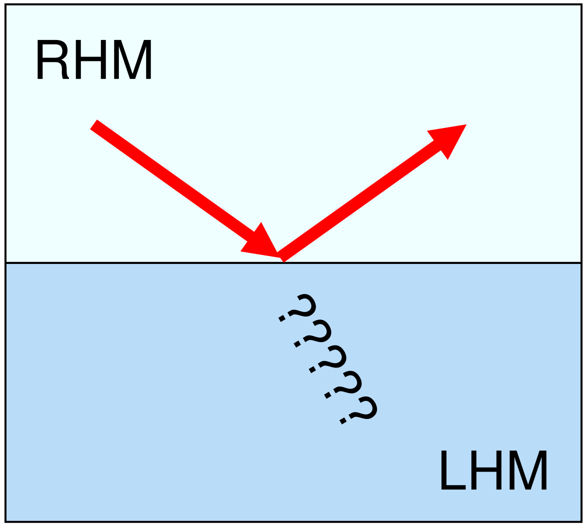
$$\vec{S}=\vec{E}\times\vec{H}$$

$$\vec{S}=-\frac{S}{k}\vec{k}$$

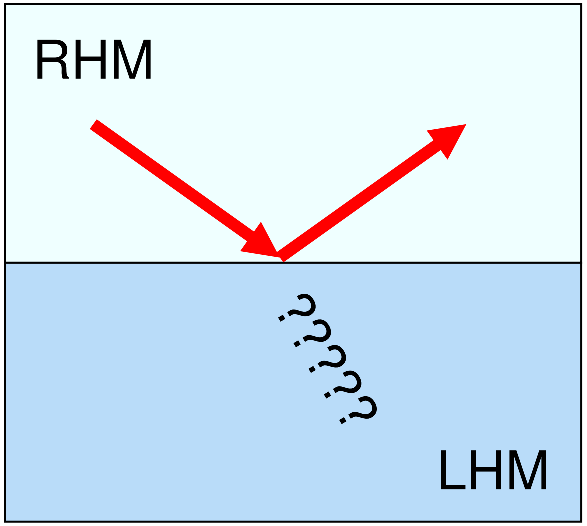
$$\vec{v}_p\cdot\vec{S}>0$$

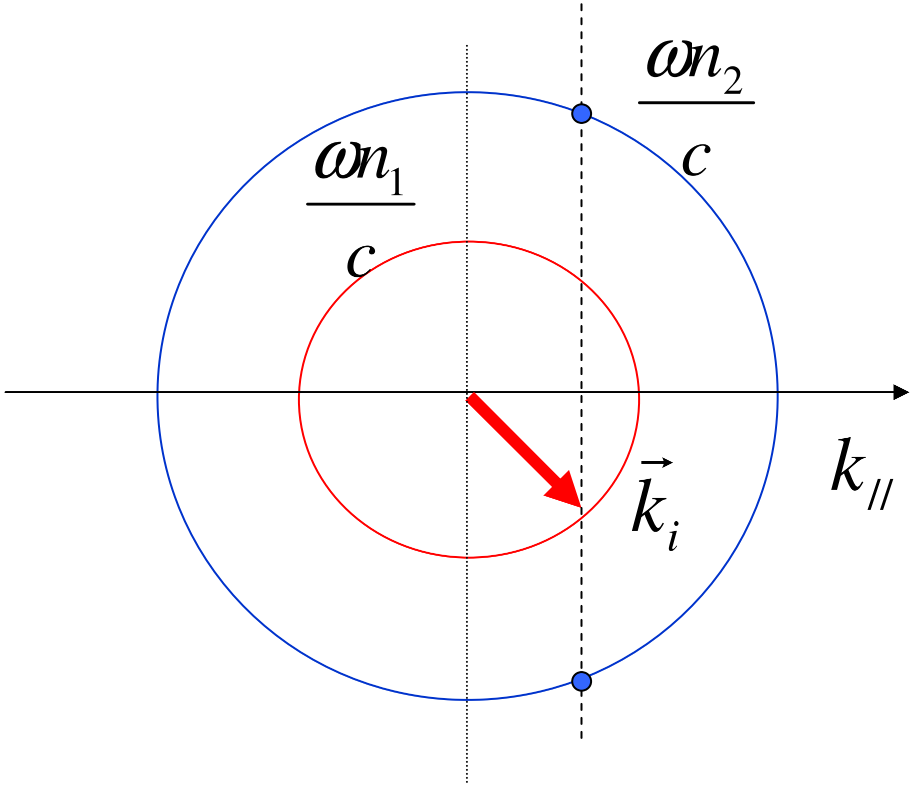
$$\vec{v}_p\cdot\vec{S}<0$$

Refraction from RHM to LHM

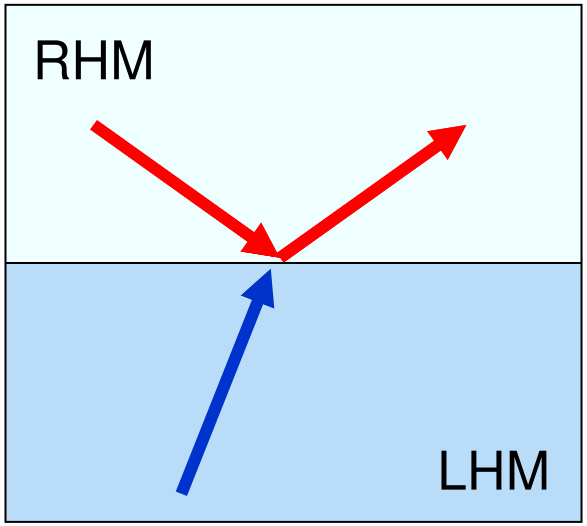


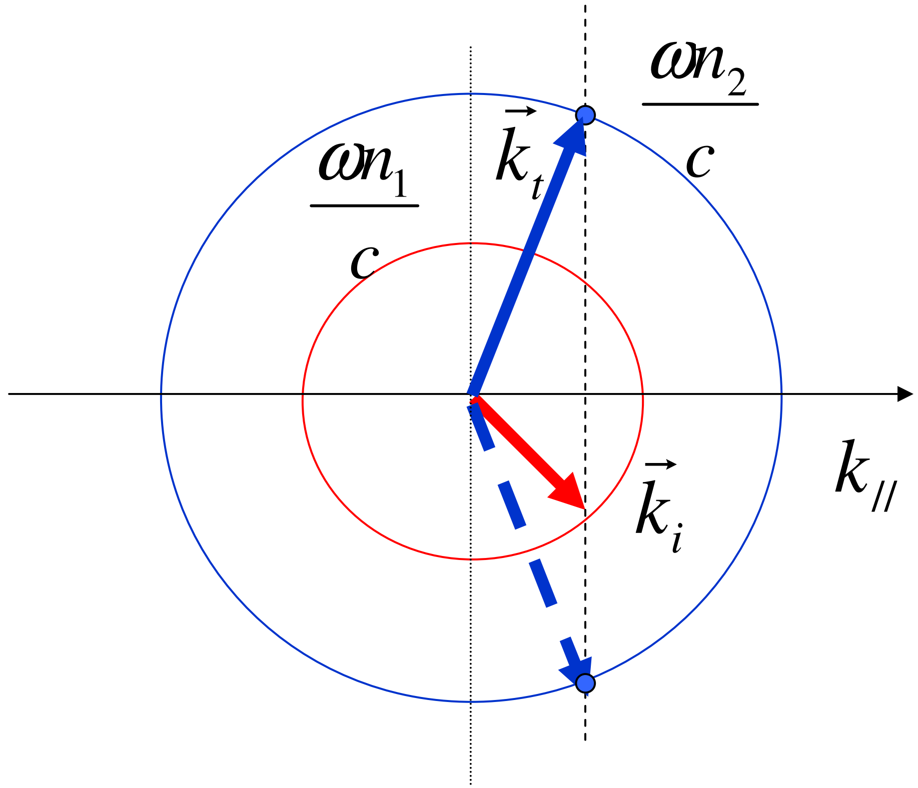
Refraction from RHM to LHM



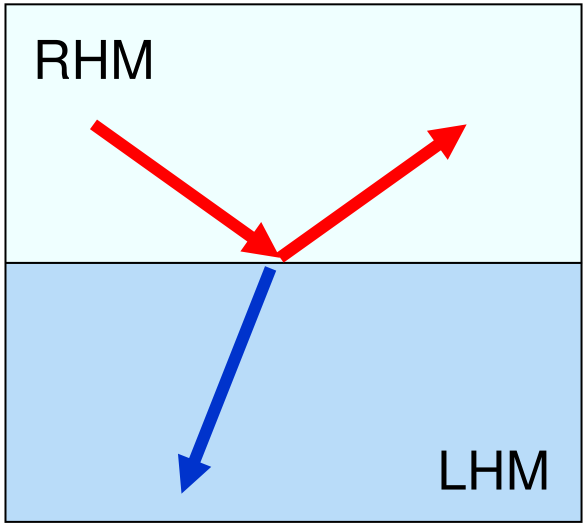


Refraction from RHM to LHM

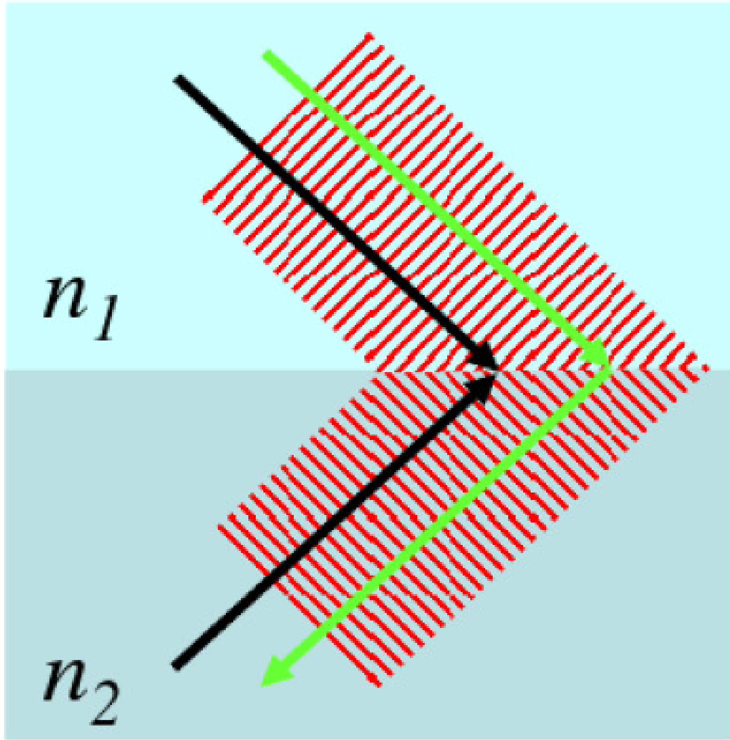




Refraction from RHM to LHM



$$\vec{S} = -\frac{S}{k}\vec{k}$$

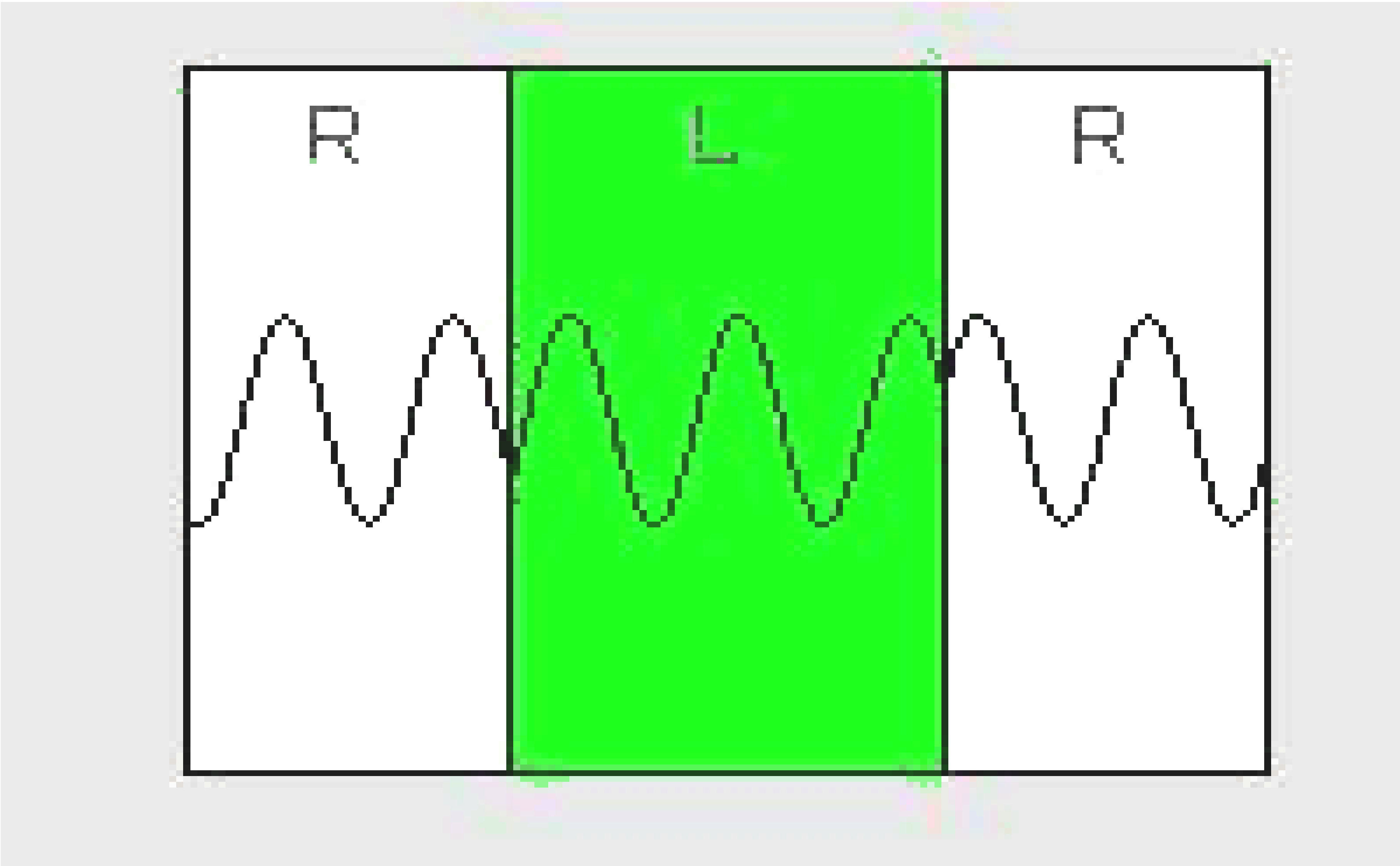


$$\sin\theta_i = \frac{n_2}{n_1}\sin\theta_t$$

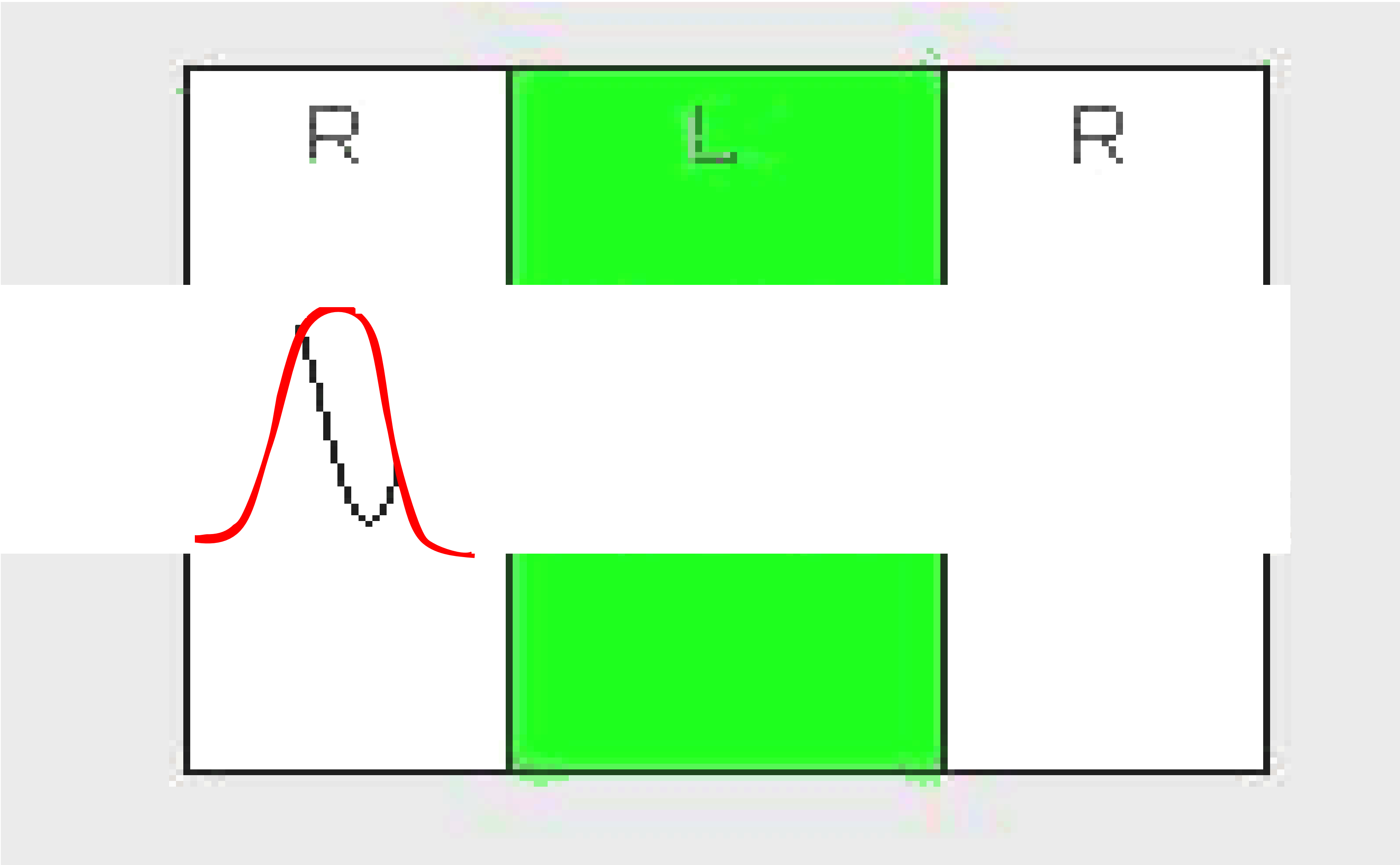
Energy refraction as if n<0

Anomalous propagation

$\vec{v}_p \cdot \vec{v}_g < 0$



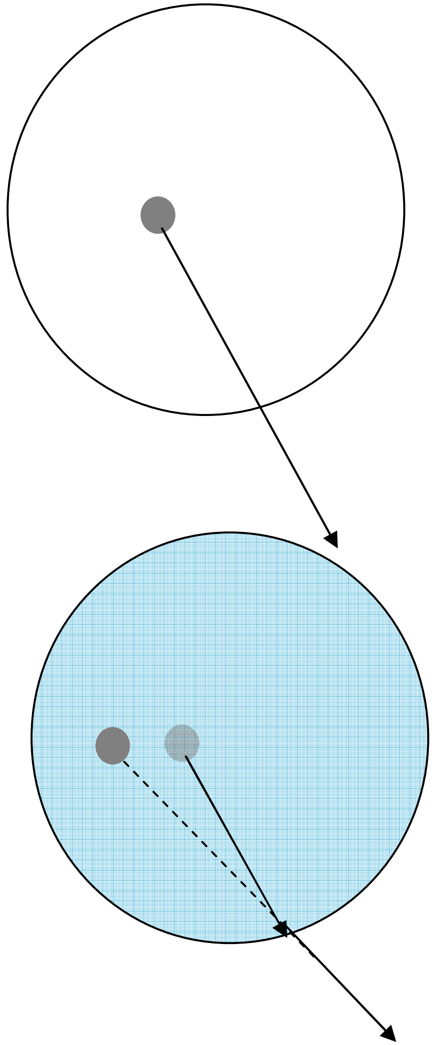
Propagazione "anormale" (n<0) $\vec{v}_p \cdot \vec{v}_g < 0$



A metal rod in an empty drinking glass

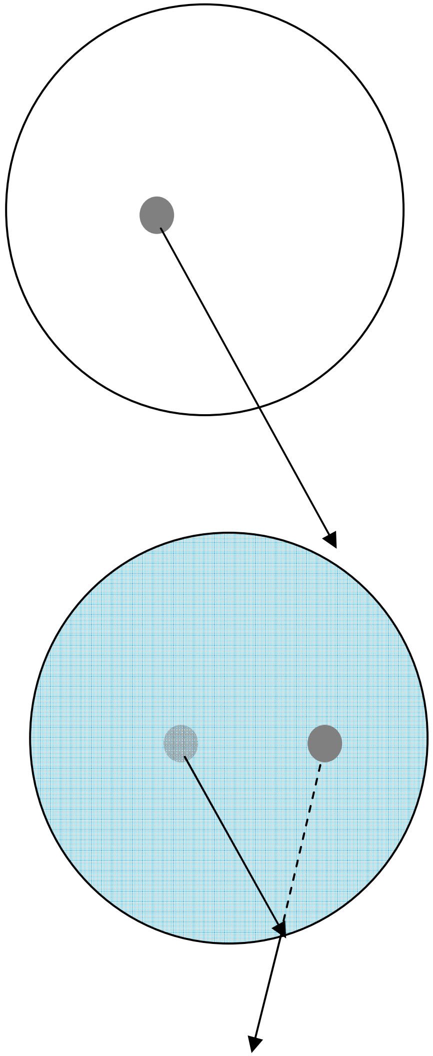
Fill the glass with blueberry juice (n = 1.3)…

Positive Refraction



Negative Refraction

A metal rod in an empty drinking glass



Now try the new recipe: negative refraction

A metal rod in an empty drinking glass

Fill the glass with blueberry juice (n = 1.3)…

Now try the new recipe: negative refraction

These pictures are NOT quoted from science fictions; they are computer simulations published in renowned peer-reviewed scientific journals!

G. Dolling, et al., "Photorealistic images of objects in effective negative-index materials," Opt. Express **14**, 1842-1849 (2006).

Superlens