Equazioni Maxwell semplificate nella materia

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M} = \frac{\vec{B}}{\mu}$$

Equazioni onde

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E} \right) = -\mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Onde piane

$$\vec{k} \times \left(\vec{k} \times \vec{E} \right) = -\omega^2 \mu \varepsilon \vec{E}$$

$$\mu \varepsilon = \frac{n^2}{c^2}$$

Dielettrici anisotropi

$$\vec{D} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zx} \end{pmatrix} \vec{E} \qquad \varepsilon_{ij} = \varepsilon_{ji}$$

Assi principali

$$\vec{D} = \begin{pmatrix} \varepsilon_{x} & 0 & 0 \\ 0 & \varepsilon_{y} & 0 \\ 0 & 0 & \varepsilon_{z} \end{pmatrix} \vec{E} \qquad D_i = \varepsilon_i E_i$$

Equazioni Onde mezzi anisotropi

$$\vec{k} \times \left(\vec{k} \times \vec{E}\right) = -\omega^2 \mu \varepsilon \vec{E}$$

$$\begin{pmatrix} \omega^2\mu\varepsilon_x - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_y k_x & \omega^2\mu\varepsilon_y - k_x^2 - k_z^2 & k_y k_z \\ k_x k_z & k_y k_z & \omega^2\mu\varepsilon_z - k_x^2 - k_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

Materiali uniassici

$$\varepsilon_x = \varepsilon_y = \varepsilon_0 n_o^2 \qquad \varepsilon_z = \varepsilon_0 n_e^2$$

Equazioni Onde mezzi uniassici ($k_x=0$)

$$\begin{pmatrix} \frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 & 0 & 0 \\ 0 & \frac{\omega^2 n_o^2}{c^2} - k_z^2 & k_y k_z \\ 0 & k_y k_z & \frac{\omega^2 n_e^2}{c^2} - k_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$$\left(\frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 \right) \left[\left(\frac{\omega^2 n_o^2}{c^2} - k_z^2 \right) \left(\frac{\omega^2 n_e^2}{c^2} - k_y^2 \right) - k_y^2 k_z^2 \right] = 0$$

Equazioni Onde mezzi uniassici ($k_x = 0$)

Onda ordinaria

$$\left(\frac{\omega^2 n_o^2}{c^2} - k_y^2 - k_z^2 \right) = 0 \qquad \vec{E} = (E_x, 0, 0)$$

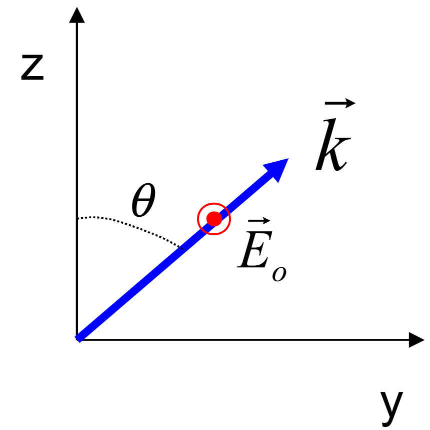
Onda straordinaria

$$\left[\left(\frac{\omega^2 n_o^2}{c^2} - k_z^2 \right)\left(\frac{\omega^2 n_e^2}{c^2} - k_y^2 \right) - k_y^2 k_z^2 \right] = 0 \qquad \vec{E} = (0, E_y, E_z)$$

Equazioni Onde mezzi uniassici ($k_x=0$)

Onda ordinaria (TE)

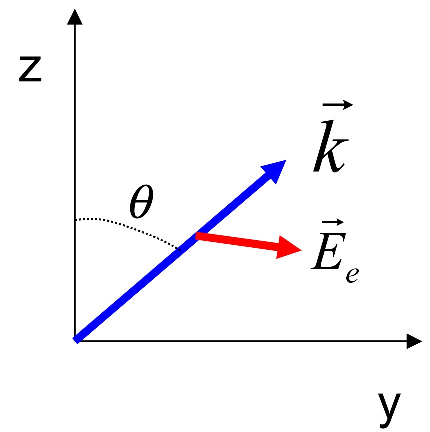
$$k^2 = \frac{\omega^2 n_o^2}{c^2} \qquad \vec{E}_o = (E_x, 0, 0)$$



Onda straordinaria (TM)

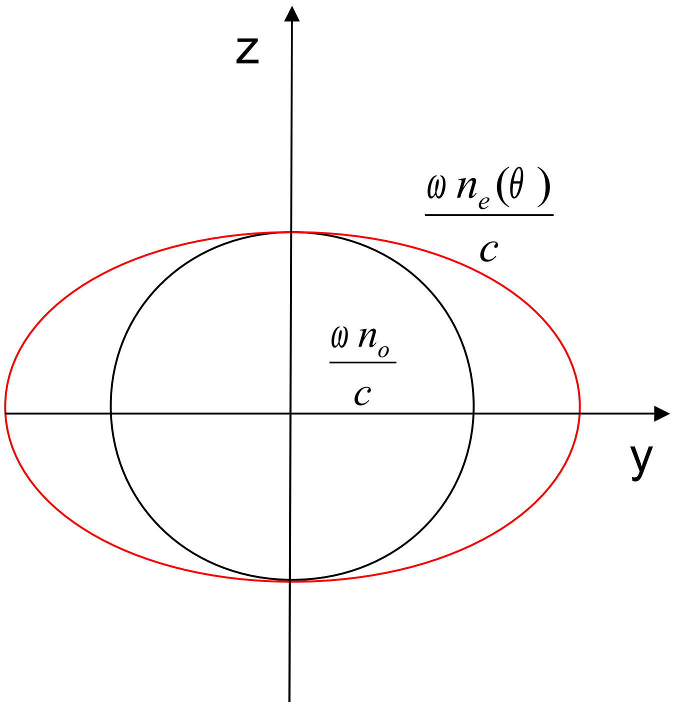
$$k^2 = \frac{\omega^2 n_e(\theta)^2}{c^2} \qquad \vec{E}_e = (0, E_y, E_z)$$

$$\frac{1}{n_e(\theta)^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$



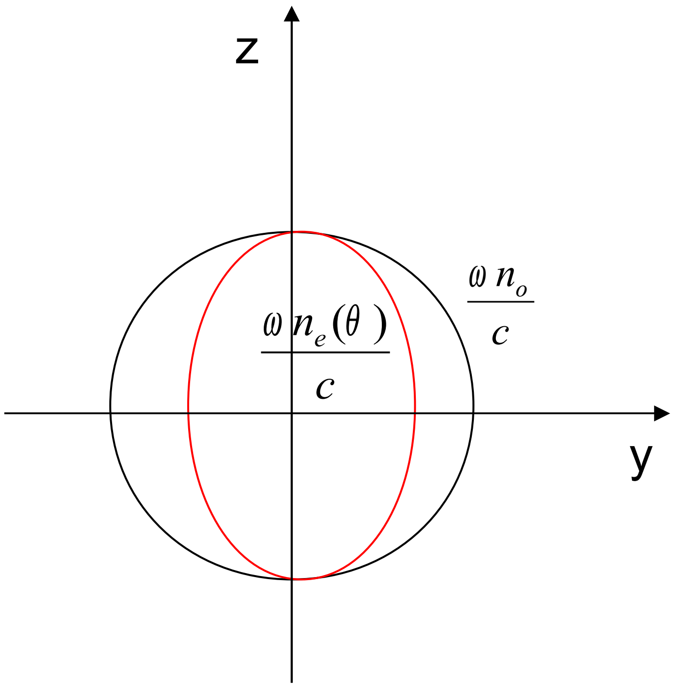
Superfici isofrequenza

$$n_e > n_o$$



Ghiaccio	1.309	1.310
Quarzo	1.544	1.553
ZnS	2.354	2.358

$$n_e < n_o$$



Tormalina	1.638	1.618
Calcite	1.658	1.486
KDP	1.507	1.467

Equazioni Maxwell in onde piane

$$\vec{k} \cdot \vec{D} = 0 \qquad \vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \cdot \vec{B} = 0 \qquad \vec{k} \times \vec{H} = -\omega \vec{D}$$

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu_o}$$

$$\vec{k} \cdot \vec{D} = 0 \qquad \vec{D} = \varepsilon \vec{E}$$

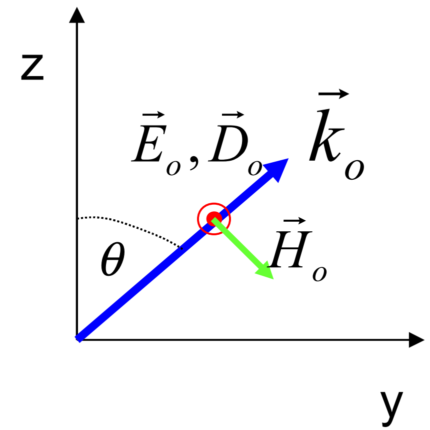
$$\vec{k} \times \vec{H} = -\omega \vec{D}$$

Onda ordinaria

$$\vec{k}_o = \frac{\omega n_o}{c}\left(0, \sin\theta, \cos\theta\right)$$

$$\vec{D}_o = D(1,0,0)$$

$$\vec{E}_o = \frac{D}{\varepsilon_x}(1,0,0) \qquad \vec{H}_o = \frac{c}{n_o} D(0, \cos\theta, -\sin\theta)$$

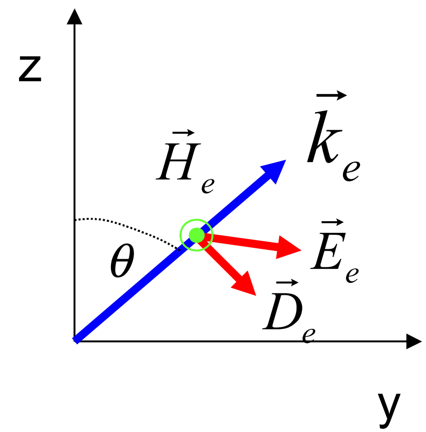


Onda straordinaria

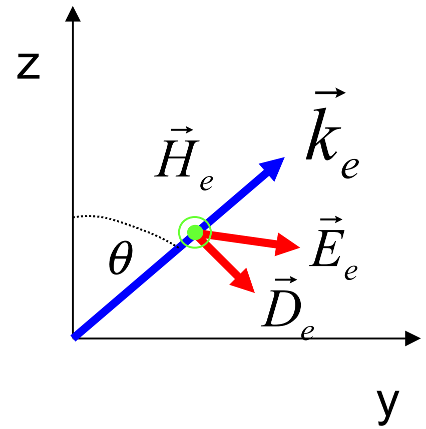
$$\vec{k}_e = \frac{\omega n_e(\theta)}{c}\left(0, \sin\theta, \cos\theta\right)$$

$$\vec{D}_e = D(0, \cos\theta, -\sin\theta)$$

$$\vec{E}_e = D\left(0, \frac{\cos\theta}{\varepsilon_x}, -\frac{\sin\theta}{\varepsilon_z}\right) \qquad \vec{H}_o = \frac{c}{n_e(\theta)} D(1,0,0)$$



Onda straordinaria:
Componente longitudinale



$$\vec{k}_e = \frac{\omega\, n_e(\theta)}{c}\left(0, \sin\theta, \cos\theta\right)$$

$$\vec{D}_e = D(0, \cos\theta, -\sin\theta)$$

$$\vec{E}_e = D\left(0, \frac{\cos\theta}{\varepsilon_x}, -\frac{\sin\theta}{\varepsilon_z}\right) \qquad \vec{H}_o = \frac{c}{n_e(\theta)} D(1,0,0)$$

$$\vec{E}_{e,long} = \frac{\vec{k}_e \cdot \vec{E}_e}{\left|\vec{k}_e\right|} = D\left(\frac{\cos\theta\ \sin\theta}{\varepsilon_x} - \frac{\sin\theta\ \cos\theta}{\varepsilon_z}\right) =$$

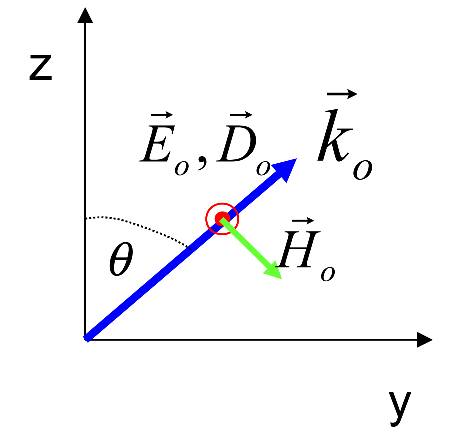
$$= \frac{D\cos\theta\ \sin\theta}{\varepsilon_0}\,\frac{n_e^{\,2} - n_o^{\,2}}{n_o^{\,2} n_e^{\,2}} = \boxed{\frac{D\cos\theta\ \sin\theta}{\varepsilon_0}\,\frac{\left(n_e - n_o\right)\left(n_e + n_o\right)}{n_o^{\,2} n_e^{\,2}}}$$

Onda ordinaria

$$\vec{k}_o = \frac{\omega n_o}{c}\left(0, \sin\theta, \cos\theta\right)$$

$$\vec{E}_o = \frac{D}{\varepsilon_x}(1,0,0) \qquad \vec{H}_o = \frac{c}{n_o} D(0, \cos\theta, -\sin\theta)$$

$$\vec{S} = \frac{c}{\varepsilon_x n_o}|D|^2(0, \sin\theta, \cos\theta) \quad // \vec{k}$$

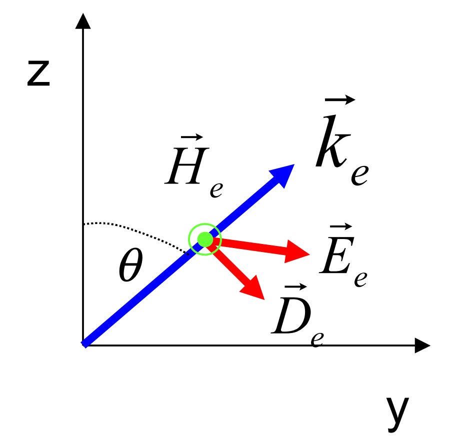


Onda straordinaria

$$\vec{k}_e = \frac{\omega n_e(\theta)}{c}\left(0, \sin\theta, \cos\theta\right)$$

$$\vec{E}_e = D\left(0, \frac{\cos\theta}{\varepsilon_x}, -\frac{\sin\theta}{\varepsilon_z}\right) \qquad \vec{H}_e = -\frac{c}{n_e(\theta)} D(1,0,0)$$

$$\vec{S} = \frac{c}{n_e(\theta)}|D|^2\left(0, \frac{\sin\theta}{\varepsilon_x}, \frac{\cos\theta}{\varepsilon_z}\right) \quad \not\parallel \vec{k}$$



Velocità di gruppo

$$\omega^2 = \frac{k^2c^2}{n_e(\theta)^2} = k^2c^2\left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right) = c^2\left(\frac{k_z^2}{n_o^2} + \frac{k_y^2}{n_e^2}\right)$$

$$\omega = c\sqrt{\frac{k_z^2}{n_o^2} + \frac{k_y^2}{n_e^2}}$$

$$\vec{v}_g = \vec{\nabla}_k\omega = \frac{c^2}{\omega}\left(0, \frac{k_y}{n_e^2}, \frac{k_z}{n_o^2}\right) = n_e(\theta)c\varepsilon_0\left(0, \frac{\sin\theta}{\varepsilon_z}, \frac{\cos\theta}{\varepsilon_x}\right)$$

$$\vec{S} = \frac{c}{n_e(\theta)}|D|^2\left(0, \frac{\sin\theta}{\varepsilon_z}, \frac{\cos\theta}{\varepsilon_x}\right) = \frac{1}{\varepsilon_0 n_e^2(\theta)}|D|^2\vec{v}_g$$

Velocità energia

$$\vec{S} \equiv U_{em}\vec{v}_e$$

$$\vec{D}_e = D(0,\cos\theta, -\sin\theta)$$

$$\vec{E}_e = D\left(0, \frac{\cos\theta}{\varepsilon_x}, -\frac{\sin\theta}{\varepsilon_z}\right) \qquad \vec{H}_e = \frac{c}{n_e(\theta)}D(1,0,0)$$

$$U_{em} = \vec{E}_e \cdot \vec{D}_e^{\,*} = |D|^2\left(\frac{\cos^2\theta}{\varepsilon_x} + \frac{\sin^2\theta}{\varepsilon_z}\right) =$$

$$= \frac{|D|^2}{\varepsilon_0}\left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right) = \frac{|D|^2}{\varepsilon_0 n_e^2(\theta)}$$

$$\vec{S} = \frac{1}{\varepsilon_0 n_e(\theta)}|D|^2\vec{v}_g = U_{em}\vec{v}_g \qquad \vec{v}_e = \vec{v}_g$$

Velocità energia=Velocità gruppo

$$\vec{k} \times \vec{E} = \omega \mu \vec{H} \qquad \vec{k} \times \vec{H} = -\omega \varepsilon \vec{E}$$

$$\begin{cases} \delta\vec{k} \times \vec{E} + \vec{k} \times \delta\vec{E} = \delta\omega \mu \vec{H} + \omega \mu \delta\vec{H} \\ \delta\vec{k} \times \vec{H} + \vec{k} \times \delta\vec{H} = -\delta\omega \varepsilon \vec{E} - \omega \varepsilon \delta\vec{E} \end{cases}$$

$$\begin{cases} \vec{H} \cdot \left(\delta\vec{k} \times \vec{E} + \vec{k} \times \delta\vec{E}\right) = \vec{H} \cdot \left(\delta\omega \mu \vec{H} + \omega \mu \delta\vec{H}\right) \\ \vec{E} \cdot \left(\delta\vec{k} \times \vec{H} + \vec{k} \times \delta\vec{H}\right) = -\vec{E} \cdot \left(\delta\omega \varepsilon \vec{E} + \omega \varepsilon \delta\vec{E}\right) \end{cases}$$

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \vec{B} \cdot \left(\vec{C} \times \vec{A}\right) = \vec{C} \cdot \left(\vec{A} \times \vec{B}\right)$$

Velocità energia=Velocità gruppo

$$\begin{cases} \delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)+\vec{k}\cdot\left(\delta\vec{E}\times\vec{H}\right)=\delta\omega\left(\vec{H}\cdot\mu\vec{H}\right)+\omega\left(\vec{H}\cdot\mu\,\delta\vec{H}\right) \\ \delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)+\vec{k}\cdot\left(\delta\vec{H}\times\vec{E}\right)=-\delta\omega\left(\vec{E}\cdot\varepsilon\vec{E}\right)-\omega\left(\vec{E}\cdot\varepsilon\,\delta\vec{E}\right) \end{cases}$$

$$\vec{E}\cdot\varepsilon\,\delta\vec{E}=\delta\vec{E}\cdot\varepsilon\vec{E} \qquad \vec{H}\cdot\mu\,\delta\vec{H}=\delta\vec{H}\cdot\mu\vec{H}$$

$$2\delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)+\vec{k}\cdot\left(\delta\vec{E}\times\vec{H}\right)-\vec{k}\cdot\left(\delta\vec{H}\times\vec{E}\right)=$$
$$\delta\omega\left(\vec{H}\cdot\mu\vec{H}+\vec{E}\cdot\varepsilon\vec{E}\right)+\omega\left(\delta\vec{H}\cdot\mu\vec{H}+\delta\vec{E}\cdot\varepsilon\vec{E}\right)$$

$$2\delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)-\delta\omega\left(\vec{H}\cdot\mu\vec{H}+\vec{E}\cdot\varepsilon\vec{E}\right)=$$
$$-\vec{k}\cdot\left(\delta\vec{E}\times\vec{H}\right)+\vec{k}\cdot\left(\delta\vec{H}\times\vec{E}\right)+\omega\left(\delta\vec{H}\cdot\mu\vec{H}+\delta\vec{E}\cdot\varepsilon\vec{E}\right)$$

Velocità energia=Velocità gruppo

$$2\delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)-\delta\omega\left(\vec{H}\cdot\mu\vec{H}+\vec{E}\cdot\varepsilon\vec{E}\right)=$$
$$\delta\vec{E}\cdot\left(\vec{k}\times\vec{H}\right)-\delta\vec{H}\cdot\left(\vec{k}\times\vec{E}\right)+\omega\left(\delta\vec{H}\cdot\mu\vec{H}+\delta\vec{E}\cdot\varepsilon\vec{E}\right)$$

$$2\delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)-\delta\omega\left(\vec{H}\cdot\mu\vec{H}+\vec{E}\cdot\varepsilon\vec{E}\right)=$$
$$\delta\vec{E}\cdot\left(\vec{k}\times\vec{H}+\omega\varepsilon\vec{E}\right)-\delta\vec{H}\cdot\left(\vec{k}\times\vec{E}-\omega\mu\vec{H}\right)=0$$

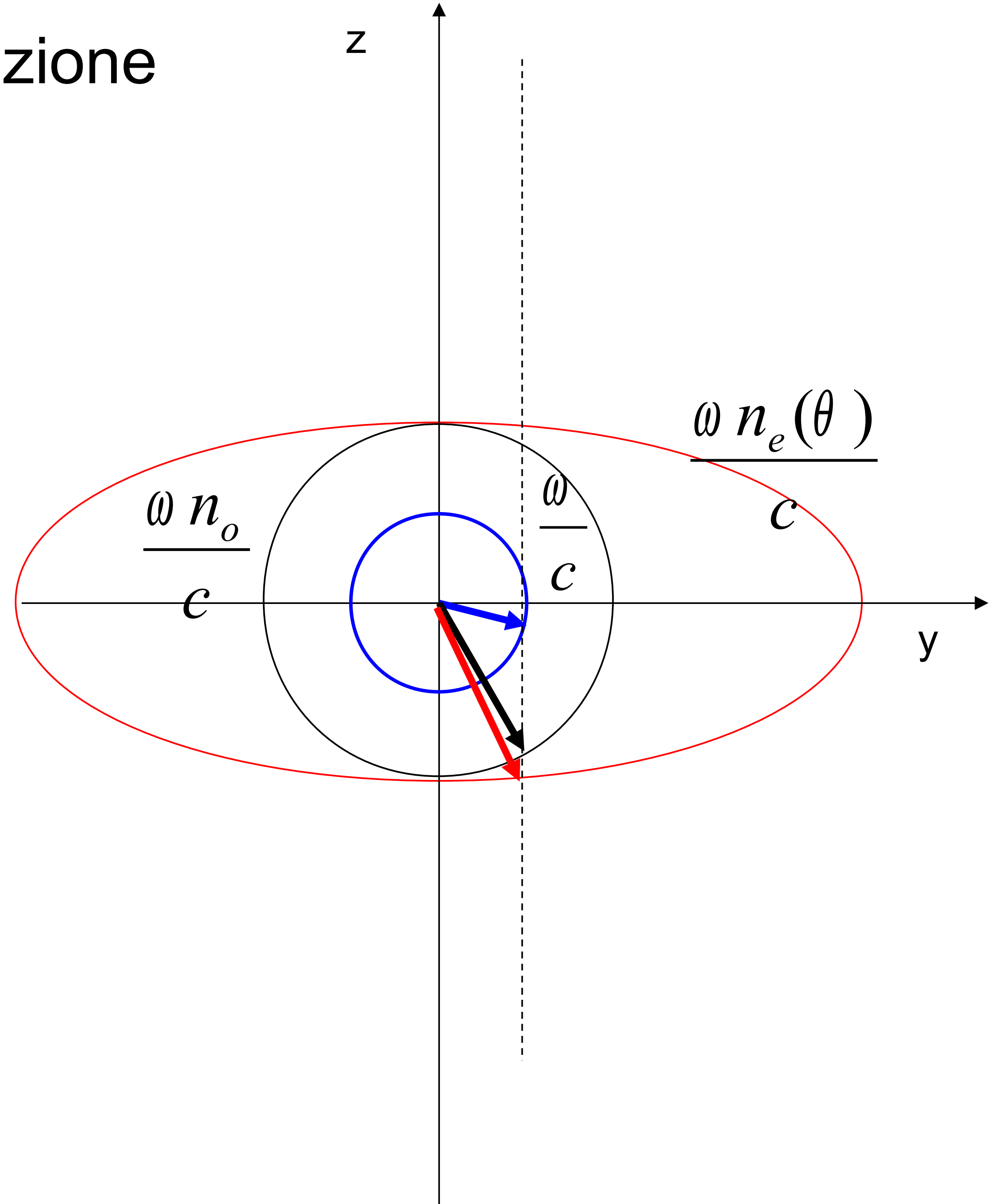
$$\delta\vec{k}\cdot\left(\vec{E}\times\vec{H}\right)=\delta\omega\,\frac{1}{2}\left(\vec{H}\cdot\mu\vec{H}+\vec{E}\cdot\varepsilon\vec{E}\right)$$

$$\delta\omega=\delta\vec{k}\cdot\frac{\vec{S}}{U_{el}}\qquad\qquad\delta\omega=\delta\vec{k}\cdot\vec{\nabla}_k\omega\left(\vec{k}\right)\qquad\therefore$$

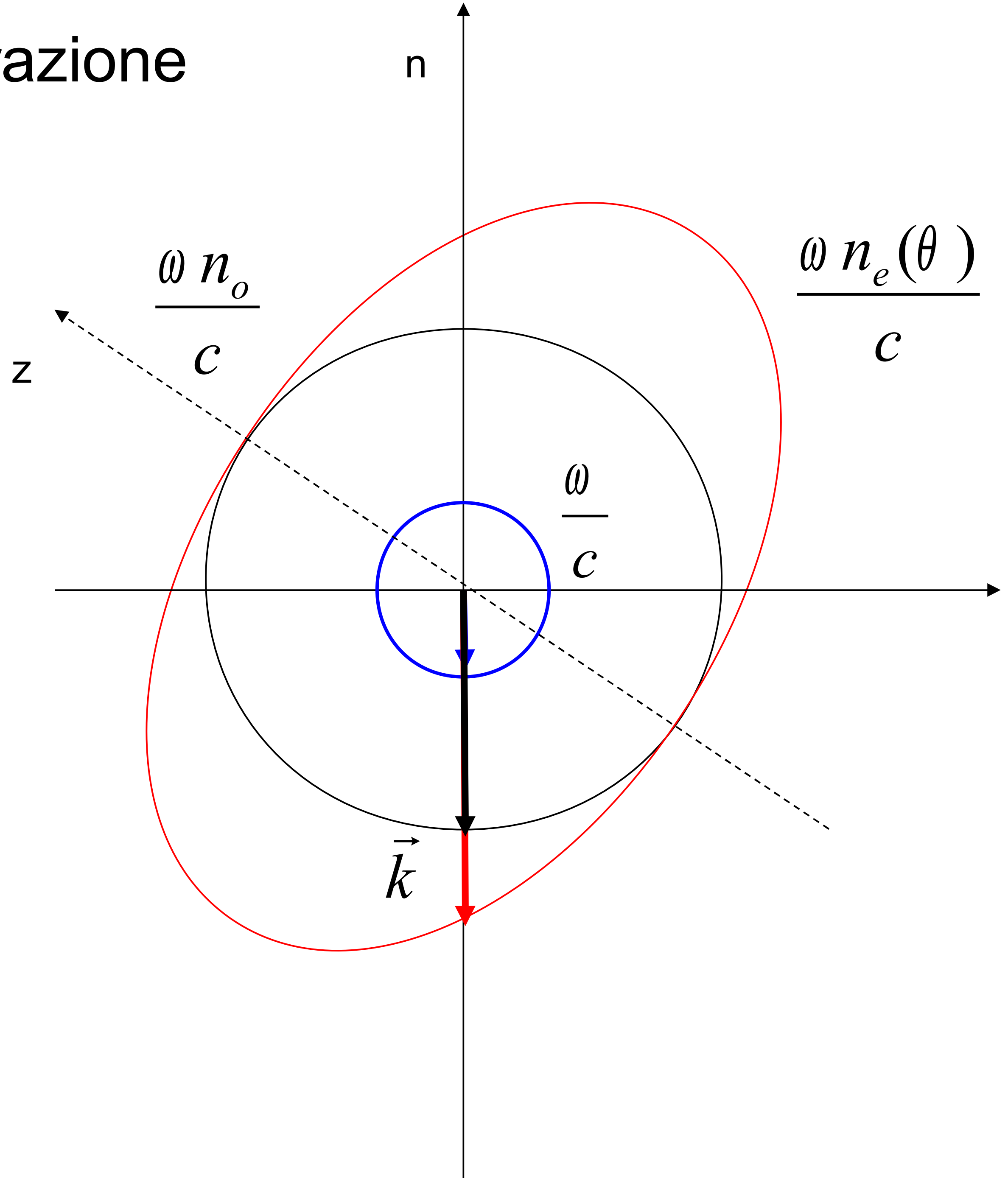
Doppia rifrazione

NOTA: effetto anche ad incidenza normale

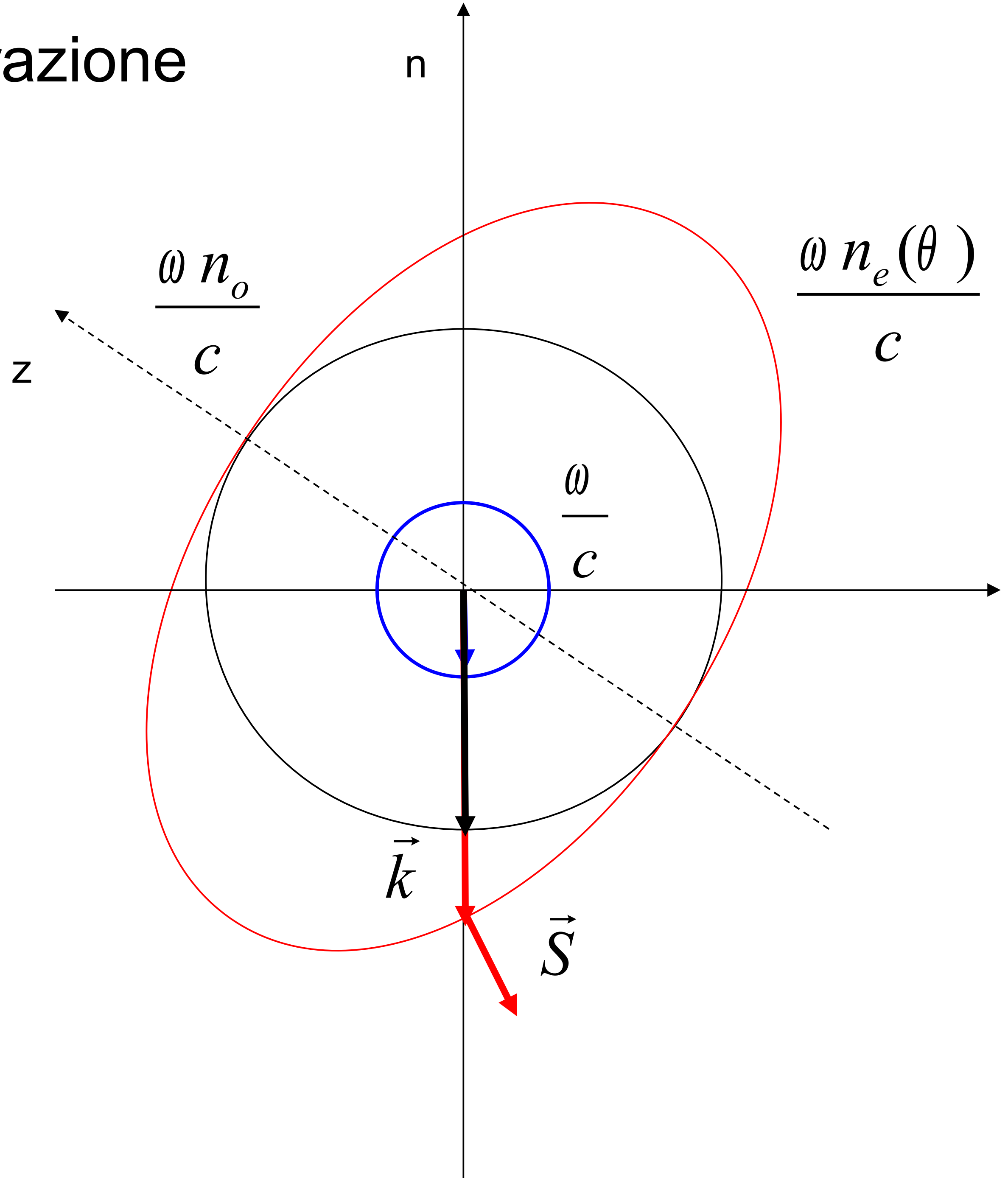
Doppia rifrazione



Doppia rifrazione

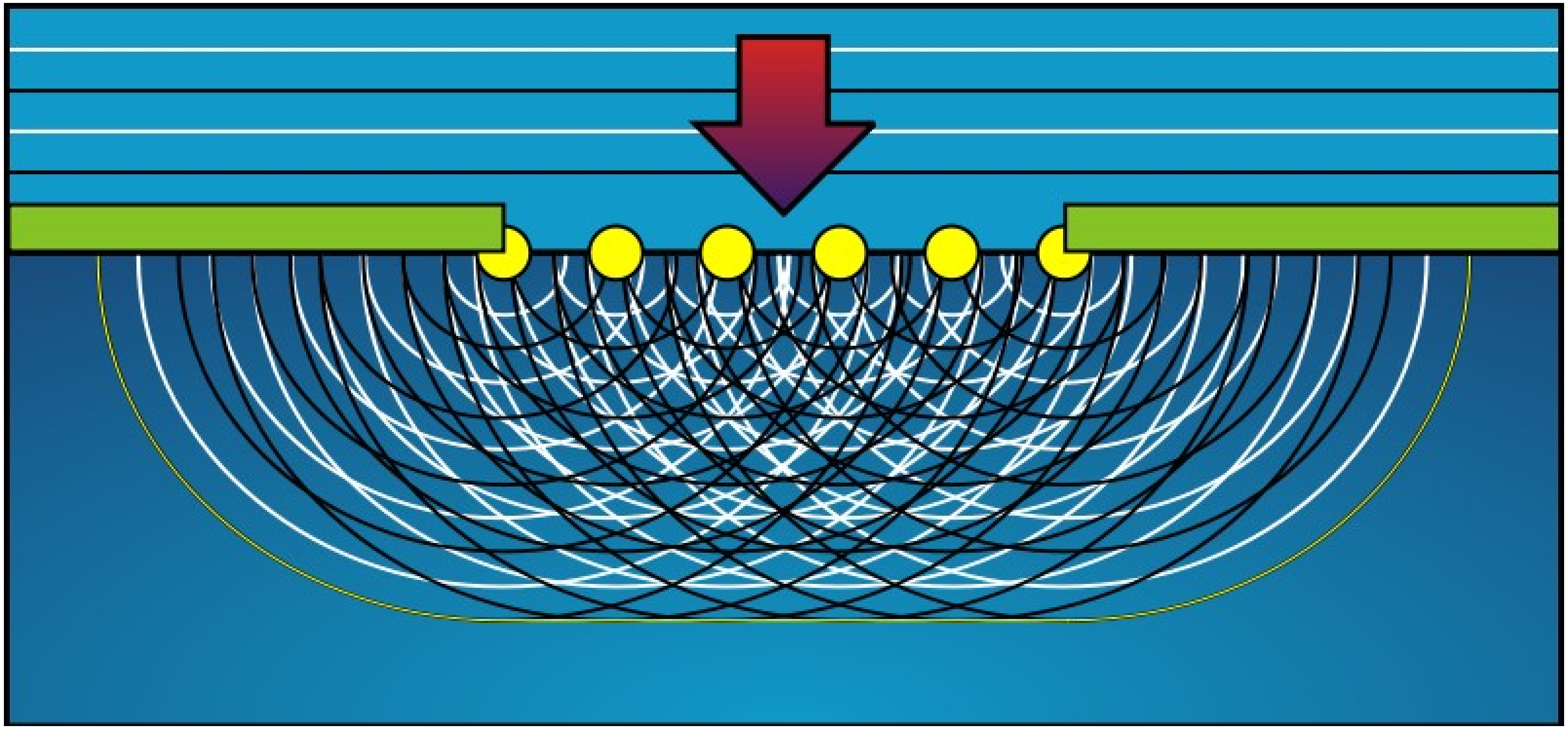


Doppia rifrazione

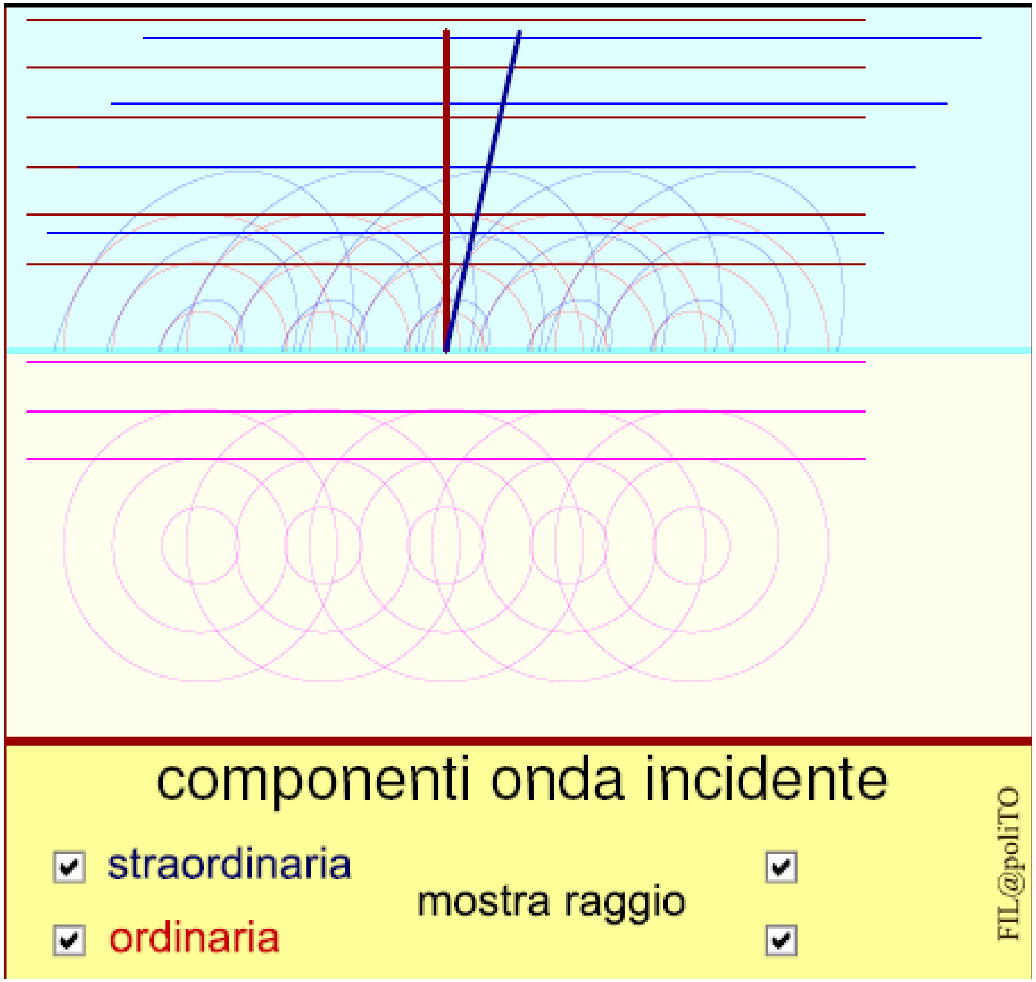


Principio di Huygens

$$\frac{u_0}{i\lambda}\int_\Sigma \frac{e^{ikr}}{r} f(\theta)\mathrm{d}\Sigma$$



Principio di Huygens

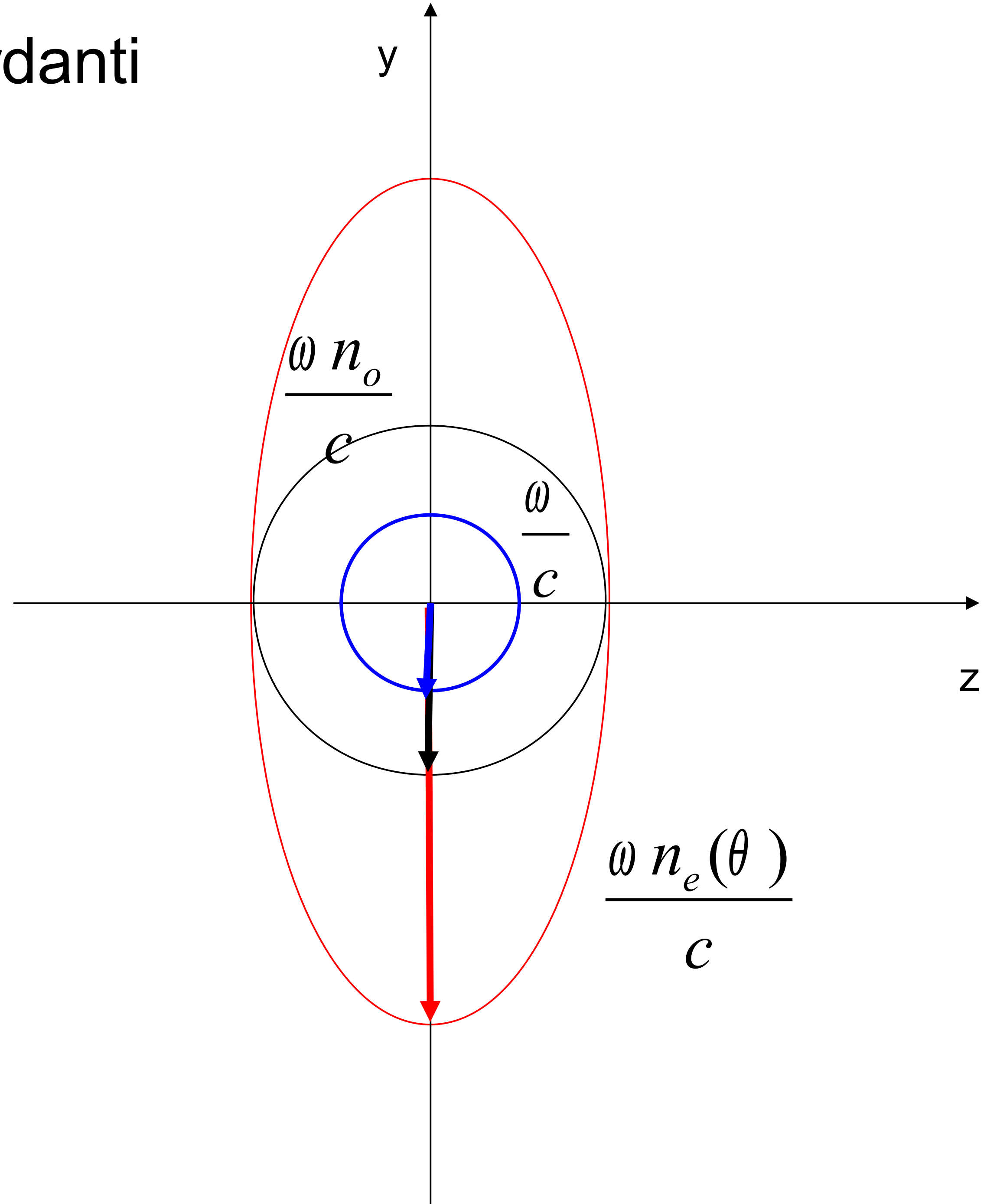


Per vedere immagine in movimento:
http://www2.polito.it/ricerca/qdbf/fil/indicegenerale/ottica/ottica_fisica/birifrangenza.htm

Lamine ritardanti

$$\vec{k}_e = \frac{\omega n_e}{c} \hat{e}_y$$

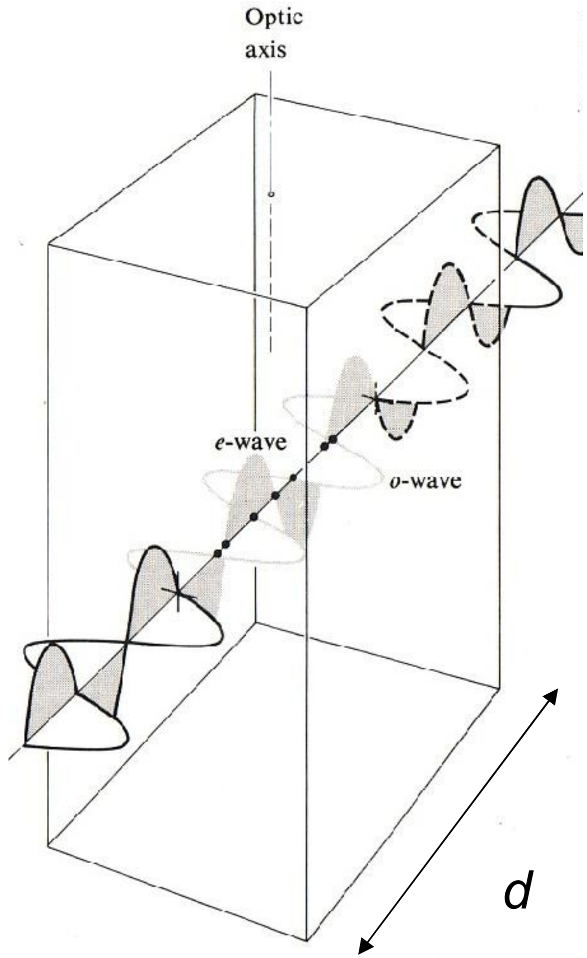
$$\vec{k}_o = \frac{\omega n_o}{c} \hat{e}_y$$

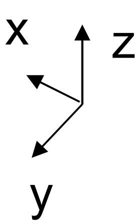


Lamine ritardanti

$$\vec{k}_e = \frac{\omega\, n_e}{c}\hat{e}_y$$

$$\vec{k}_o = \frac{\omega\, n_o}{c}\hat{e}_y$$





$$\varphi_e = \frac{\omega\, n_e}{c} d = \frac{2\pi\, n_e}{\lambda} d$$

$$\varphi_o = \frac{\omega\, n_o}{c} d = \frac{2\pi\, n_o}{\lambda} d$$

$$E_{in} = E_0\left(\hat{e}_x + \hat{e}_z\right)e^{j(ky-\omega t)}$$

$$E_{out} = E_0\left(\hat{e}_x e^{j\varphi_o} + \hat{e}_z e^{j\varphi_e}\right)e^{j(ky-\omega t)} =$$

$$= E_0\left(\hat{e}_x + \hat{e}_z e^{j(\varphi_e-\varphi_o)}\right)e^{j\varphi_o}e^{j(ky-\omega t)}$$

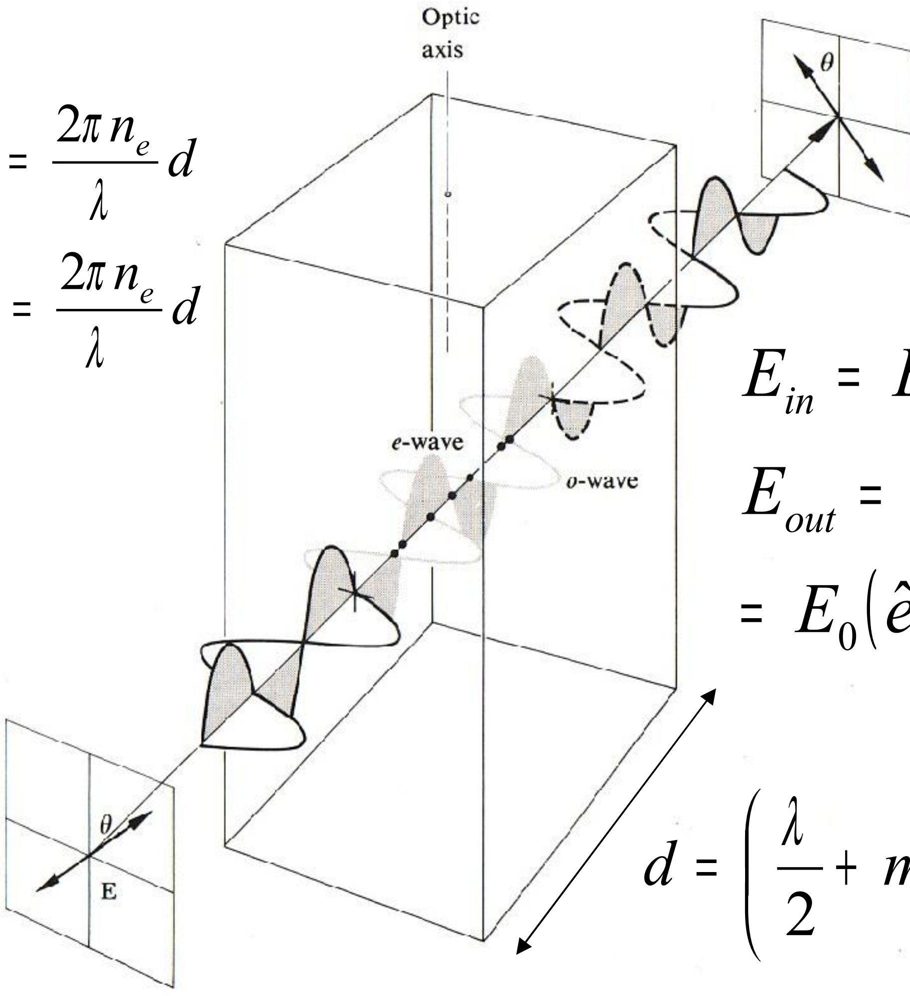
$$\varphi_e - \varphi_o = \frac{2\pi}{\lambda} d\left(n_e - n_o\right)$$

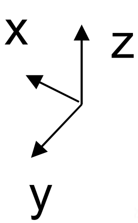
Lamine λ/2

$$\varphi_e - \varphi_o = \pi\,(2m+1)$$

$$\varphi_e = \frac{\omega\, n_e}{c} d = \frac{2\pi\, n_e}{\lambda} d$$

$$\varphi_o = \frac{\omega\, n_o}{c} d = \frac{2\pi\, n_e}{\lambda} d$$





$$E_{in} = E_0\left(\hat{e}_x + \hat{e}_z\right)e^{j(ky-\omega t)}$$

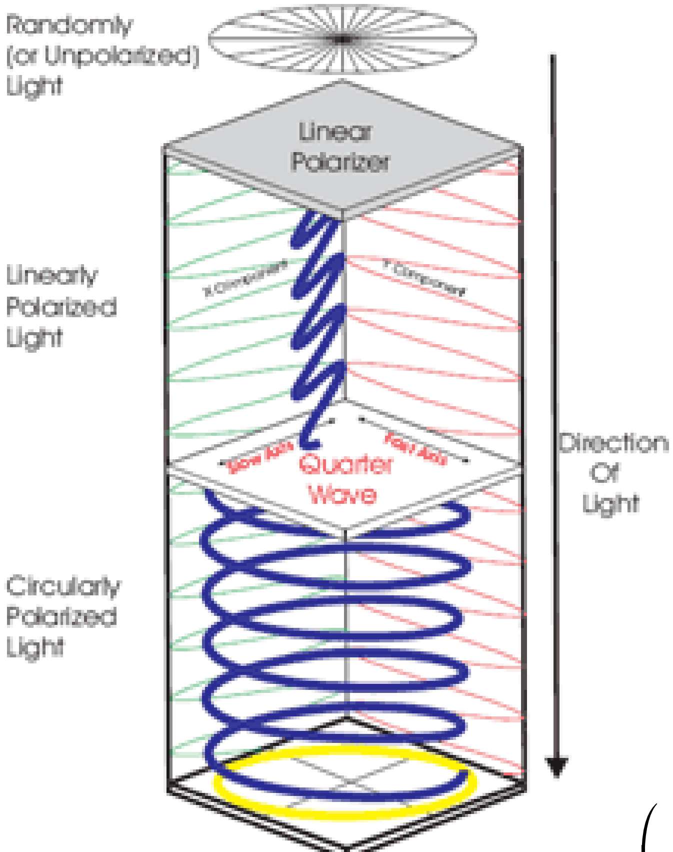
$$E_{out} = E_0\left(\hat{e}_x + \hat{e}_z e^{j\pi}\right)e^{j\varphi_o}e^{j(ky-\omega t)} =$$

$$= E_0\left(\hat{e}_x - \hat{e}_z\right)e^{j\varphi_o}e^{j(ky-\omega t)}$$

$$d = \left(\frac{\lambda}{2} + m\lambda\right)\frac{1}{n_e - n_o}$$

Lamine λ/4

$$\varphi_e - \varphi_o = \pi\left(2m + \frac{1}{2}\right)$$



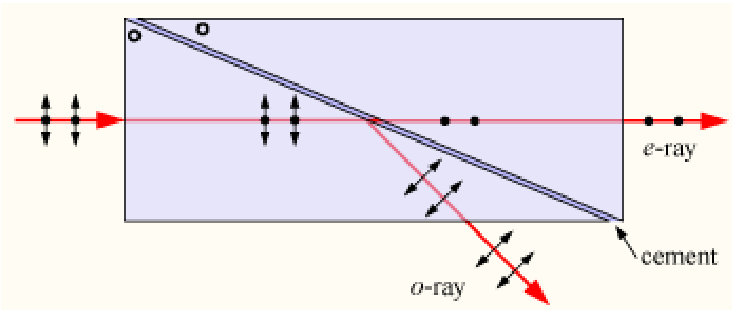
$$E_{in} = E_0\left(\hat{e}_x + \hat{e}_z\right)e^{j(ky - \omega t)}$$

$$E_{out} = E_0\left(\hat{e}_x + \hat{e}_z e^{j\frac{\pi}{2}}\right)e^{j\varphi_o}e^{j(ky-\omega t)} =$$

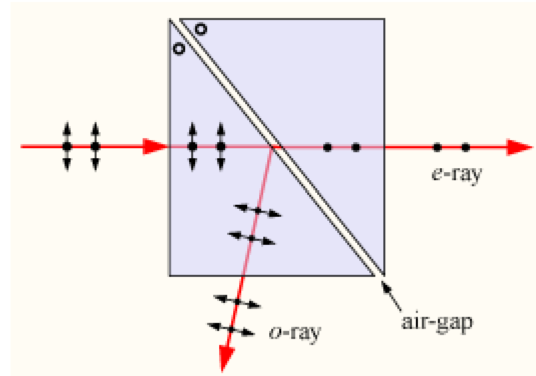
$$= E_0\left(\hat{e}_x + j\hat{e}_z\right)e^{j\varphi_o}e^{j(ky-\omega t)}$$

$$d = \left(\frac{\lambda}{4} + m\lambda\right)\frac{1}{n_e - n_o}$$

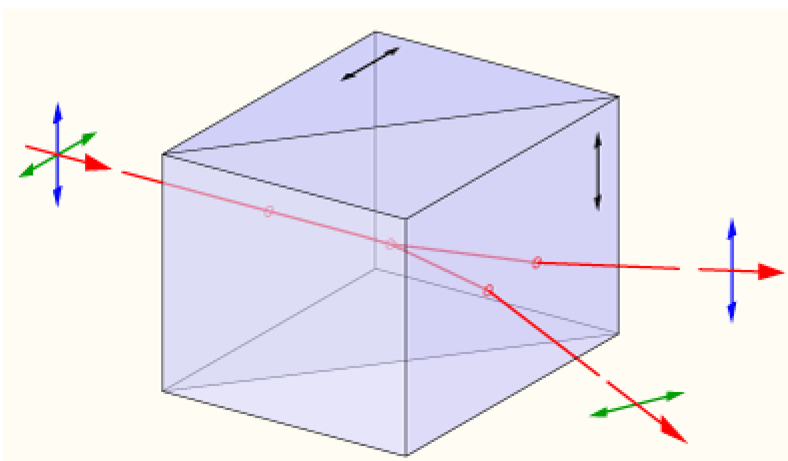
Polarizer prisms



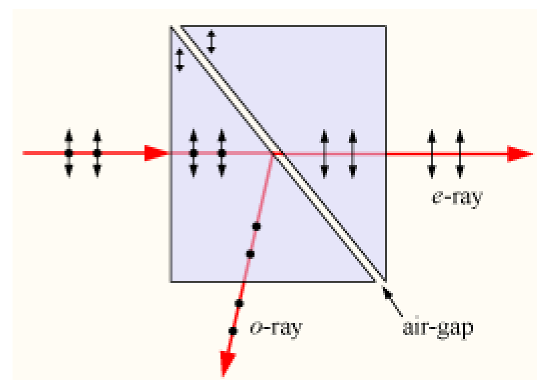
Glan Thompson



Glan Foucault



Wollaston



Glan Taylor