

# Rifrazione fra vuoto e mezzo dielettrico

# Equazioni Maxwell: visione indice rifrazione

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \epsilon_r \frac{\partial \vec{E}}{\partial t}$$

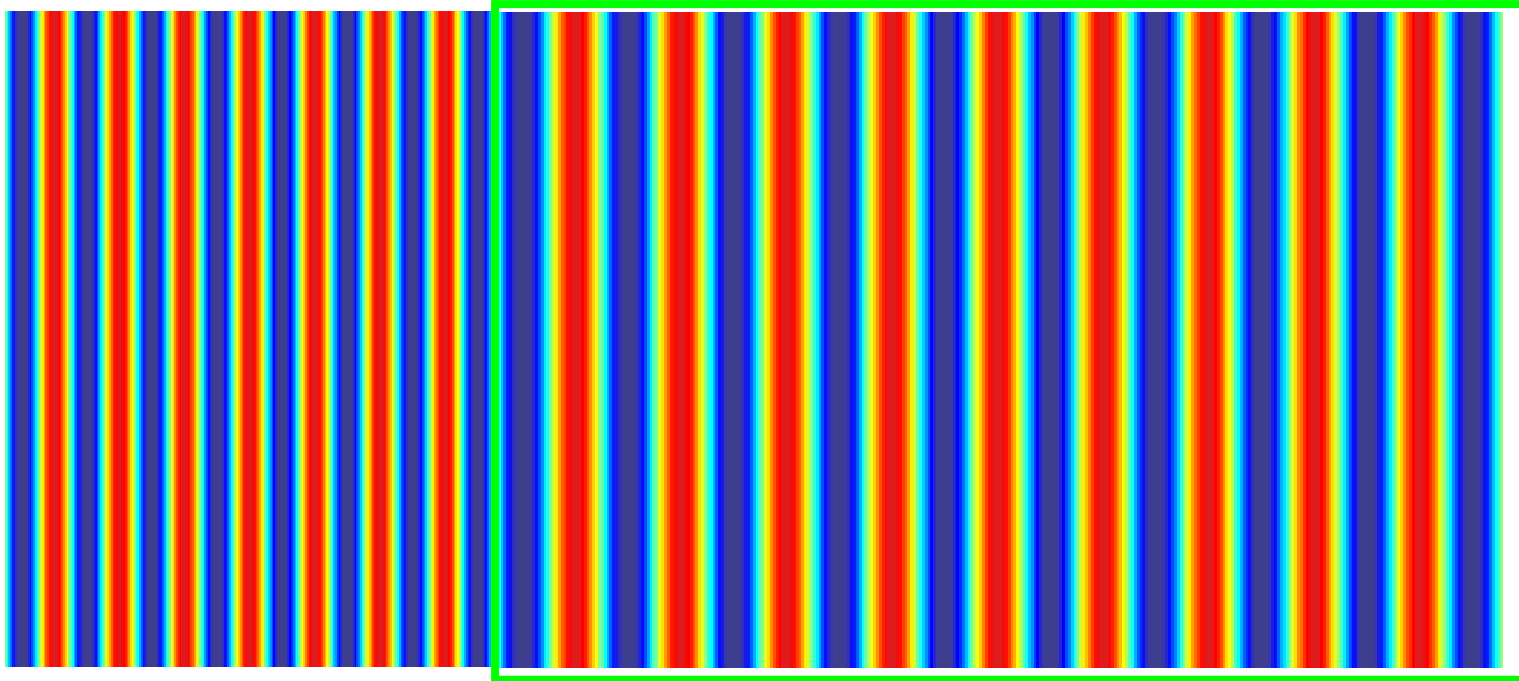
$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \epsilon_r \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Equazione  
delle onde

$$n^2 = \epsilon_r$$

# Rifrazione visione indice di rifrazione

$$n^2 = \epsilon_r$$



$$\vec{E}_o e^{ik_o z}$$

$$\vec{E} e^{ikz}$$

$$\vec{E} = \frac{2}{n+1} \vec{E}_o \quad \text{Fresnel}$$

$$k = nk_o$$

## Equazioni Maxwell: visione sorgenti

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

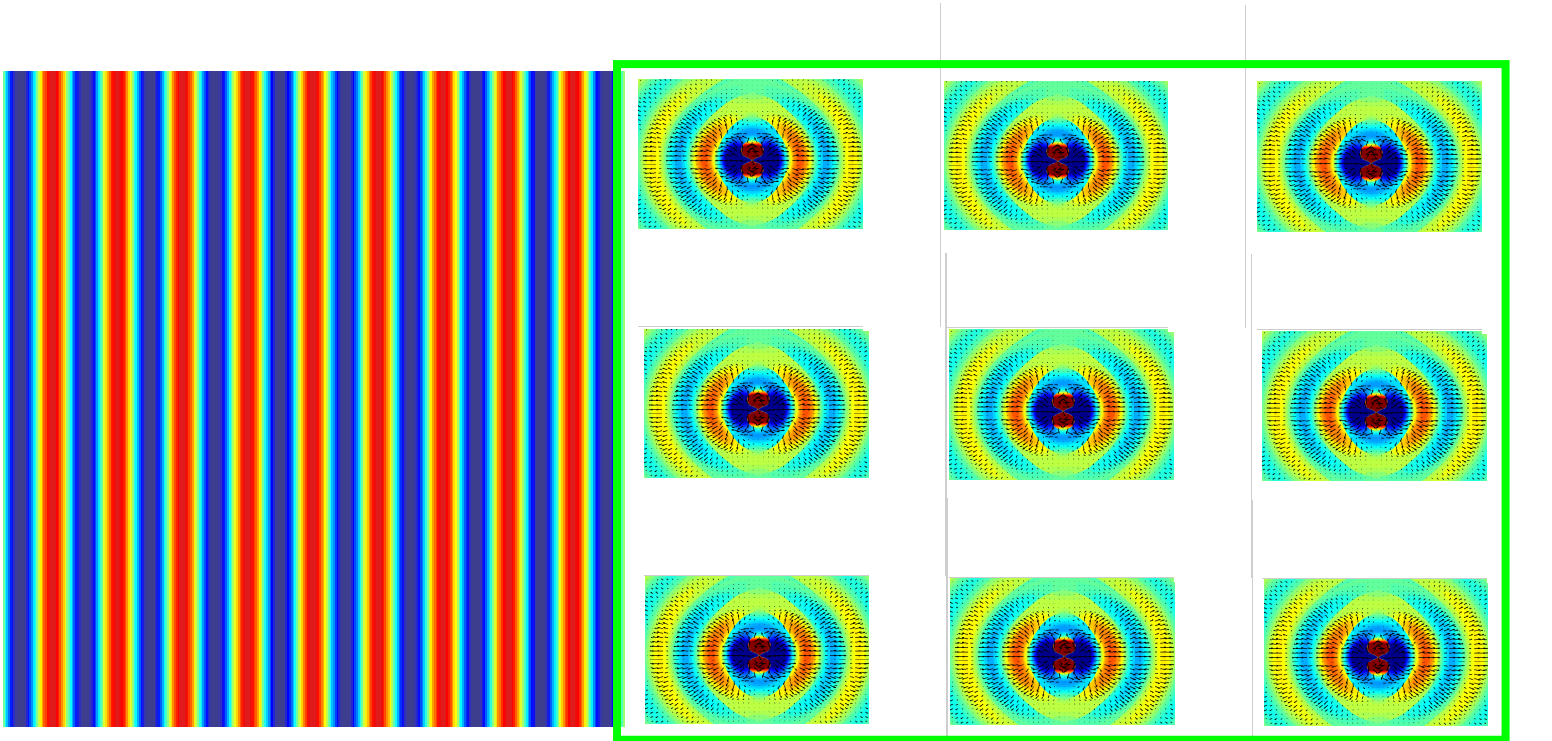
$$\vec{\nabla} \cdot \vec{E} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \frac{\partial \vec{P}}{\partial t}$$

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

Equazione  
delle onde  
con sorgenti

# Rifrazione visione sorgenti



$$\vec{P} = \frac{\sum_i \vec{p}_i}{\Delta V}$$

$$\vec{p}_i = \alpha \vec{E}_i$$

# Relazioni fenomenologiche

$$\frac{\rho\alpha}{3\varepsilon_0} = \frac{n^2 - 1}{n^2 + 2}$$

Lorentz-Lorenz

$$n^2 = \frac{1 + \frac{2\rho\alpha}{3\varepsilon_0}}{1 - \frac{\rho\alpha}{3\varepsilon_0}}$$

Clausius-Mossotti

# Teorema ottico o di estinzione Ewald-Oseen

Visione sorgenti

Equazioni Maxwell nel vuoto

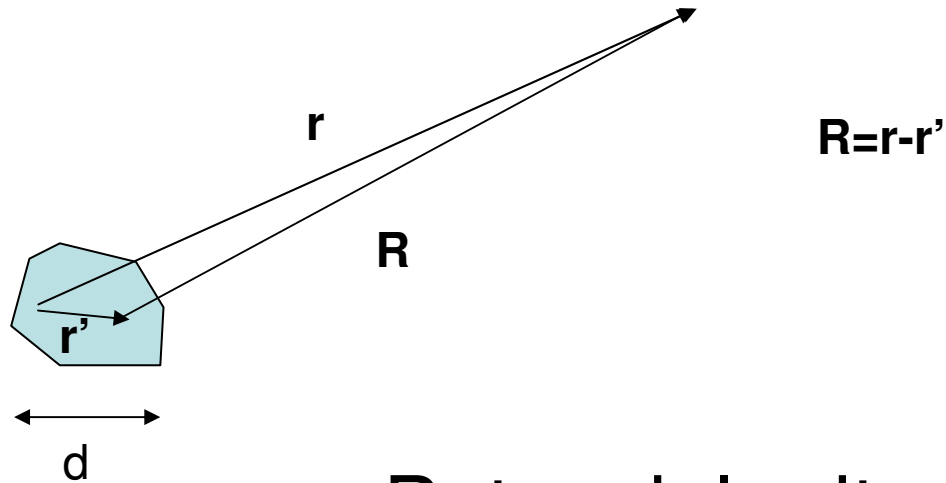
$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = i \frac{c^2}{\omega} \vec{\nabla} \times \vec{B} = i \frac{c^2}{\omega} \vec{\nabla} \times (\vec{\nabla} \times \vec{A})$$

Tutto è ricavabile a partire dal potenziale vettore



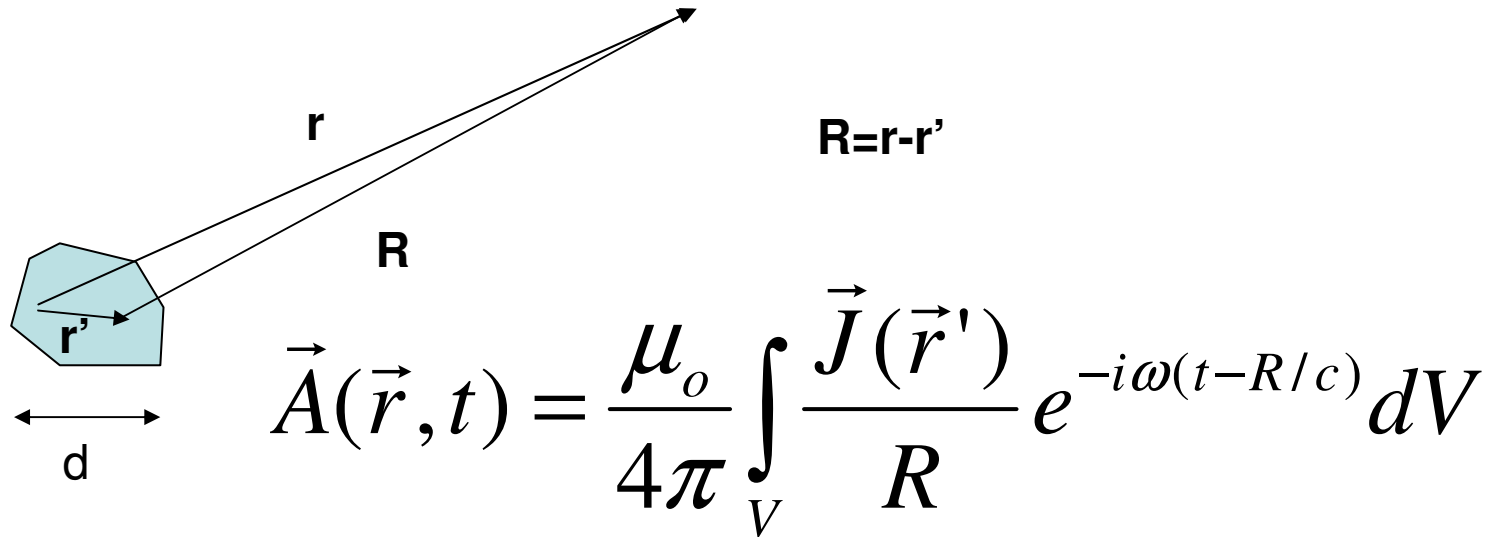
# Corrente localizzata variabile nel tempo



Potenziale ritardato

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t - R/c)}{R} dV$$

# Corrente localizzata variabile nel tempo

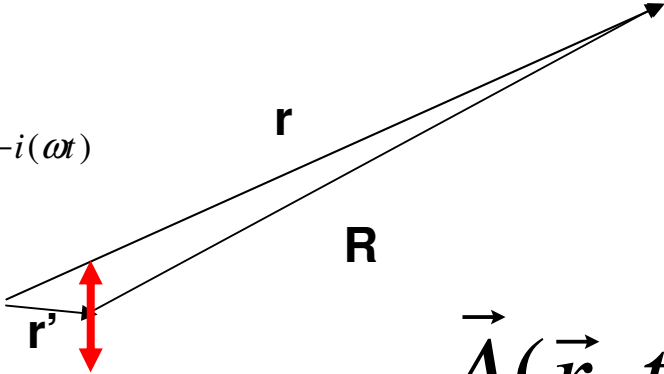


$d \ll \lambda$

$$\vec{A}(\vec{r}, t) = \frac{\mu_o}{4\pi} \frac{e^{i(k_o R - \omega t)}}{R} \int_V \vec{J}(\vec{r}') dV$$

$$= \frac{\mu_o}{4\pi} \frac{e^{i(k_o R - \omega t)}}{R} I_o \hat{e}$$

# Dipolo elettrico variabile nel tempo



$p(t) = p_o e^{-i(\omega t)}$

$\vec{A}(\vec{r}, t) = \frac{\mu_o}{4\pi} \frac{e^{i(k_o R - \omega t)}}{R} I_o \hat{e}$

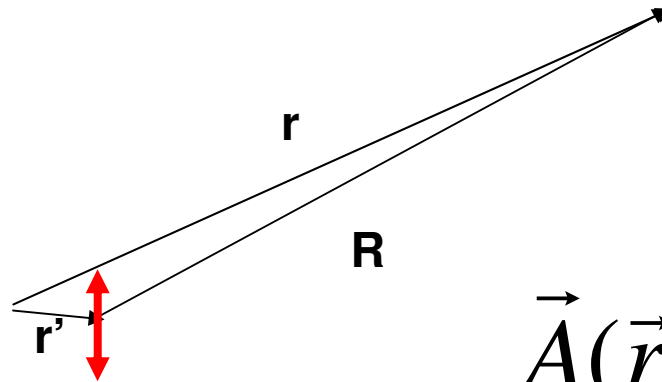
$R = r - r'$

$d \ll \lambda$

$$q(t) = \int I dt = \int I_o e^{-i(\omega t)} dt = \frac{I_o e^{-i(\omega t)}}{-i\omega}$$

$$p(t) = p_o e^{-i(\omega t)} = q(t) d \quad I_o = -i\omega p_o$$

# Dipolo elettrico variabile nel tempo



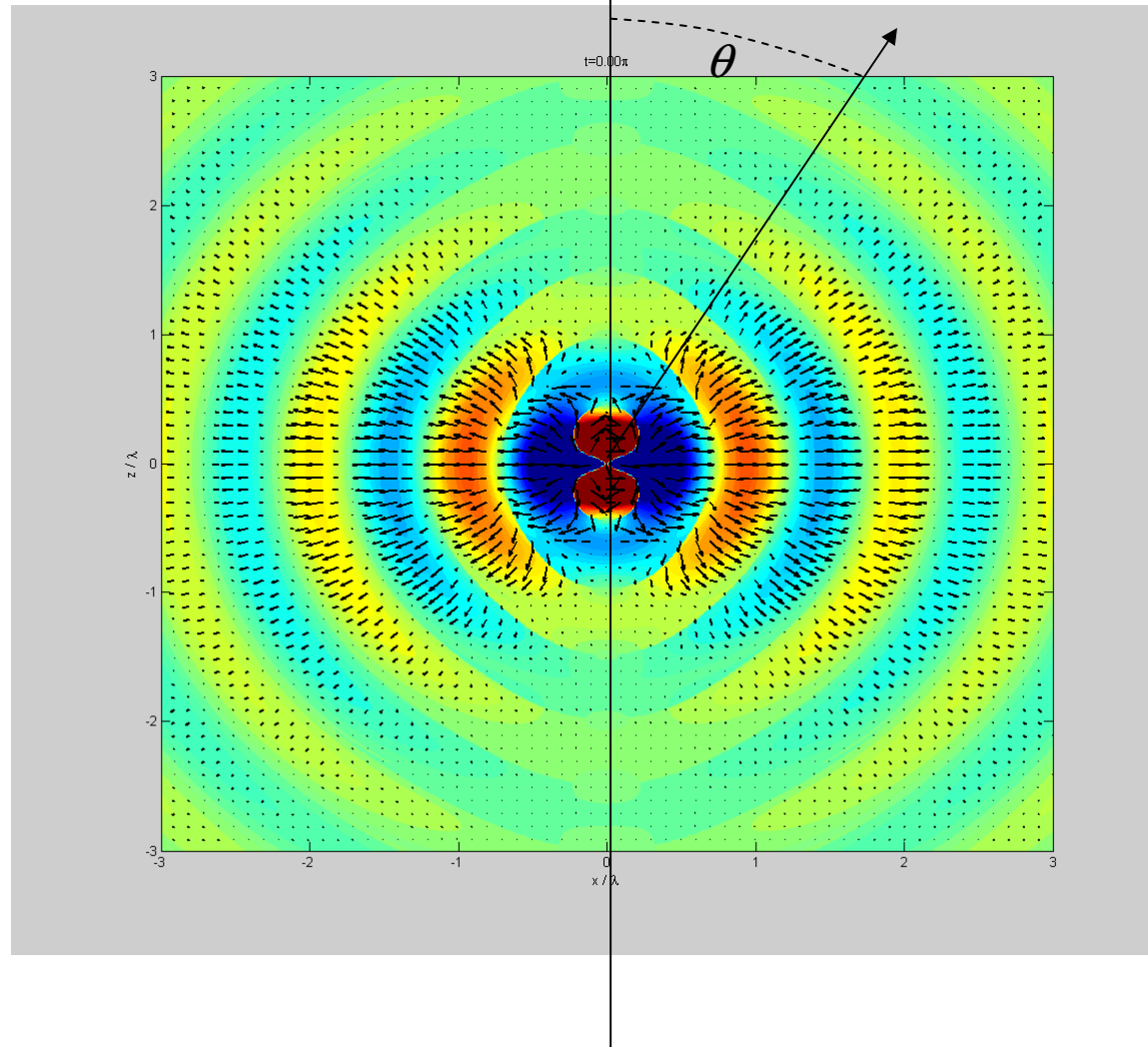
$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{e^{i(k_0 R - \omega t)}}{R} \dot{\vec{p}}$$

$d \ll \lambda$

$$\vec{E} = i \frac{c^2}{\omega} \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \begin{cases} E_r = \frac{2 \cos \theta}{4\pi \epsilon_0} \frac{\dot{p}}{cr} \\ E_\theta = \frac{\sin \theta}{4\pi \epsilon_0} \frac{\ddot{p}}{c^2 r} \\ E_\phi = 0 \end{cases}$$

Vettore di Poynting

$$\vec{S} = \frac{p^2 \omega^4 \sin^2 \theta}{32\pi^2 \epsilon_0 c^3 r^2} \hat{r}$$



Campo come sovrapposizione del contributo dei dipoli

$$\vec{E}_i(\vec{r}_i) = \vec{E}_o(\vec{r}_i) + \sum_{k \neq i} \vec{E}_k(\vec{r}_i)$$

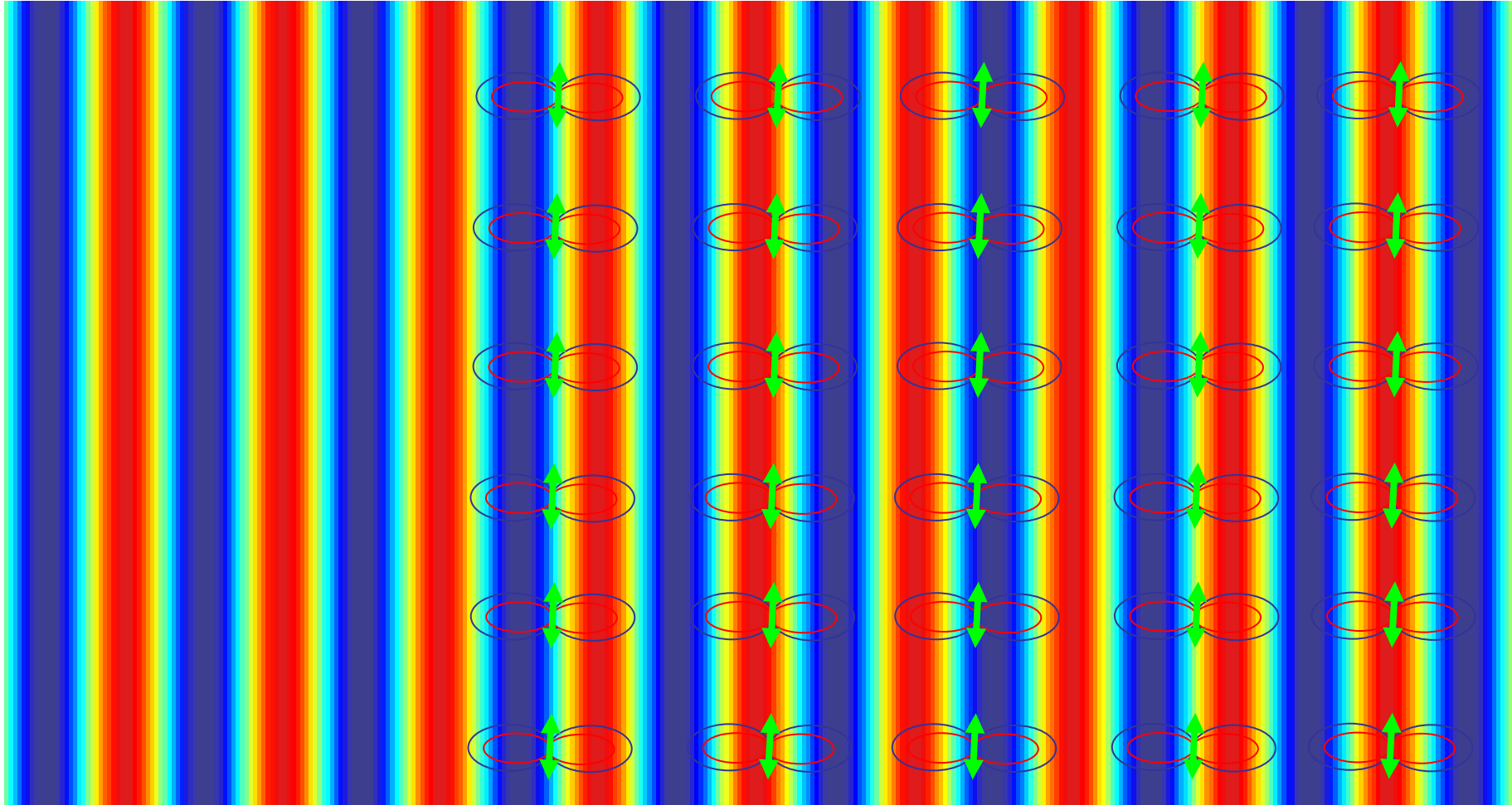
$$\vec{E}_k = i \frac{c^2}{\omega} \vec{\nabla} \times \left( \vec{\nabla} \times \vec{A}_k \right)$$

$$\vec{A}_k = \frac{\mu_o}{4\pi} \frac{e^{i(k_o R)}}{R} i \omega \vec{p}_k$$

$$\vec{p}_k = \alpha \vec{E}_k(\vec{r}_k)$$

Dipolo forzato da  
campo locale

# Sistema denso di dipoli forzati: rifrazione



Campo come sovrapposizione del contributo dei dipoli

$$\vec{E}_i(\vec{r}_i) = \vec{E}_o(\vec{r}_i) + c^2 \frac{\mu_o}{4\pi} \sum_{k \neq i} \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ik_o R}}{R} \vec{p}_k \right)$$

$$\vec{p}_k = \alpha \vec{E}_k(\vec{r}_k)$$

$$\vec{E}_i(\vec{r}_i) = \vec{E}_o(\vec{r}_i) + c^2 \alpha \frac{\mu_o}{4\pi} \sum_{k \neq i} \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ik_o R}}{R} \vec{E}_k(\vec{r}_k) \right)$$

$$\sum_{k \neq i} ( ) \Rightarrow \int_{V-V_i} ( ) \rho dV$$



# Campo come sovrapposizione del contributo dei dipoli

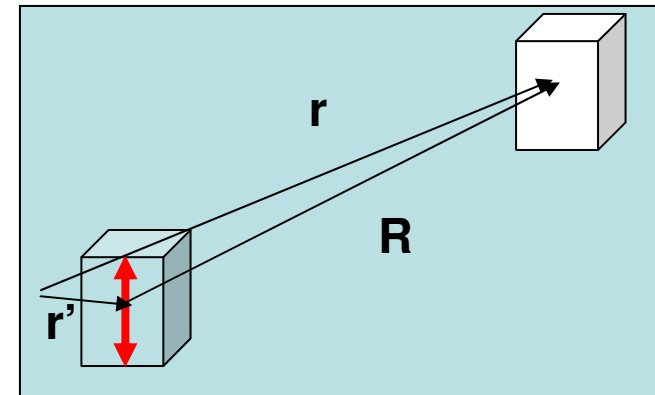
Campo incidente

Campo locale

$$\vec{E}(\vec{r}) = \vec{E}_o(\vec{r}) + c^2 \alpha \frac{\mu_o}{4\pi} \rho \int_{V-\Delta V(\vec{r})} \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') \right) dV$$

$$\vec{E}(\vec{r}) = \vec{E}_o(\vec{r}) + c^2 \alpha \frac{\mu_o}{4\pi} \rho \int_V \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') \right) dV +$$

$$-\frac{8\pi}{3} c^2 \alpha \frac{\mu_o}{4\pi} \rho \vec{E}(\vec{r})$$



Campo come sovrapposizione del contributo dei dipoli

$$\int_V \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') \right) dV = \vec{\nabla} \times \vec{\nabla} \times \int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV$$

$$\text{Hp } \vec{E}_o(\vec{r}) = \vec{E}_o e^{ik_o z} \quad \vec{E}(\vec{r}') = \vec{E}_1 e^{ikz'} \quad \text{Ansatz}$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{(x')^2 + (y')^2 + (z-z')^2}$$

$$\text{Coordinate cilindriche} \quad R = \sqrt{(z-z')^2 + \rho'^2}$$

$$\int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV = \vec{E}_1 \int_0^{2\pi} d\varphi \int_0^{\infty} dz' e^{ikz'} \int_0^{\infty} d\rho' \frac{e^{ik_o R}}{R} \rho'$$

Campo come sovrapposizione del contributo dei dipoli

$$R = \sqrt{(z - z')^2 + \rho'^2} \quad RdR = \rho' d\rho'$$

$$\begin{aligned} \int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV &= \vec{E}_1 \int_0^{2\pi} d\varphi \int_0^\infty dz' e^{ikz'} \int_0^\infty d\rho' \frac{e^{ik_o R}}{R} \rho' = \\ &= 2\pi \vec{E}_1 \int_0^\infty dz' e^{ikz'} \int_{|z-z'|}^\infty dR e^{ik_o R} = 2\pi \vec{E}_1 \int_0^\infty dz' e^{ikz'} i \frac{e^{ik_o |z-z'|}}{k_o} = \\ &= \frac{2\pi i \vec{E}_1}{k_o} \left( \int_0^z dz' e^{ikz'} e^{ik_o(z-z')} + \int_z^\infty dz' e^{ikz'} e^{ik_o(z'-z)} \right) = \\ &= \frac{2\pi i \vec{E}_1}{k_o} e^{ikz} \left( \int_0^z dz' e^{ik(z'-z)} e^{ik_o(z-z')} + \int_z^\infty dz' e^{ik(z'-z)} e^{ik_o(z'-z)} \right) \end{aligned}$$

Campo come sovrapposizione del contributo dei dipoli

$$\begin{aligned}
 & \int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV = \\
 & = \frac{2\pi i \vec{E}_1}{k_o} e^{ikz} \left( \int_0^z dz' e^{i(k-k_o)(z'-z)} + \int_z^\infty dz' e^{i(k+k_o)(z'-z)} \right) = \\
 & = \frac{2\pi i \vec{E}_1}{k_o} e^{ikz} \left( \frac{1 - e^{-i(k-k_o)z}}{i(k-k_o)} - \frac{1}{i(k+k_o)} \right) = \\
 & = \frac{2\pi i \vec{E}_1}{k_o} \left( \frac{e^{ikz} - e^{ik_o z}}{i(k-k_o)} - \frac{e^{ikz}}{i(k+k_o)} \right) = \\
 & = \frac{2\pi \vec{E}_1}{k_o} \left( 2k_o \frac{e^{ikz}}{(k^2 - k_o^2)} - \frac{e^{ik_o z}}{(k - k_o)} \right)
 \end{aligned}$$

Campo come sovrapposizione del contributo dei dipoli

$$\vec{\nabla} \times \vec{\nabla} \times \int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV$$

Campo trasverso

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}_1 e^{ikz}) = ik\hat{z} \times (ik\hat{z} \times \vec{E}_1 e^{ikz}) = k^2 \vec{E}_1 e^{ikz}$$

$$\vec{\nabla} \times \vec{\nabla} \times \int_V \frac{e^{ik_o R}}{R} \vec{E}(\vec{r}') dV = \frac{2\pi \vec{E}_1}{k_o} \left( 2k_o \frac{k^2 e^{ikz}}{k^2 - k_o^2} - \frac{k_o^2 e^{ik_o z}}{k - k_o} \right)$$

Campo come sovrapposizione del contributo dei dipoli

$$\vec{E}(\vec{r}) = \vec{E}_o(\vec{r}) + c^2 \alpha \frac{\mu_o}{4\pi} \rho \int_V \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ikR}}{R} \vec{E}(\vec{r}) \right) dV +$$

$$-\frac{8\pi}{3} c^2 \alpha \frac{\mu_o}{4\pi} \rho \vec{E}(\vec{r})$$

$$\vec{E}_1 e^{ikz} = \vec{E}_o e^{ik_o z} + c^2 \alpha \frac{\mu_o}{4\pi} \rho 2\pi \vec{E}_1 \left( \frac{2k^2 e^{ikz}}{k^2 - k_o^2} - \frac{k_o e^{ik_o z}}{k - k_o} \right) +$$

$$-\frac{2}{3} c^2 \alpha \mu_o \rho \vec{E}_1 e^{ikz}$$

Campo come sovrapposizione del contributo dei dipoli

$$\vec{E}(\vec{r}) = \vec{E}_o(\vec{r}) + \alpha c^2 \frac{\mu_o}{4\pi} \rho \int_V \vec{\nabla} \times \left( \vec{\nabla} \times \frac{e^{ikR}}{R} \vec{E}(\vec{r}) \right) dV +$$

$$-\frac{2}{3} \alpha c^2 \mu_o \rho \vec{E}(\vec{r})$$

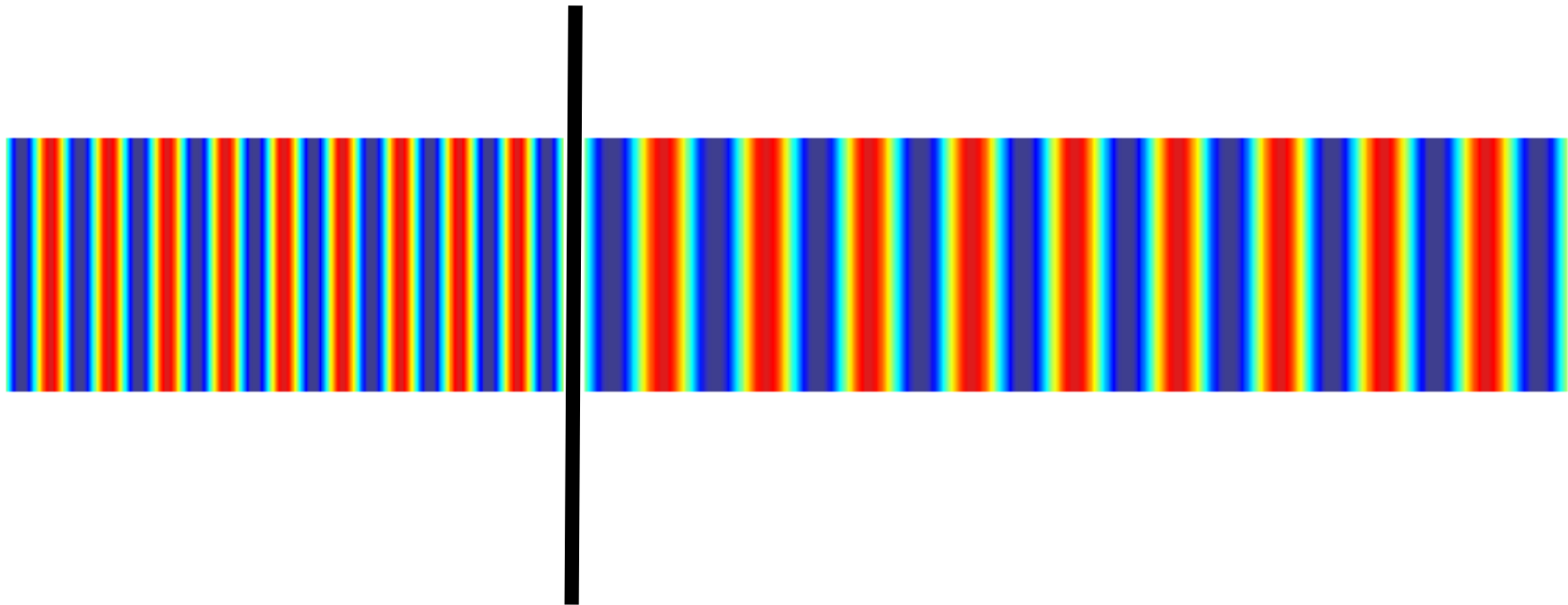
$$\vec{E}_1 e^{ikz} = \vec{E}_o e^{ik_o z} + \alpha \frac{1}{4\pi \epsilon_o} \rho 2\pi \vec{E}_1 \left( \frac{2k^2 e^{ikz}}{k^2 - k_o^2} - \frac{k_o e^{ik_o z}}{k - k_o} \right) +$$

$$-\frac{2}{3} \frac{\alpha}{\epsilon_o} \rho \vec{E}_1 e^{ikz}$$

Campo come sovrapposizione del contributo dei dipoli

$$\left\{ \begin{array}{l} \vec{E}_1 e^{ikz} \left( 1 - \frac{\alpha}{\epsilon_o} \rho \frac{k^2}{k^2 - k_o^2} + \frac{2}{3} \alpha \frac{1}{\epsilon_o} \rho \right) = 0 \\ \vec{E}_o e^{ik_o z} - \alpha \frac{1}{4\pi\epsilon_o} \rho 2\pi \vec{E}_1 \frac{k_o e^{ik_o z}}{k - k_o} = 0 \end{array} \right.$$

Ewald-Oseen  
Teorema  
estinzione





Campo come sovrapposizione del contributo dei dipoli

$$\left\{ \begin{array}{l} \vec{E}_1 e^{ikz} \left( 1 - \frac{\alpha}{\epsilon_o} \rho \frac{k^2}{k^2 - k_o^2} + \frac{2}{3} \alpha \frac{1}{\epsilon_o} \rho \right) = 0 \\ \vec{E}_o e^{ik_o z} - \alpha \frac{1}{4\pi\epsilon_o} \rho 2\pi \vec{E}_1 \frac{k_o e^{ik_o z}}{k - k_o} = 0 \end{array} \right.$$

Ewald-Oseen  
Teorema  
estinzione

$$k = nk_o$$

$$\left( 1 - \frac{\alpha}{\epsilon_o} \rho \frac{n^2}{n^2 - 1} + \frac{2}{3} \alpha \frac{1}{\epsilon_o} \rho \right) = 0$$

$$\frac{\alpha}{\epsilon_o} \rho \left( \frac{n^2}{n^2 - 1} - \frac{2}{3} \right) = 1$$

$$\frac{\rho\alpha}{3\epsilon_o} = \frac{n^2 - 1}{n^2 + 2}$$

Lorentz-  
Lorenz

## Ulteriori conseguenze

$$\frac{\rho\alpha}{3\varepsilon_o} = \frac{n^2 - 1}{n^2 + 2} \quad \text{Lorentz-Lorenz}$$

$$\frac{\rho\alpha}{3\varepsilon_o} (n^2 + 2) = n^2 - 1$$

$$n^2 \left( \frac{\rho\alpha}{3\varepsilon_o} - 1 \right) = -\frac{2\rho\alpha}{3\varepsilon_o} - 1$$

$$n^2 = \frac{1 + \frac{2\rho\alpha}{3\varepsilon_o}}{1 - \frac{\rho\alpha}{3\varepsilon_o}}$$

Clausius-Mossotti

Ulteriori conseguenze  $\vec{E}_o = \alpha \frac{1}{2\varepsilon_o} \rho \vec{E}_1 \frac{k_o}{k - k_o} \quad k = nk_o$

$$\vec{E}_o = \alpha \frac{1}{2\varepsilon_o} \rho \vec{E}_1 \frac{1}{n-1}$$

$$\vec{E}_1 = \frac{2\varepsilon_o}{\rho\alpha} (n-1) \vec{E}_o$$

$$\vec{P} = \rho\alpha \vec{E}_1$$

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon_o n^2 \vec{E}$$

$$\vec{E}_1 = \frac{\varepsilon_o (n^2 - 1)}{\rho\alpha} \vec{E}$$

$$\frac{2\varepsilon_o}{\rho\alpha} (n-1) \vec{E}_o = \frac{\varepsilon_o (n^2 - 1)}{\rho\alpha} \vec{E}$$

$\vec{E}_o =$  Campo incidente

$\vec{E}_1 =$  Campo locale

$\vec{E} =$  Campo macroscopico

$$\vec{E} = \frac{2}{n+1} \vec{E}_o$$

Fresnel