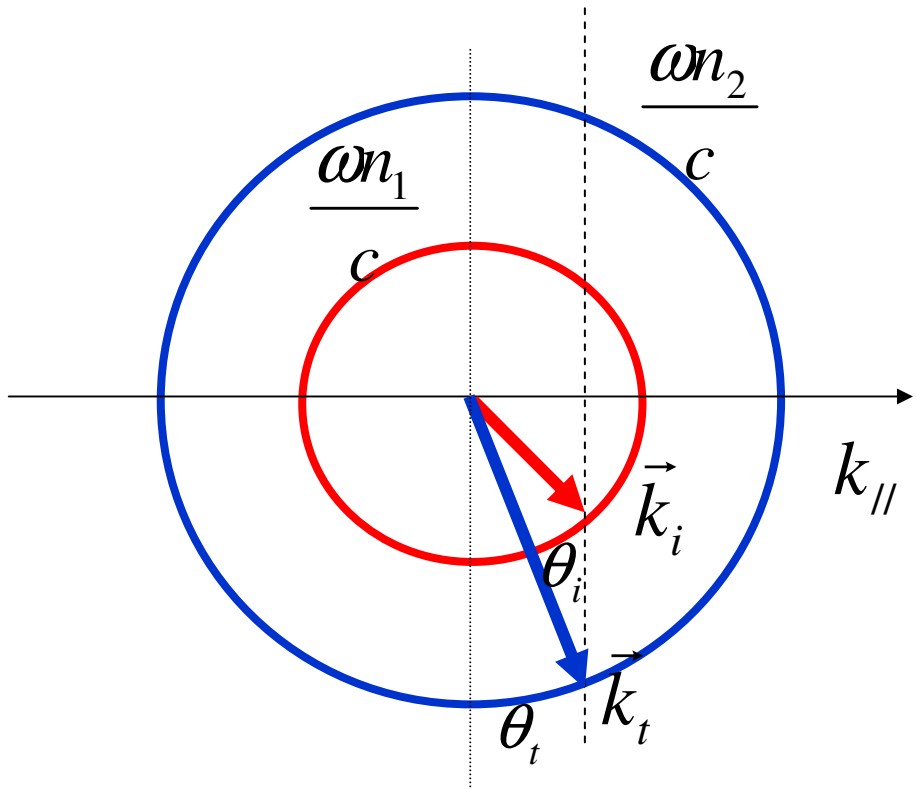
Metodo grafico per rifrazione $n_2 > n_1$

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

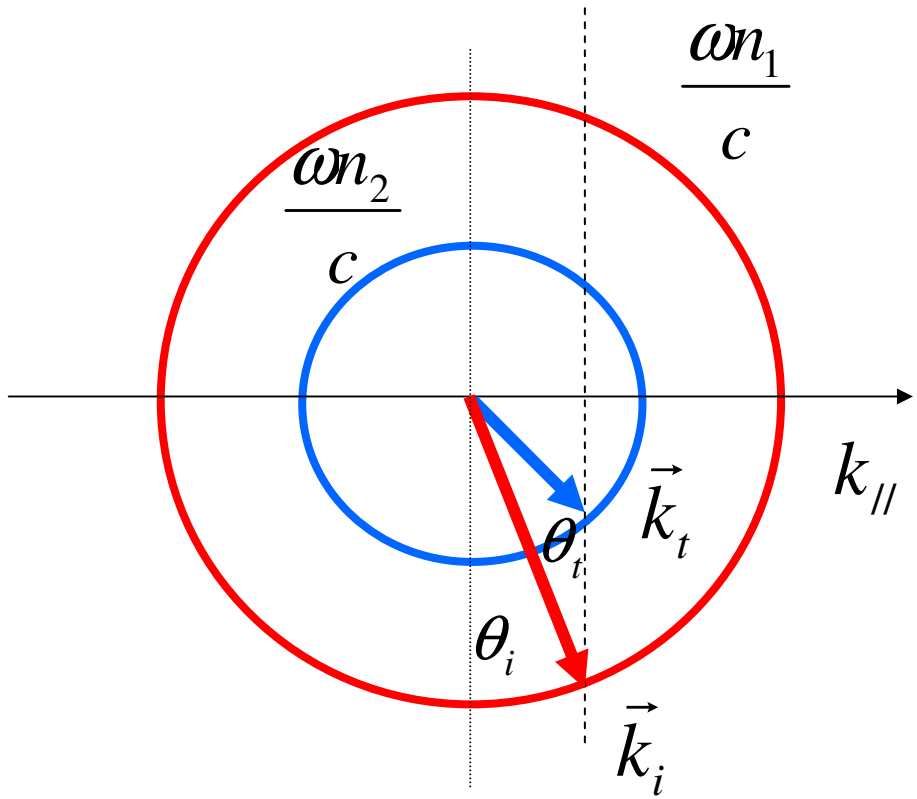


Metodo grafico per rifrazione $n_2 < n_1$

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$



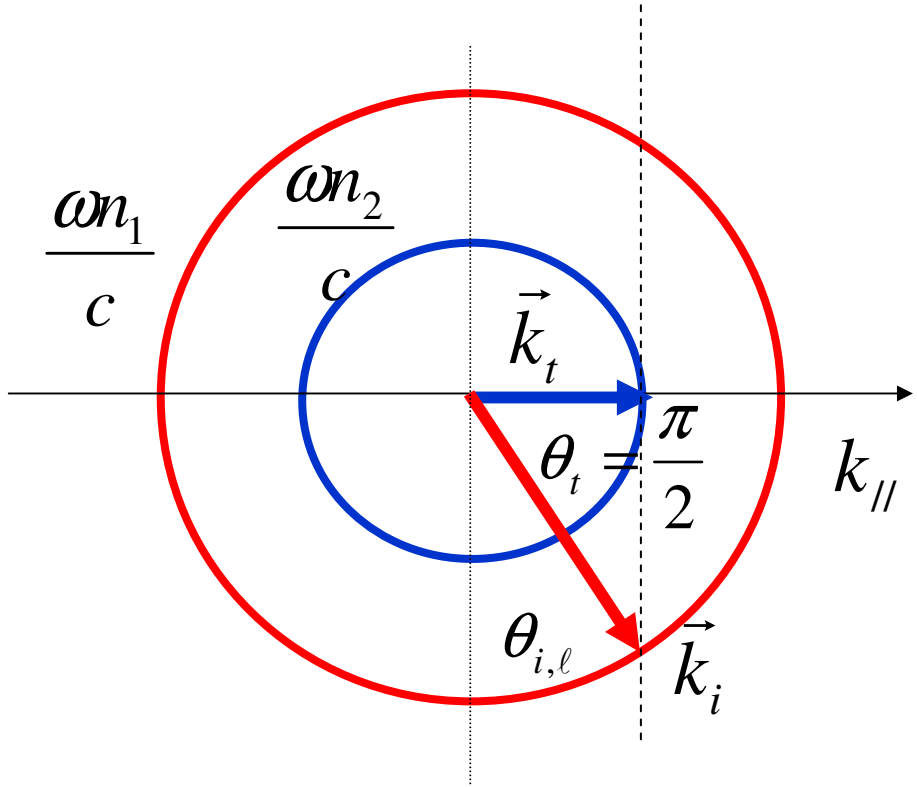
Metodo grafico per rifrazione $n_1 > n_2$

$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Angolo limite

$$\boxed{\sin\theta_{i,\ell} = \frac{n_2}{n_1}}$$



Metodo grafico per rifrazione $n_1 > n_2$

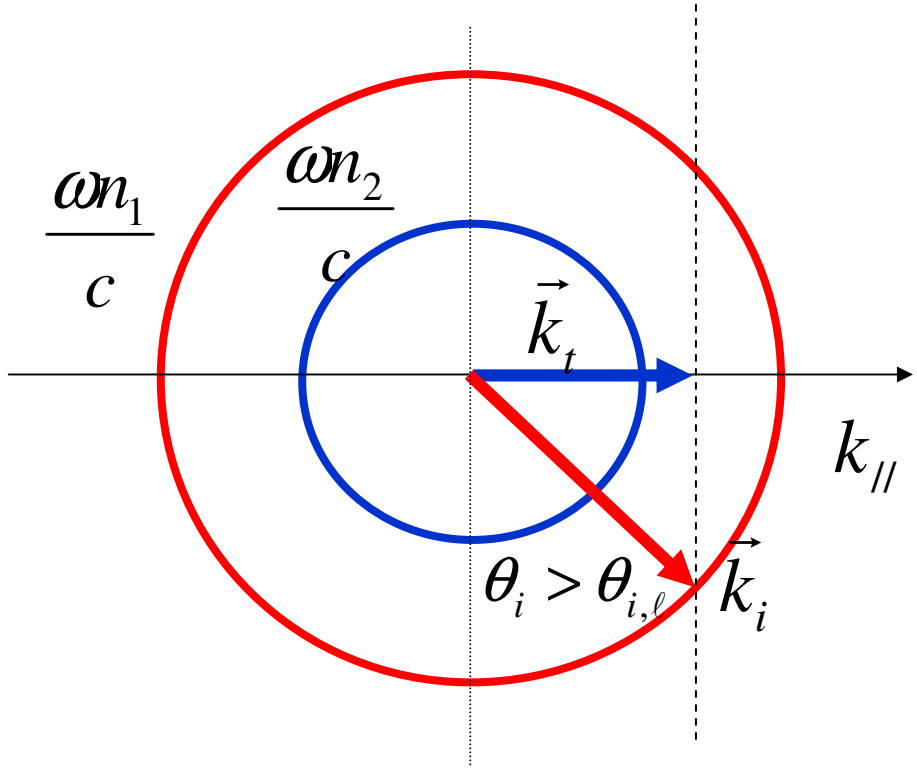
$$\vec{k}_{i,//} = \vec{k}_{r,//} = \vec{k}_{t,//}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Oltre angolo limite

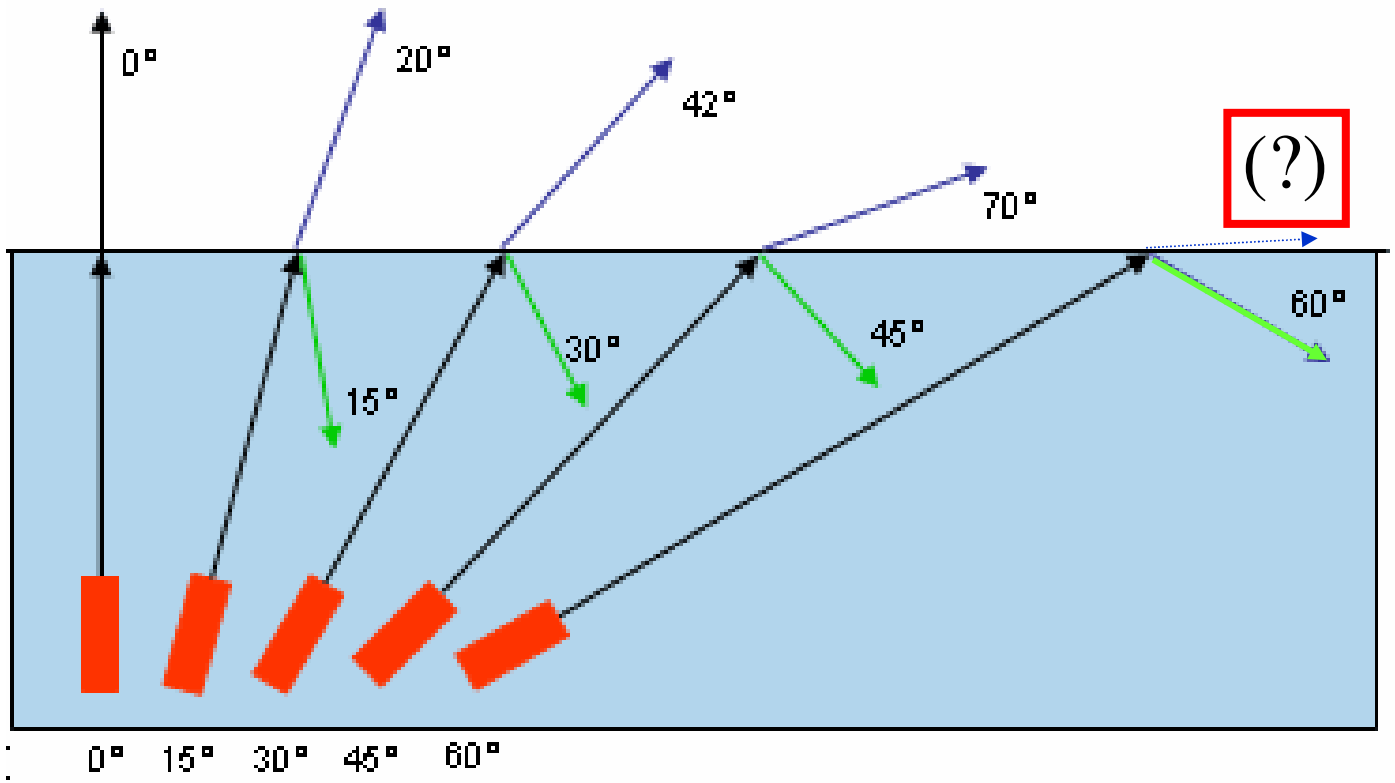
$$\boxed{\sin\theta_i > \frac{n_2}{n_1}}$$

$$\left|\vec{k}_{t,//}\right| > \frac{\omega n_2}{c} \quad (?)$$



Rifrazione oltre angolo limite

$$n_1 \sin\theta_i = n_2 \sin\theta_t \qquad n_1 > n_2$$



Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\begin{cases} \left|\vec{k}_{t,//}\right| = \left|\vec{k}_{i,//}\right| = \dfrac{\omega n_1}{c}\sin\theta_i \\ \vec{k}_t = \vec{k}_{t,//} + \vec{k}_{t,z} \\ \left|\vec{k}_t\right| = \dfrac{\omega n_2}{c} \end{cases}$$

$$Se \quad \sin\theta_i > \frac{n_2}{n_1}$$

$$\left|\vec{k}_{t,z}\right|^2 = \left|\vec{k}_t\right|^2 - \left|\vec{k}_{i,//}\right|^2 < 0$$

Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\left|\vec{k}_{t,z}\right|^2 = \left|\vec{k}_t\right|^2 - \left|\vec{k}_{i,//}\right|^2 < 0 \qquad Se \quad \sin\theta_i > \frac{n_2}{n_1}$$

$$\left|\vec{k}_{t,z}\right| = j\beta = \frac{\omega n_2}{c}\cos\theta_t \qquad \boxed{\cos\theta_t = ja}$$

$$\boxed{\vec{k}_t = \vec{k}_{i,//} + j\frac{\omega n_2}{c}a\hat{e}_z = \vec{k}_{i,//} + j\beta\hat{e}_z}$$

Rifrazione oltre angolo limite

$$n_1 > n_2$$

$$\vec{k}_t = \vec{k}_{i,//} + j\frac{\omega n_2}{c} a\hat{e}_z = \vec{k}_{i,//} + j\beta\hat{e}_z$$

$$\left(\frac{\omega n_2}{c}\right)^2 = \left(\frac{\omega n_1}{c}\sin\theta_i\right)^2 - \left(\frac{\omega n_2}{c}\right)^2 a^2$$

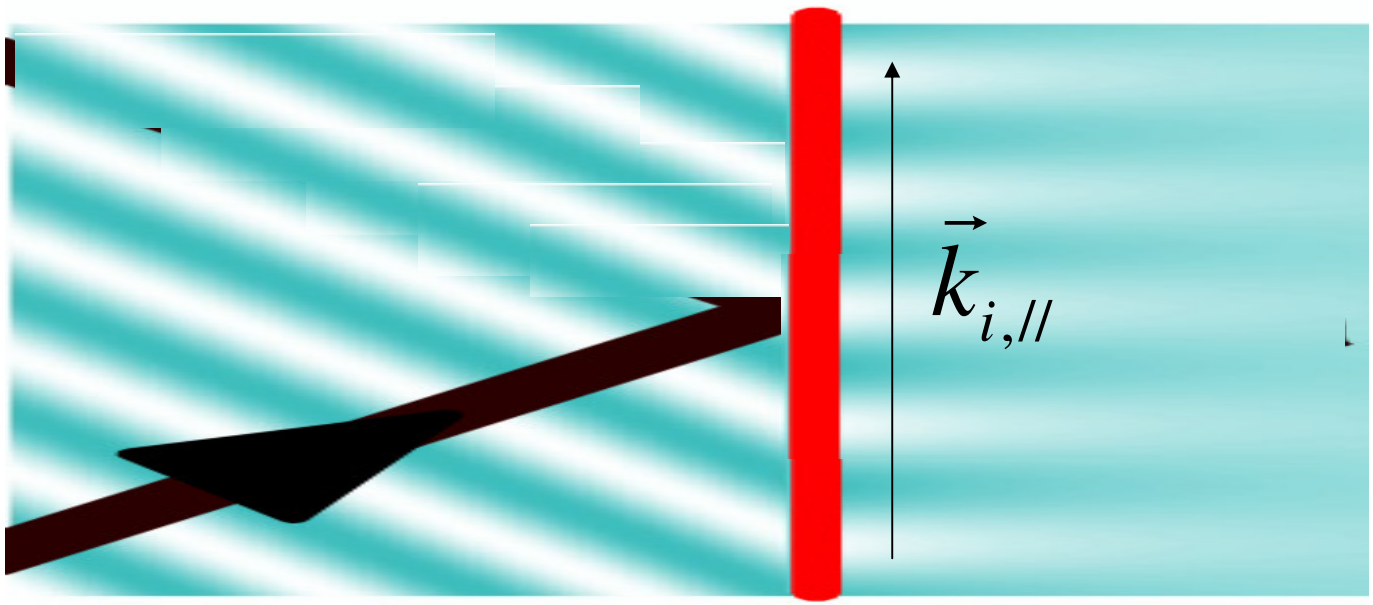
$$a^2 = \left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1$$

$$\boxed{\cos\theta_t = ja}$$

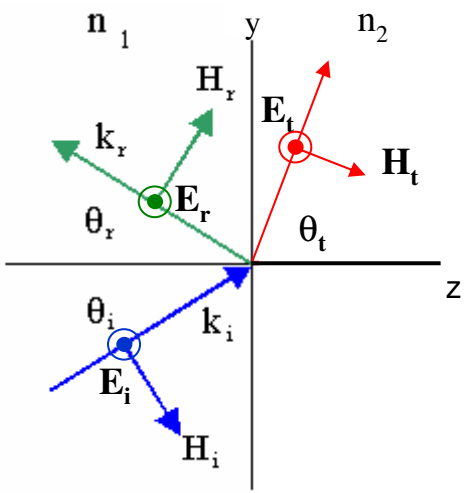
Onda evanescente

$$\vec{E}_t(\vec{r},t) = E_t e^{j(\vec{k}_{i,//}\cdot\vec{r}_{//} + j\beta z - \omega_i t)}\hat{e}$$

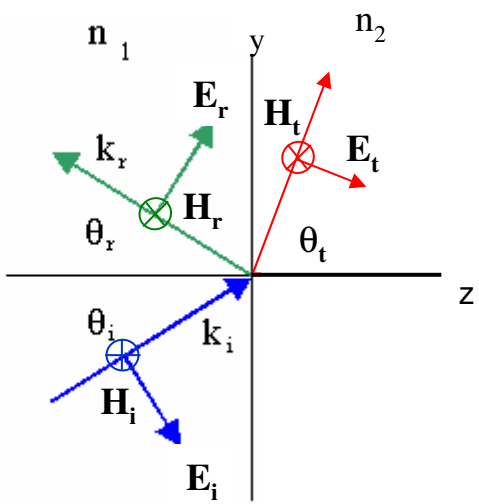
$$= E_t e^{j(\vec{k}_{i,//}\cdot\vec{r}_{//} - \omega_i t)} e^{-\beta z}\hat{e}$$



Relazioni di Fresnel



$$n_1 > n_2$$

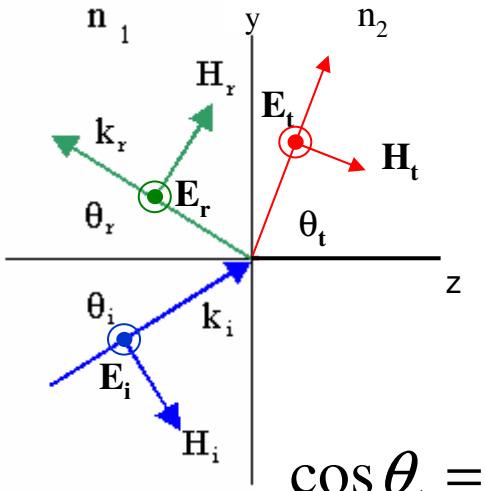


$$\begin{cases} E_r = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \\ E_t = \dfrac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_i \end{cases}$$

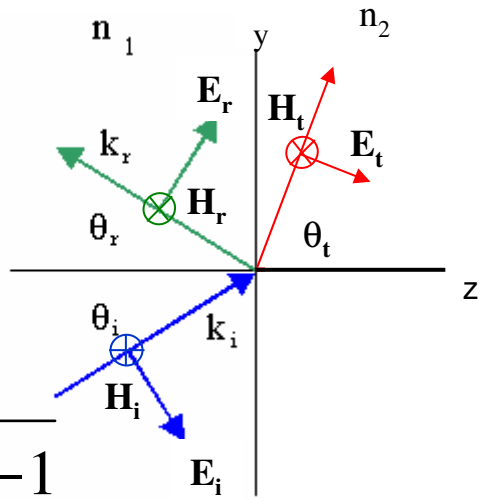
$$\begin{cases} E_r = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \\ E_t = \dfrac{2 n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_i \end{cases}$$

$$H_i = \frac{n_1}{\mu_1 c} E_i \qquad H_r = \frac{n_1}{\mu_1 c} E_r \qquad H_t = \frac{n_2}{\mu_2 c} E_t$$

Relazioni di Fresnel



$$n = \frac{n_2}{n_1} < 1$$



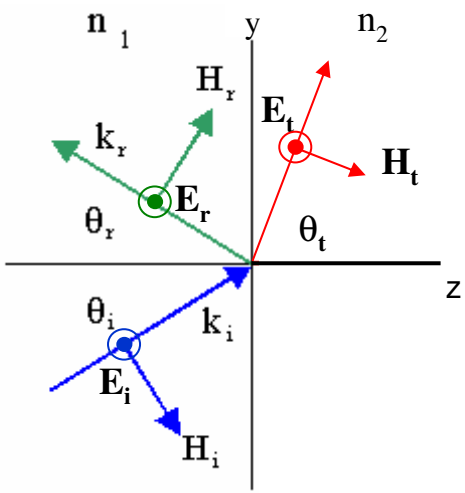
$$\cos\theta_t = ja = j\sqrt{(n\sin\theta_i)^2 - 1}$$

$$\begin{cases} E_r = \dfrac{\cos\theta_i - jna}{\cos\theta_i + jna} E_i = r_\perp E_i \\ E_t = \dfrac{2\cos\theta_i}{\cos\theta_i + jna} E_i = t_\perp E_i \end{cases}$$

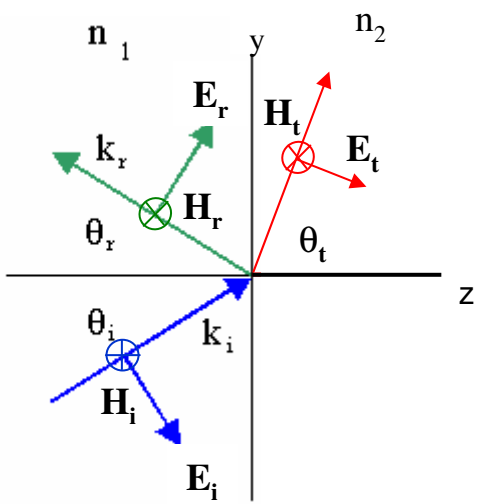
$$\begin{cases} E_r = \dfrac{n\cos\theta_i - ja}{n\cos\theta_i + ja} E_i = r_{//} E_i \\ E_t = \dfrac{2n\cos\theta_i}{n\cos\theta_i + ja} E_i = t_{//} E_i \end{cases}$$

$$H_i = \frac{n_1}{\mu_1 c} E_i \qquad H_r = \frac{n_1}{\mu_1 c} E_r \qquad H_t = \frac{n_2}{\mu_2 c} E_t$$

Riflessione e rifrazione



$$n = \frac{n_2}{n_1} < 1$$



$$r_\perp = e^{j\varphi_\perp}$$

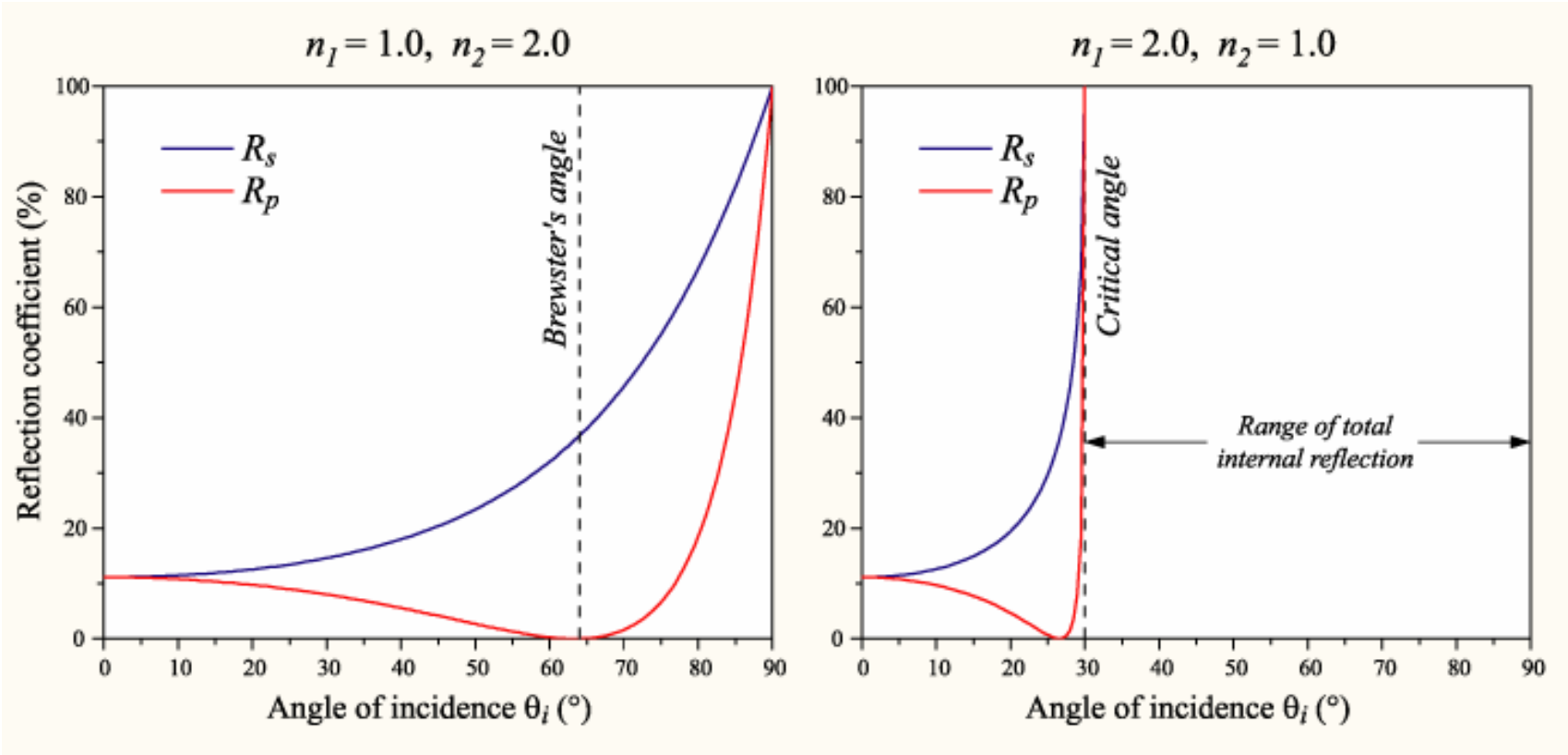
$$r_{//} = e^{j\varphi_{//}}$$

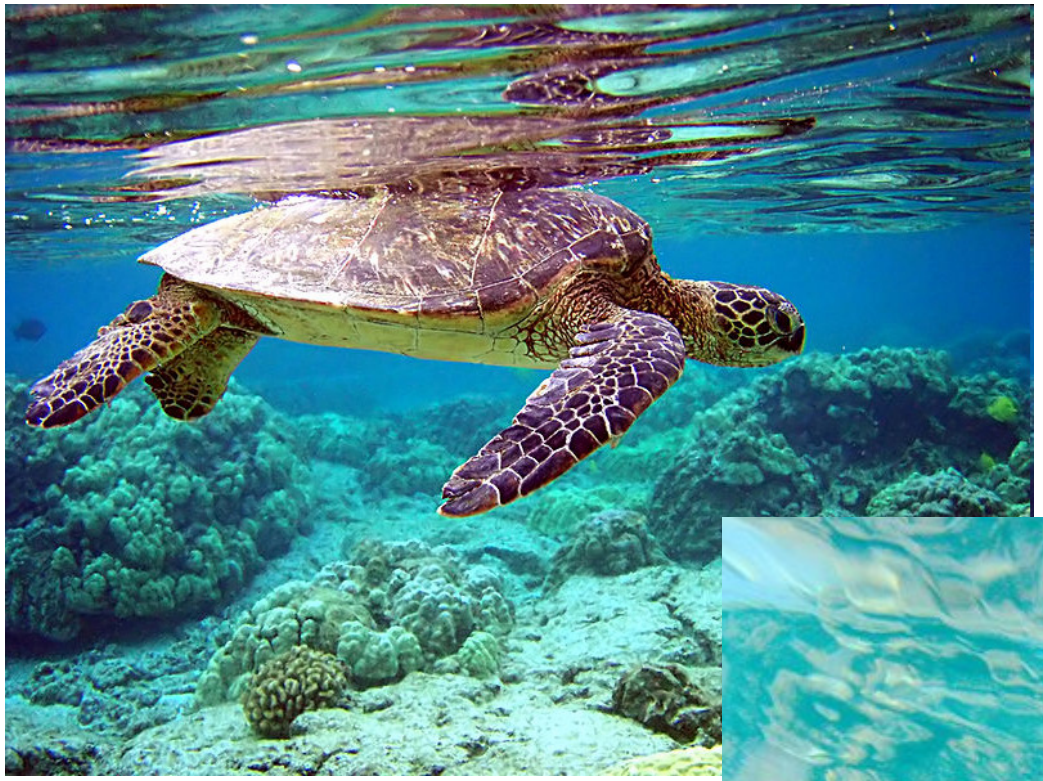
$$\tan\frac{\varphi_\perp}{2} = \frac{-\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}$$

$$\tan\frac{\varphi_{//}}{2} = \frac{-\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i}$$

$$\boxed{\left|\vec{S}_r\right| = \left|\vec{S}_i\right|}$$

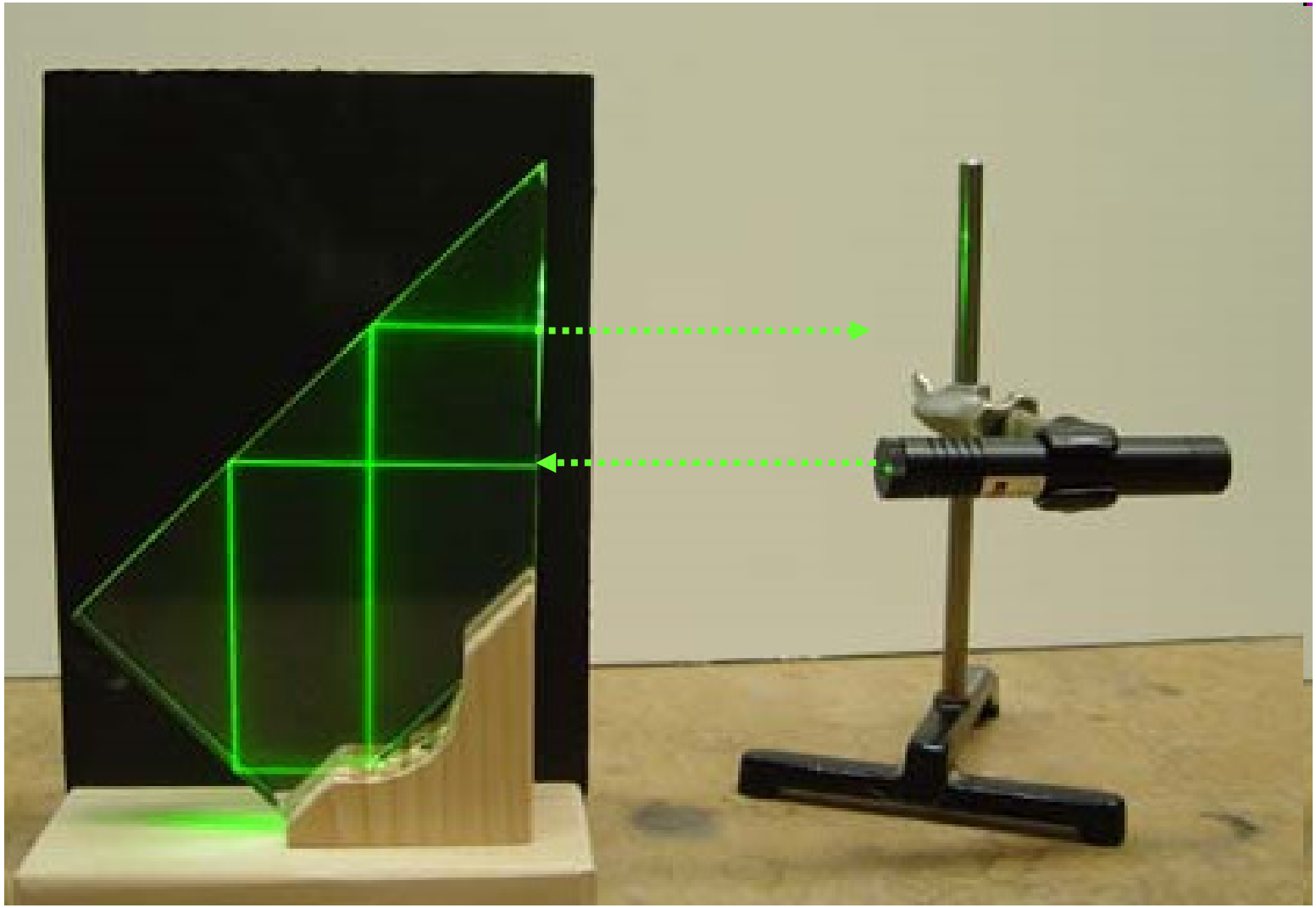
Riflessione e rifrazione





Riflessione totale

Riflessione totale



Prismi a riflessione totale

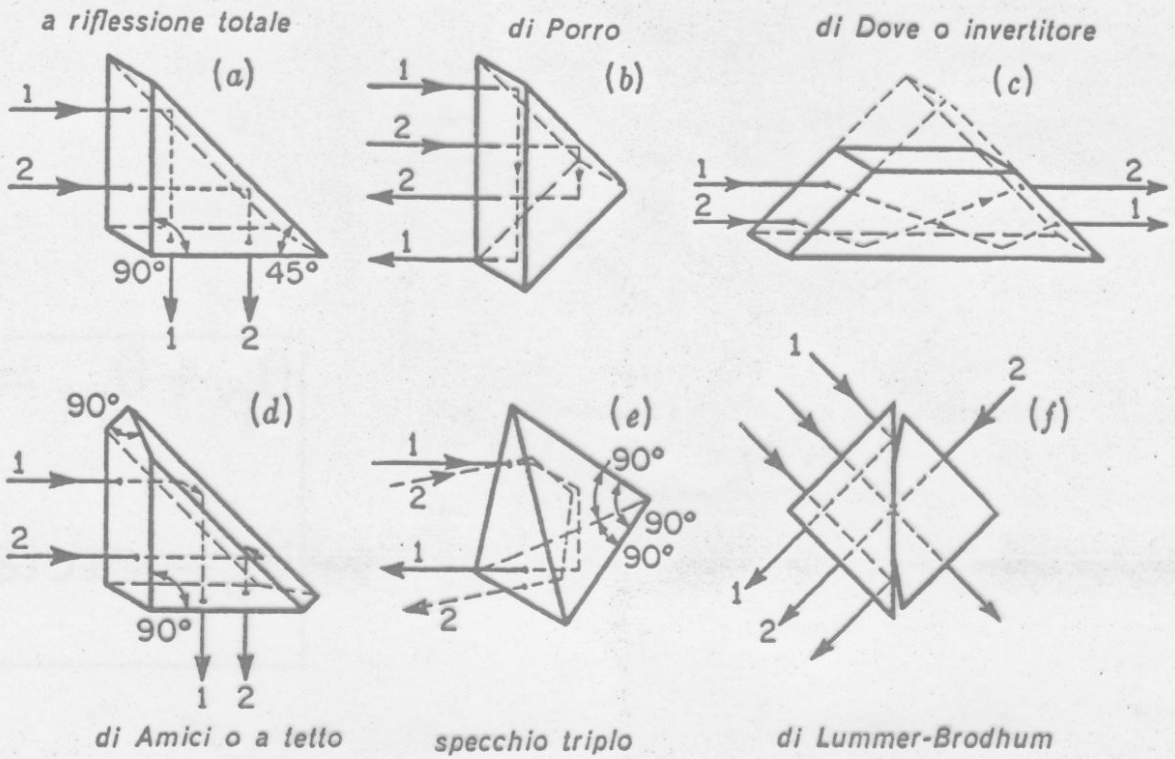
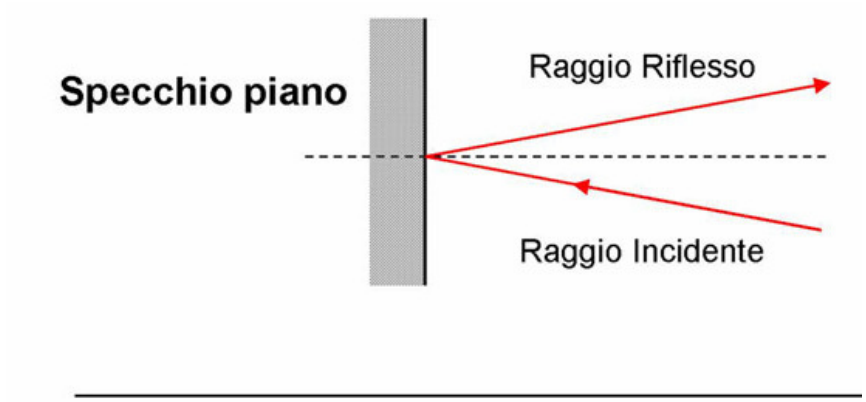
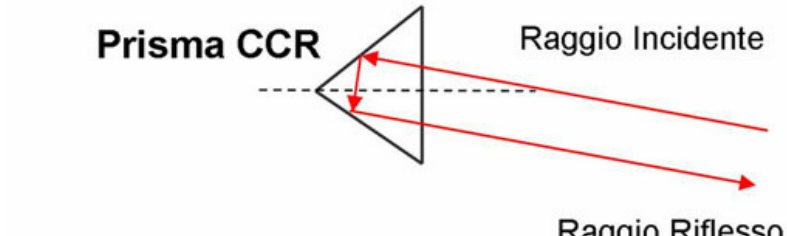
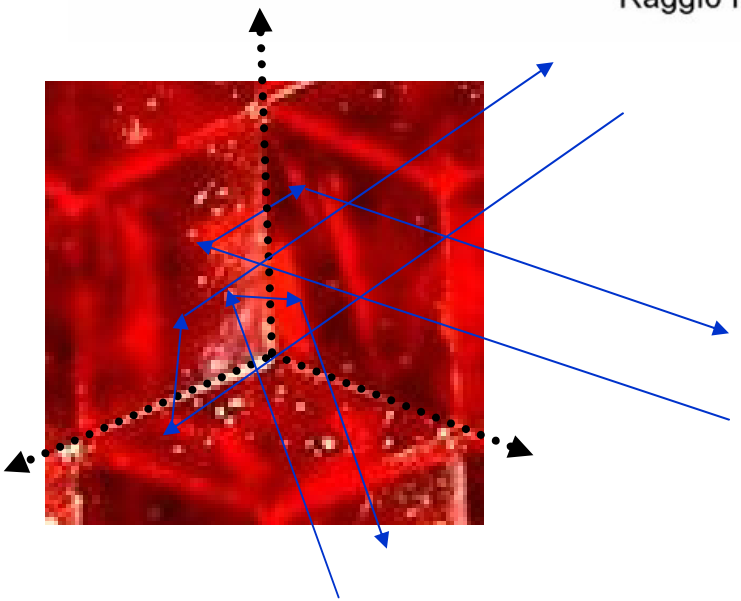


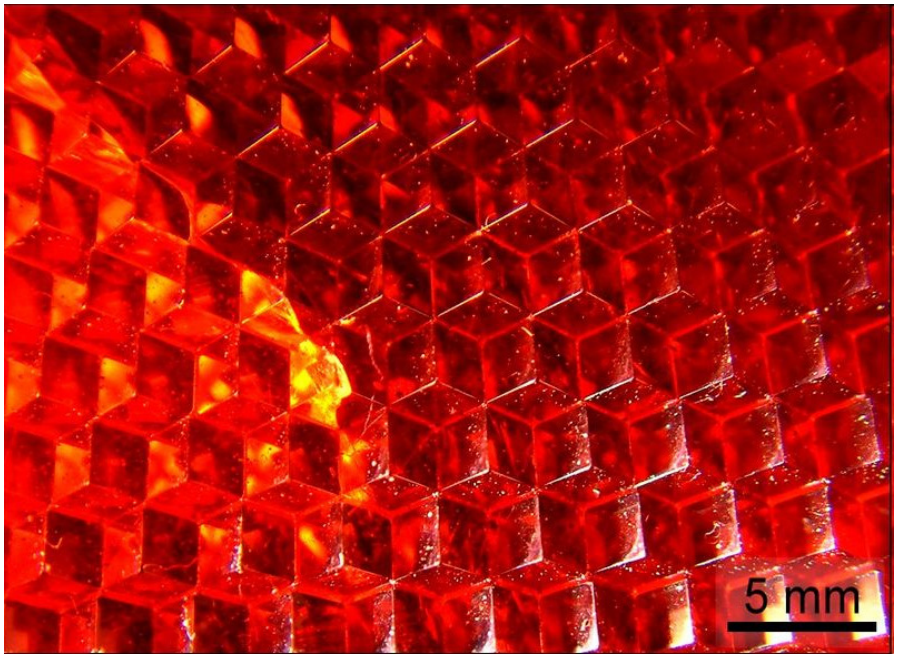
FIG. 2.3 – Prismi riflettenti che utilizzano il principio della riflessione totale.

Retroriflettori e Catadiottri





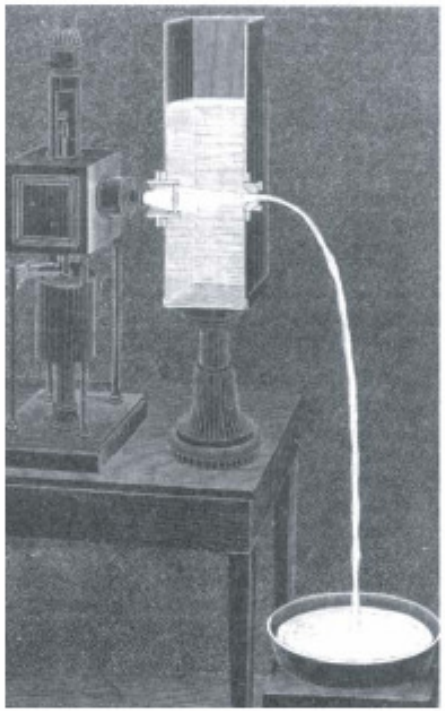




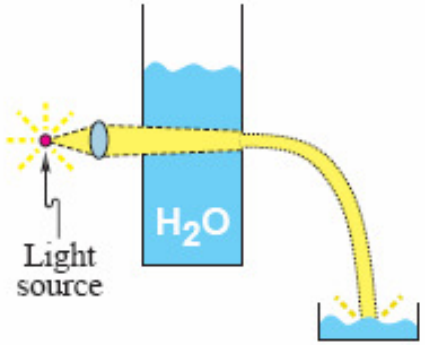
The Guiding of Light

Total internal reflection: Johannes Kepler (before 1611)
Laws of reflection and refraction: Willebrord Snell (between 1621 and 1625)
Guiding of light by total internal reflection: Daniel Colladon (1842)
(*Comptes Rendus*, Vol. **15**, p. 800, Oct. 24, 1842)

Daniel Colladon (1802 – 1893)
"Father of light guiding"



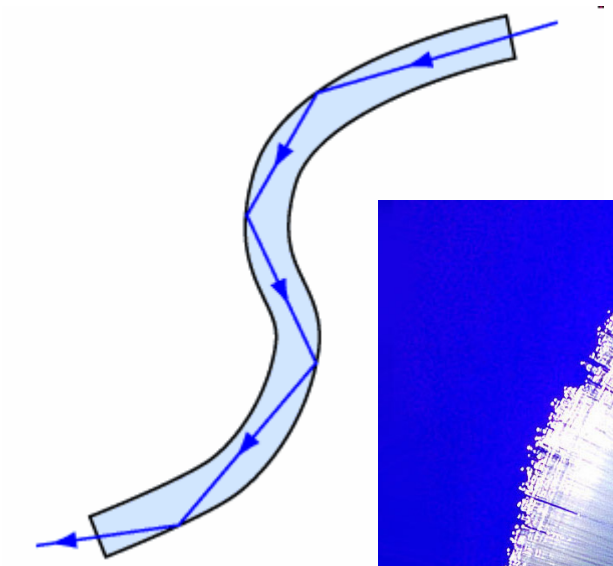
Daniel Colladon's apparatus

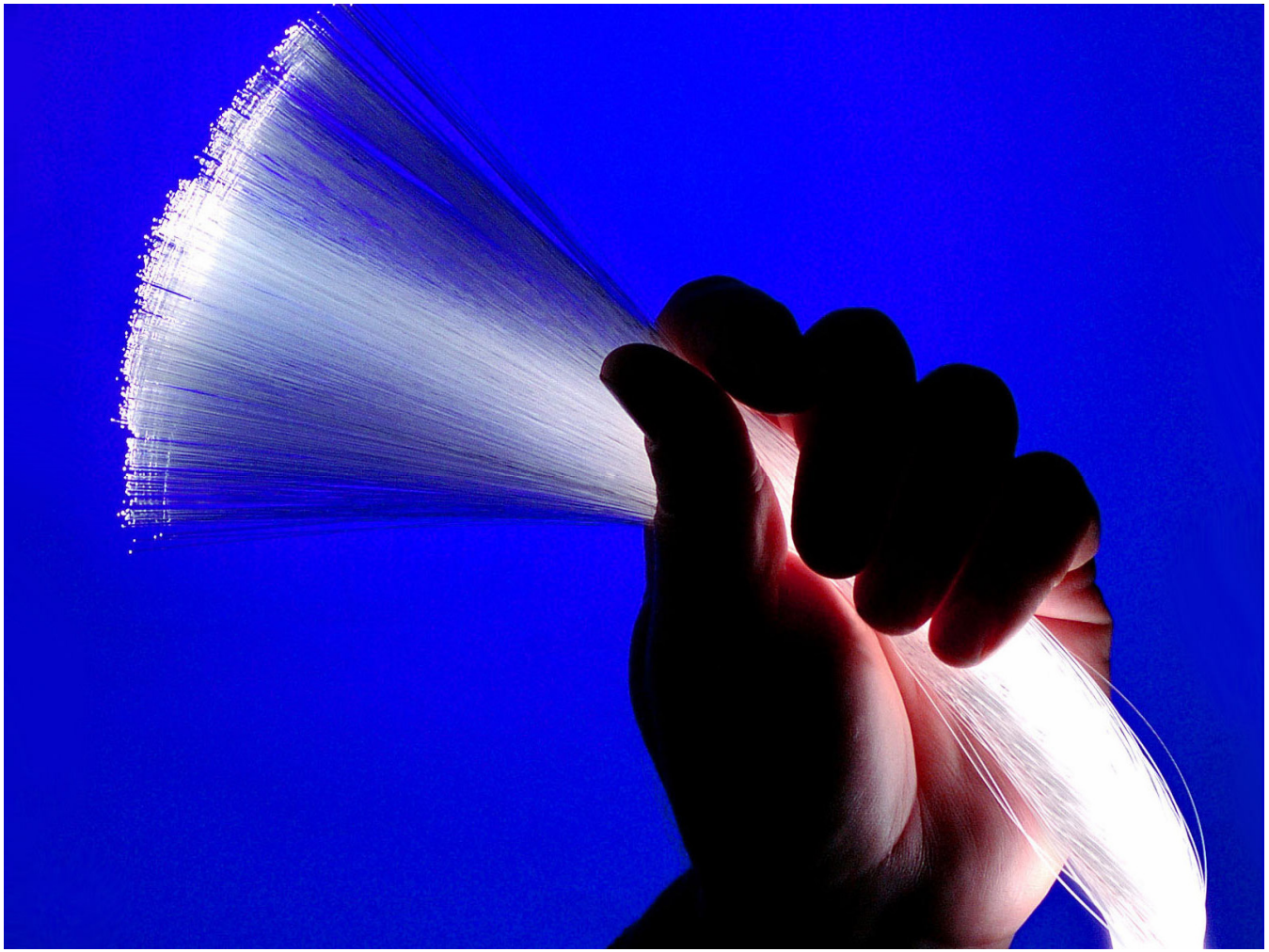


"... one of the most beauitful experiments one can perform in a course on optics ..."

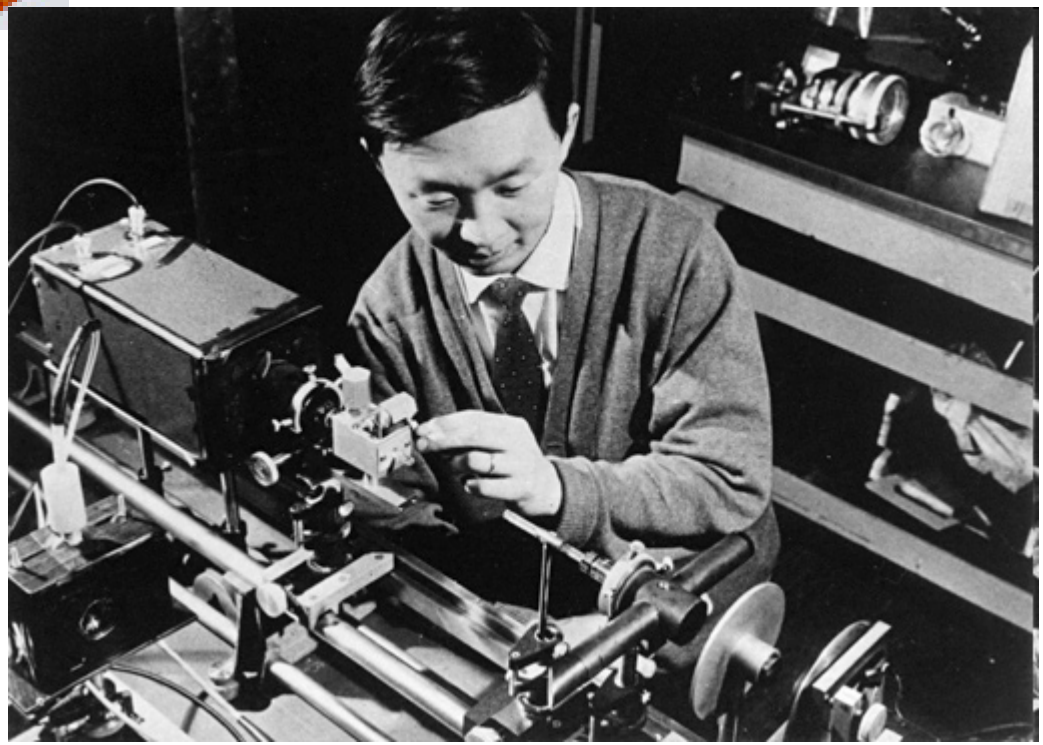
(after J. Hecht, Optics & Photonics News, Oct. 1999)

Fibre ottiche





The Nobel Prize in Physics 2009



Charles K. Kao

"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"

Fase dell'onda riflessa totalmente

$$\tan\frac{\varphi_\perp}{2} = \frac{-\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}$$

$$\tan\frac{\varphi_{//}}{2} = \frac{-\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i}$$

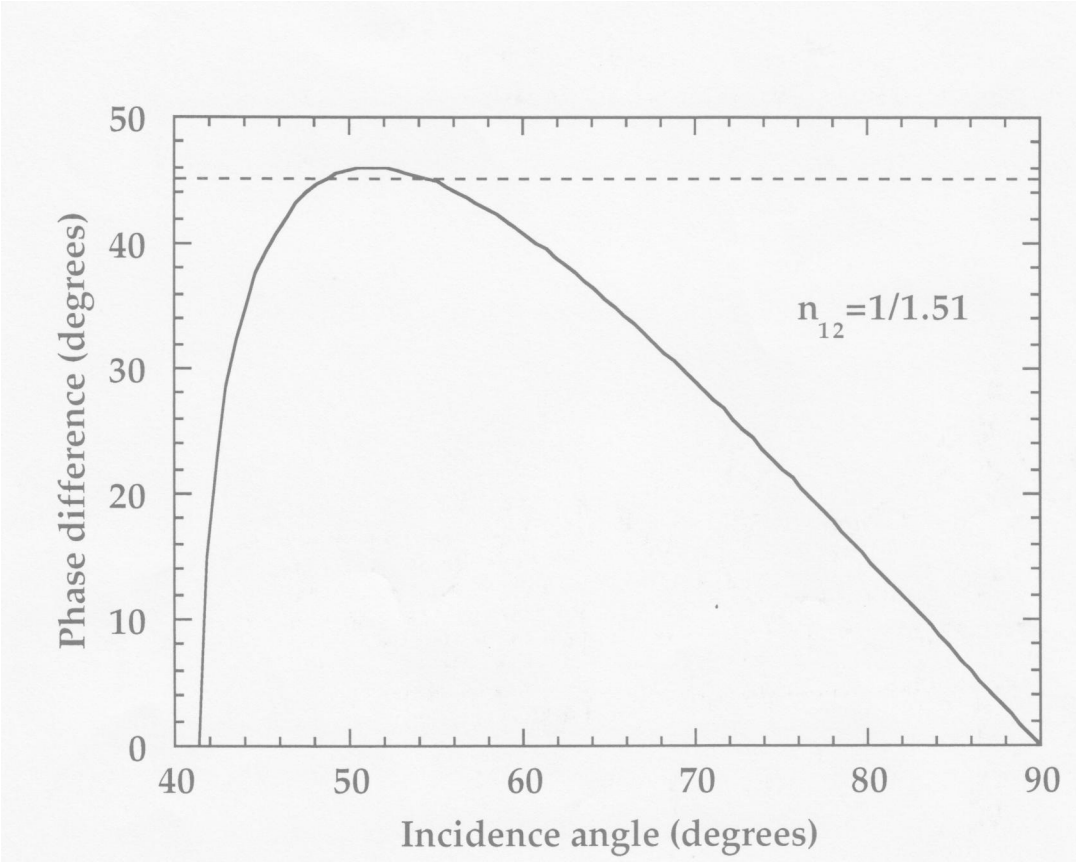
$$\tan\frac{\varphi_\perp - \varphi_{//}}{2} = \frac{\tan\frac{\varphi_\perp}{2} - \tan\frac{\varphi_{//}}{2}}{1 + \tan\frac{\varphi_\perp}{2}\tan\frac{\varphi_{//}}{2}} =$$

$$= \frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}\left(\frac{1}{n^2} - 1\right)\frac{1}{1 + \frac{\sin^2\theta_i - n^2}{n^2\cos^2\theta_i}} =$$

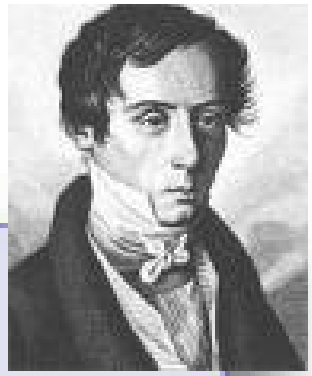
$$= \sqrt{\sin^2\theta_i - n^2}\,\frac{\cos\theta_i}{\sin^2\theta_i}$$

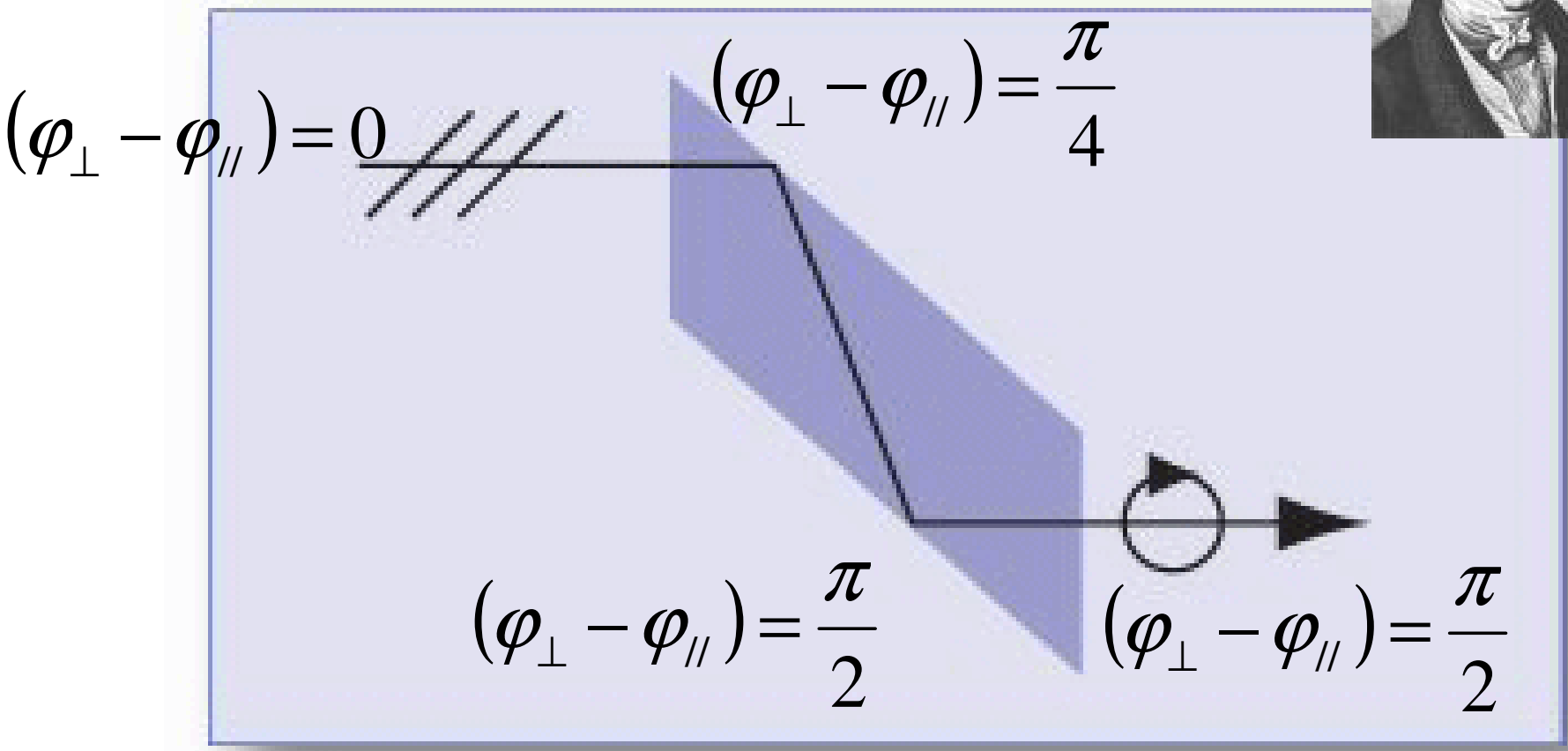
Fase dell'onda riflessa totalmente

$$\varphi_{\perp} - \varphi_{//} = 2\arctan\left(\sqrt{\sin^2\theta_i - n^2}\,\frac{\cos\theta_i}{\sin^2\theta_i}\right)$$

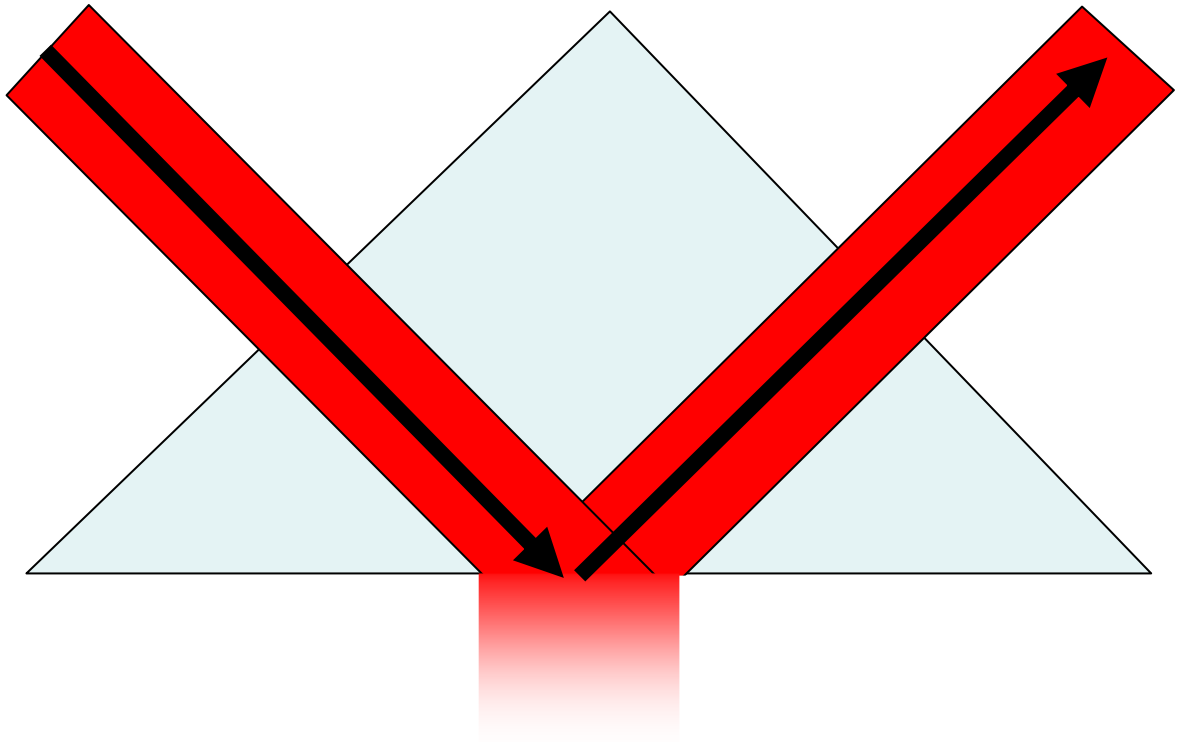


Rombo di Fresnel

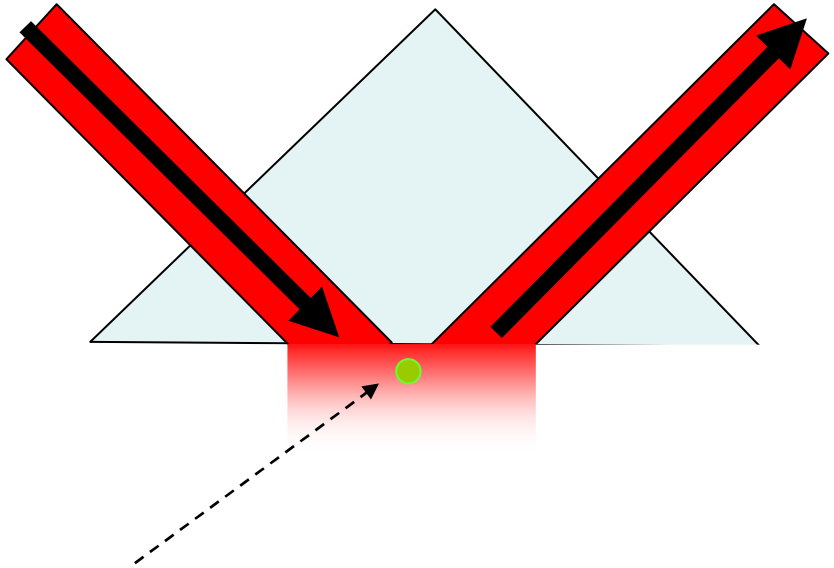




Onda evanescente

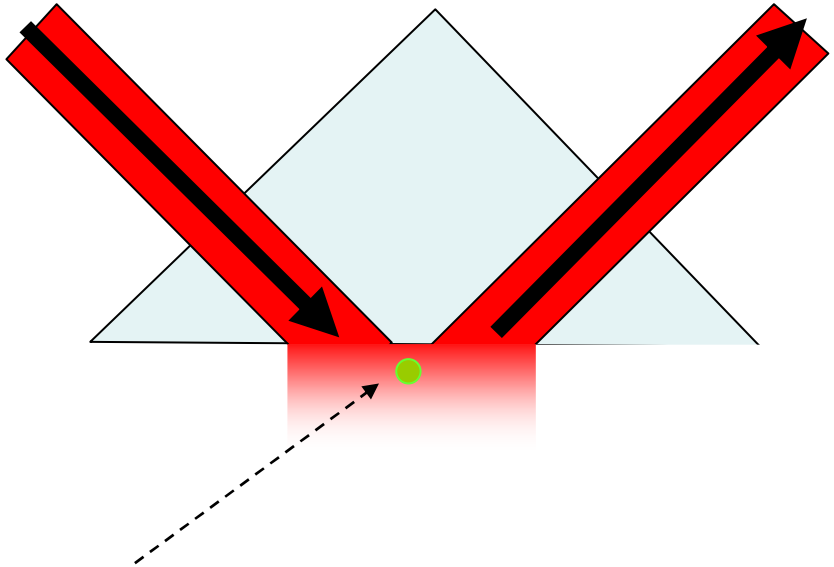


Frustrated total attenuated reflection

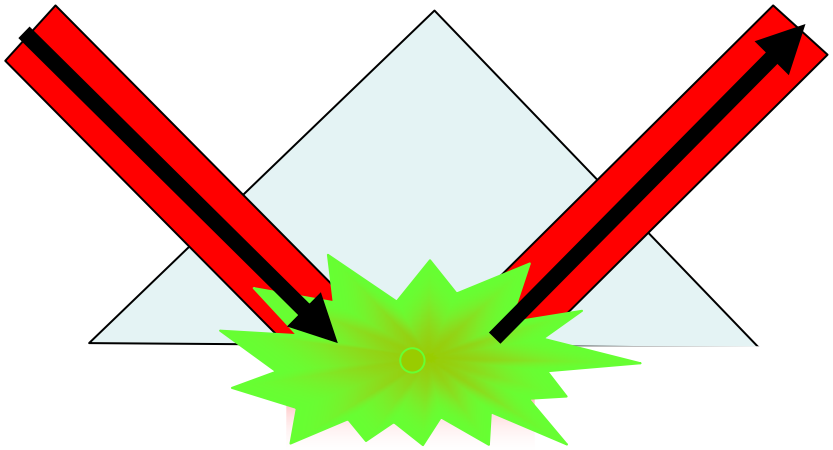


Atomo, molecola,
centro di scatterning

Frustrated total attenuated reflection



Atomo, molecola,
centro di scatterning



EMISSIONE RIEMESSA

$$\Gamma \propto \left| \vec{d} \cdot \vec{E} \right|^2$$

Onda evanescente esiste !

Total Internal Reflection Fluorescence Microscopy (TIRFM)

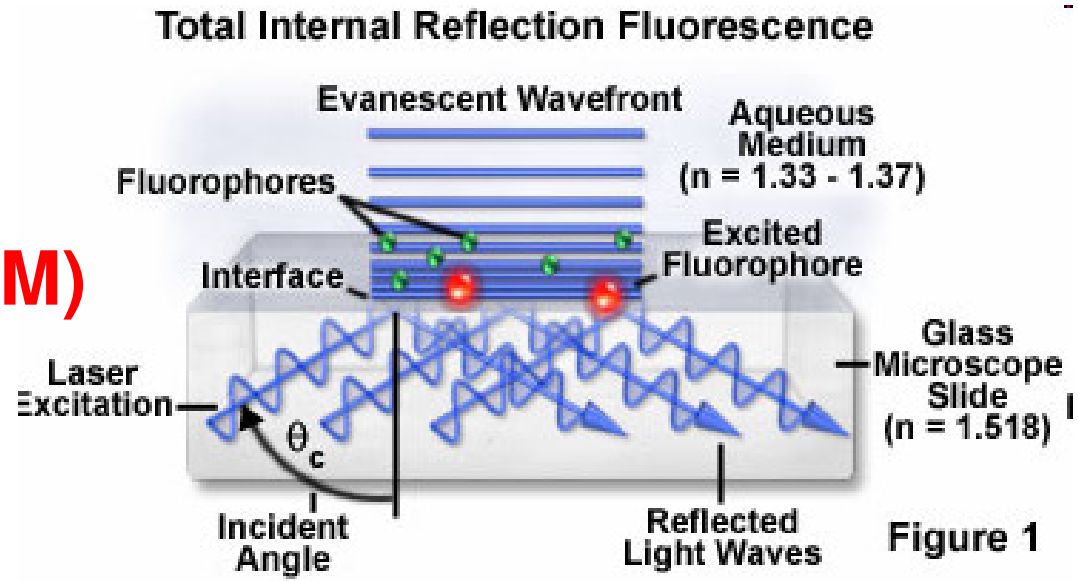


Figure 1

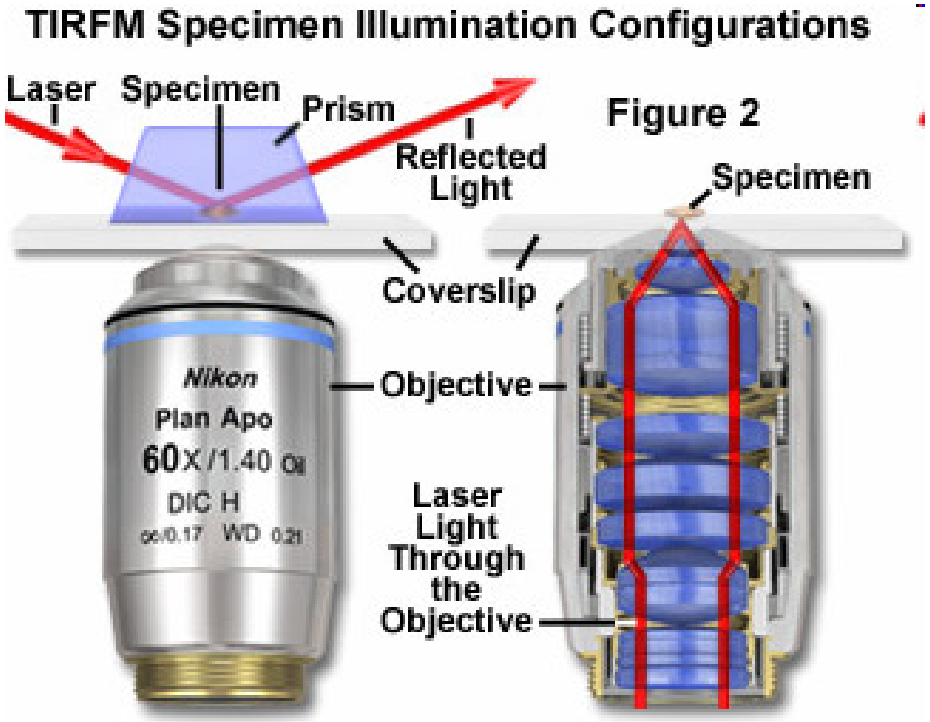


Figure 2

TIFRM measurements on fluorescent microsphere

TIFRM Histogram

TIFRM Image

WideField Histogram

Widefield Image

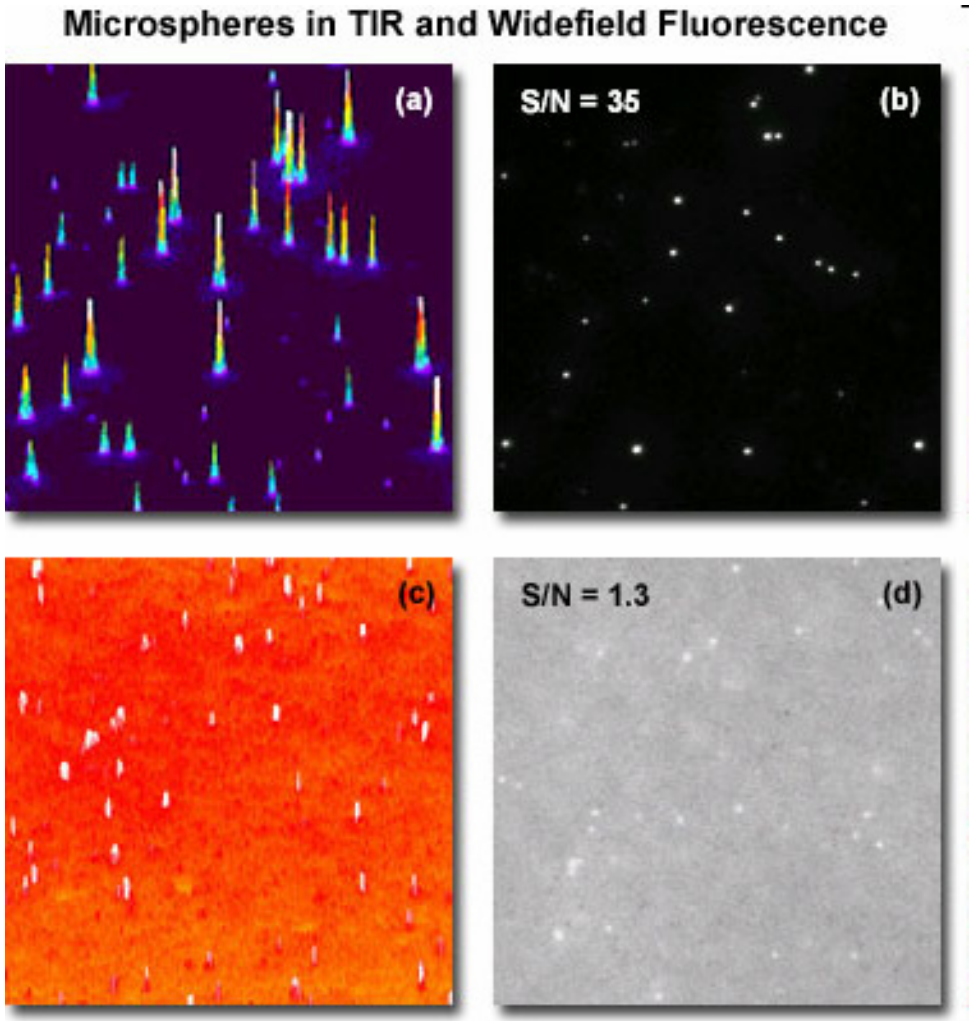
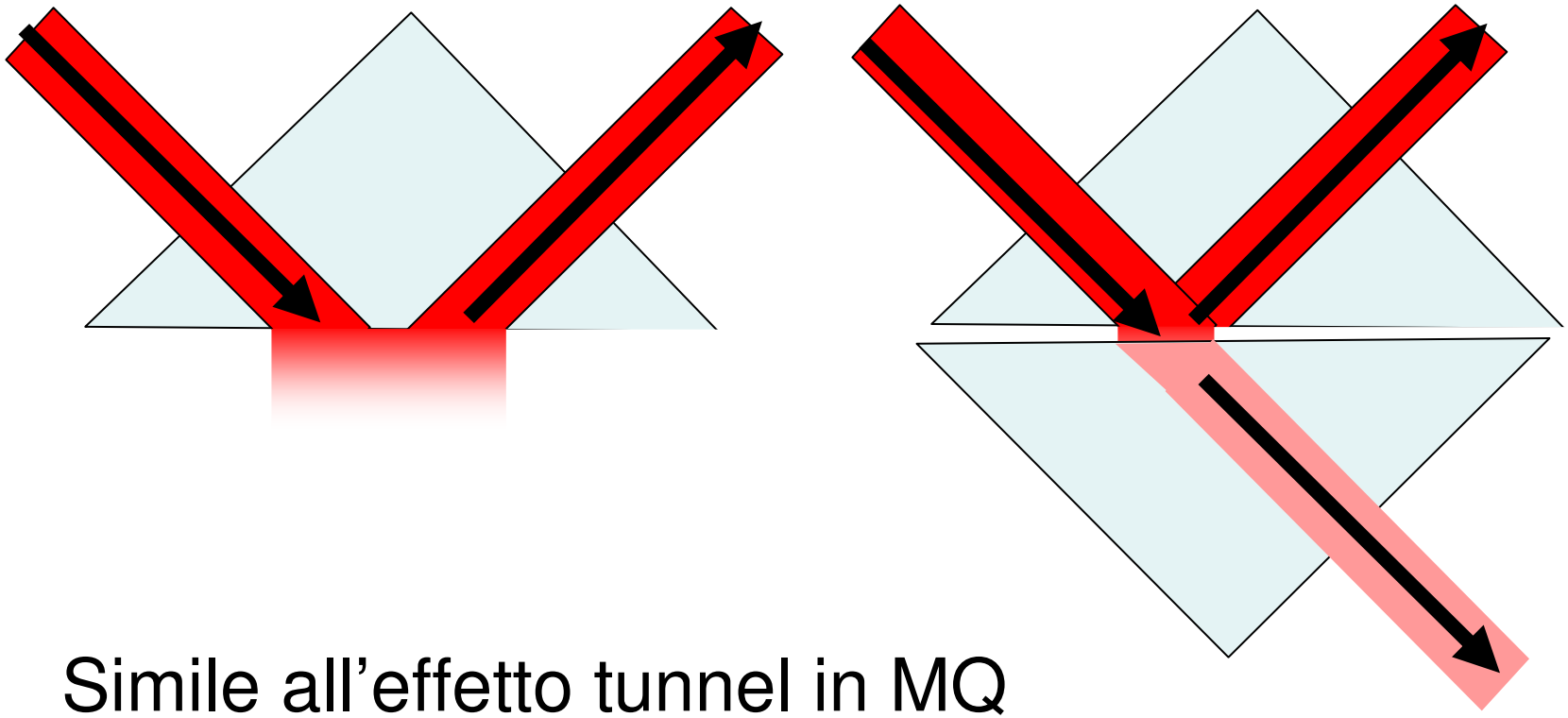


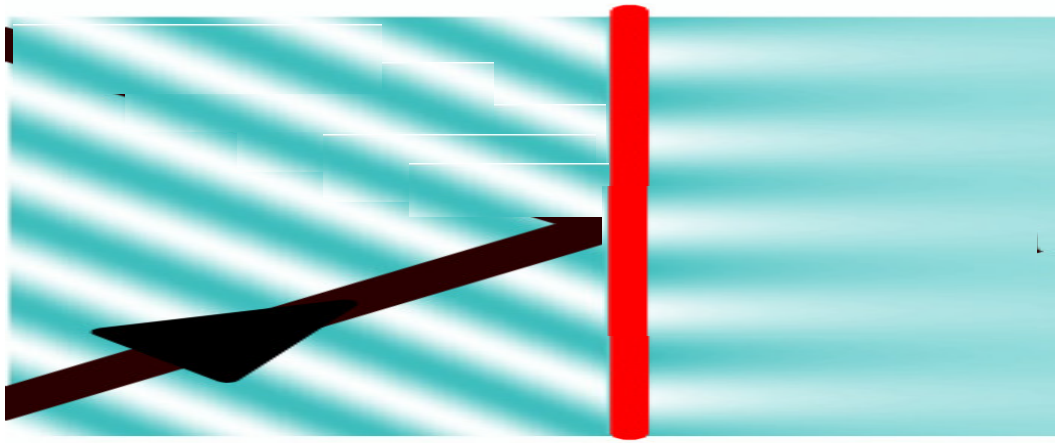
Figure 4

Frustrated total attenuated reflection



Simile all’effetto tunnel in MQ

Onda trasmessa



$$n = \frac{n_2}{n_1} < 1$$

$$\cos\theta_t = ja = j\sqrt{(n\sin\theta_i)^2 - 1}$$

TE

$$\begin{cases} E_r = \dfrac{\cos\theta_i - jna}{\cos\theta_i + jna} E_i = r_\perp E_i \\ E_t = \dfrac{2\cos\theta_i}{\cos\theta_i + jna} E_i = t_\perp E_i \end{cases}$$

TM

$$\begin{cases} E_r = \dfrac{n\cos\theta_i - ja}{n\cos\theta_i + ja} E_i = r_{//} E_i \\ E_t = \dfrac{2n\cos\theta_i}{n\cos\theta_i + ja} E_i = t_{//} E_i \end{cases}$$

$$H_i = \frac{n_1}{\mu_1 c} E_i \qquad H_r = \frac{n_1}{\mu_1 c} E_r \qquad H_t = \frac{n_2}{\mu_2 c} E_t$$

Relazioni di Fresnel

TE

$$\vec{E}_t = \frac{2\cos\theta_i}{\cos\theta_i + jna} E_i \hat{e}_x$$

$$\vec{H}_t = \frac{1}{\omega\mu_2} \vec{k}_t \times \vec{E}_t$$

TM

$$\vec{H}_t = \frac{2n\cos\theta_i}{n\cos\theta_i + ja} \frac{n_2}{\mu_2 c} E_i \hat{e}_x$$

$$\vec{E}_t = -\mu_2 c^2 \vec{k}_t \times \vec{H}_t$$

$$\vec{k}_t = \vec{k}_{t,//} + j\beta\hat{e}_z$$

$$\vec{E}_t(\vec{r},t) = \vec{E}_t e^{j(\vec{k}_{t,//}\cdot\vec{r}_{//} - \omega_i t)} e^{-\beta z}$$

$$\vec{H}_t(\vec{r},t) = \vec{H}_t e^{j(\vec{k}_{t,//}\cdot\vec{r}_{//} - \omega_i t)} e^{-\beta z}$$

$$\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t) \times \vec{H}(\vec{r},t)$$

Vettore di Poynting

$$\vec{S}(\vec{r},t) = \frac{1}{4}\left(\vec{E}(\vec{r},t) + \vec{E}^*(\vec{r},t)\right) \times \left(\vec{H}(\vec{r},t) + \vec{H}^*(\vec{r},t)\right)$$

$$\left\langle \vec{S}(\vec{r},t) \right\rangle = \frac{1}{4}\left(\left\langle \vec{E}(\vec{r},t) \times \vec{H}^*(\vec{r},t) \right\rangle + \left\langle \vec{E}^*(\vec{r},t) \times \vec{H}(\vec{r},t) \right\rangle\right)$$

$$\left\langle \vec{S}(\vec{r},t) \right\rangle = \frac{1}{2}\Re e\left[\left\langle \vec{E}(\vec{r},t) \times \vec{H}^*(\vec{r},t) \right\rangle\right]$$

$$\boxed{\left\langle \vec{S}(\vec{r},t) \right\rangle = \left\langle \vec{E}(\vec{r},t) \times \vec{H}^*(\vec{r},t) \right\rangle}$$

Relazioni di Fresnel

$$\vec{k}_t = \vec{k}_{i,//} + j\beta\hat{e}_z$$

TE

$$\vec{E}_t = \frac{2\cos\theta_i}{\cos\theta_i + jna} E_i\hat{e}_x$$

$$\vec{H}_t = \frac{1}{\omega\mu_2}\vec{k}_t \times \vec{E}_t$$

TM

$$\vec{H}_t = \frac{2n\cos\theta_i}{n\cos\theta_i + ja}\frac{n_2}{\mu_2 c} E_i\hat{e}_x$$

$$\vec{E}_t = -\frac{\mu_2 c^2}{\omega n^2}\vec{k}_t \times \vec{H}_t$$

$$\left\langle \vec{S}_{t,\perp}(\vec{r},t) \right\rangle = \left[\frac{|t_\perp|^2 |E_i|^2}{\omega\mu_2}\vec{k}_t \right] e^{-2\beta z}$$

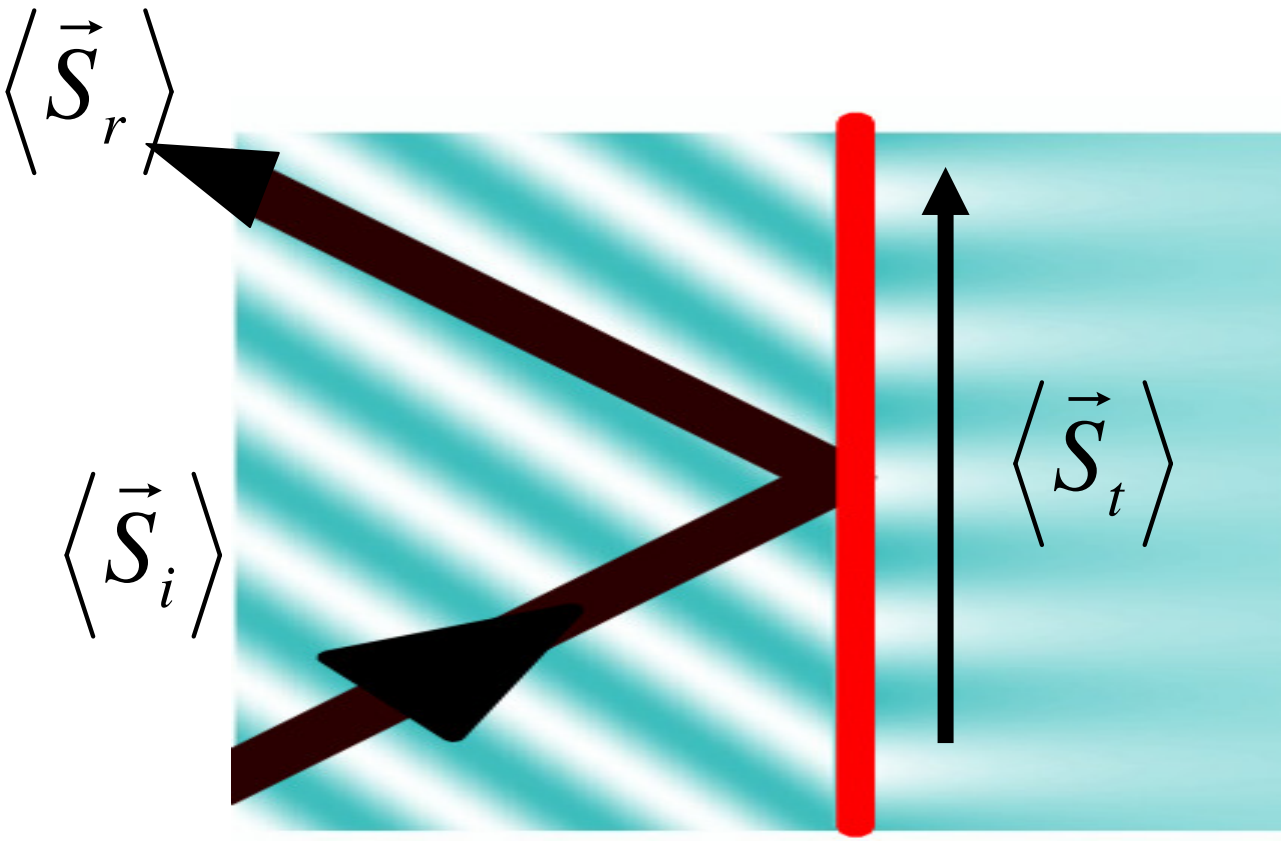
$$\left\langle \vec{S}_{t,//}(\vec{r},t) \right\rangle = \left[\frac{|t_{//}|^2 |E_i|^2}{\omega\mu_2}\vec{k}_t \right] e^{-2\beta z}$$

$$\boxed{\left\langle \vec{S}_{t,n}(\vec{r},t) \right\rangle = \left[\frac{|t_n|^2 |E_i|^2}{\omega\mu_2}\left(\vec{k}_{i,//} + j\beta\hat{e}_z\right) \right] e^{-2\beta z} \qquad n = \perp, //}$$

Onda evanescente $\beta = \frac{\omega n_2}{c} a = \frac{2\pi n_2}{\lambda}\sqrt{n^2 \sin^2\theta_i - 1}$

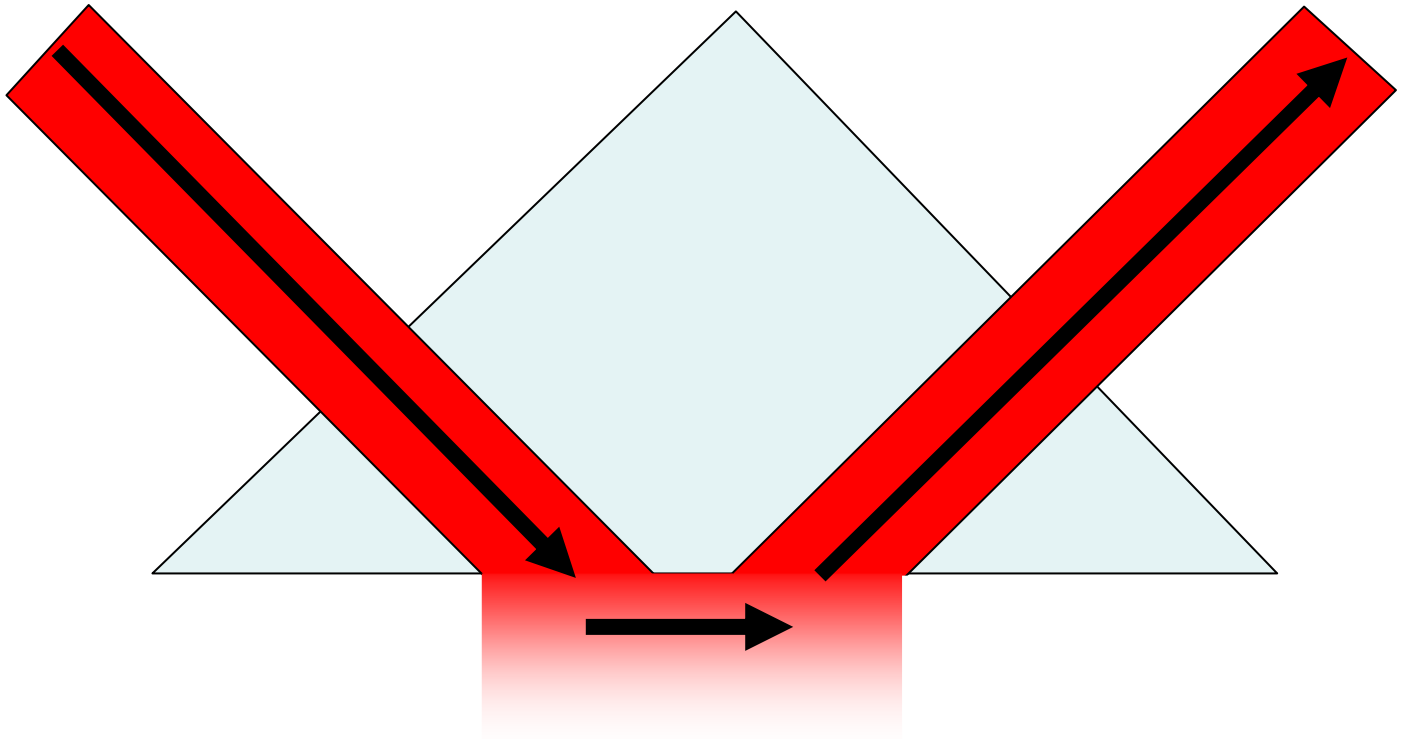
$$\boxed{\left\langle \vec{S}_n(\vec{r},t)\right\rangle = \left[\frac{|t_n|^2 |E_i|^2}{\omega\mu_2}\left(\vec{k}_{i,//} + j\beta\hat{e}_z\right)\right] e^{-2\beta z} \qquad n = \perp, //}$$

Bilancio energia ?

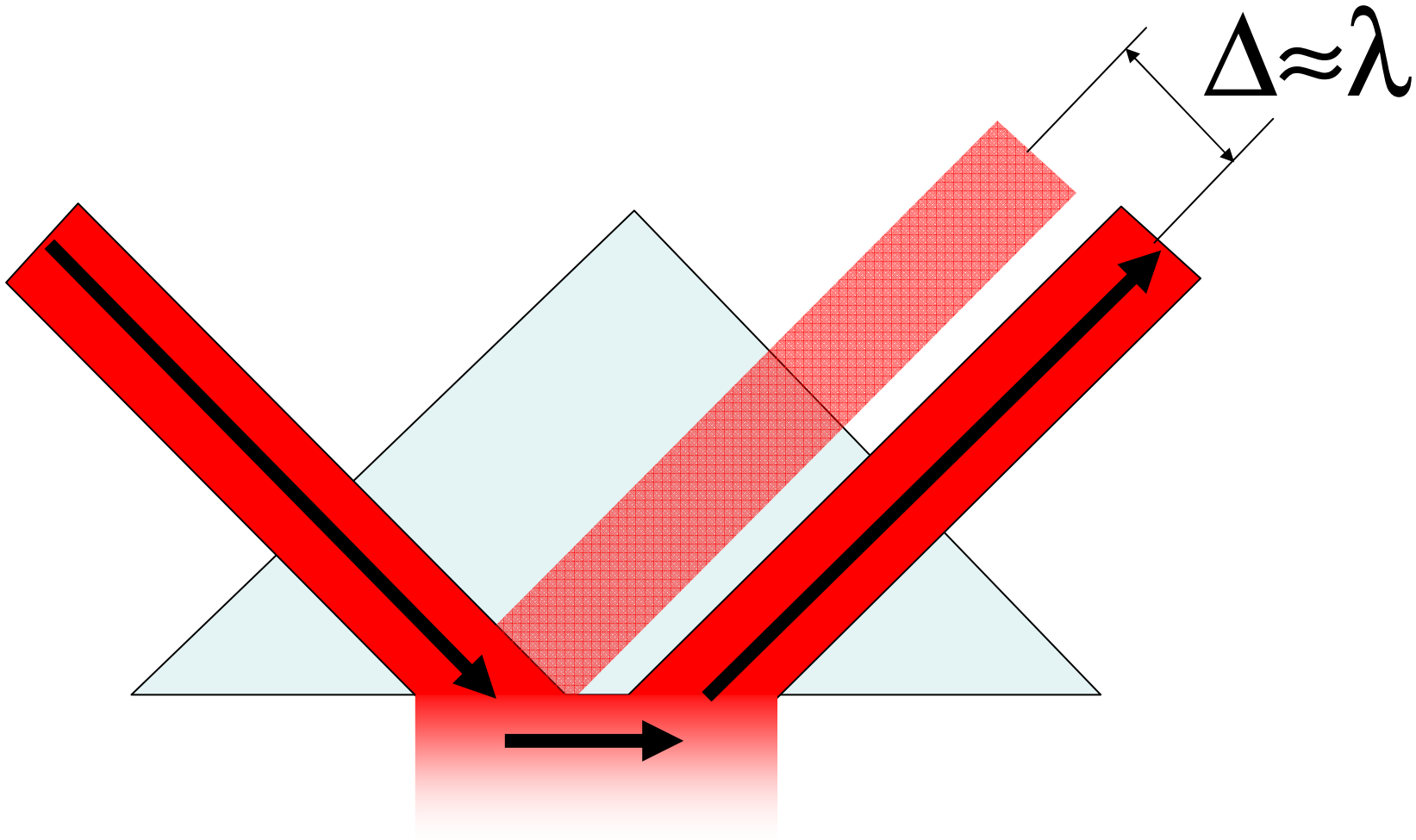


$$\langle \vec{S}_i \rangle = \langle \vec{S}_r \rangle \qquad \langle \vec{S}_t \rangle = ??$$

Goos Hanchen shift



Goos Hanchen shift



Goos Hanchen shift

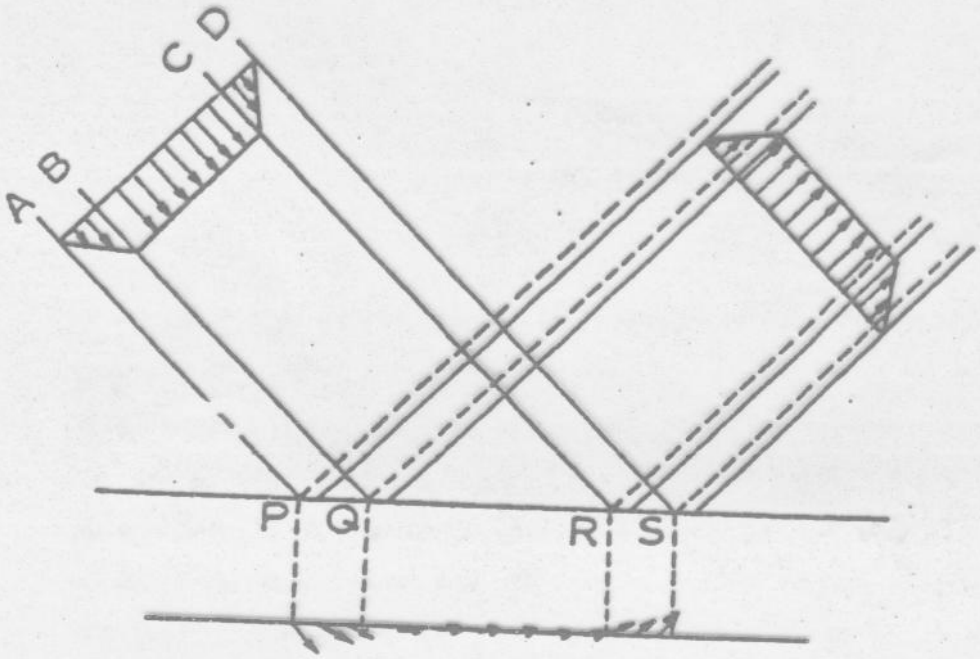
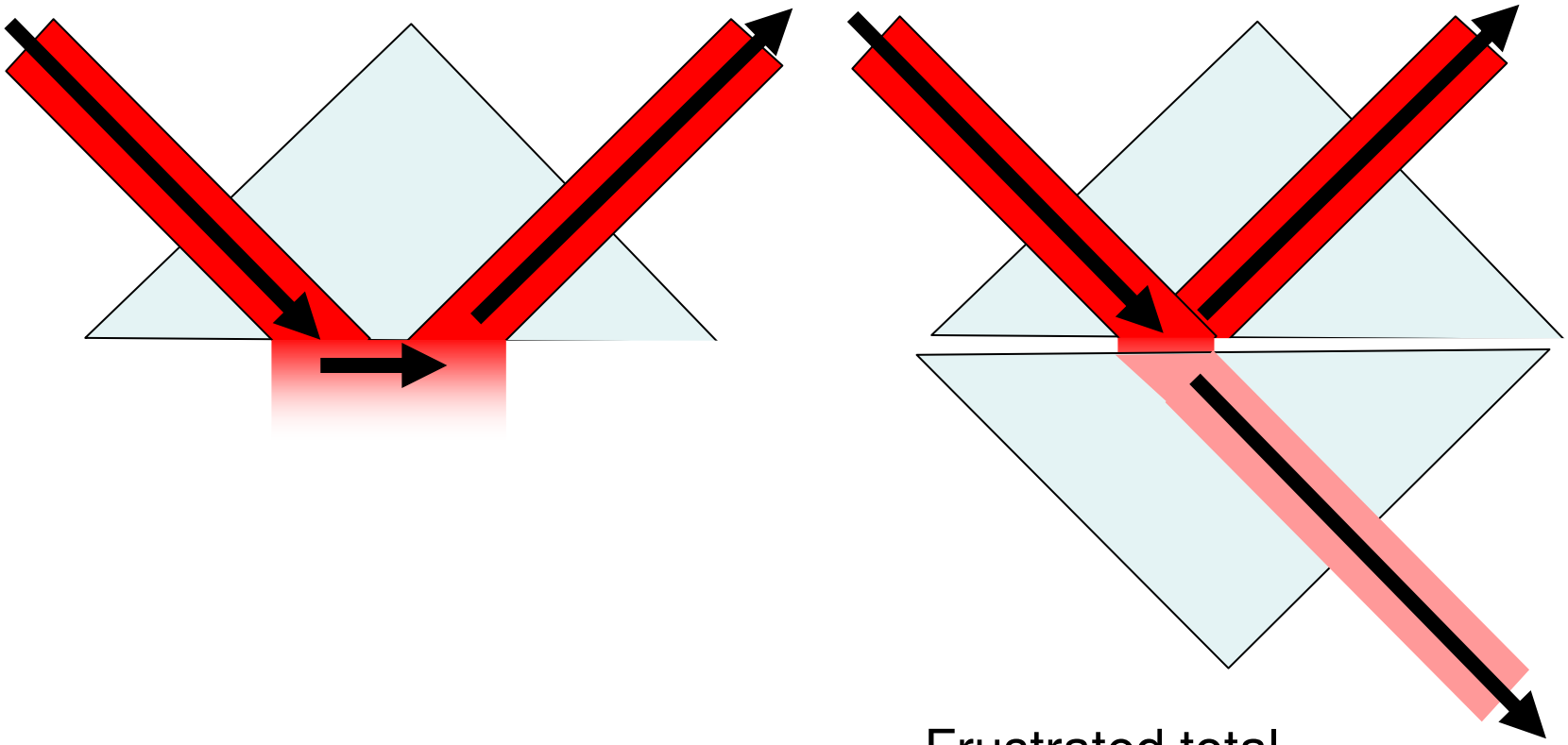


Fig. 11.4. The Poynting vector direction in different parts of a beam *ABCD*. In the region *PQ*, energy enters the rarer medium; in the region *QR*, energy travels parallel to the surface; and in the region *RS*, energy travels out of the rarer medium (shown by arrows in the lower part of the figure).

Misura del Goos Hanchen shift



Frustrated total
attenuated reflection

Misura del Goos Hanchen shift

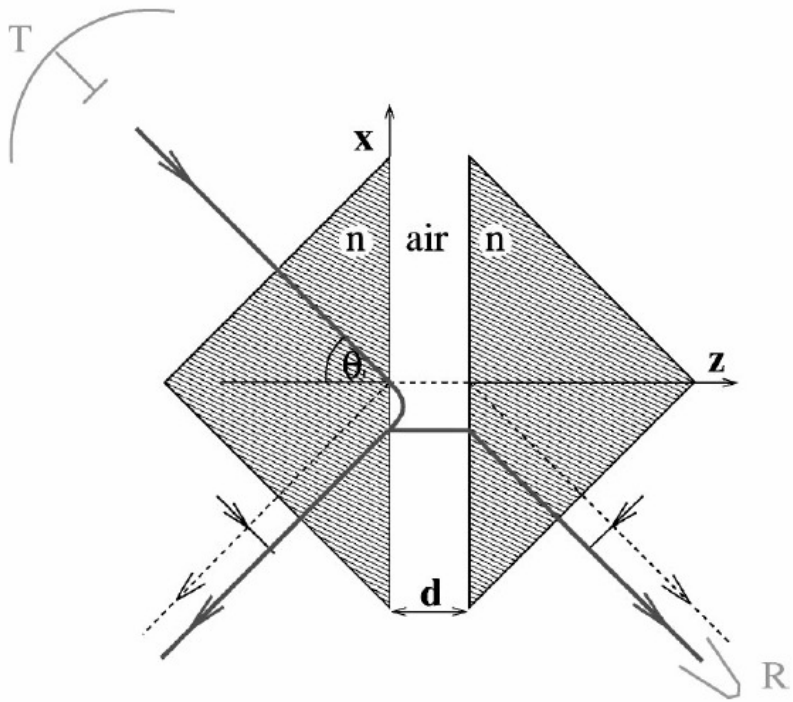
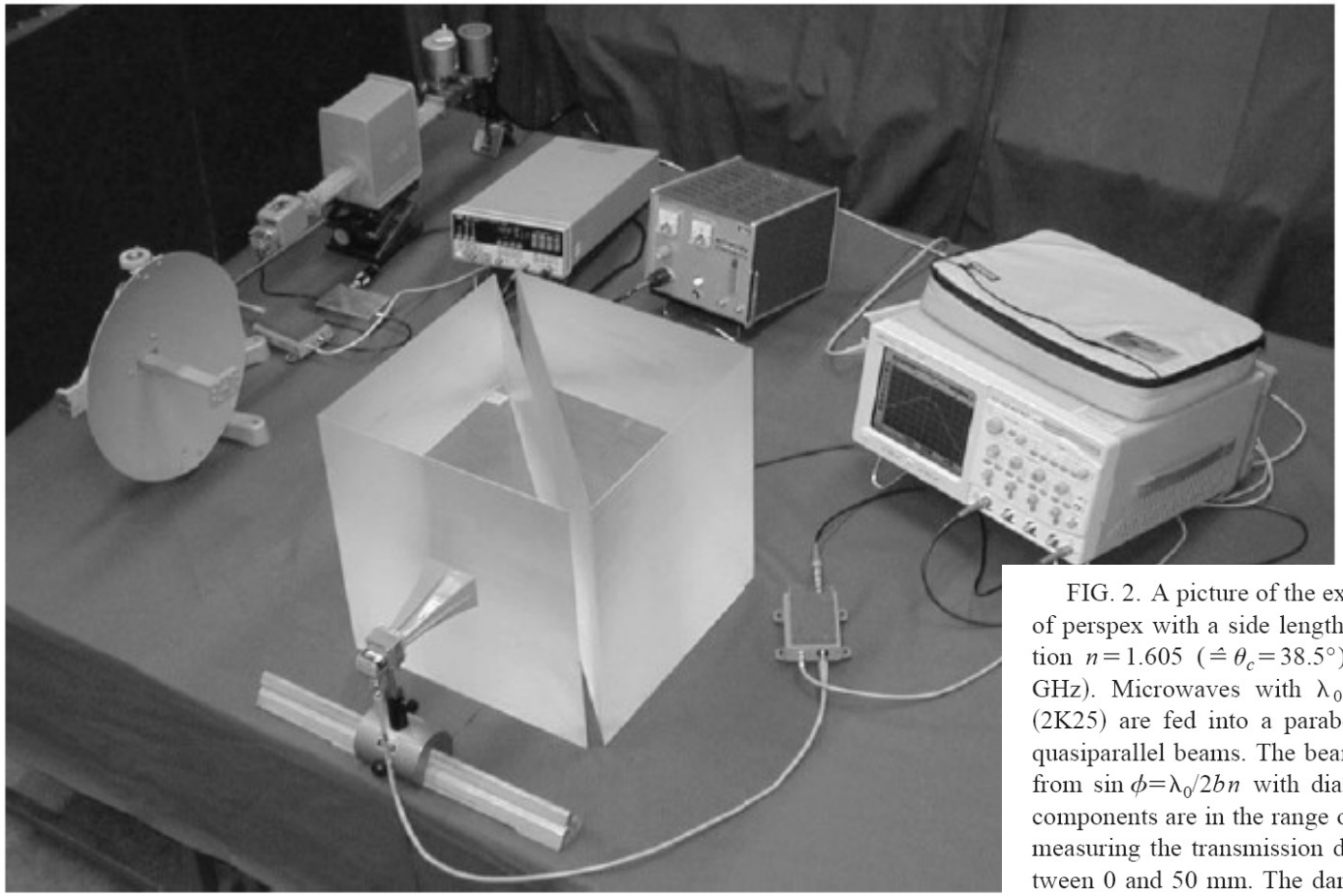


FIG. 1. Sketch of the experimental apparatus showing the parabolic transmitting antenna (T), the prisms, the air gap of width d, the horn antenna used as receiver (R), and the symmetrical shifts of the reflected/transmitted beams, where $\theta_i > \theta_c = \arcsin 1/n$ is the angle of incidence. θ_c is the critical angle of total reflection. The shift of the evanescent wave parallel to the surface in air represents the Goos-Hänchen shift D.

Misura del Goos Hanchen shift



λ=32.8 mm

FIG. 2. A picture of the experiment. The prisms, cut from a cube of perspex with a side length of 400 mm, have an index of refraction $n=1.605$ ($\hat{=}\,\theta_c=38.5°$) at the frequency in question (9.15 GHz). Microwaves with $\lambda_0=32.8$ mm, generated in a klystron (2K25) are fed into a parabolic transmitter antenna guaranteeing quasiparallel beams. The beam spreading is less than 2° as follows from $\sin\phi=\lambda_0/2bn$ with diameter $b_{\text{antenna}}=350$ mm and all beam components are in the range of total reflection. This was verified by measuring the transmission damping depending on the air gap between 0 and 50 mm. The damping would be 1.8 dB in the case of normal reflection compared with our measured 36 dB for the case of $\theta_i=45°$ and a 50-mm gap. The measured value of 7.2 dB/10 mm is in agreement with the theoretical transmission Ref. [17]. The signals have been picked up by a microwave horn and fed across an amplifier to an oscilloscope (HP 54825A). A metallic reflector placed at the base of the first prism to determine the position of the reflected beam in the case of geometrical optics. The results presented here are averaged values of several runs with error bars. (For the photo we put the various components near together to present all of them in one picture.)

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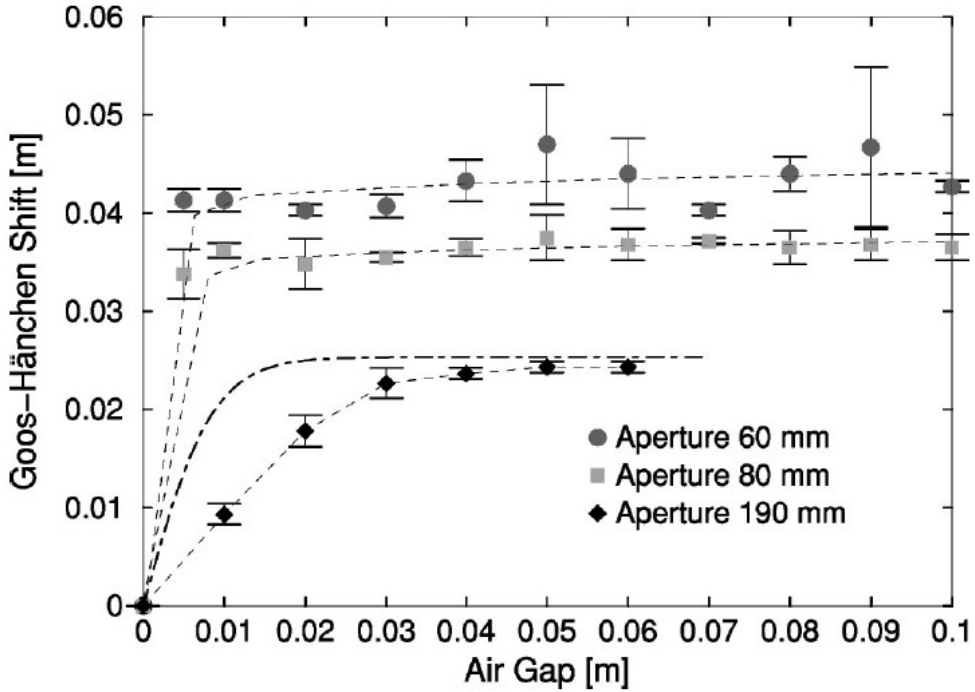
Misura del Goos Hanchen shift



λ=32.8 mm

FIG. 4. The Goos-Hänchen shift vs air gap for different beam diameters in TM polarization and for $\theta_i = 45°$: the shift measured for the large beams (no aperture or an aperture of 190 mm) is roughly in agreement with theoretical prediction (dot-dashed line) [8], while decreasing beam diameters lead to increasing shifts reaching the constant asymptotic value already for very small values of the air gap. The zero point was obtained by substituting the air gap with a metallic plate.

A. Haibel et al. Phys. Rev. E 63, 047601 (2001)