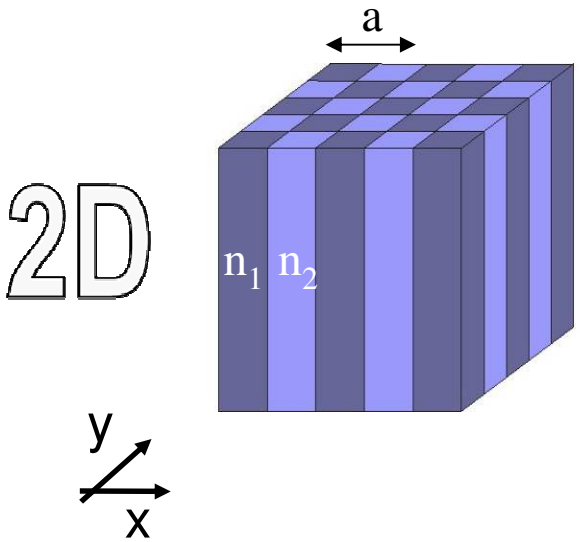
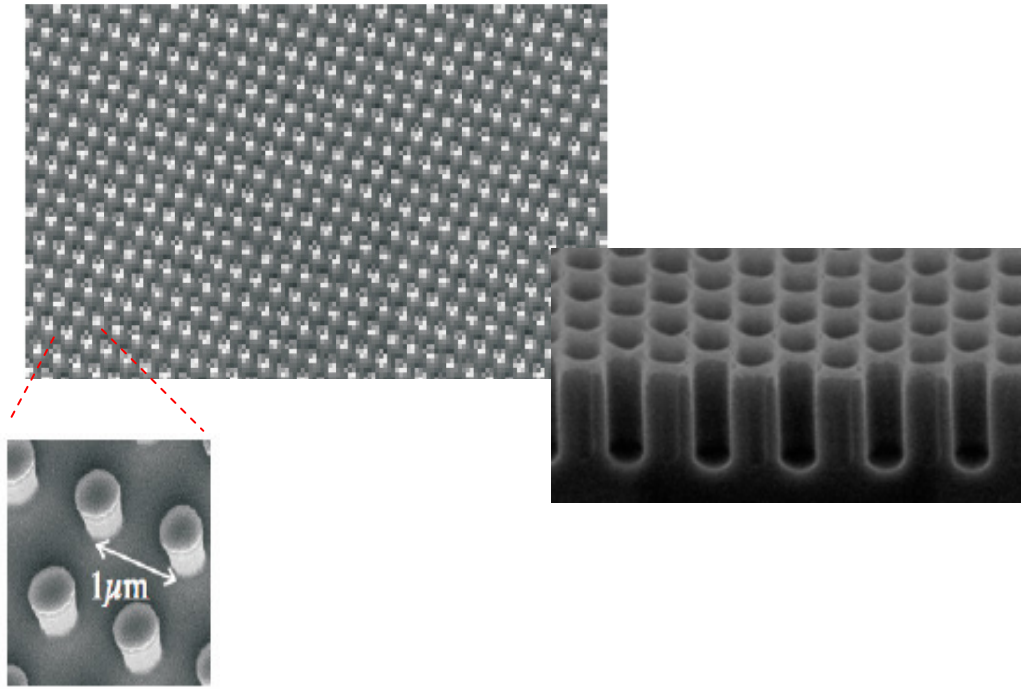
Fotonica 2D
Struttura a bande

PhC in 2D

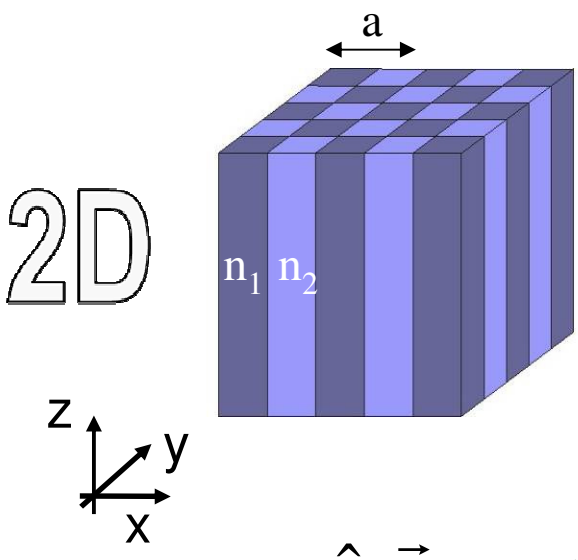




$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$

$$\vec{k} = \vec{k}_{//} + \vec{k}_z \qquad \vec{\rho} = x\hat{x} + y\hat{y}$$

Simmetria inversione



$$\hat{T}_z \varepsilon(\vec{\rho}, z) = \varepsilon(\vec{\rho}, -z) = \varepsilon(\vec{\rho}, z)$$

$z \rightarrow -z$ è una simmetria

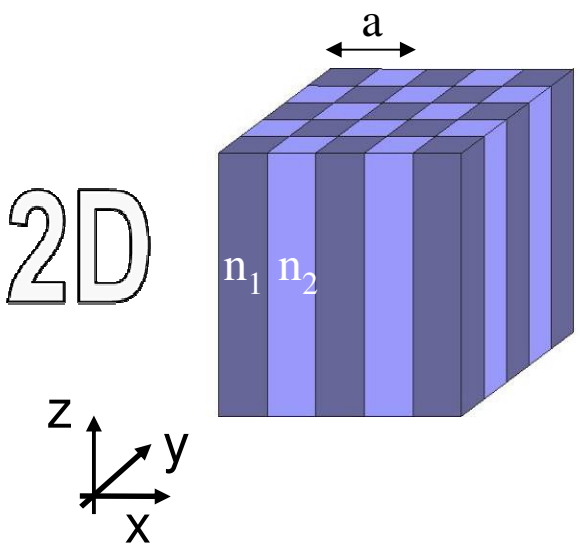
Deve corrispondere una proprietà del modo

$$\hat{\Theta} \vec{H}_{n,\vec{k}}(\vec{\rho}, z) = \frac{\omega_n(\vec{k})^2}{c^2} \vec{H}_{n,\vec{k}}(\vec{\rho}, z)$$

Autostati comuni

$$\left[\hat{T}_z, \hat{\Theta}\right] \vec{H}_{n,\vec{k}}(\vec{r}) = \hat{T}_z \hat{\Theta} \vec{H}_{n,\vec{k}}(\vec{r}) - \hat{\Theta} \hat{T}_z \vec{H}_{n,\vec{k}}(\vec{r}) = 0$$

Simmetria inversione



$$\hat{T}_z \varepsilon(\vec{\rho}, z) = \varepsilon(\vec{\rho}, -z) = \varepsilon(\vec{\rho}, z)$$

$z \rightarrow -z$ è una simmetria

Deve corrispondere una proprietà del modo

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$

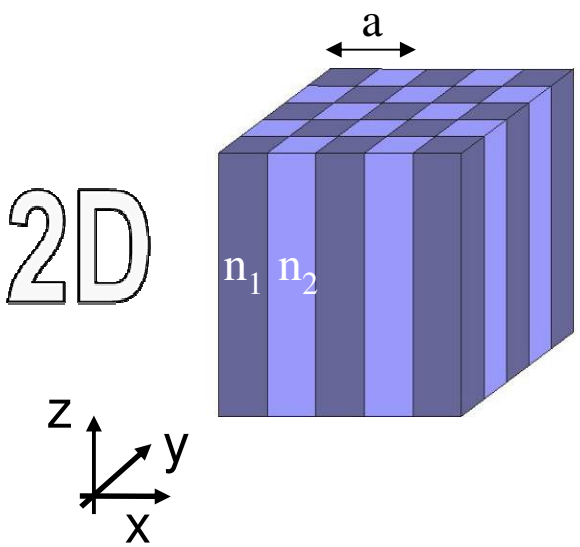
$$\vec{k} = \vec{k}_{//} + \vec{k}_z \qquad \vec{\rho} = x\hat{x} + y\hat{y}$$

$$e^{ik_z z} = \cos(k_z z) + i\sin(k_z z)$$ né pari né dispari

$z \rightarrow -z$ $\quad e^{ik_z z} \rightarrow e^{-ik_z z}$

Onda riflessa $\quad \omega(\vec{k}) = \omega(-\vec{k})$

Simmetria inversione



$$\hat{T}_z \varepsilon(\vec{\rho}, z) = \varepsilon(\vec{\rho}, -z) = \varepsilon(\vec{\rho}, z)$$

$z \rightarrow -z$ è una simmetria

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$

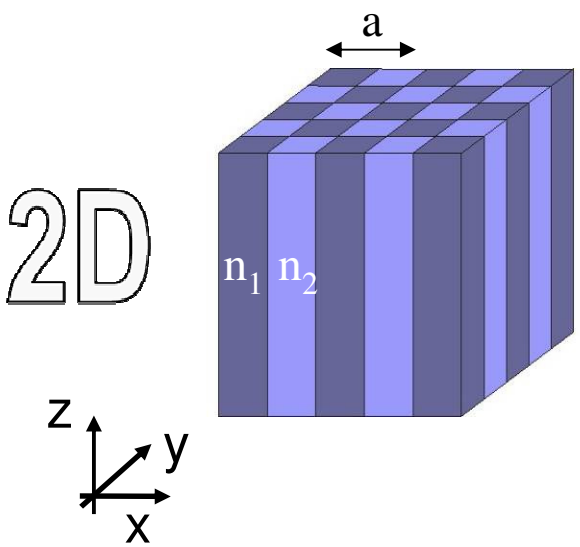
$$\vec{k} = \vec{k}_{//} + \vec{k}_z \qquad \vec{\rho} = x\hat{x} + y\hat{y}$$

$$k_z \neq 0 \quad \Rightarrow \quad \hat{T}_z \vec{k} \neq \vec{k}$$

$$\hat{T}_z \vec{H}_{n,\vec{k}}(\vec{r}) = \hat{T}_z \left(e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho}) \right) = e^{i\phi} \left(e^{-ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho}) \right)$$

Proprietà (poco utile) su onda riflessa

Simmetria inversione



$$\hat{T}_z \varepsilon(\vec{\rho}, z) = \varepsilon(\vec{\rho}, -z) = \varepsilon(\vec{\rho}, z)$$

$z \rightarrow -z$ è una simmetria

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$

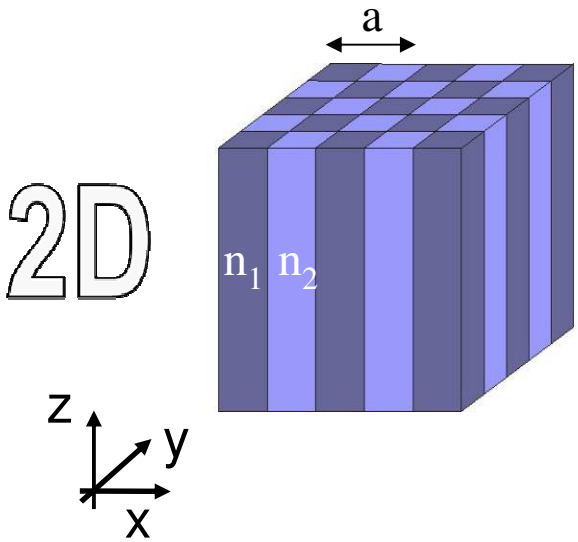
$$\vec{k} = \vec{k}_{//} + \vec{k}_z \qquad \vec{\rho} = x\hat{x} + y\hat{y}$$

$$k_z = 0 \quad \Rightarrow \quad \hat{T}_z \vec{k} = \vec{k}$$

$$\hat{T}_z \vec{H}_{n,\vec{k}}(\vec{r}) = \hat{T}_z \left(e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho}) \right) = \pm \left(e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho}) \right)$$

Classificazione in stati pari e dispari

Definizione modi pari e dispari



z→-z è una simmetria

$$k_z = 0 \quad \Rightarrow \quad \hat{T}_z \vec{k} = \vec{k}$$

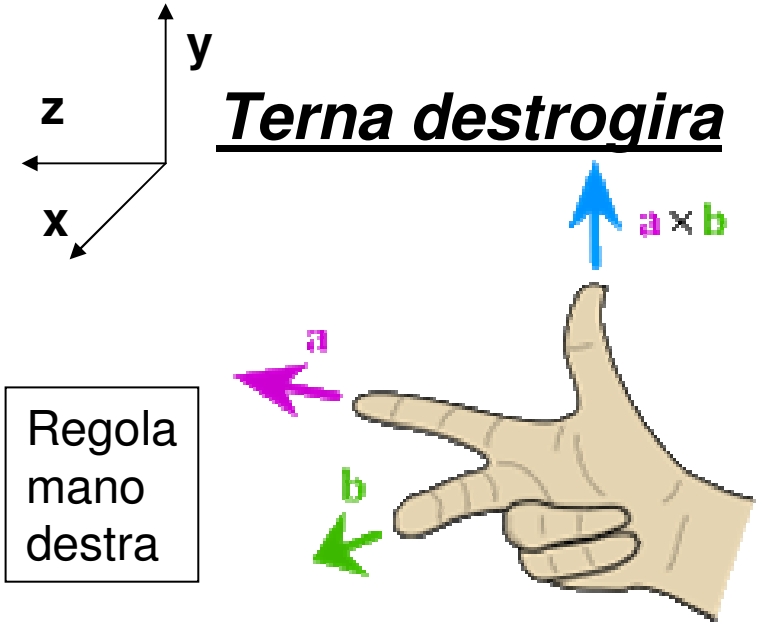
Modi pari

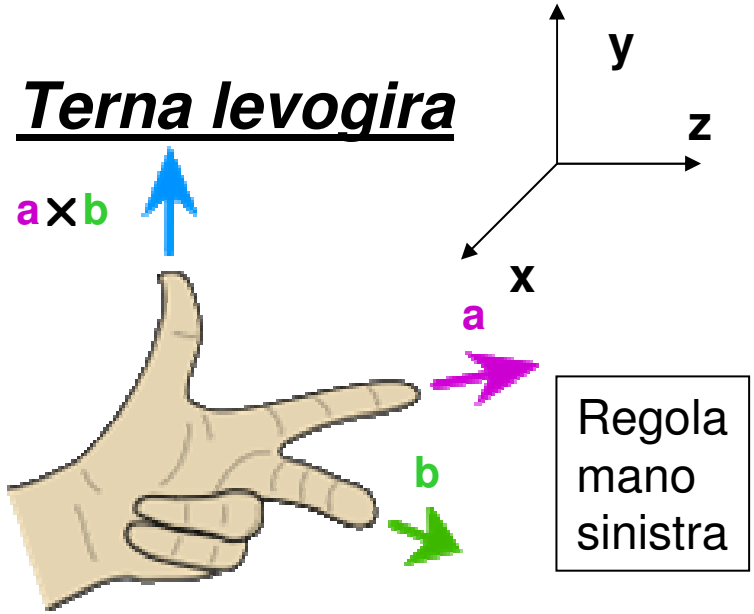
$$U_i(\vec{\rho}, -z) = U_i(\vec{\rho}, z) \qquad i = x, y, z \qquad U = E, H$$

Modi dispari

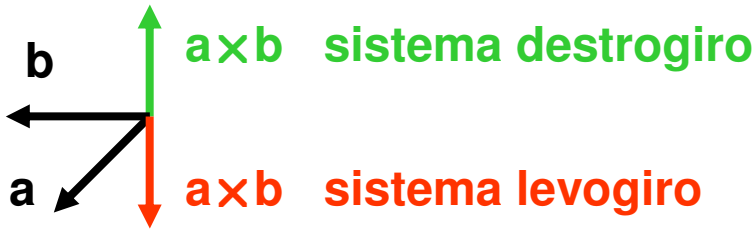
$$U_i(\vec{\rho}, -z) = -U_i(\vec{\rho}, z) \qquad i = x, y, z \qquad U = E, H$$

Nota su prodotto vettoriale (pseudovettore)





In entrambi i casi vale la regola ciclica: $x=y\times z$, $y=z\times x$, $z=x\times y$.
Quindi



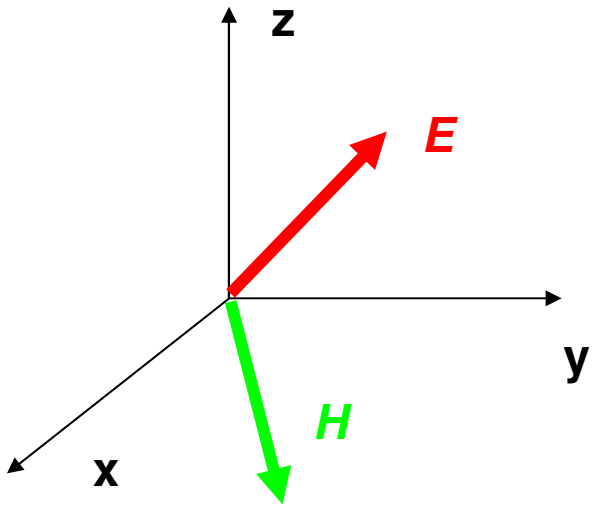
Se destrogira → levogira allora $a\times b$ cambia verso

Proprietà su riflessioni

E è un vettore

z→-z

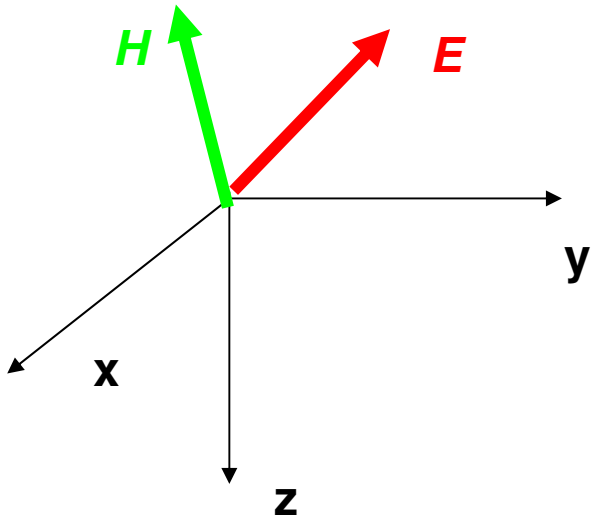
$$(E_x, E_y, E_z) \rightarrow (E_x, E_y, -E_z)$$



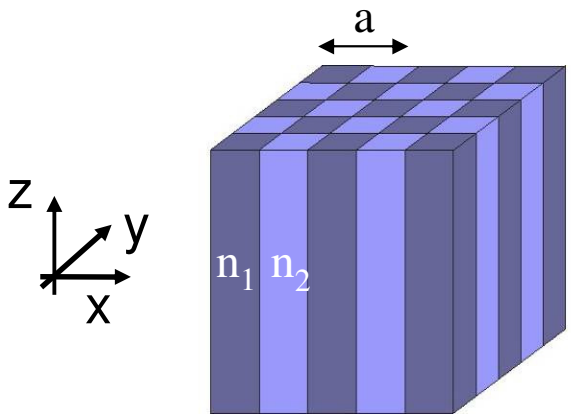
H è uno pseudovettore

z→-z

$$(H_x, H_y, H_z) \rightarrow (-H_x, -H_y, H_z)$$

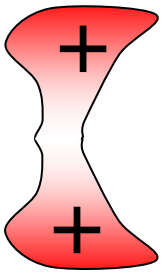


Modi pari e dispari



$$k_z = 0$$

Componenti pari

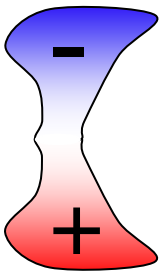


$$E_x(\vec{\rho},-z) = E_x(\vec{\rho},z)$$

$$E_y(\vec{\rho},-z) = E_y(\vec{\rho},z)$$

$$H_z(\vec{\rho},-z) = H_z(\vec{\rho},z)$$

Componenti dispari

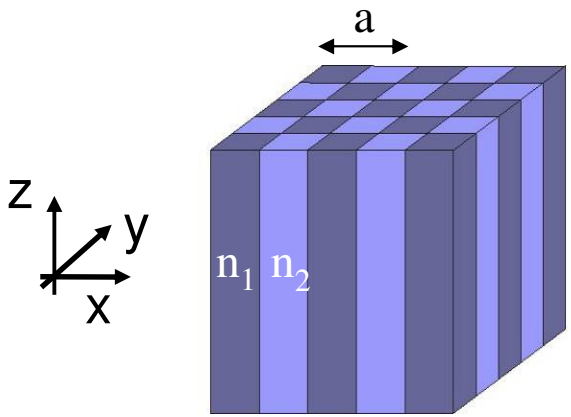


$$H_x(\vec{\rho},-z) = -H_x(\vec{\rho},z)$$

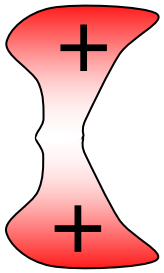
$$H_y(\vec{\rho},-z) = -H_y(\vec{\rho},z)$$

$$E_z(\vec{\rho},-z) = -E_z(\vec{\rho},z)$$

Modi pari e dispari $k_z = 0$

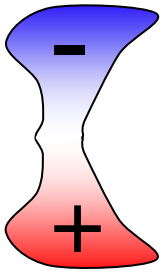


Modi pari



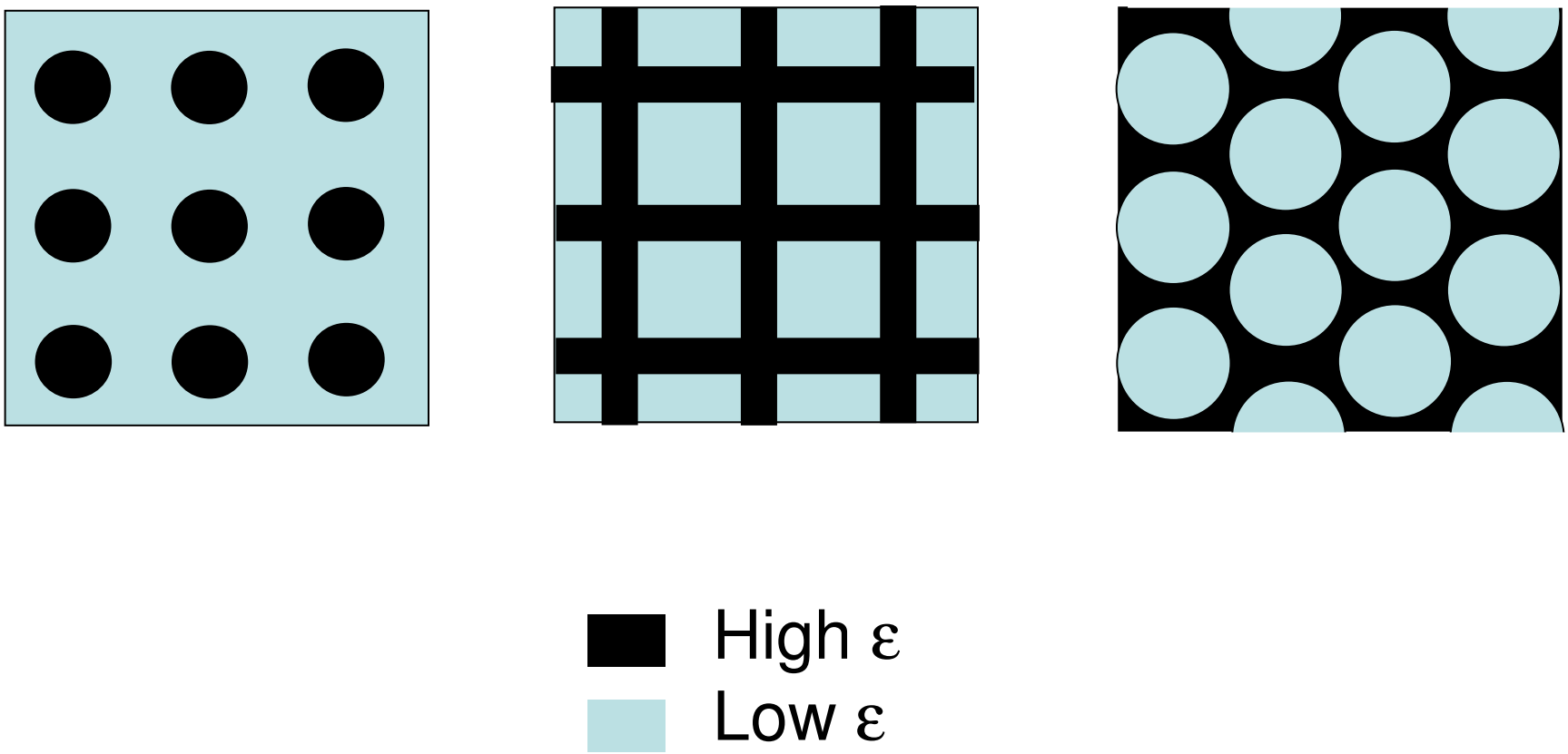
(E_x, E_y, H_z) Transverse-electric (TE) $\vec{E} \cdot \hat{z} = 0$

Modi dispari



(H_x, H_y, E_z) Transverse-magnetic (TM) $\vec{H} \cdot \hat{z} = 0$

Tre differenti PhC in 2D

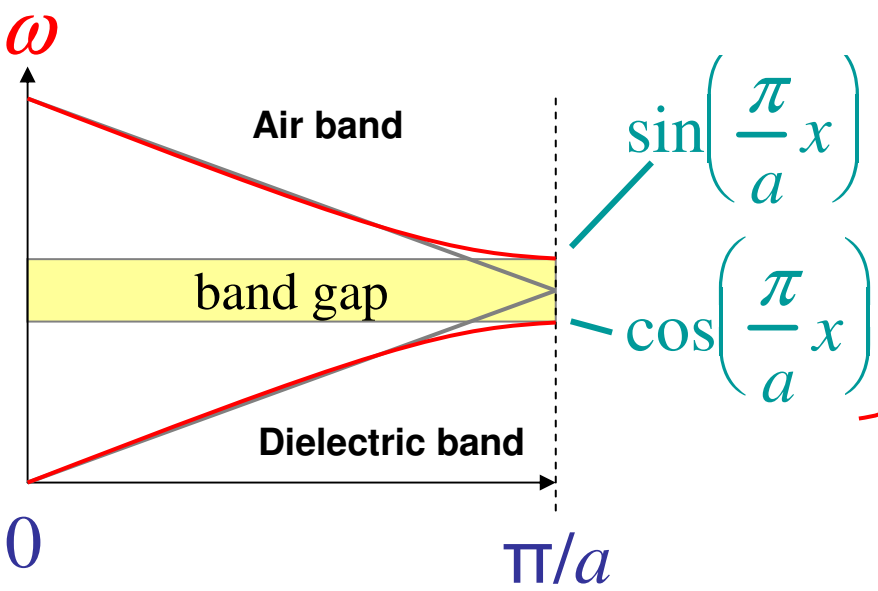


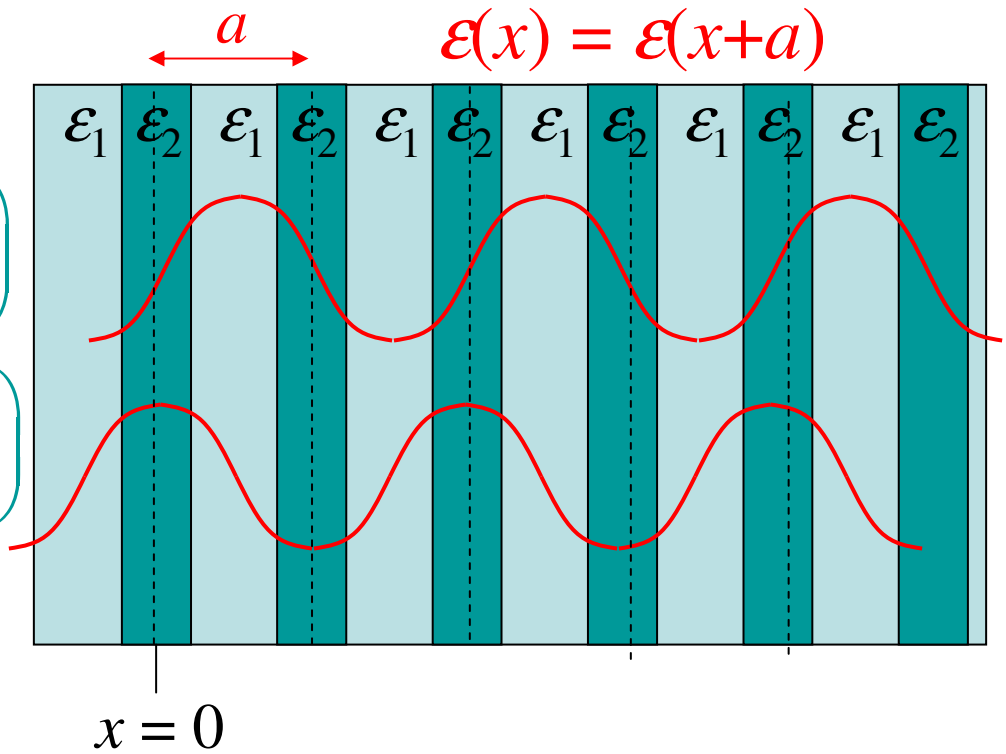
Bang gap si apre al bordo della FBZ
Richiamo esempio 1D

Aggiungiamo una piccola anisotropia

$$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$$

Splitting della degenerazione:
state concentrated in higher index (ε_2)
has lower frequency





Distribuzione spaziale dei modi

$$k_z = 0$$

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{ik_z z} e^{i\vec{k}_{//}\cdot\vec{\rho}} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$

$$\vec{k} = \vec{k}_{//} + \vec{k}_z \qquad \vec{\rho} = x\hat{x} + y\hat{y}$$

$$\vec{H}_{n,\vec{k}}(\vec{r}) = \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$ Γ-point k=0

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\frac{\pi}{a}x} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$ X-point k=(π/a,0)

$$\vec{H}_{n,\vec{k}}(\vec{r}) = e^{i\left(\frac{\pi}{a}(x+y)\right)} \vec{u}_{n,\vec{k}_{//}}(\vec{\rho})$$ M-point k=(π/a,π/a)

Distribuzione spaziale dei modi

$$\left(\vec{H}_2(\vec{r}), \vec{H}_1(\vec{r})\right) = 0$$

Ortogonalità autovettori

$$U_f(\vec{H}) \equiv \frac{\int d^3r \left|\vec{\nabla} \times \vec{E}\right|^2}{\int d^3r \varepsilon(\vec{r}) \left|\vec{E}\right|^2}$$

Principio variazionale:
i modi fotonici di più bassa frequenza hanno ampiezza concentrata nella regione ad alto dielettrico.

Distribuzione spaziale dei modi

$$\frac{\int d^3 r \varepsilon_2(\vec{r}) \left|\vec{E}\right|^2}{\int d^3 r \varepsilon(\vec{r}) \left|\vec{E}\right|^2}$$

Fattore di concentrazione

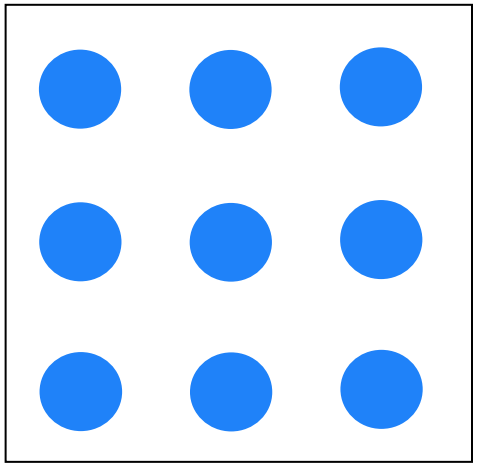
(determina i gaps)

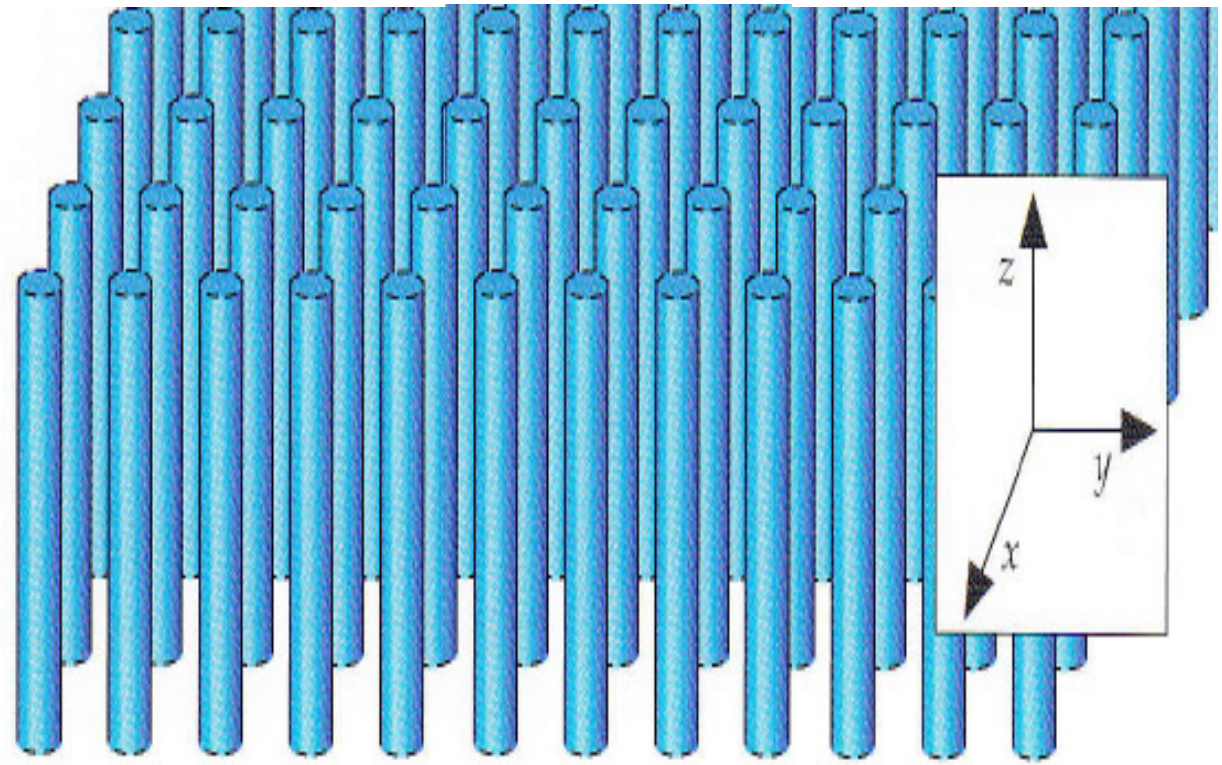
$$D_{2,n}(\vec{r}) = D_{1,n}(\vec{r})$$

$$E_{2,t}(\vec{r}) = E_{1,t}(\vec{r})$$

Boundary conditions

Sistema di colonne su reticolo quadrato



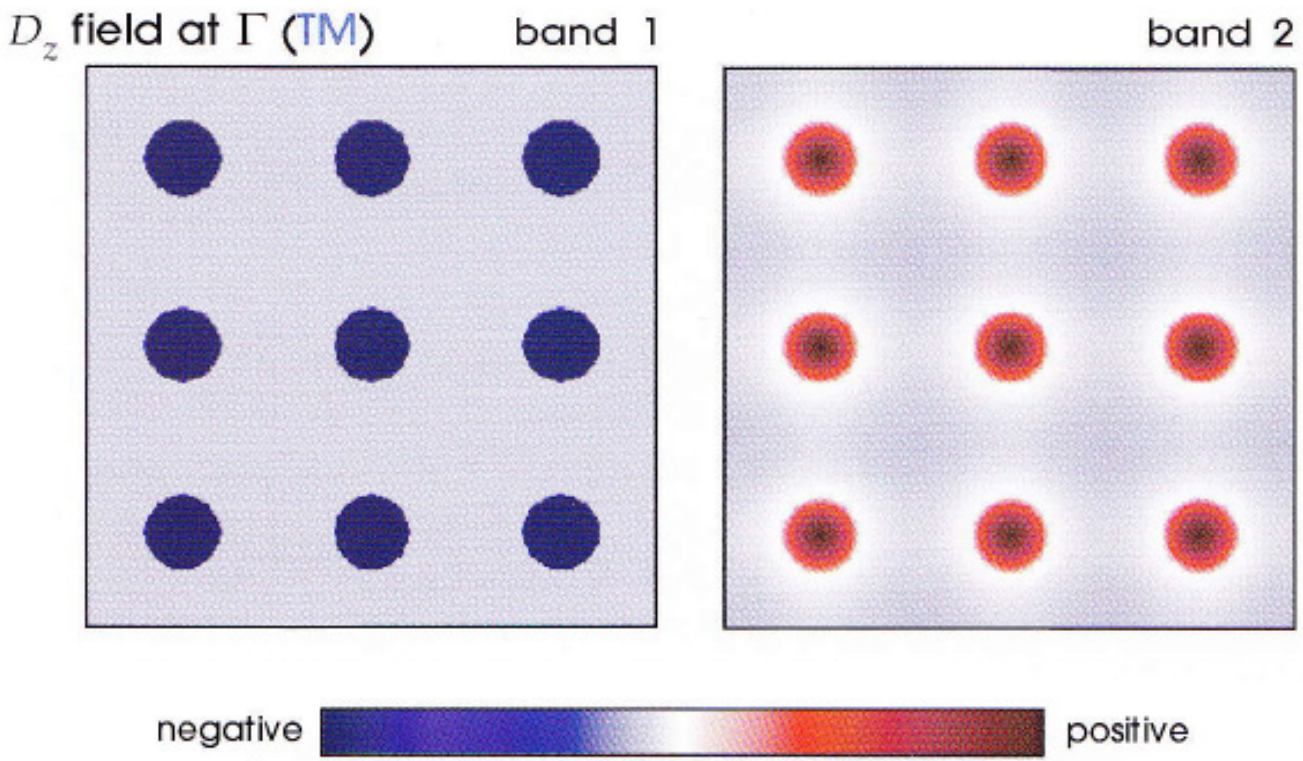


High ε

Low ε

Distribuzione spaziale dei modi (TM)

$$k_z = 0$$

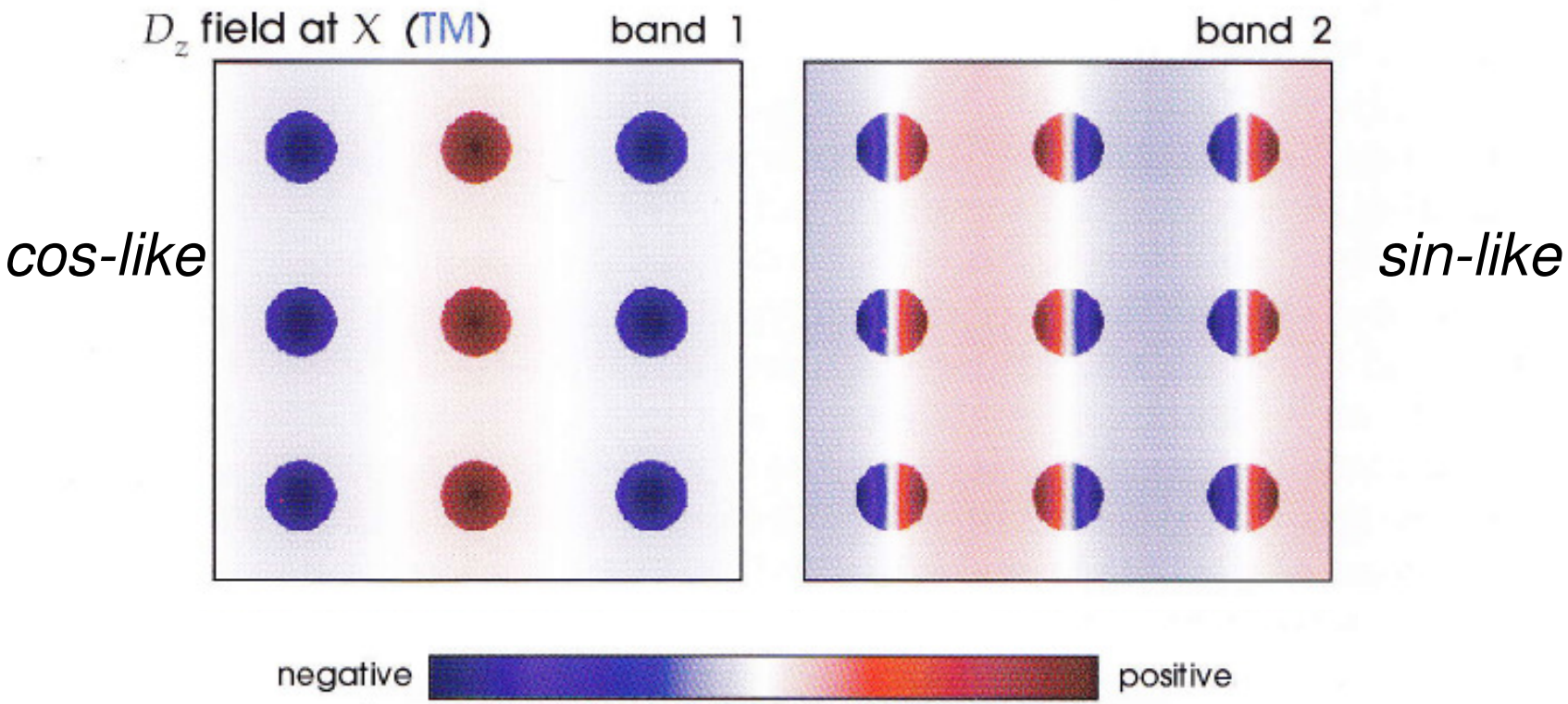


Identico in ogni cella
Discontinuo all'interfaccia
1 nodo sullo stato eccitato
Grande differenza ω

$$D_{n,\vec{k}}(\vec{r}) = u_{n,\vec{k}_{//}}(\vec{\rho})$$

Distribuzione spaziale dei modi (TM)

$$k_z = 0$$

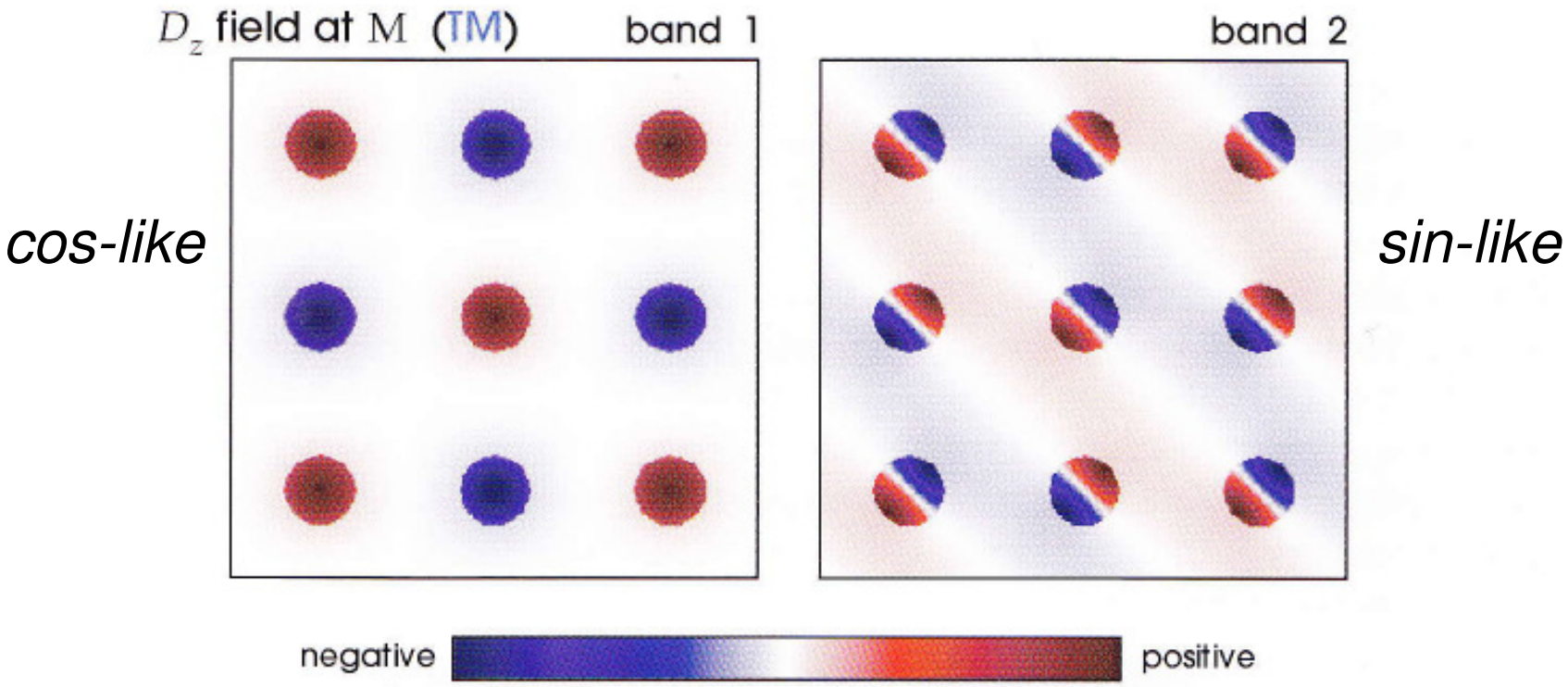


Modulazione π/a
Discontinuo all'interfaccia
Nodo stato eccitato nel dielettrico
Grande differenza ω

$$D_{n,\vec{k}}(\vec{r}) = e^{i\frac{\pi}{a}x} u_{n,\vec{k}_{//}}(\vec{\rho})$$

Distribuzione spaziale dei modi (TM)

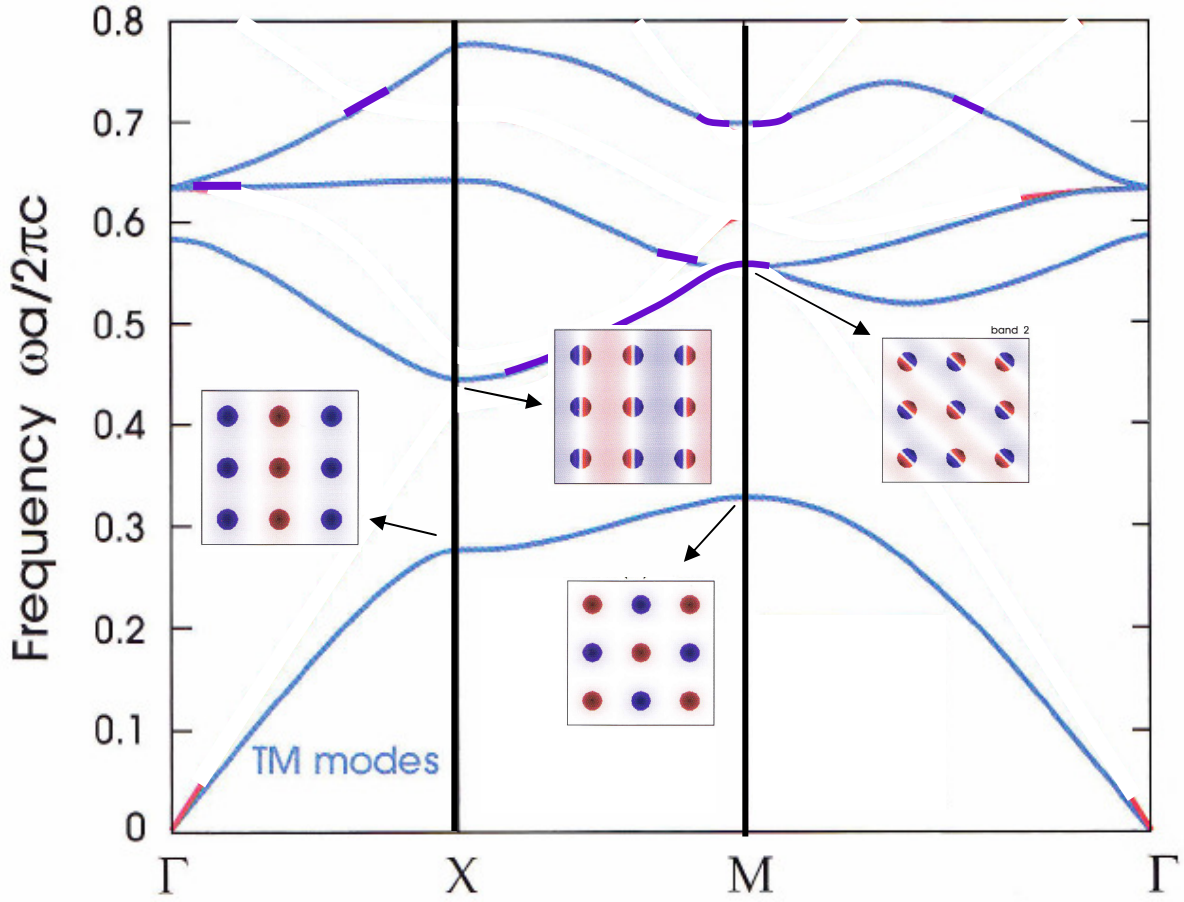
$k_z = 0$



Modulazione π/a diagonale
Discontinuo all'interfaccia
Nodo stato eccitato nel dielettrico
Grande differenza ω

$$D_{n,\vec{k}}(\vec{r}) = e^{i\left(\frac{\pi}{a}(x+y)\right)} u_{n,\vec{k}_{//}}(\vec{\rho})$$

Struttura a bande



Large TM Gap

Figure 2: The photonic band structure for a square array of dielectric columns with $r = 0.2a$. The blue bands represent TM modes and the red bands represent TE modes. The left inset shows the Brillouin zone, with the irreducible zone shaded light blue. The right inset shows a cross-sectional view of the dielectric function. The columns ($\varepsilon = 8.9$, as for alumina) are embedded in air ($\varepsilon = 1$).

Distribuzione spaziale dei modi (TE)

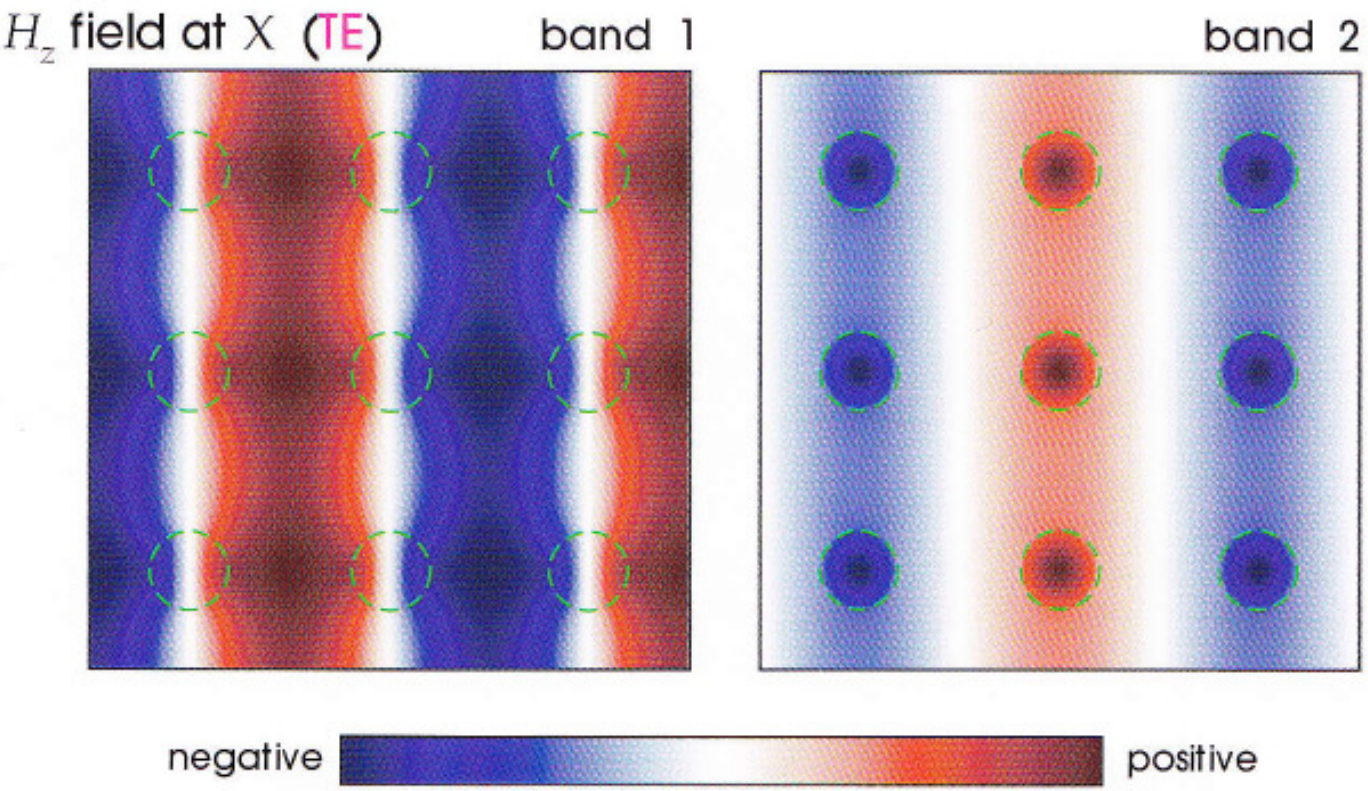


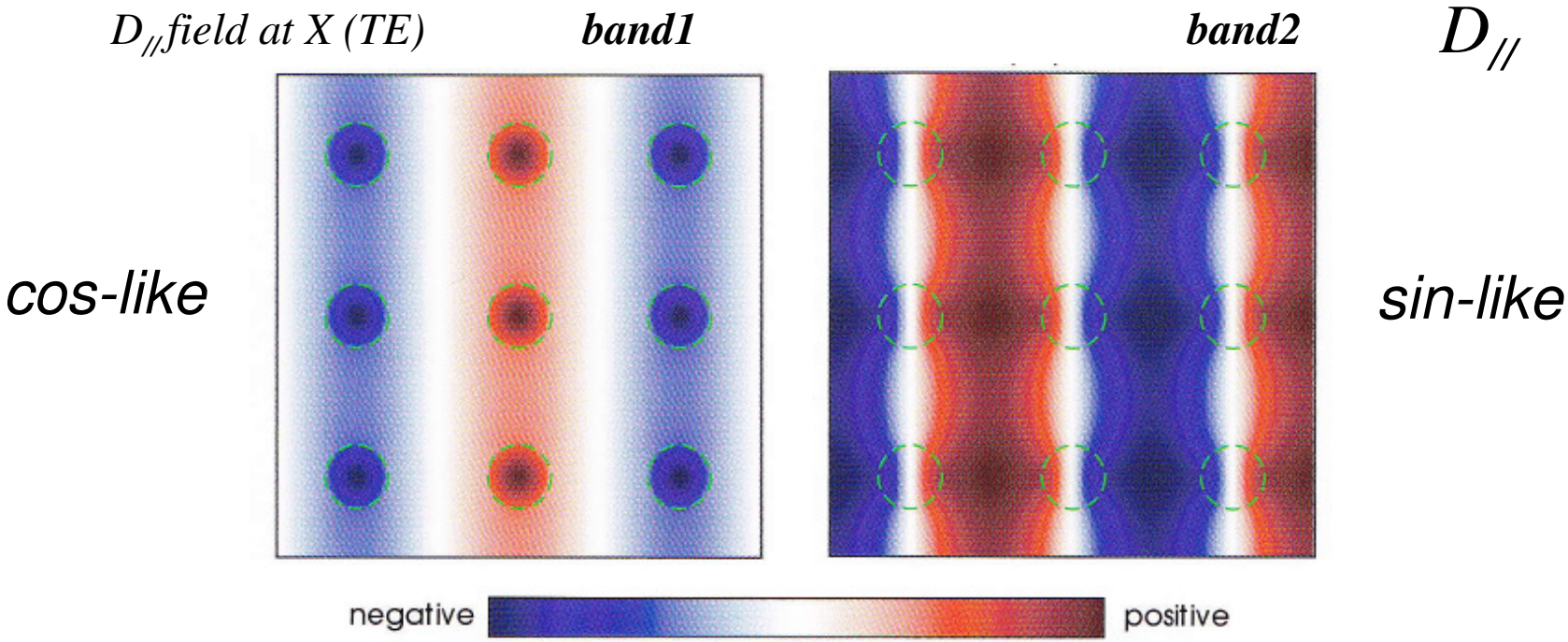
Grafico H_z che è "scalare"

$$\vec{E}(\vec{r}) = \frac{i}{\omega\varepsilon_o\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})$$

Massimi di *E* negli zeri di *H*

Distribuzione spaziale dei modi (TE)

Grafico qualitativo $D_{//}$



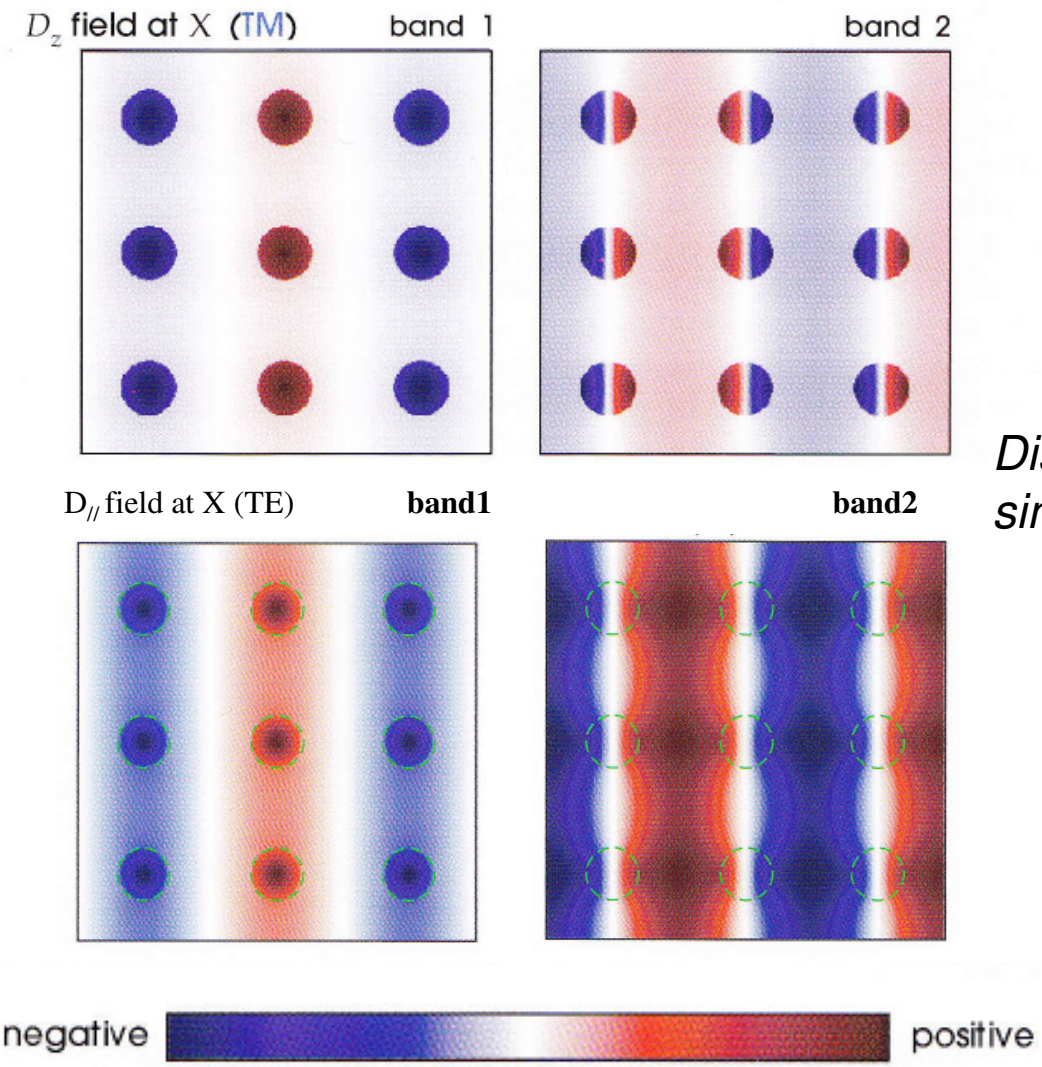
Modulazione π/a
Continuo all'interfaccia
(espulsione campo da dielettrico)
Poca concentrazione in 2
Poca differenza ω

$$D_{n,\vec{k}}(\vec{r}) = e^{i\frac{\pi}{a}x} u_{n,\vec{k}_{//}}(\vec{\rho})$$

Confronto TM vs TE

$k_z = 0$

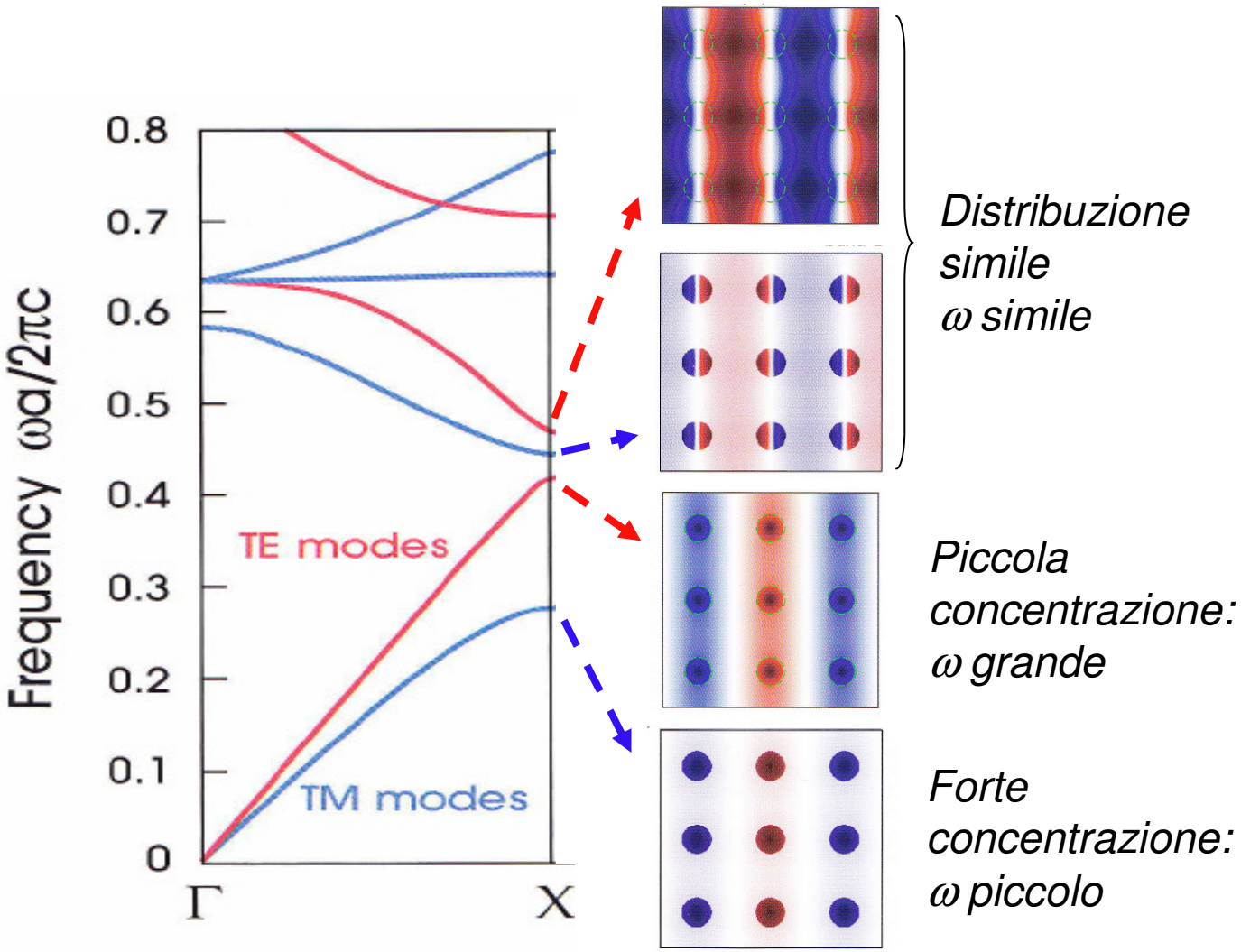
Forte concentrazione: ω piccolo



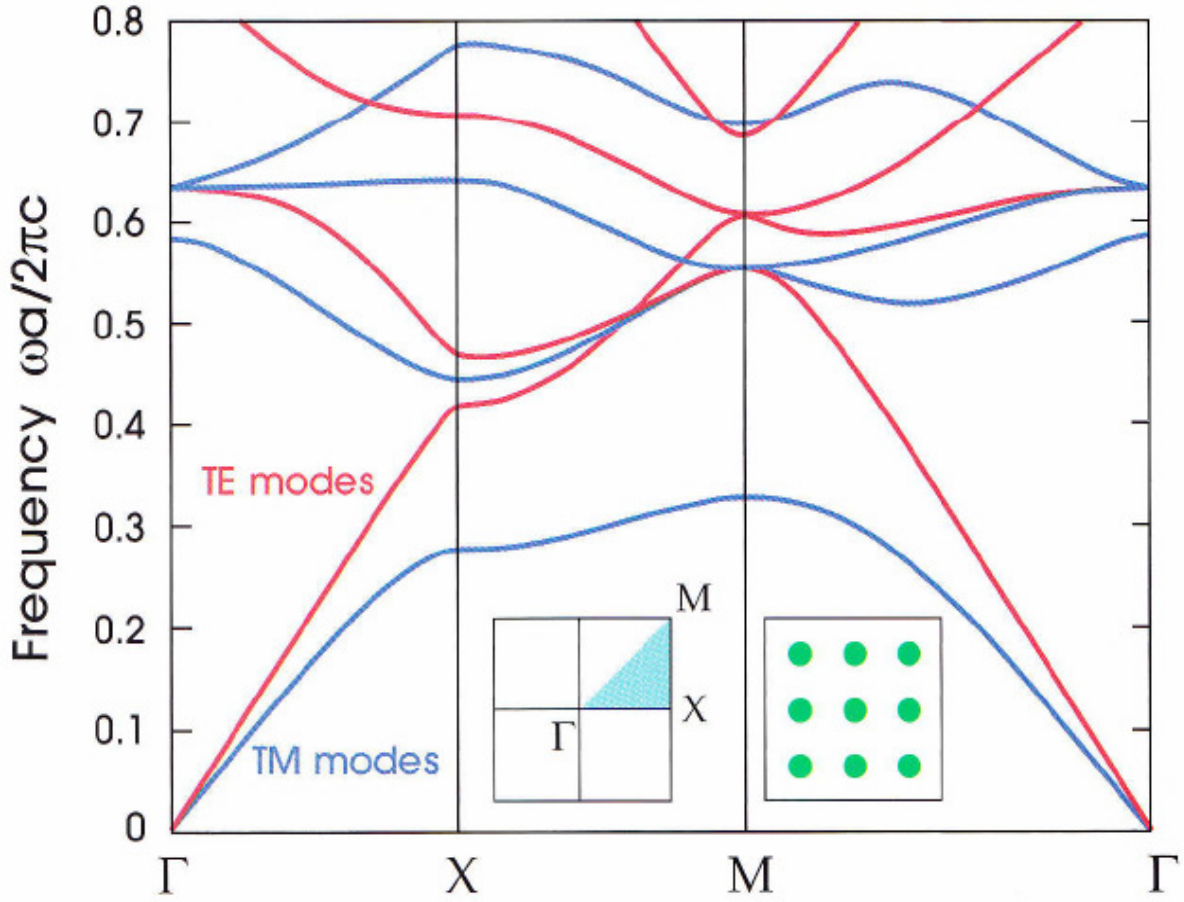
Distribuzione simile

Piccola concentrazione: ω grande

Struttura a bande



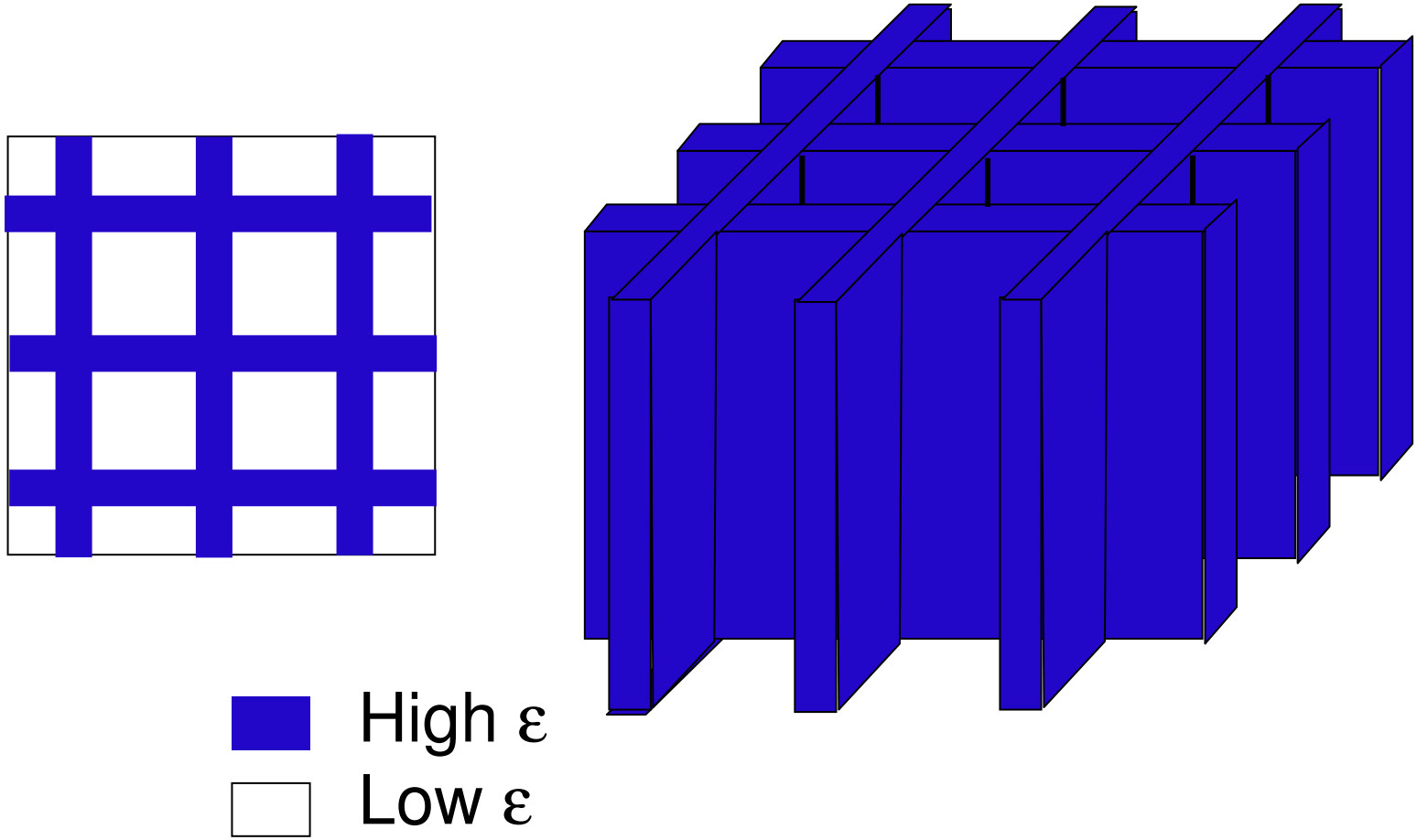
Struttura a bande



TM Gap
No TE gap

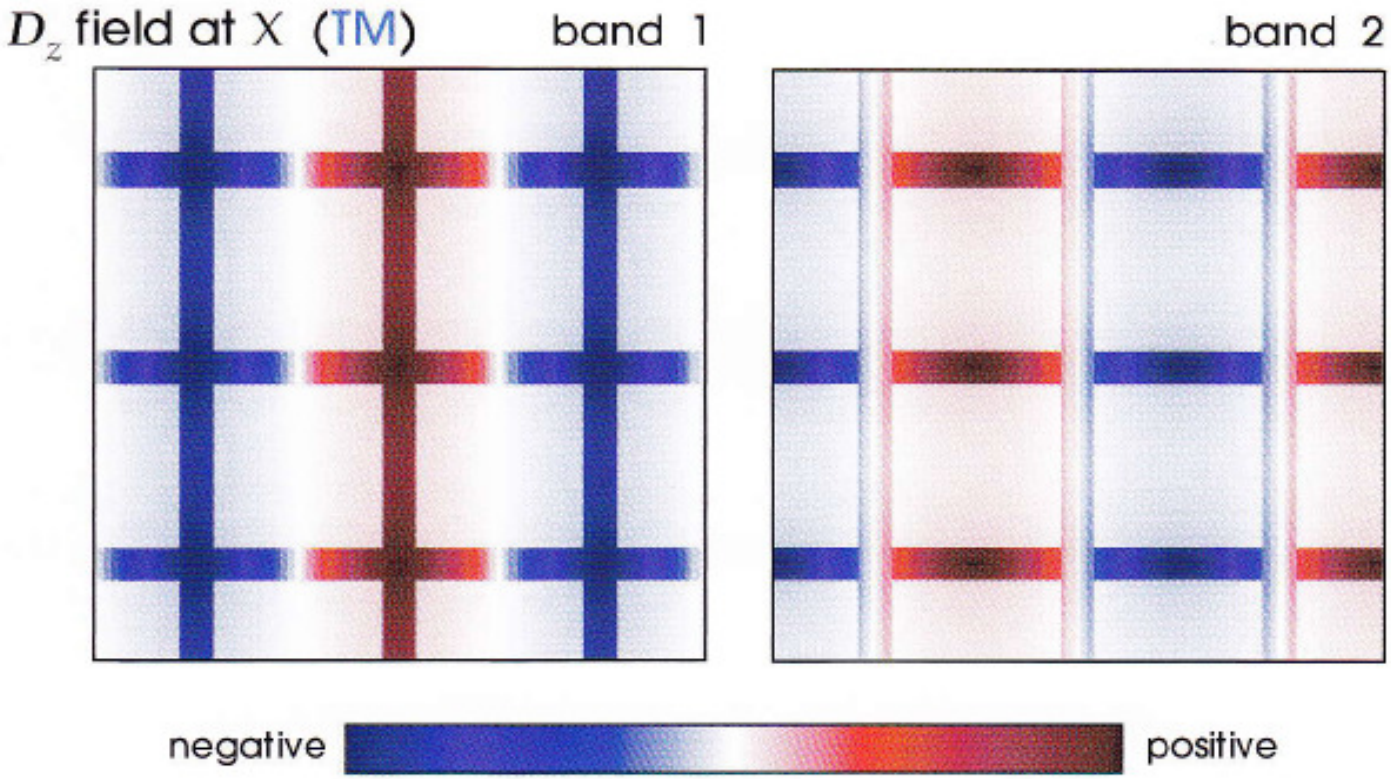
Figure 2: The photonic band structure for a square array of dielectric columns with $r = 0.2a$. The blue bands represent TM modes and the red bands represent TE modes. The left inset shows the Brillouin zone, with the irreducible zone shaded light blue. The right inset shows a cross-sectional view of the dielectric function. The columns ($\varepsilon = 8.9$, as for alumina) are embedded in air ($\varepsilon = 1$).

Sistema di “piani” su reticolo quadrato



Distribuzione spaziale dei modi (TM)

$$k_z = 0$$

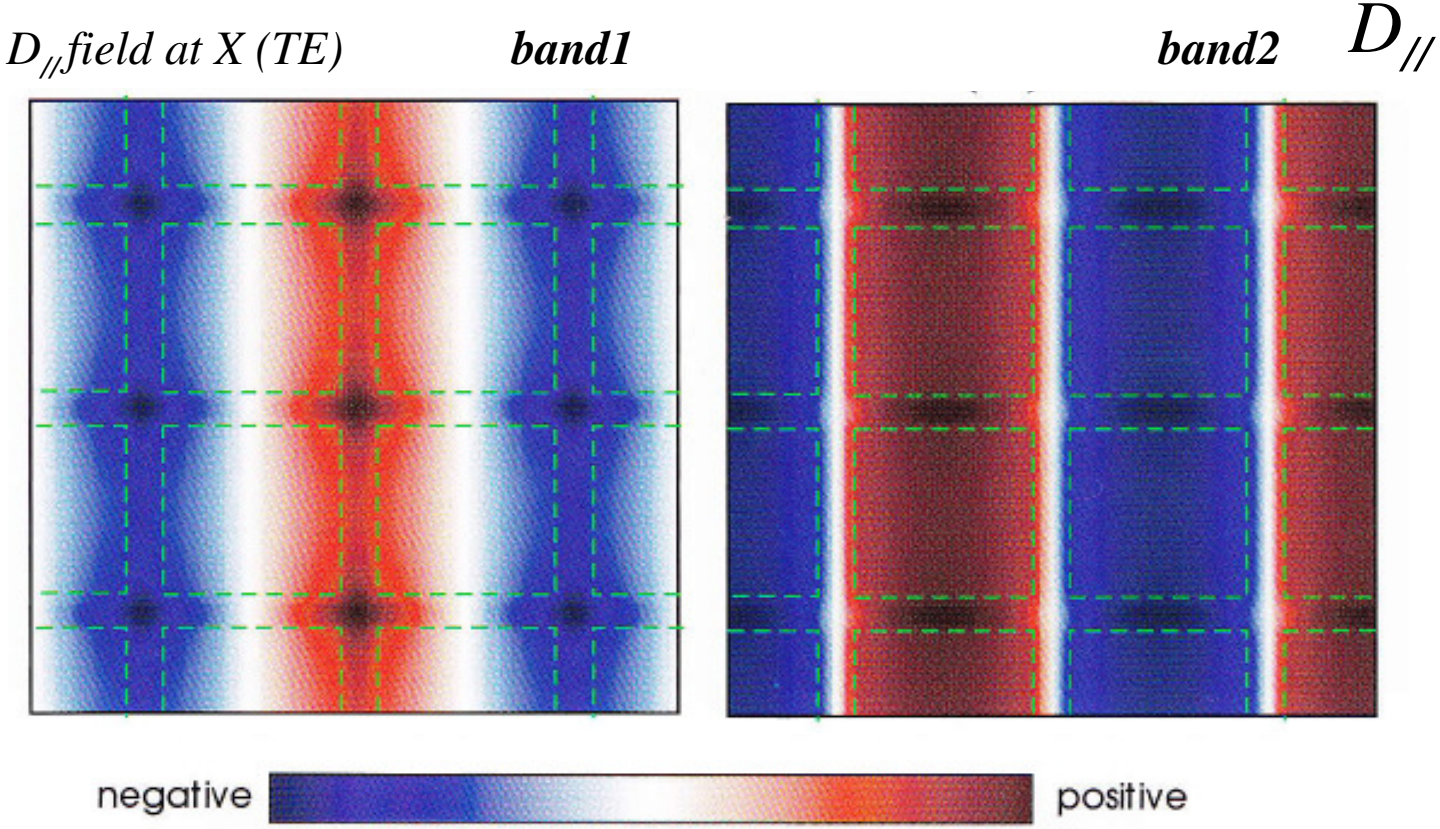


Modulazione π/a
Discontinuo all'interfaccia
Entrambi concentrati nel dielettrico
Poca differenza ω

$$D_{n,\vec{k}}(\vec{r}) = e^{i\frac{\pi}{a}x} u_{n,\vec{k}_{//}}(\vec{\rho})$$

Distribuzione spaziale dei modi (TE)

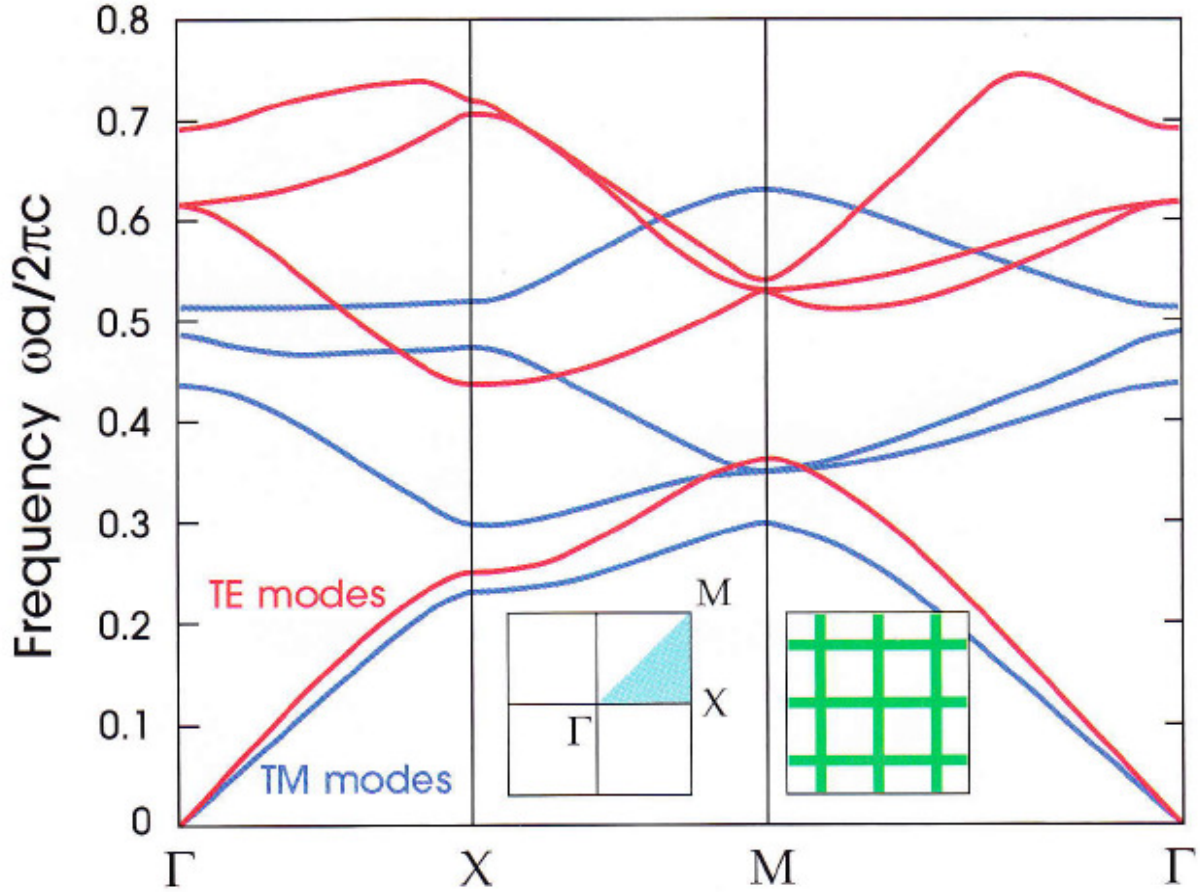
Grafico qualitativo $D_{//}$



Modulazione π/a
Continuo all'interfaccia
Differenza concentrazione
Grande differenza ω

$$D_{n,\vec{k}}(\vec{r}) = e^{i\frac{\pi}{a}x} u_{n,\vec{k}_{//}}(\vec{\rho})$$

Struttura a bande

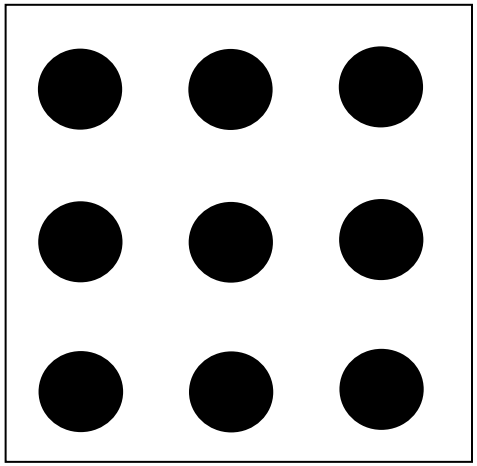


TE Gap
Small TM gap

Figure 5: The photonic band structure for the lowest-frequency modes of a square array of dielectric ($\varepsilon = 8.9$) veins (thickness $0.165a$) in air. The blue lines are TM bands and the red lines are TE bands. The left inset shows the high-symmetry points at the corners of the irreducible Brillouin zone (shaded light blue). The right inset shows a cross-sectional view of the dielectric function.

Sommario

Disgiunto

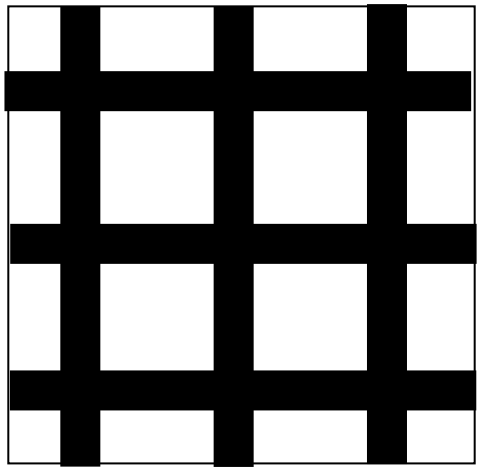


Favorisce i modi con discontinuità

Large TM gap

Small TE gap

Interconnesso

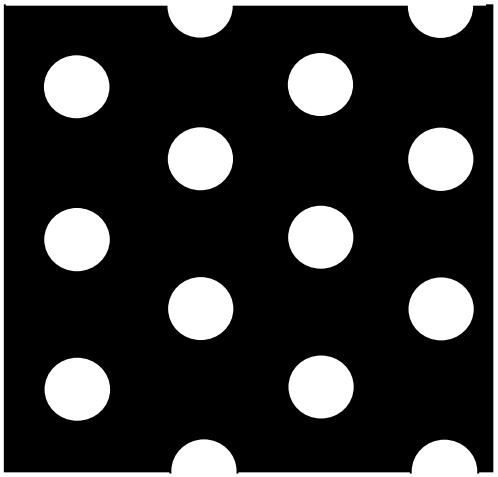


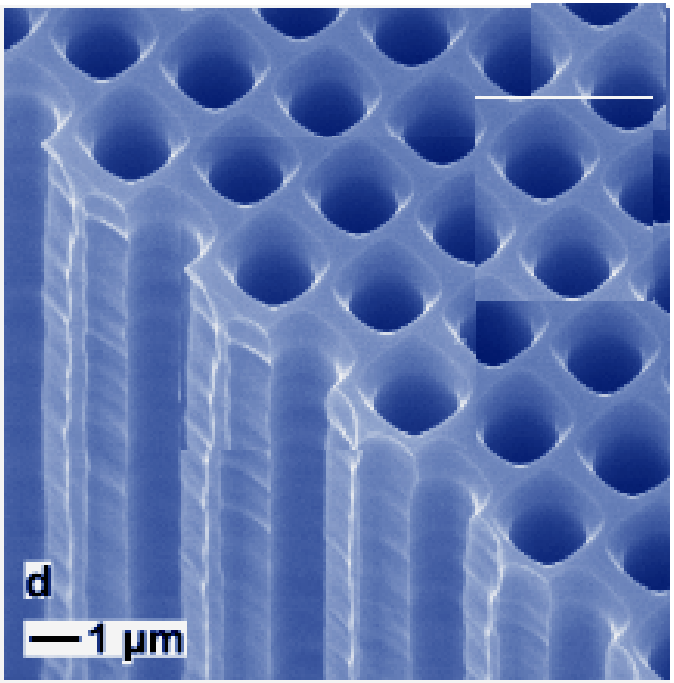
Favorisce i modi senza discontinuità

Small TM gap

Large TE gap

Sistema di “fori” su reticolo triangolare

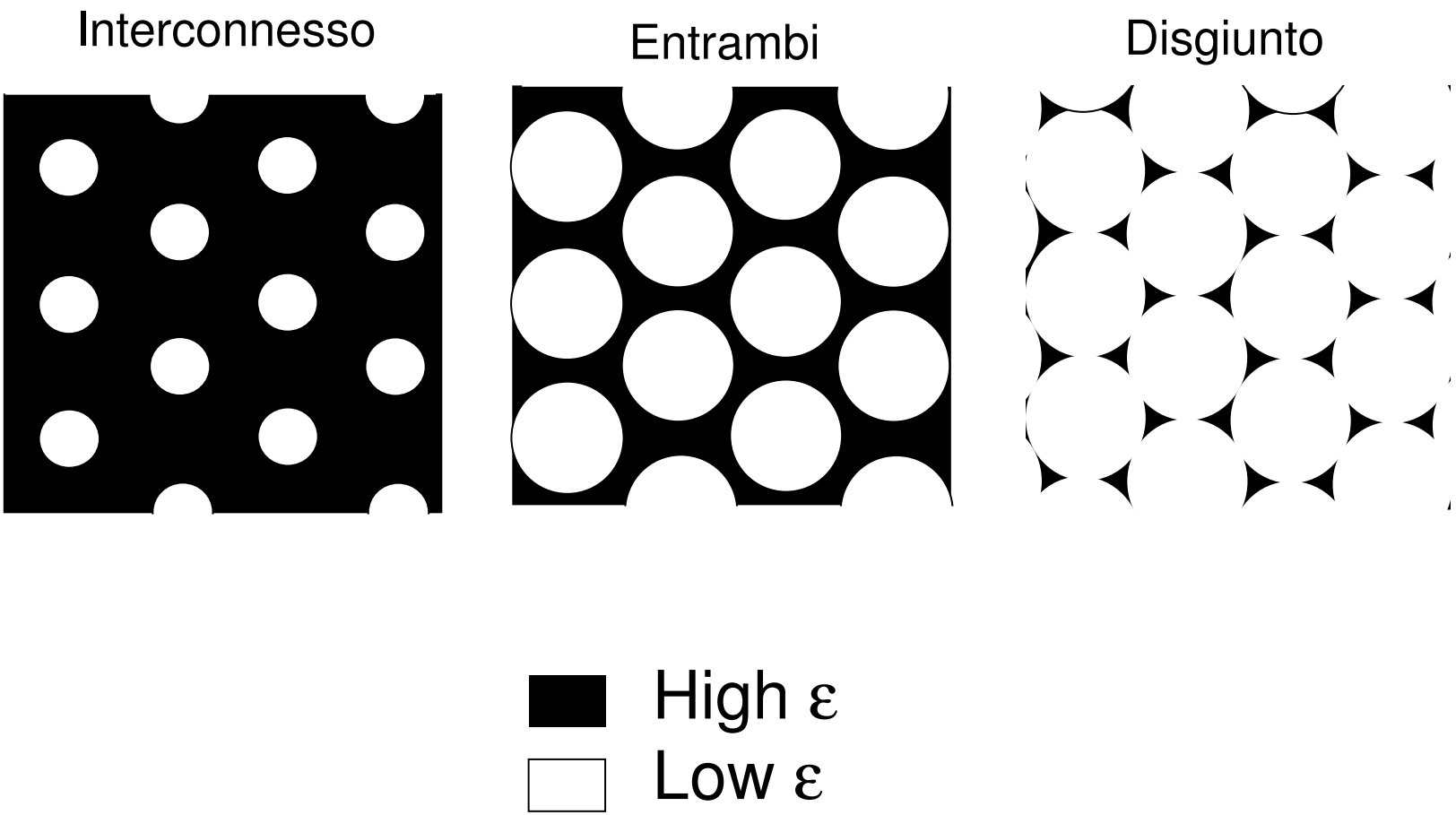




High ε

Low ε

Reticolo triangolare di fori



Struttura a bande

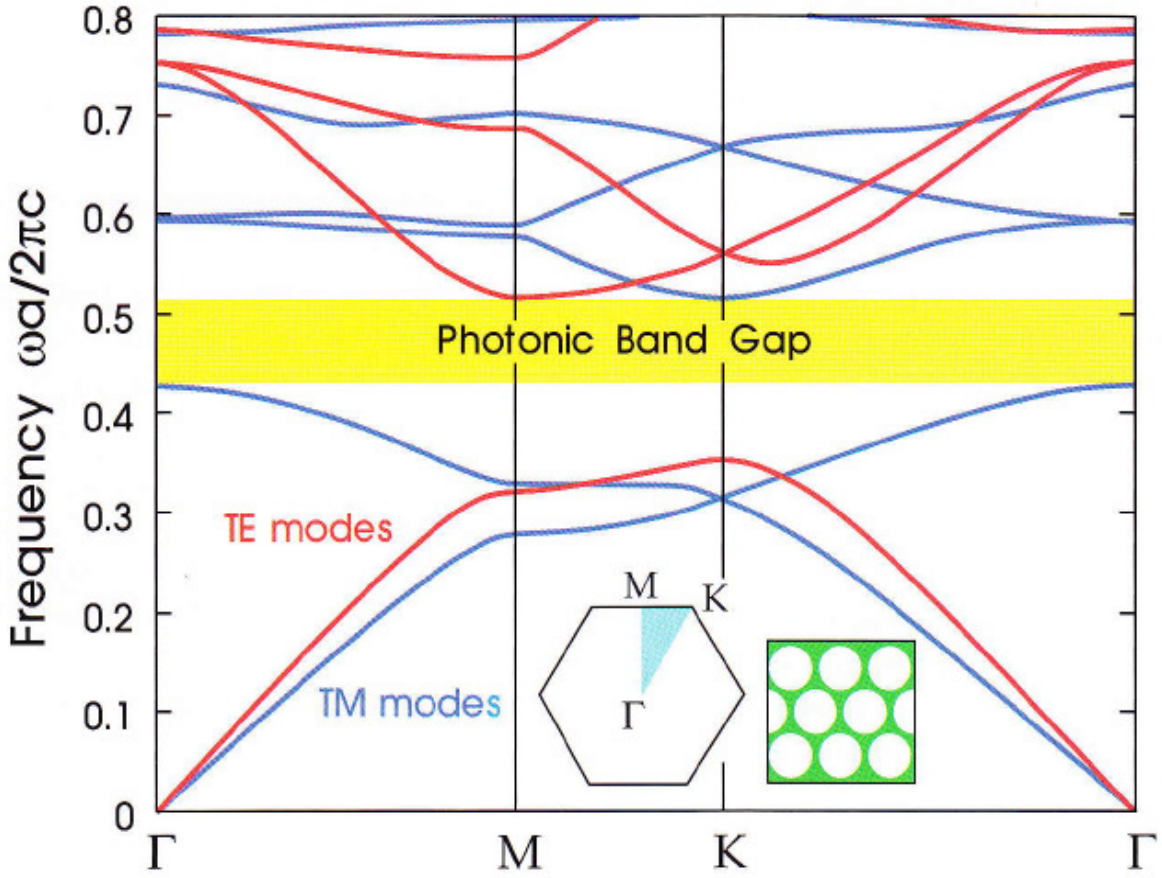


Figure 10: The photonic band structure for the modes of a triangular array of air columns drilled in a dielectric substrate ($\varepsilon = 13$). The blue lines represent TM bands and the red lines represent TE bands. The inset shows the high-symmetry points at the corners of the irreducible Brillouin zone (shaded light blue). Note the complete photonic band gap.

Propagazione lungo z

Band gap scompare

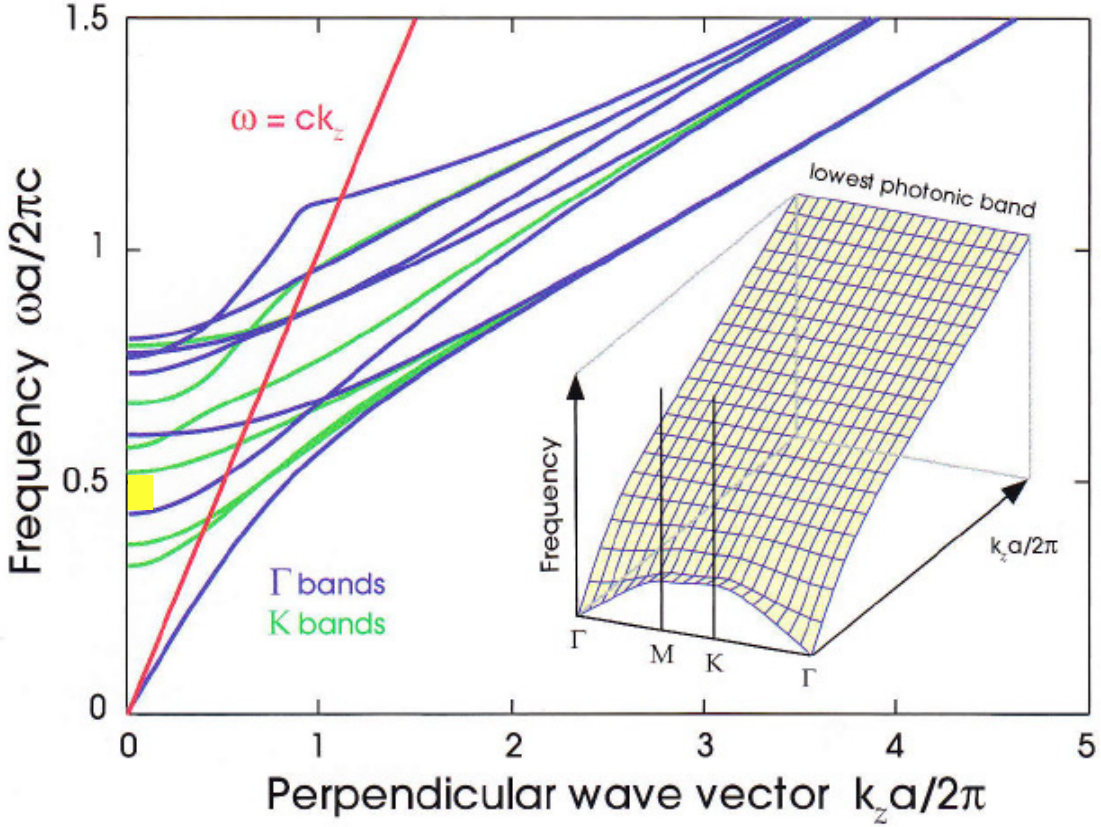
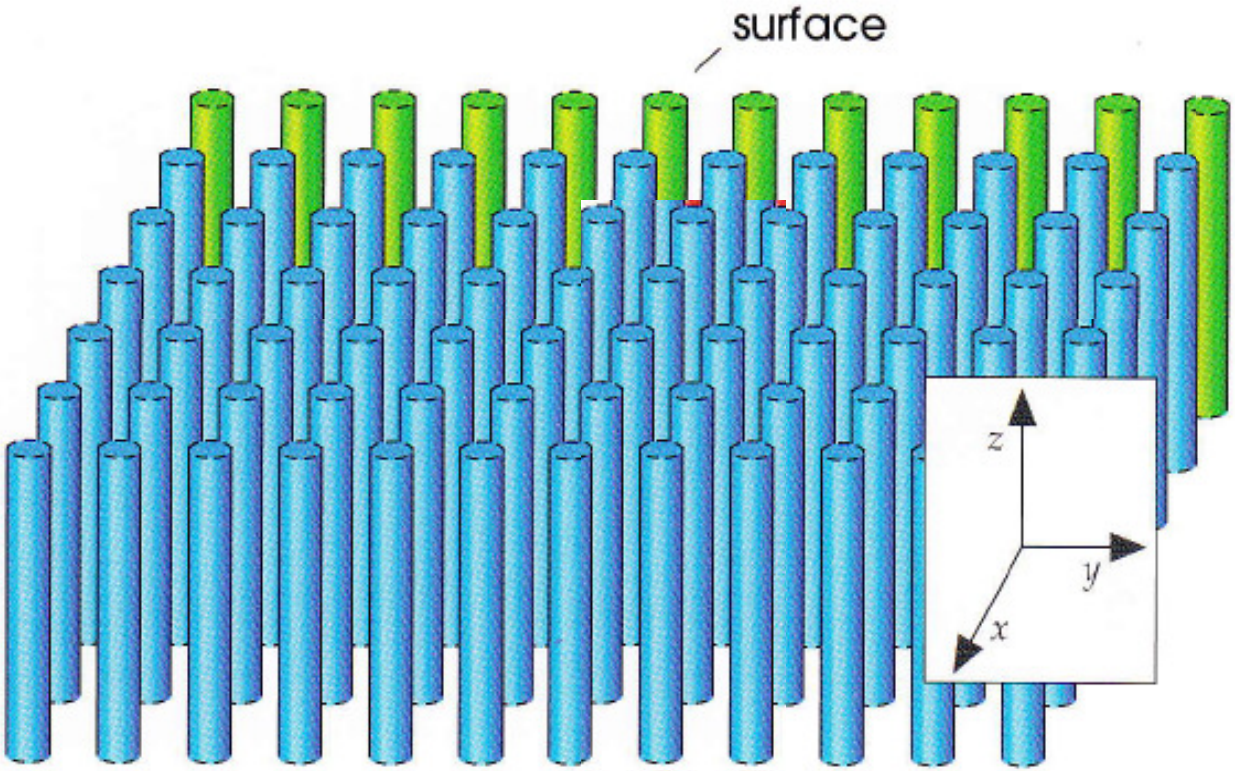


Figure 11: The out-of-plane band structure of the triangular lattice of air columns for the first few bands. The bands that start at Γ, $\omega(\Gamma, k_z)$, are plotted with blue lines, whereas the bands that start at K, $\omega(\mathrm{K}, k_z)$, are plotted with green lines. The light line $\omega = ck_z$ (red) separates the modes that are oscillatory ($\omega \geq ck_z$) in the air regions from those that are evanescent ($\omega < ck_z$) in the air regions. The inset shows the frequency dependence of the lowest band as k_z varies. Note that as k_z increases, the lowest band flattens.

Superficie



Conservazione k_y: cono di luce

Terminazione intera

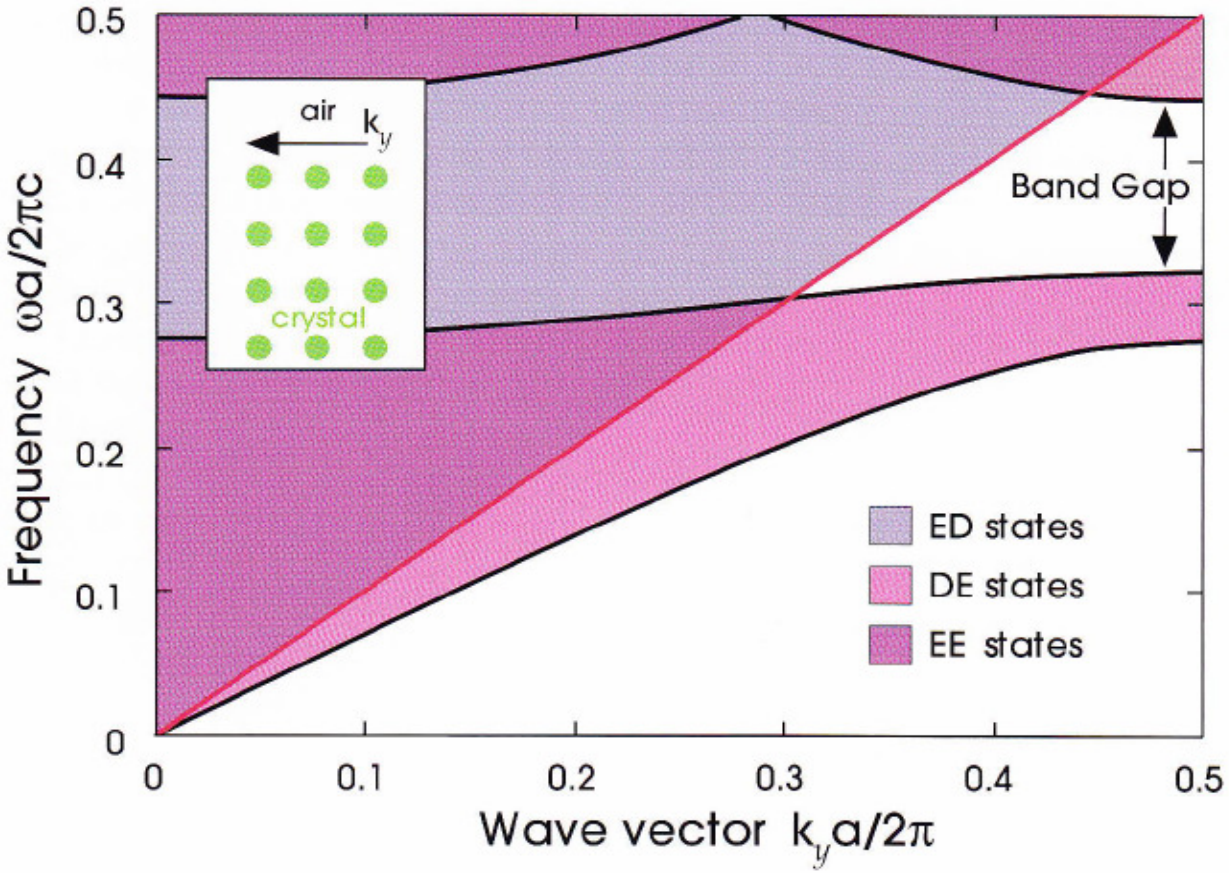


Figure 20: The projected band structure of the constant-x surface of the square lattice of alumina rods in air. The shading denotes regions in which light is transmitted (purple EE states), internally reflected (red DE states), and externally reflected (blue ED states). The crystal is terminated as shown in the inset; this termination has no surface states.

Conservazione k_y: cono di luce

Terminazione
Semi-intera

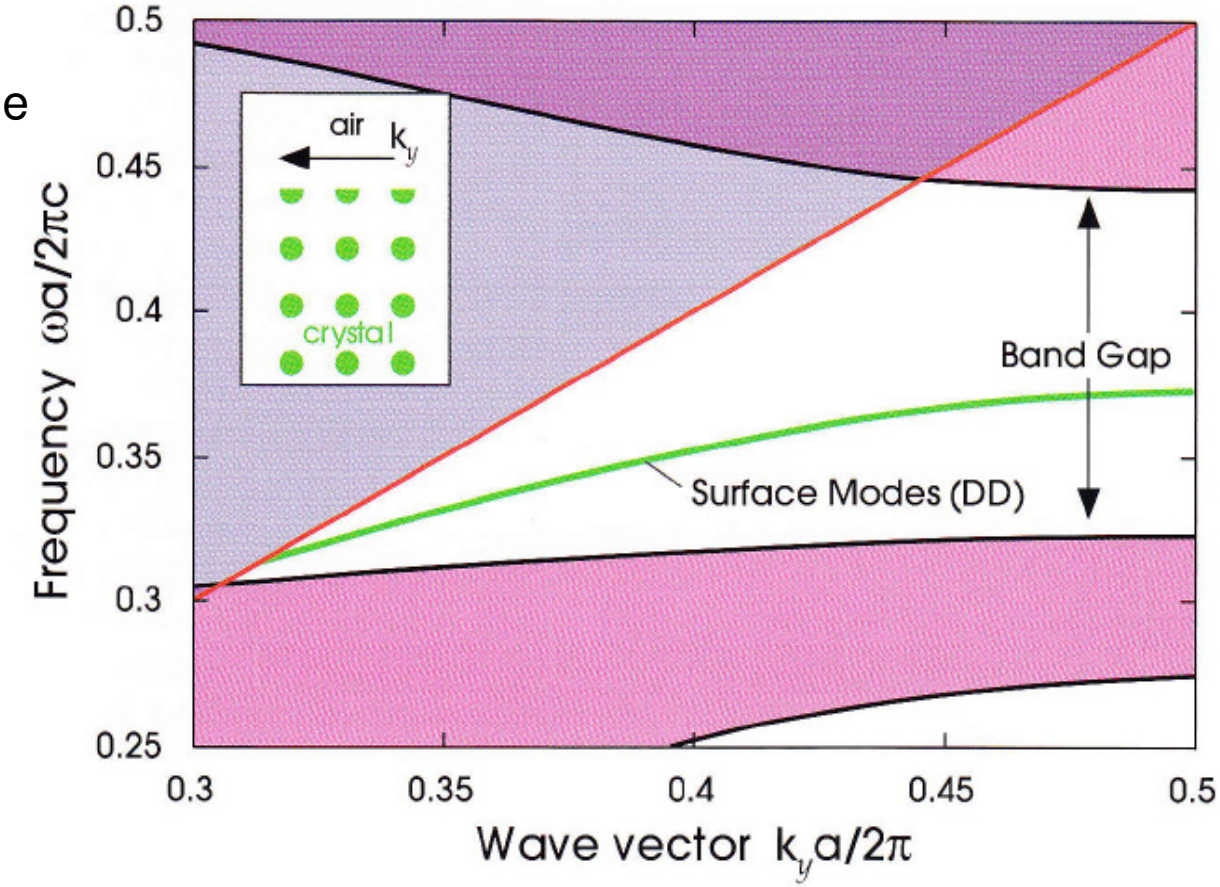


Figure 21: The projected band structure of the constant-x surface of the square lattice of alumina rods in air, like figure 20 above except with a different termination shown in the inset. The line in the gap corresponds to a surface band in which light is exponentially localized to the surface (green, DD).

Stato di superficie

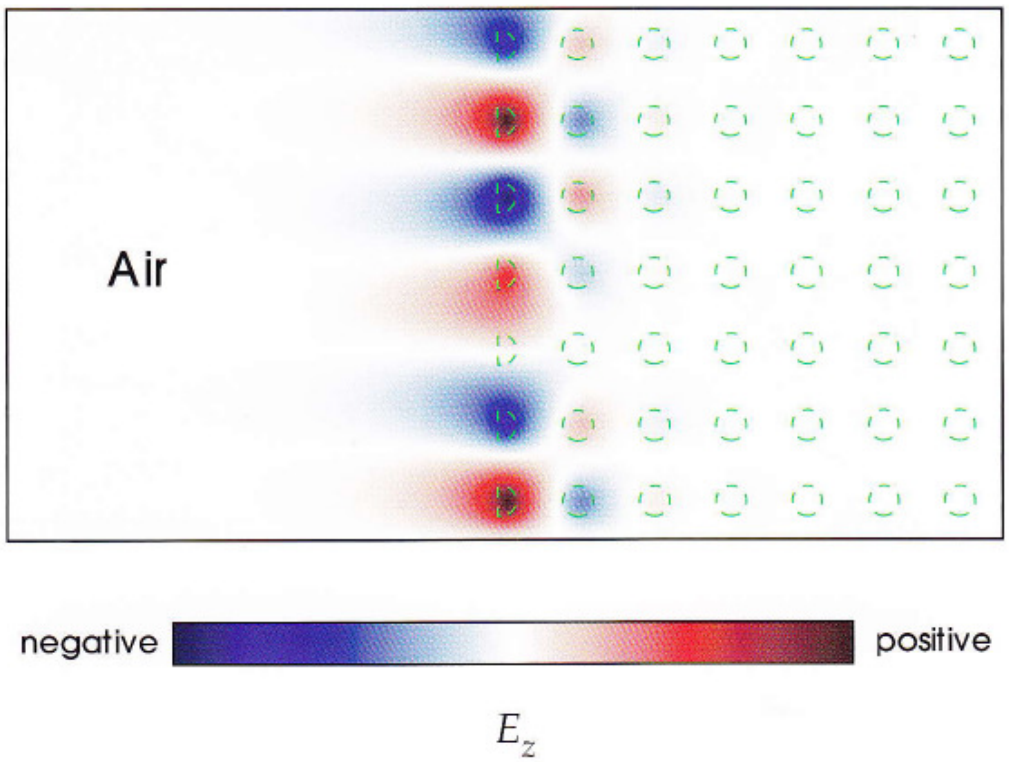


Figure 22: Electric-field (E_z) pattern associated with a surface-localized state formed by truncating a square lattice of alumina rods in air, cutting each rod in half at the boundary as in figure 21. The mode shown corresponds to a surface-parallel wave vector $k_y = 0.4\,(2\pi/a)$. The dielectric rods are shown as dashed green outlines.