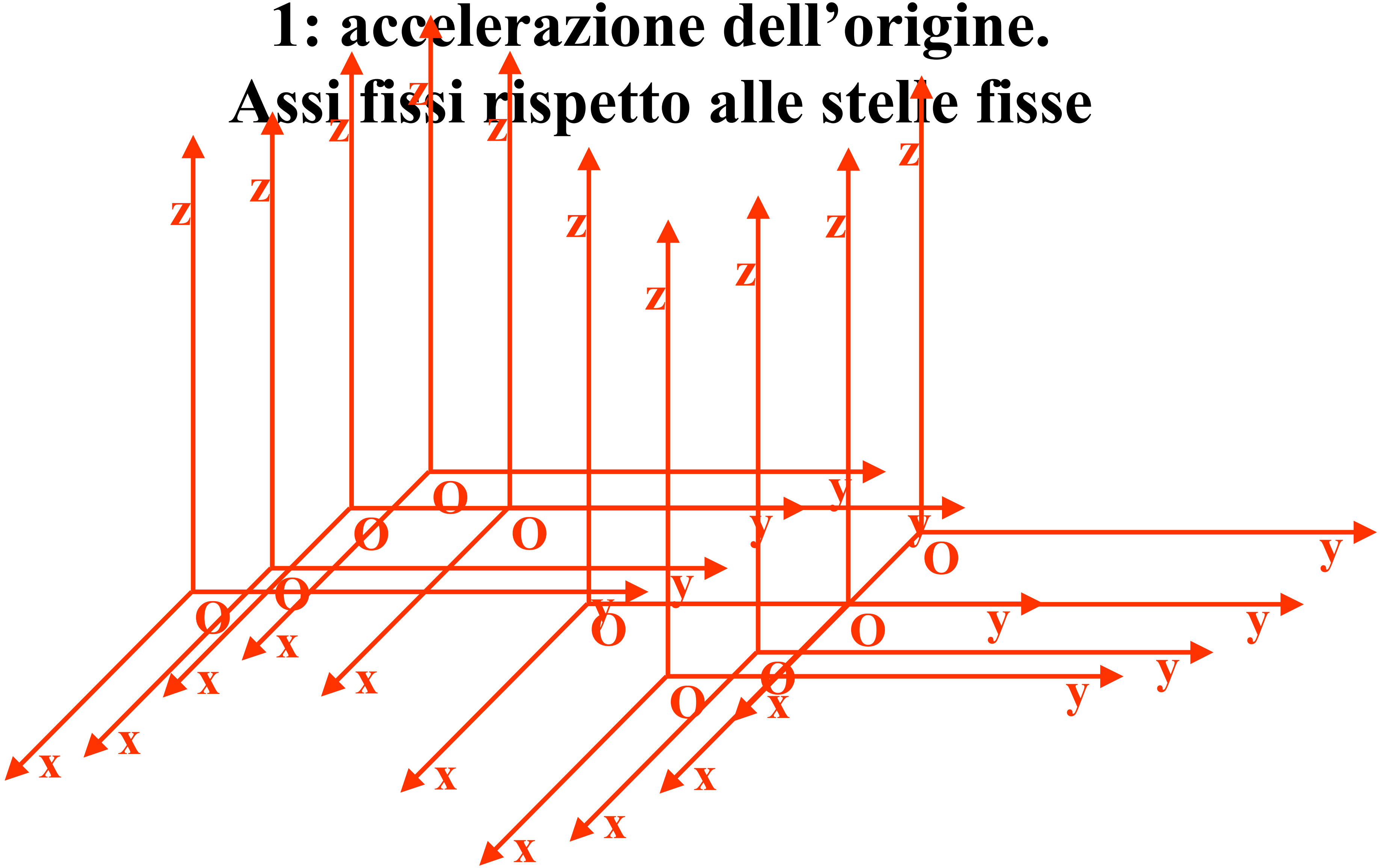
Sistemi di riferimento accelerati (rispetto ad un sistema inerziale) 1: accelerazione dell'origine. Assi fissi rispetto alle stelle fisse



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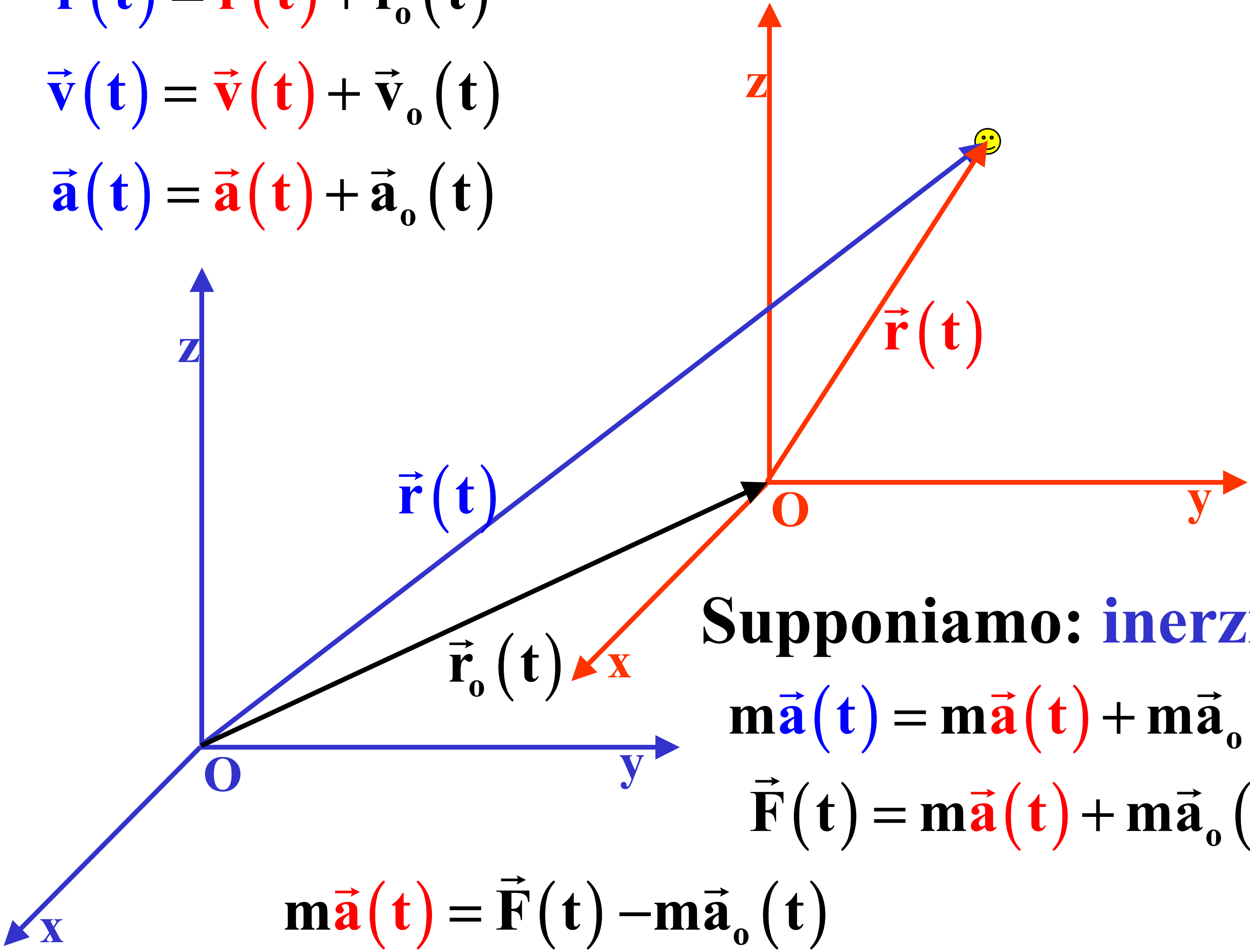
$$\vec{r}(t) = \vec{r}(t) + \vec{r}_o(t)$$

$$\vec{v}(t) = \vec{v}(t) + \vec{v}_o(t)$$

$$\vec{a}(t) = \vec{a}(t) + \vec{a}_o(t)$$



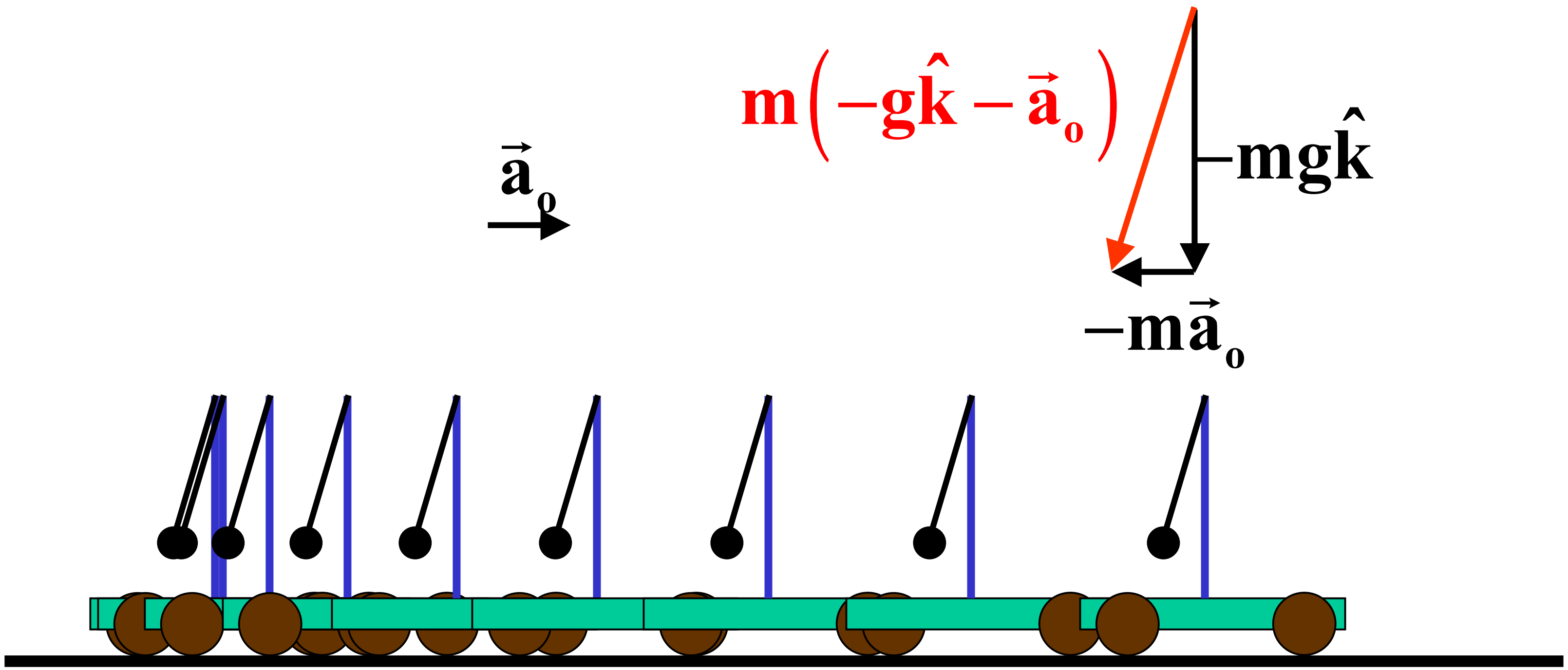
Supponiamo: inerziale

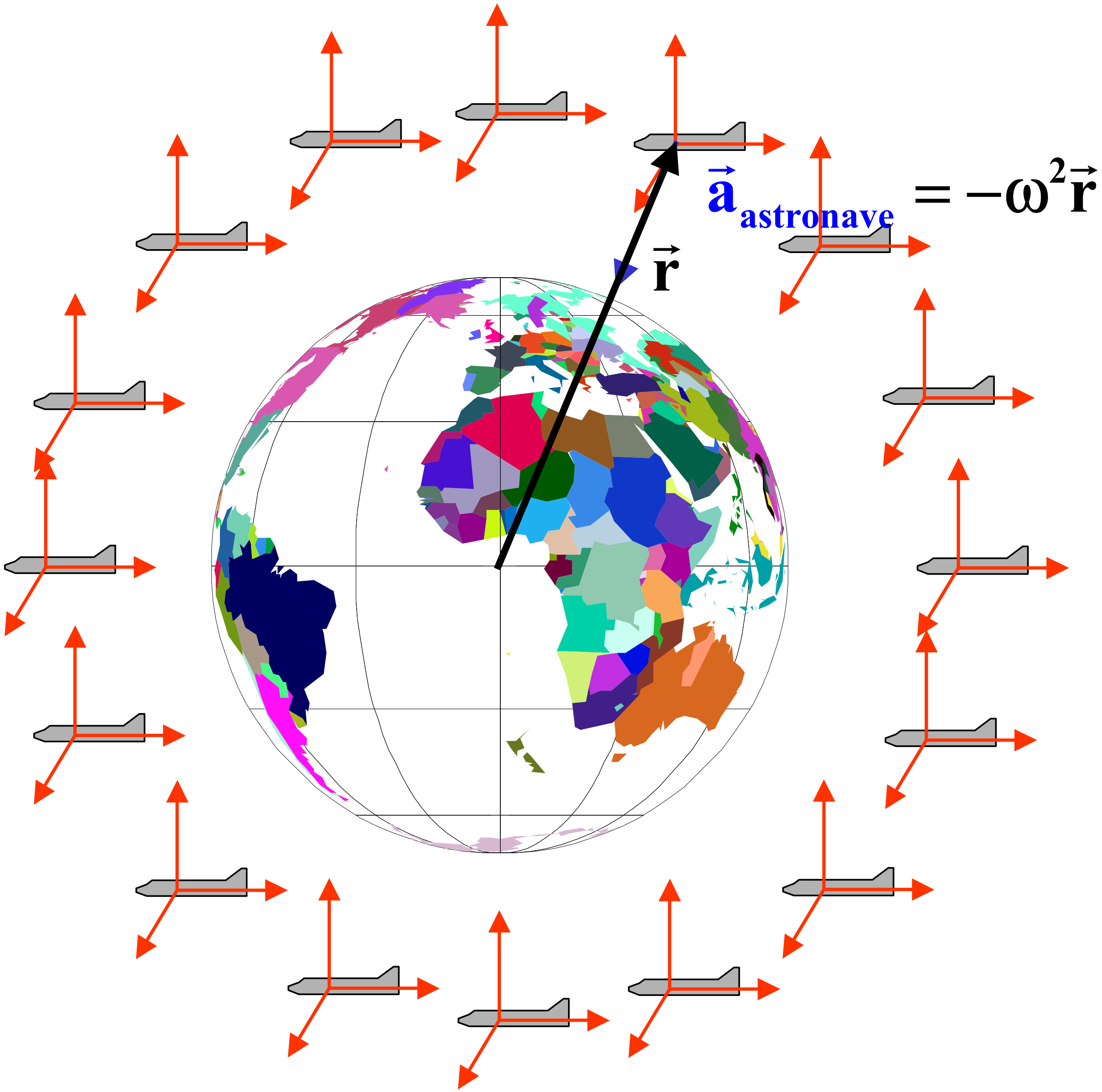
$$m\vec{a}(t) = m\vec{a}(t) + m\vec{a}_o(t)$$

$$\vec{F}(t) = m\vec{a}(t) + m\vec{a}_o(t)$$

$$m\vec{a}(t) = \vec{F}(t) \underbrace{-m\vec{a}_o(t)}_{\text{forzaapparente}}$$

La forza peso efficace





$\vec{a}_{astronave} = -\omega^2 \vec{r}$
$\vec{r}$

Ma le astronavi fanno un moto circolare uniforme ?

$$\vec{F}_{gravità} = -G\frac{M_{\oplus} m_{astronave}}{r^3}\vec{r}$$

(Le sfere si comportano come i punti)

Moto circolare uniforme:

$$\vec{F} = -m_{astronave}\omega^2\vec{r}$$

$$\text{se } \vec{F} = \vec{F}_{gravità} \quad \text{ok!}$$

$$\cancel{-}\cancel{m}_{astronave}\omega^2\cancel{\vec{r}} = \cancel{-}G\frac{M_{\oplus}\cancel{m}_{astronave}}{r^3}\cancel{\vec{r}} \quad \rightarrow \omega^2 = G\frac{M_{\oplus}}{r^3}$$

Un punto materiale nel sistema di riferimento dell'astronave

$$\vec{F}_{tot} = \vec{F}_{gravità} - m\vec{a}_{astronave}$$

$$\vec{a}_{astronave} = -\omega^2 \vec{r} = -G\frac{M_{\oplus}}{r^3}\vec{r}$$

$$\vec{F}_{tot} = -G\frac{M_{\oplus}m}{r_p^3}\vec{r}_p - m\vec{a}_{astronave} \approx -G\frac{M_{\oplus}m}{r^3}\vec{r} - m\vec{a}_{astronave}$$

$$\vec{F}_{tot} \approx -G\frac{M_{\oplus}m}{r^3}\vec{r} + G\frac{M_{\oplus}m}{r^3}\vec{r} = 0$$

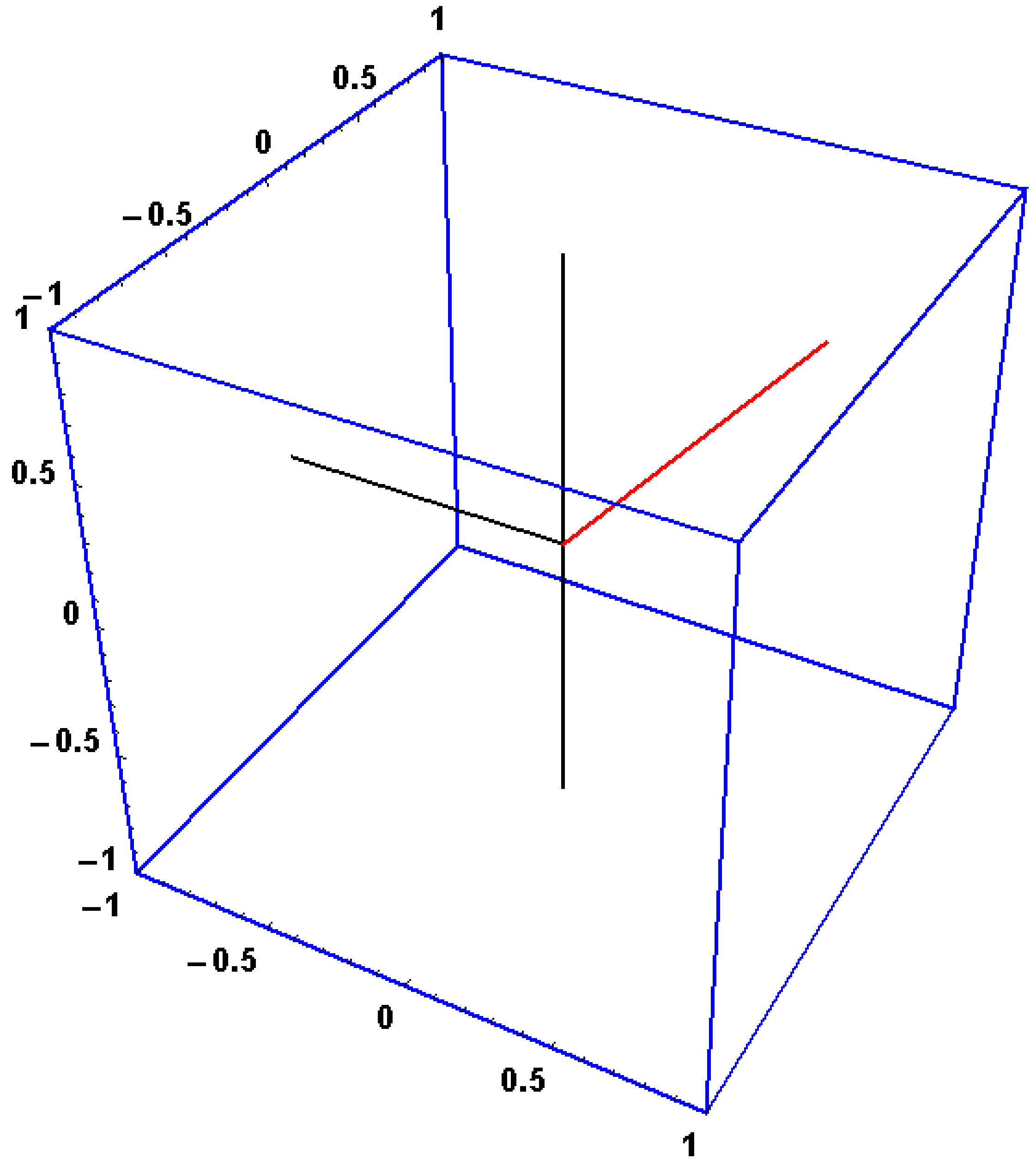
!

L'astronauta non sente il peso

Segue la stessa traiettoria dell'astronave

Rotazione degli assi

Rotazione simultanea di più vettori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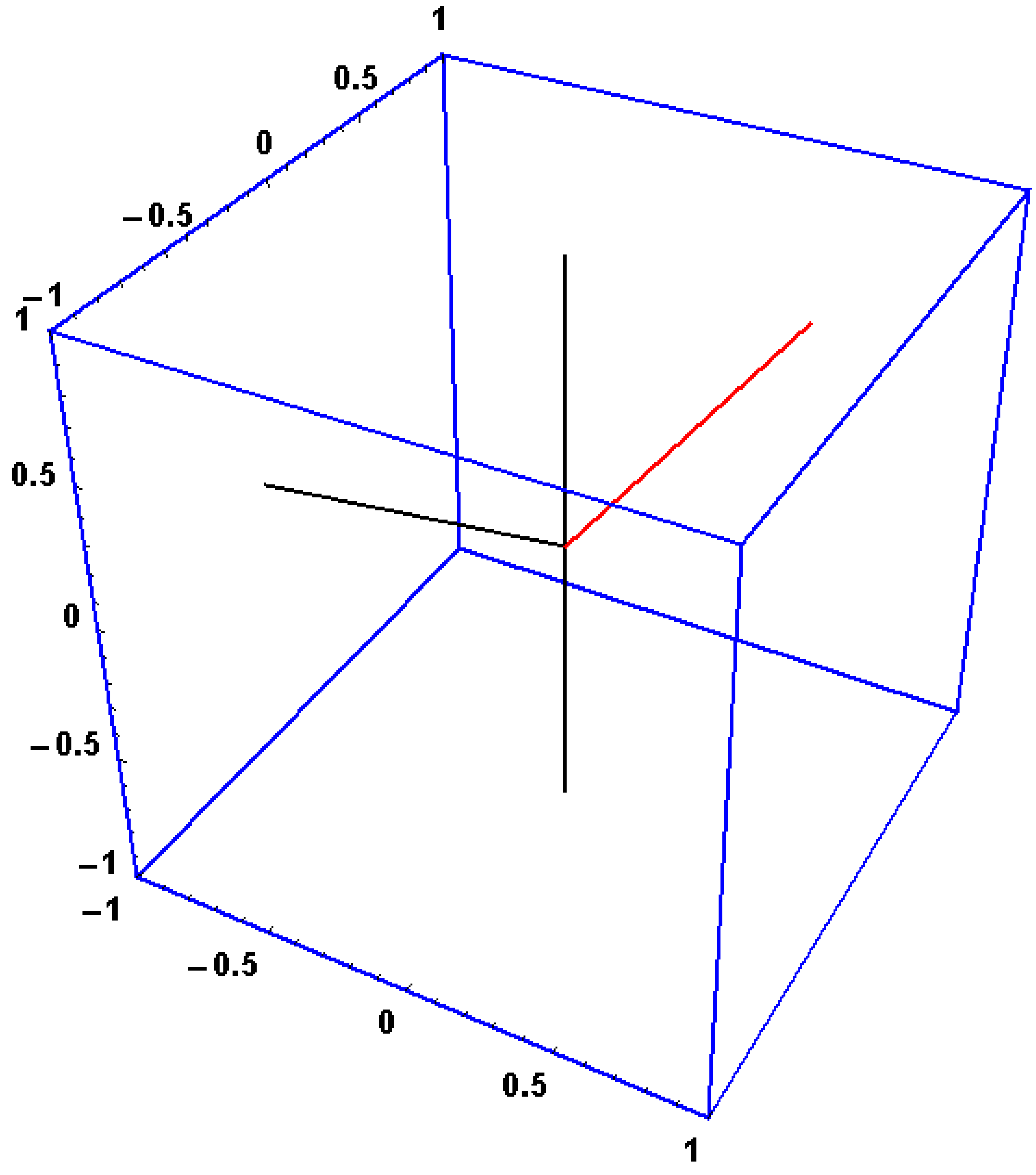
0.5
0
– 0.5
– 1
1
0.5
0
– 0.5
– 1
– 1
– 0.5
0
0.5
1


1
0.5
0
– 0.5
– 1
1
0.5
0
– 0.5
– 1
– 1
– 0.5
0
0.5
1

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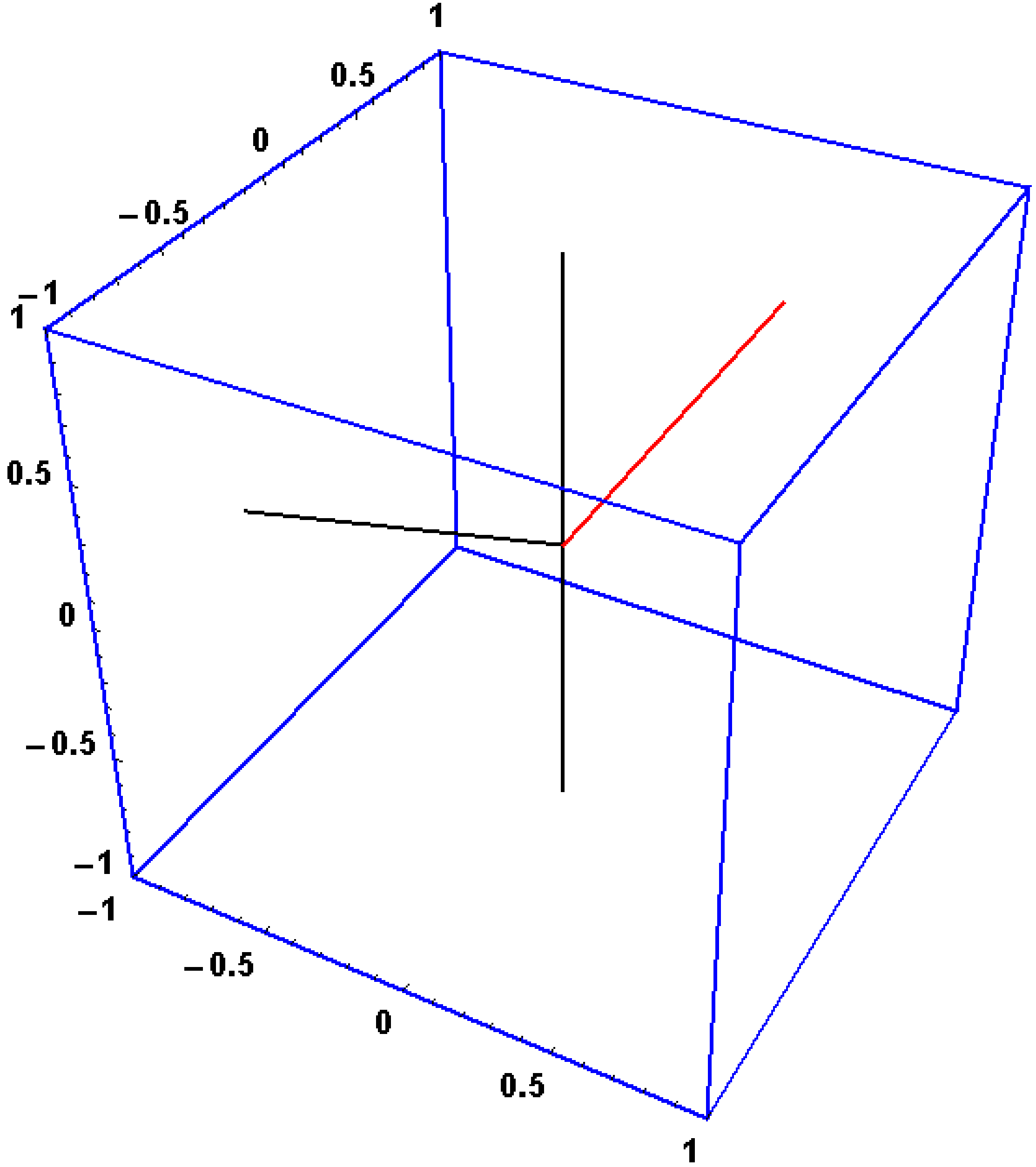

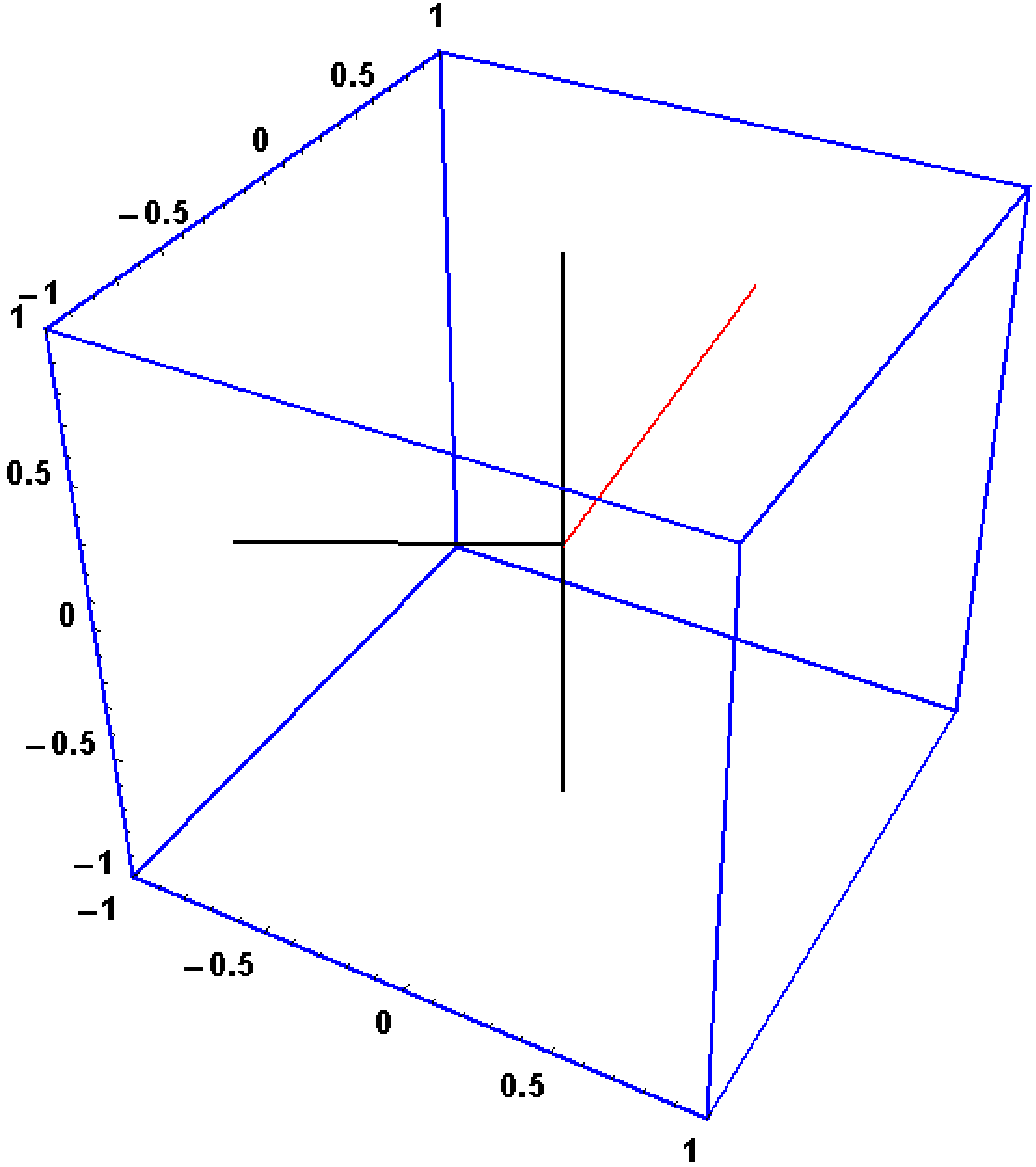
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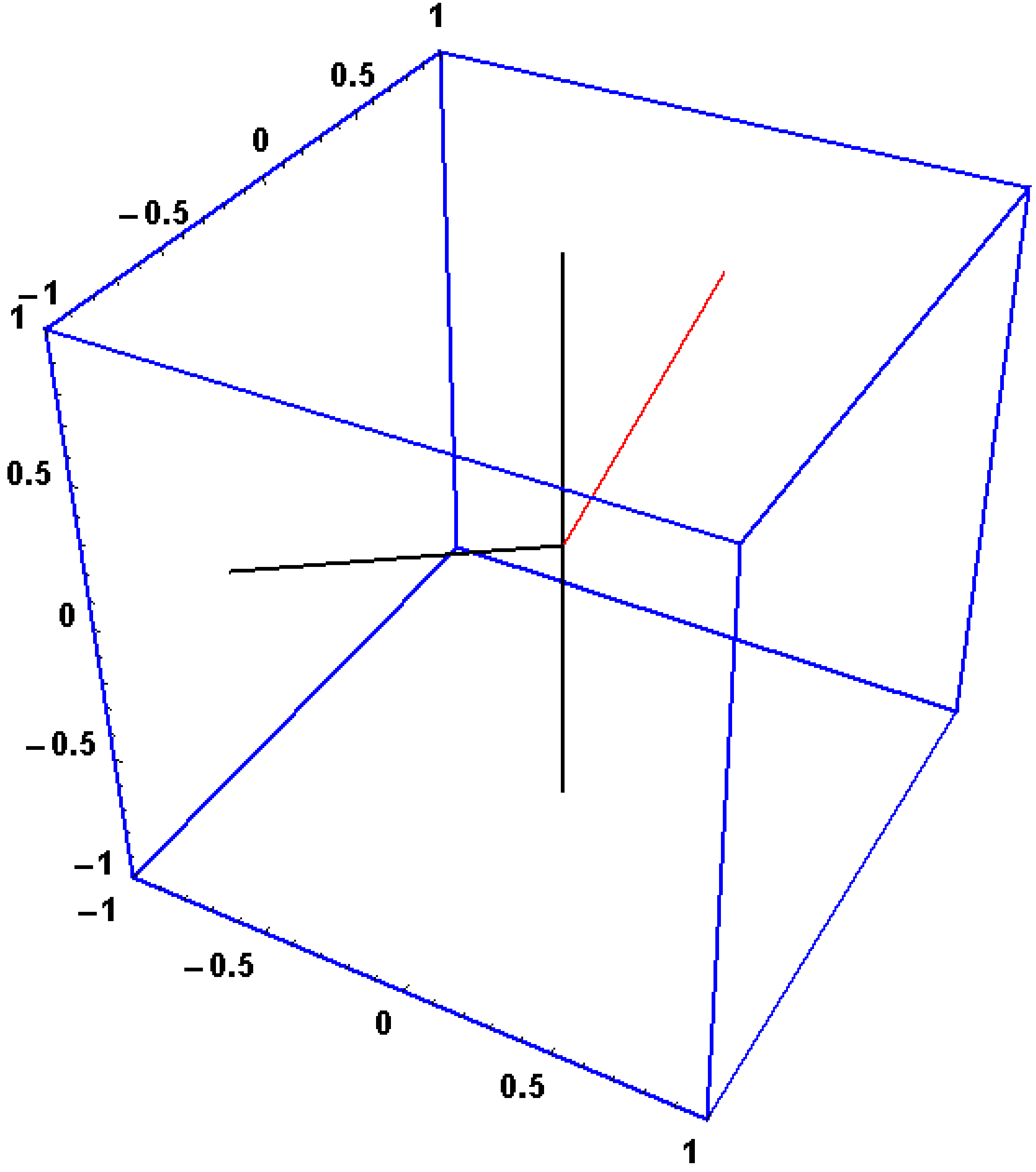
−1
−1
−0.5
0
0.5
1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-1
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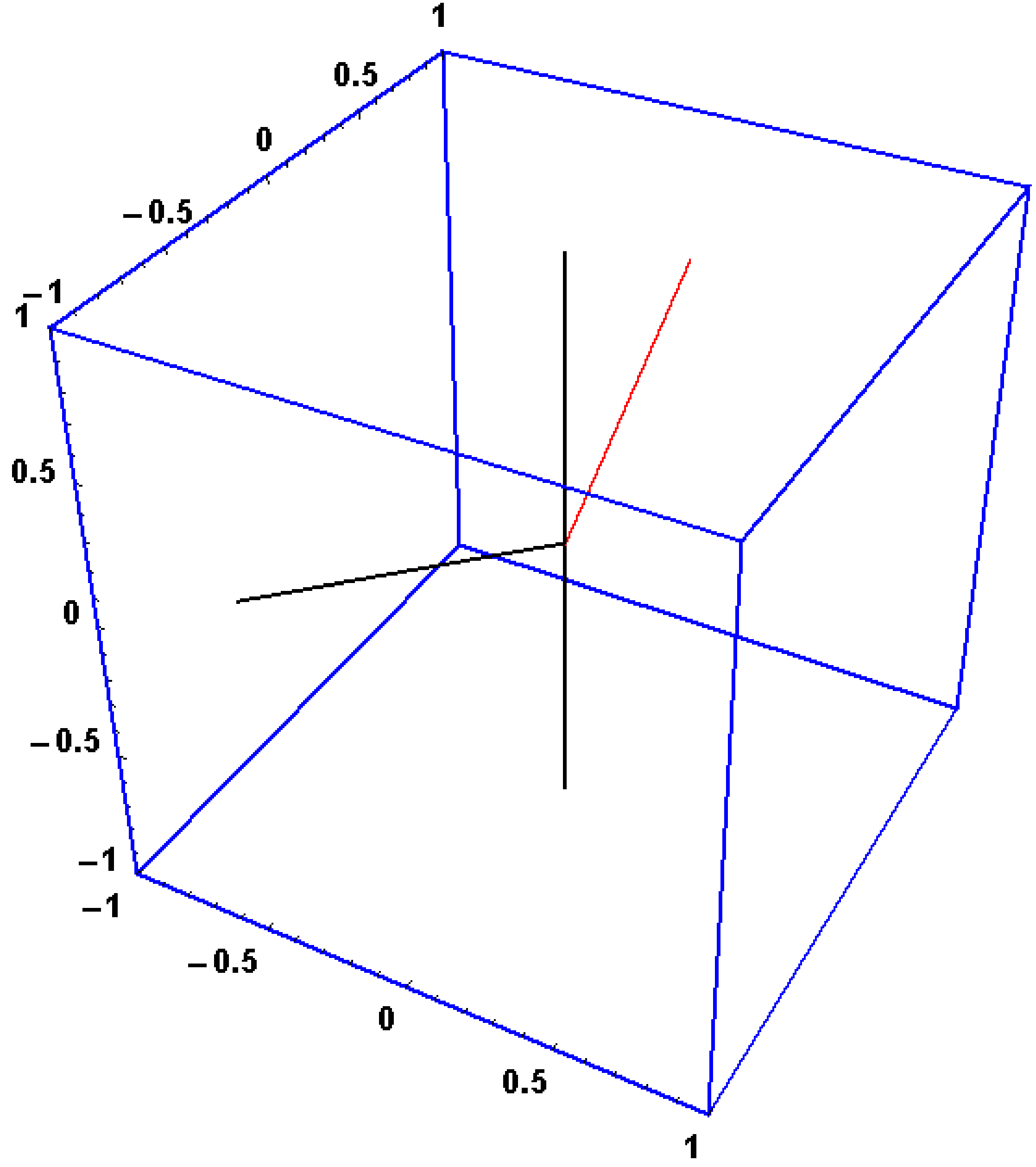
1



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-1
1
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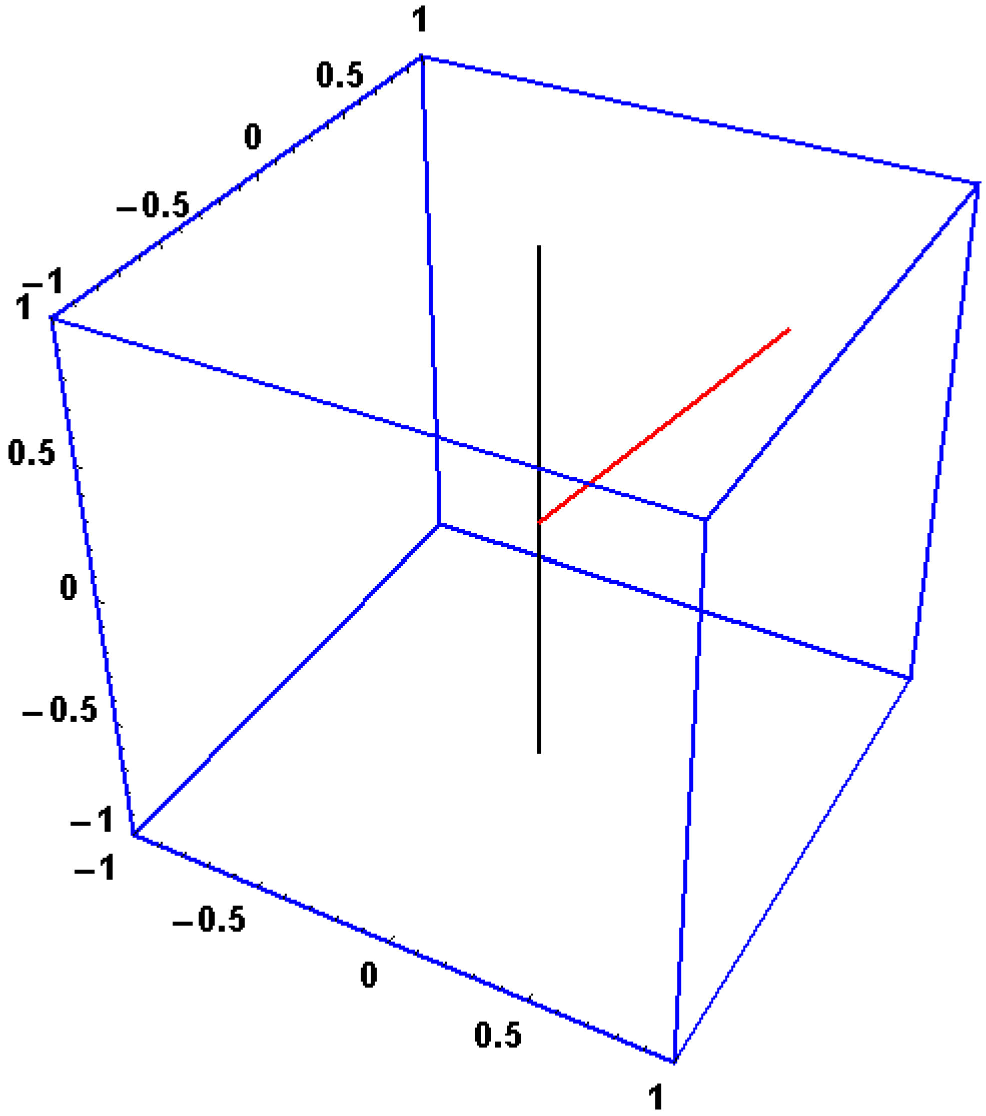
0
-0.5
-1
-1
-0.5
0
0.5
1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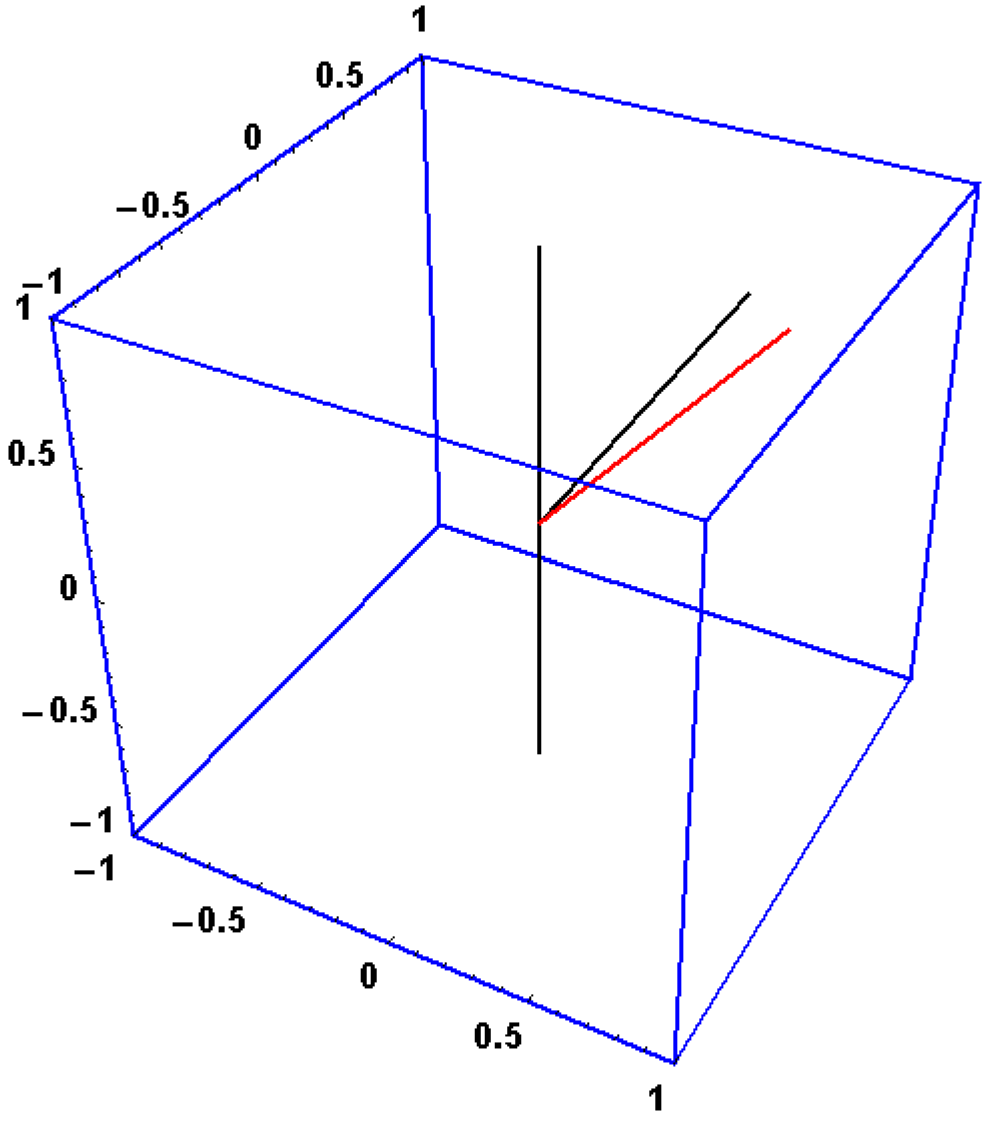
−1
−1
−0.5
0
0.5
1



1
0.5
0
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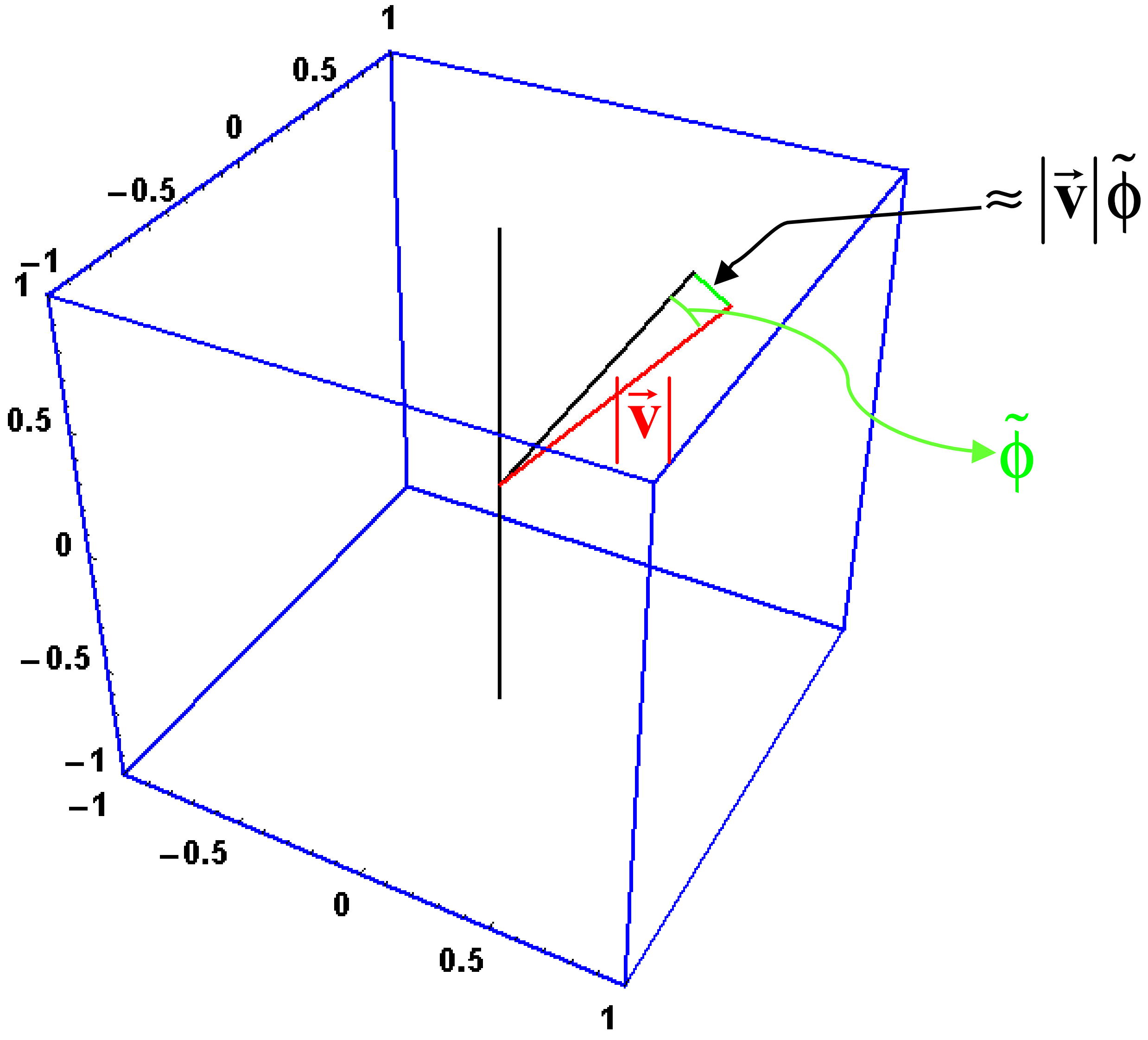
-0.5
0
0.5
1


1
0.5
0
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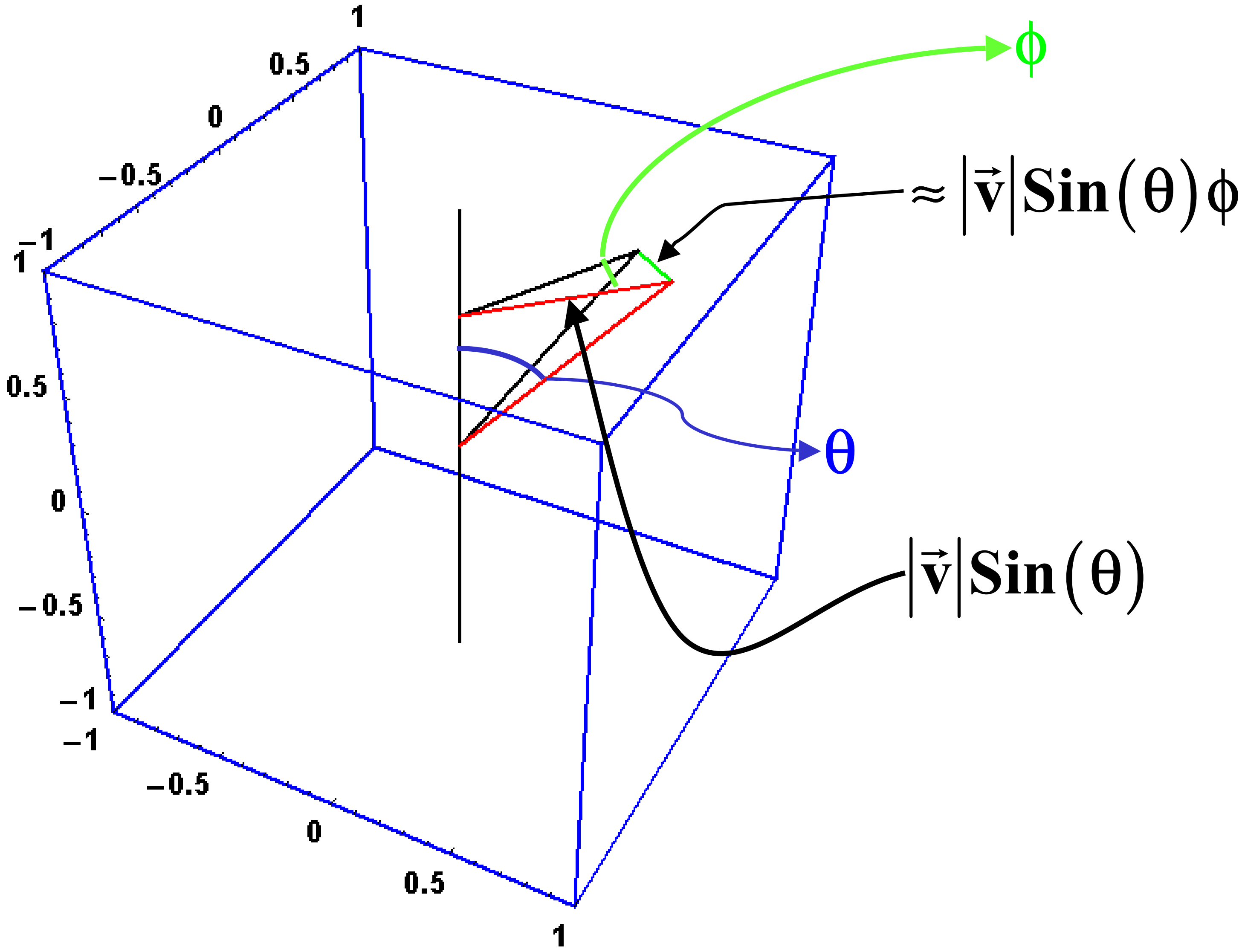
-1
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
$\approx \left|\vec{v}\right|\tilde{\phi}$
$\left|\vec{v}\right|$
$\tilde{\phi}$


1
0.5
0
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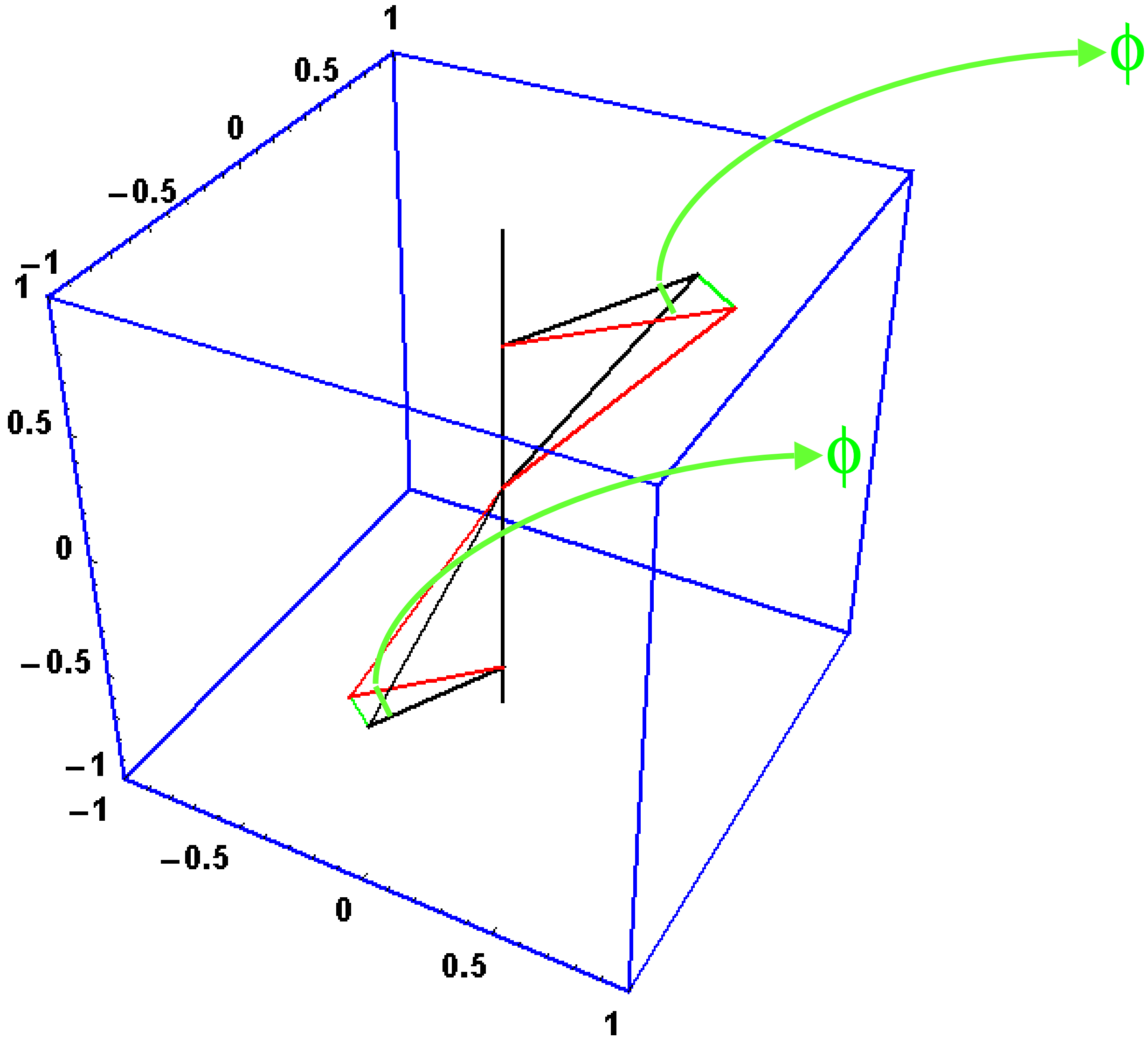
−1
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
ϕ
θ
$\approx |\vec{v}| \mathbf{Sin}(\theta)\phi$
$|\vec{v}| \mathbf{Sin}(\theta)$



1
0.5
0
−0.5
−1
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
ϕ
ϕ

Nella rotazione simultanea di più vettori intorno allo stesso asse:

$$\vec{v}_i(t+\Delta t)-\vec{v}_i(t)\perp\vec{v}_i(t)$$

$$\left|\vec{v}_i(t+\Delta t)-\vec{v}_i(t)\right|\approx\left|\vec{v}_i(t)\right|\mathrm{Sin}(\theta_i)\,\phi$$

$$\vec{v}_i(t+\Delta t)-\vec{v}_i(t)\perp\hat{\phi}$$

Limite $\quad \Delta t\to 0 \quad \phi\to 0 \quad \dfrac{\phi}{\Delta t}\to\dfrac{d\phi}{dt}$

$$\frac{d\vec{v}_i}{dt}\perp\vec{v}_i(t) \qquad \left|\frac{d\vec{v}_i}{dt}\right|\approx\left|\vec{v}_i(t)\right|\mathrm{Sin}(\theta_i)\frac{d\phi}{dt} \qquad \frac{d\vec{v}_i}{dt}\perp\hat{\phi}$$

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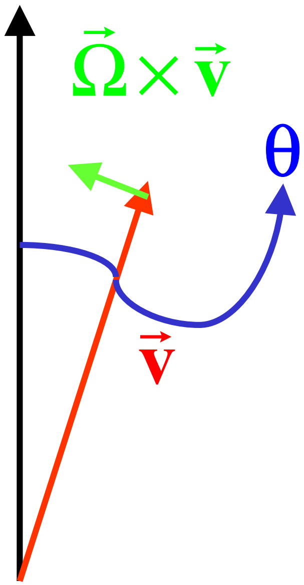
$$\vec{\Omega} \equiv \hat{\phi}\frac{d\phi}{dt}$$



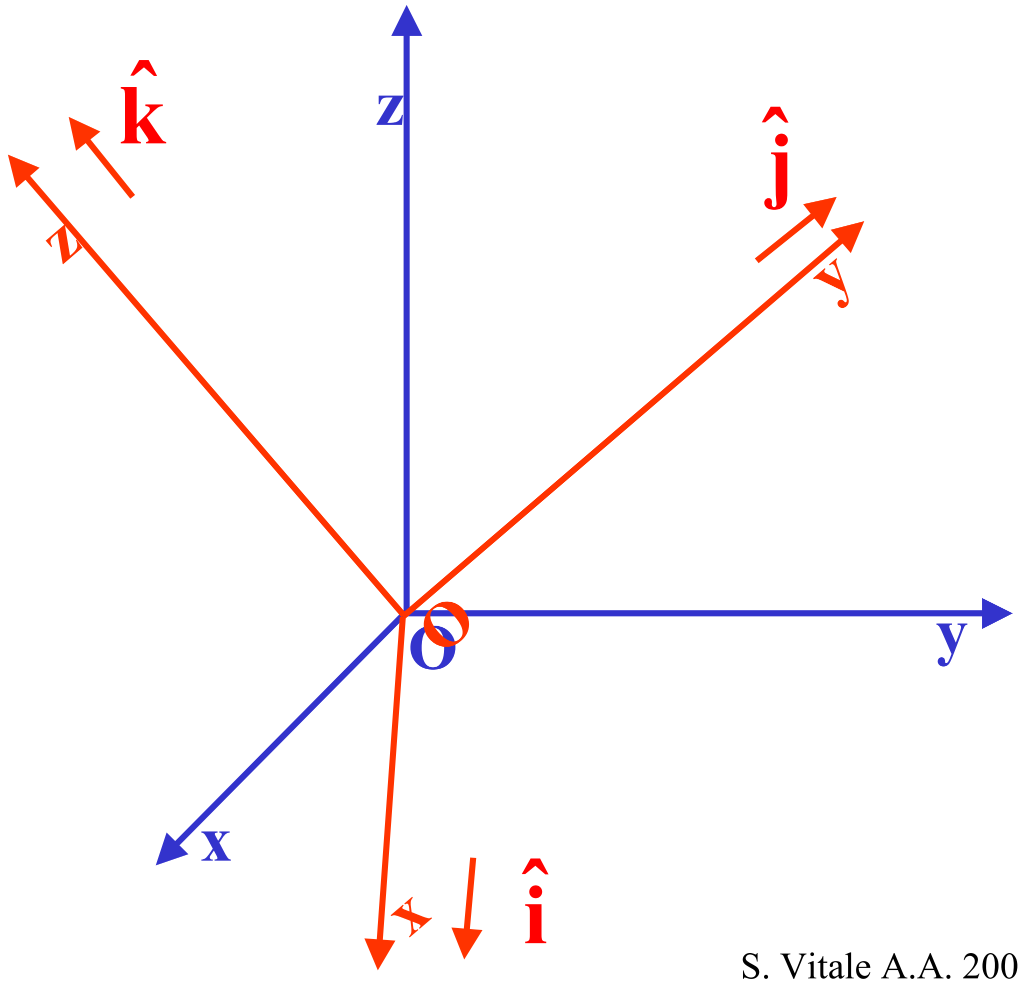
$$\vec{\Omega} \times \vec{v} \perp \vec{v}$$

$$\vec{\Omega} \times \vec{v} \perp \vec{\Omega}$$

$$\left|\vec{\Omega} \times \vec{v}\right| \perp \left|\vec{v}\right|\left|\vec{\Omega}\right| \mathrm{Sin}(\theta)$$

$$\frac{d\vec{v}}{dt} = \vec{\Omega} \times \vec{v}$$

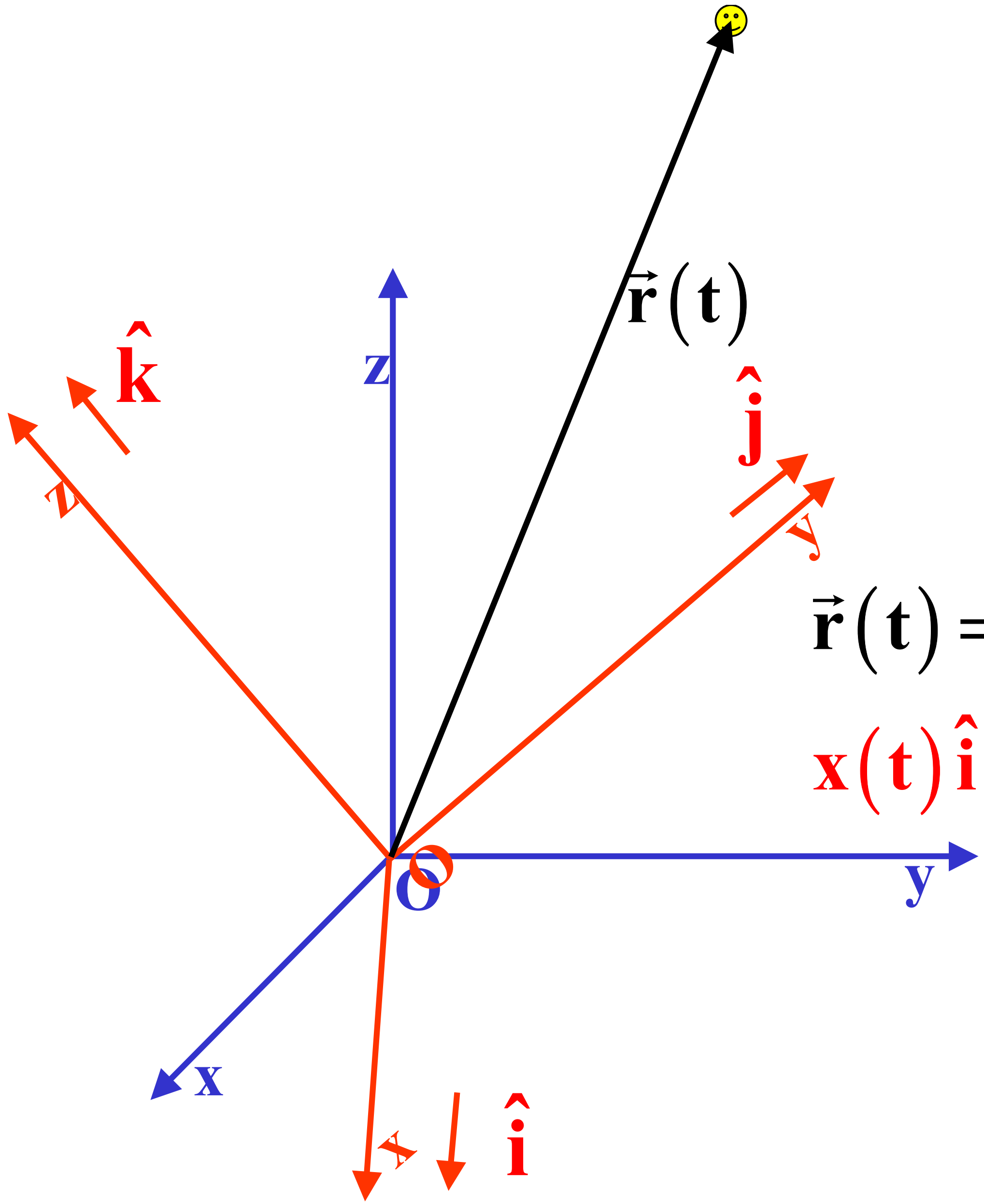
Visti dall'osservatore blu



$$\frac{d\hat{i}}{dt} = \vec{\Omega} \times \hat{i}$$

$$\frac{d\hat{j}}{dt} = \vec{\Omega} \times \hat{j}$$

$$\frac{d\hat{z}}{dt} = \vec{\Omega} \times \hat{z}$$



$$\vec{r}(t) =$$

$$x(t)\hat{i}(t) + y(t)\hat{j}(t) + z(t)\hat{k}(t) =$$

$$x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

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$$\vec{r}(t) = x(t)\,\hat{i}(t) + y(t)\,\hat{j}(t) + z(t)\,\hat{k}(t) =$$

$$x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

$$\vec{v}(t) = \frac{dx(t)}{dt}\hat{i}(t) + \frac{dy(t)}{dt}\hat{j}(t) + \frac{dz(t)}{dt}\hat{k}(t) +$$

$$x(t)\frac{d\hat{i}(t)}{dt} + y(t)\frac{d\hat{j}(t)}{dt} + z(t)\frac{d\hat{k}(t)}{dt}$$

$$= \underbrace{\frac{dx(t)}{dt}\hat{i} + \frac{dy(t)}{dt}\hat{j} + \frac{dz(t)}{dt}\hat{k}}_{\vec{v}(t)}$$

$$\vec{v}(t) = \underbrace{v_x(t)\hat{i}(t) + v_y(t)\hat{j}(t) + v_z(t)\hat{k}(t)}_{\vec{v}(t)} +$$

$$x(t)\left[\vec{\Omega}\times\hat{i}(t)\right] + y(t)\left[\vec{\Omega}\times\hat{j}(t)\right] + z(t)\left[\vec{\Omega}\times\hat{k}(t)\right]$$

$$= \vec{v}(t)$$

$$+\left[\vec{\Omega}\times x(t)\hat{i}(t)\right] + \left[\vec{\Omega}\times y(t)\hat{j}(t)\right] + \left[\vec{\Omega}\times z(t)\hat{k}(t)\right]$$

$$= \vec{v}(t)$$

$$+\vec{\Omega}\times\left[x(t)\hat{i}(t) + y(t)\hat{j}(t) + z(t)\hat{k}(t)\right]$$

$$= \vec{v}(t) + \vec{\Omega}\times\vec{r}(t)$$

$$\vec{v}(t) = \vec{v}(t) + \vec{\Omega} \times \vec{r}(t)$$

Vale per qualunque vettore

$$\frac{d\vec{A}(t)}{dt} = \frac{d\vec{A}(t)}{dt} + \vec{\Omega} \times \vec{A}(t)$$

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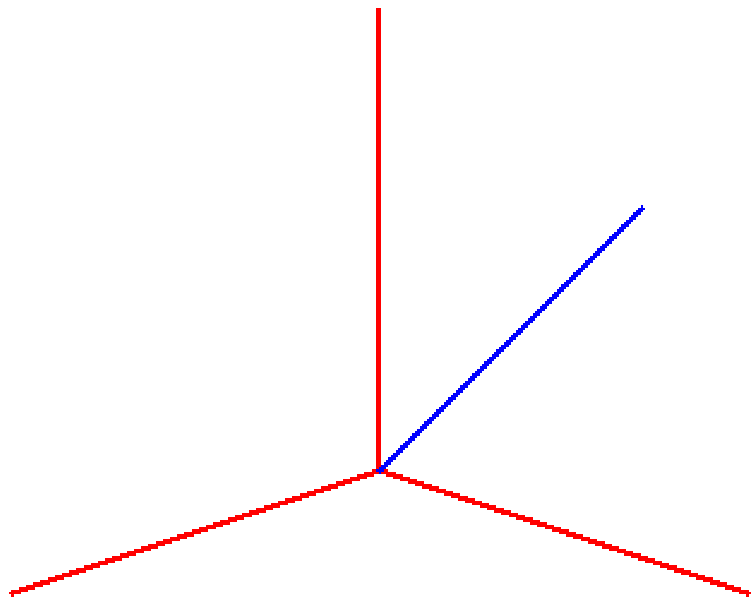
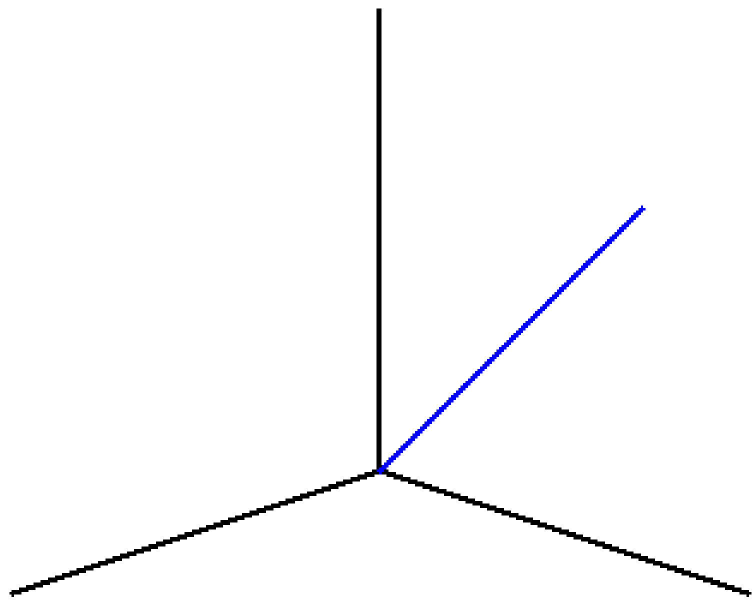


Vettore blu fermo nel sistema nero



Vettore blu visto nel sistema rosso

La derivata di un vettore dipende dal sistema di riferimento rispetto al quale viene calcolata

$$\frac{d\vec{A}(t)}{dt}=\frac{d\vec{A}(t)}{dt}+\vec{\Omega}\times\vec{A}(t)$$

$$\frac{d\vec{A}(t)}{dt}=\frac{d\vec{A}(t)}{dt}+(-\vec{\Omega})\times\vec{A}(t)$$

Un eccezione

$$\frac{d\vec{\Omega}}{dt}=\frac{d\vec{\Omega}}{dt}+\vec{\Omega}\times\vec{\Omega}=\frac{d\vec{\Omega}}{dt}$$

Accelerazione

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d\vec{r}(t)}{dt} + \vec{\Omega}\times\vec{r}(t) = \vec{v}(t) + \vec{\Omega}\times\vec{r}(t)$$

$$\frac{d\vec{v}(t)}{dt} = \frac{d\vec{v}(t)}{dt} + \vec{\Omega}\times\vec{v}(t)$$

$$\frac{d\vec{v}(t)}{dt} = \frac{d\vec{v}(t)+\vec{\Omega}\times\vec{r}(t)}{dt} + \vec{\Omega}\times\left[\vec{v}(t)+\vec{\Omega}\times\vec{r}(t)\right]$$

$$= \frac{d\vec{v}(t)}{dt} + \frac{d\vec{\Omega}}{dt}\times\vec{r}(t) + \vec{\Omega}\times\vec{v}(t) + \vec{\Omega}\times\vec{v}(t) + \vec{\Omega}\times\left[\vec{\Omega}\times\vec{r}(t)\right]$$

$$= \frac{d\vec{v}(t)}{dt} + \frac{d\vec{\Omega}}{dt}\times\vec{r}(t) + 2\vec{\Omega}\times\vec{v}(t) + \vec{\Omega}\times\left[\vec{\Omega}\times\vec{r}(t)\right]$$

$$\frac{d\vec{v}(t)}{dt} = \frac{d\vec{v}(t)}{dt} + \frac{d\vec{\Omega}}{dt}\times\vec{r}(t) + 2\vec{\Omega}\times\vec{v}(t) + \vec{\Omega}\times\left[\vec{\Omega}\times\vec{r}(t)\right]$$

$$\vec{a}(t) = \vec{a}(t) + \vec{r}(t)\times\frac{d\vec{\Omega}}{dt} + 2\vec{v}(t)\times\vec{\Omega} + \vec{\Omega}\times\left[\vec{r}(t)\times\vec{\Omega}\right]$$

$$m\vec{a}(t) = m\vec{a}(t) + m\left\{\vec{r}(t)\times\frac{d\vec{\Omega}}{dt} + 2\vec{v}(t)\times\vec{\Omega} + \vec{\Omega}\times\left[\vec{r}(t)\times\vec{\Omega}\right]\right\}$$

$$m\vec{a}(t) = \vec{F}_{reale} + \underbrace{m\left\{\vec{r}(t)\times\frac{d\vec{\Omega}}{dt} + 2\vec{v}(t)\times\vec{\Omega} + \vec{\Omega}\times\left[\vec{r}(t)\times\vec{\Omega}\right]\right\}}_{\text{Forza apparente}}$$

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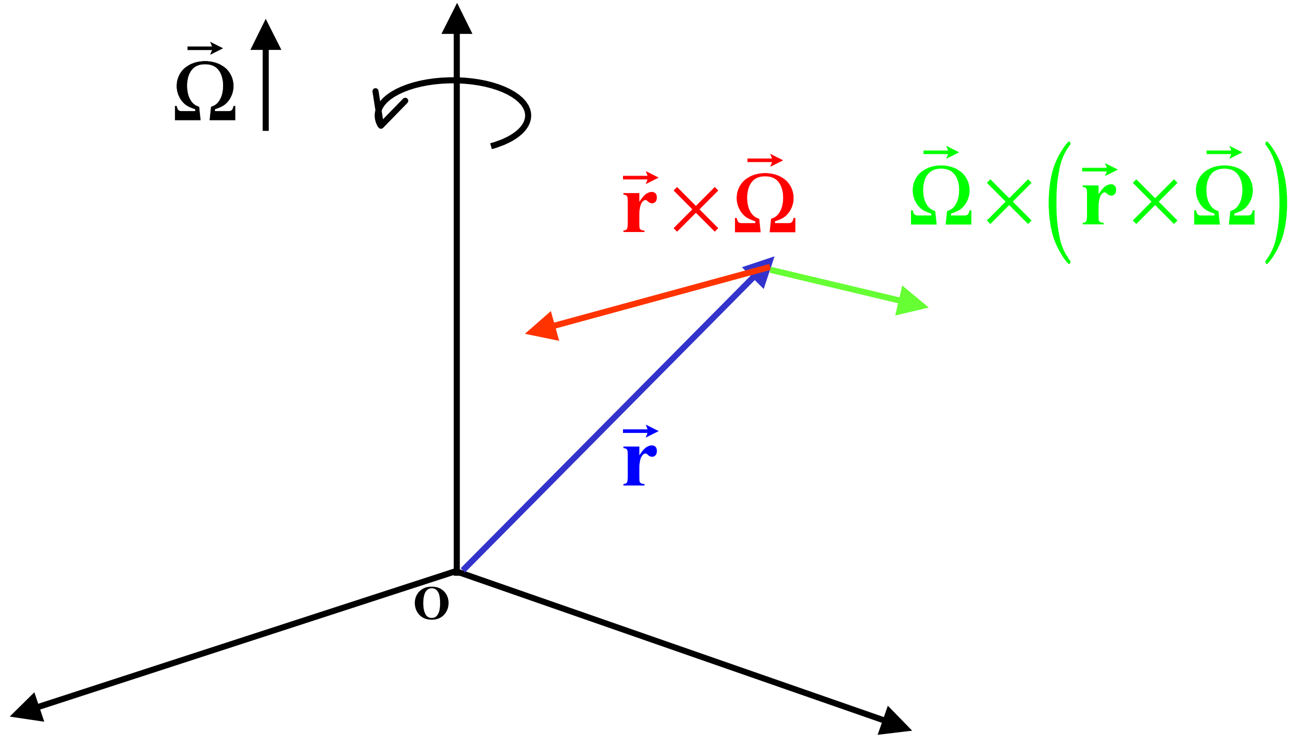
$$m\vec{a}(t) = \vec{F}_{reale} +$$

$$+\underbrace{m\vec{r}(t)\times\frac{d\vec{\Omega}}{dt}}_{\text{Forza tangenziale}}$$

$$+\underbrace{m2\vec{v}(t)\times\vec{\Omega}}_{\text{Forza di Coriolis}}$$

$$\underbrace{m\vec{\Omega}\times\left[\vec{r}(t)\times\vec{\Omega}\right]}_{\text{Forza centrifuga}}$$

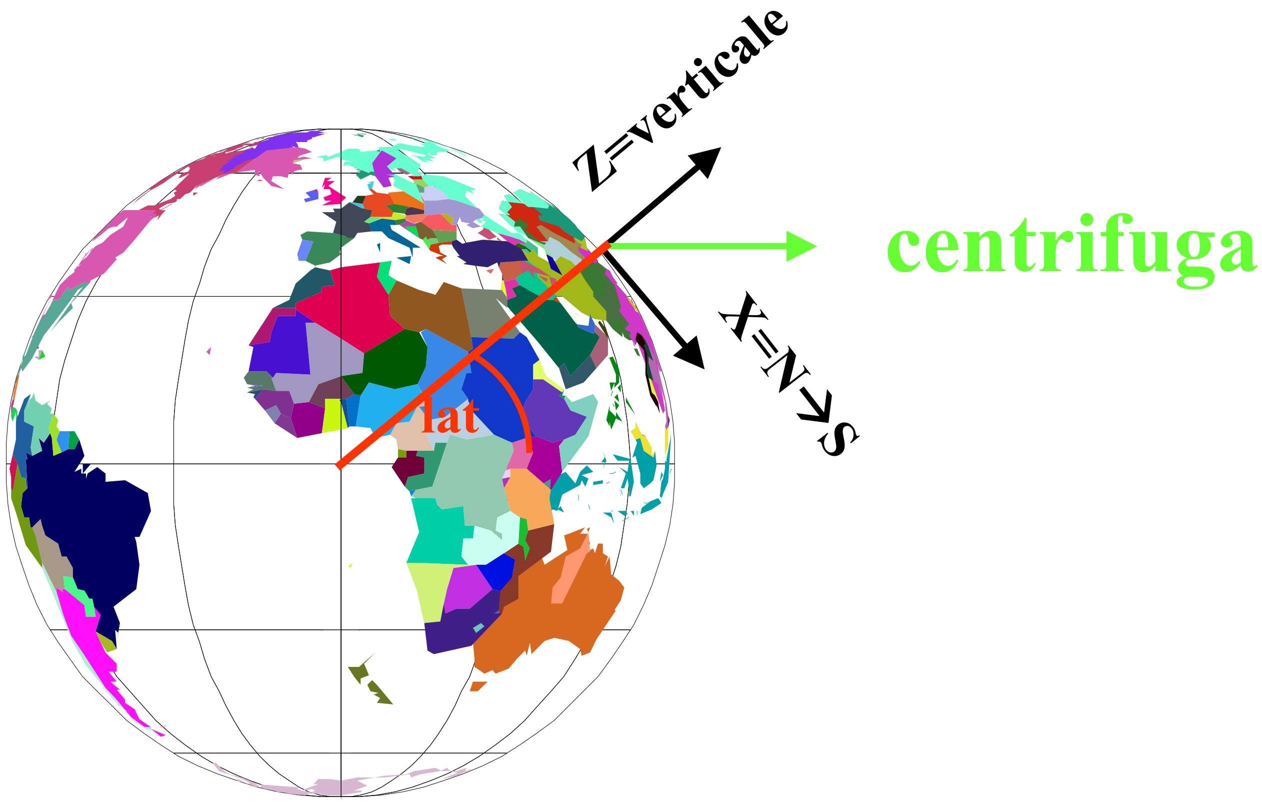
Centrifuga



$$\vec{r}\times\vec{\Omega}=\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\times\Omega\hat{k}=-x\Omega\hat{j}+y\Omega\hat{i}$$

$$\vec{\Omega}\times\left(\vec{r}\times\vec{\Omega}\right)=\Omega\hat{k}\times\left(-x\Omega\hat{j}+y\Omega\hat{i}\right)=\Omega^2\left(x\hat{i}+y\hat{j}\right)$$

Correzione centrifuga alla gravità

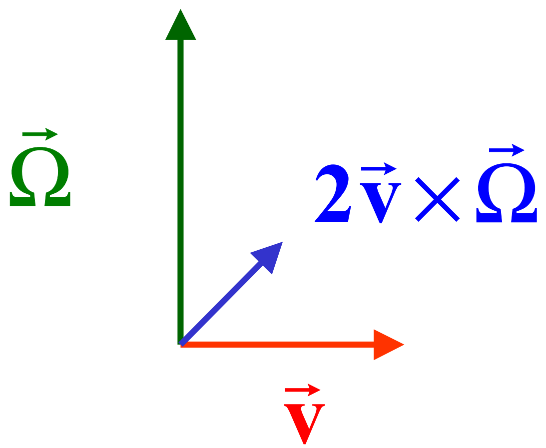


$$\left[\vec{\Omega}\times\left(\vec{r}\times\vec{\Omega}\right)\right]_{vert} = \Omega^2 R_\oplus \mathrm{Cos}\left(\mathrm{lat}\right) \approx .023\,\mathrm{m/s^2} \ll g$$

$$\left[\vec{\Omega}\times\left(\vec{r}\times\vec{\Omega}\right)\right]_{N\to S} = \Omega^2 R_\oplus \mathrm{Sin}\left(\mathrm{lat}\right) \approx .023\,\mathrm{m/s^2}$$

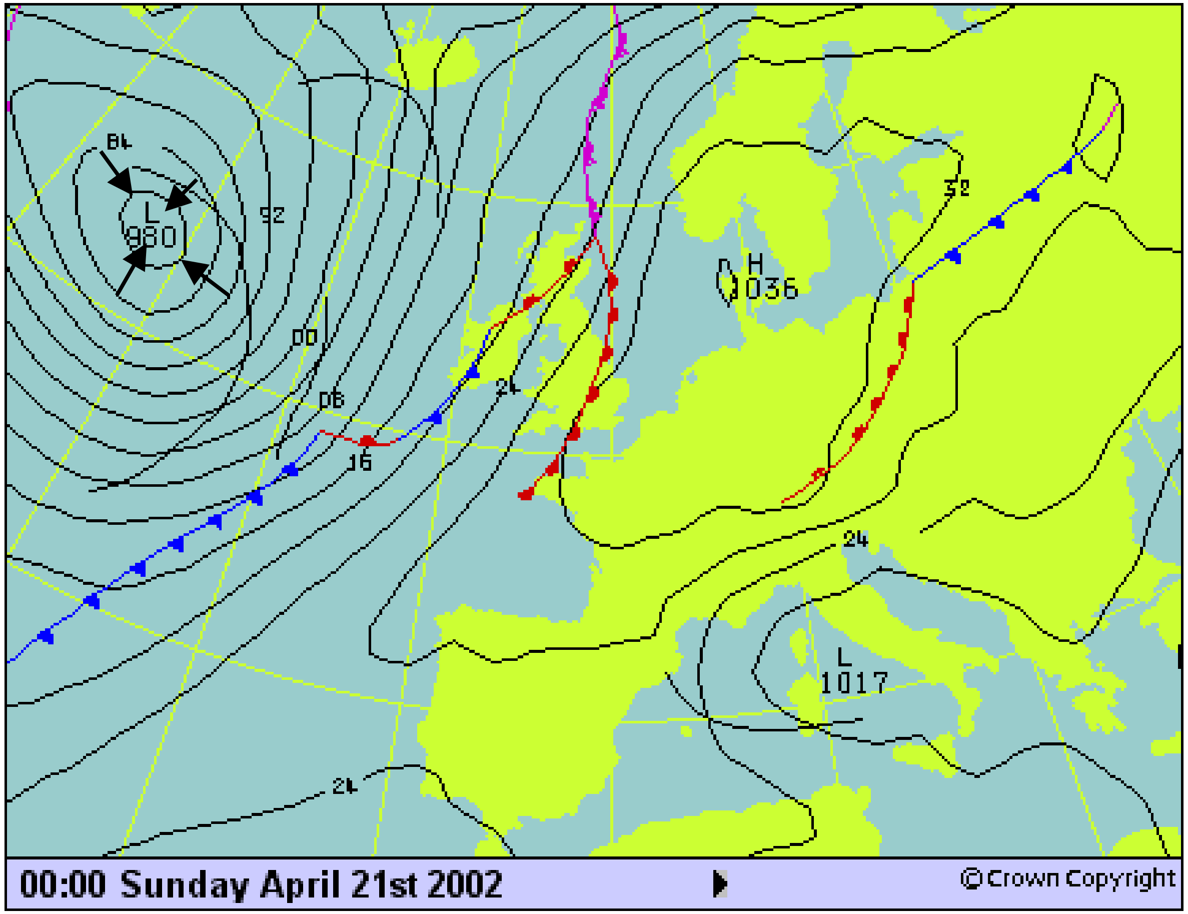
Scostamento dalla verticale .023/10 rad≈0.1°

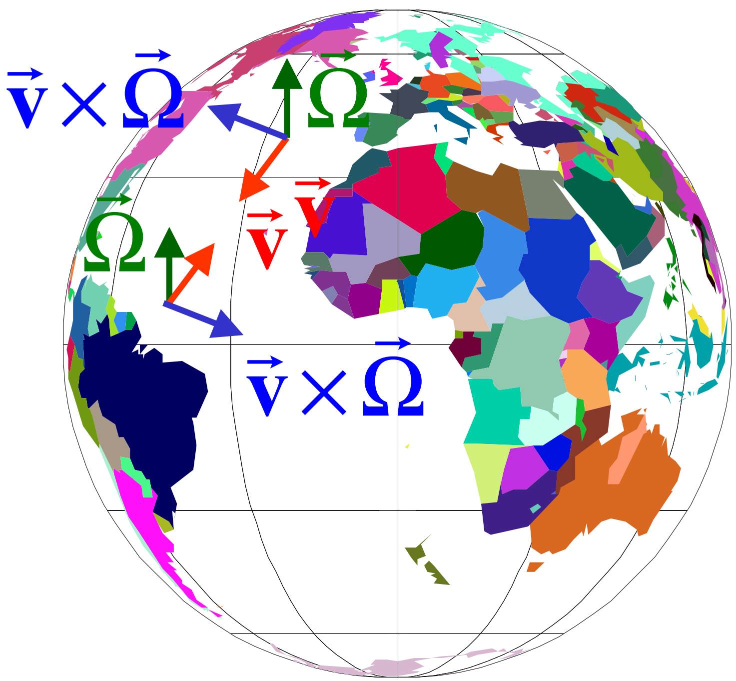
Coriolis



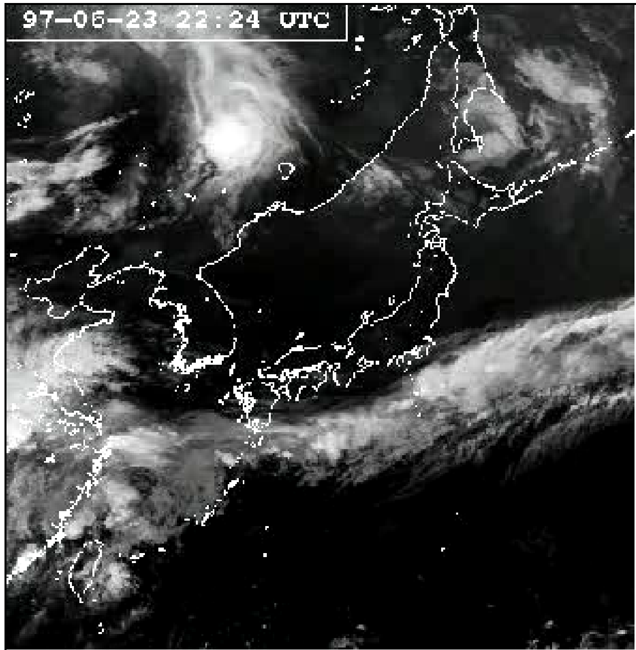
Un fenomeno importante: la circolazione atmosferica

Direzione del vento senza forza di Coriolis

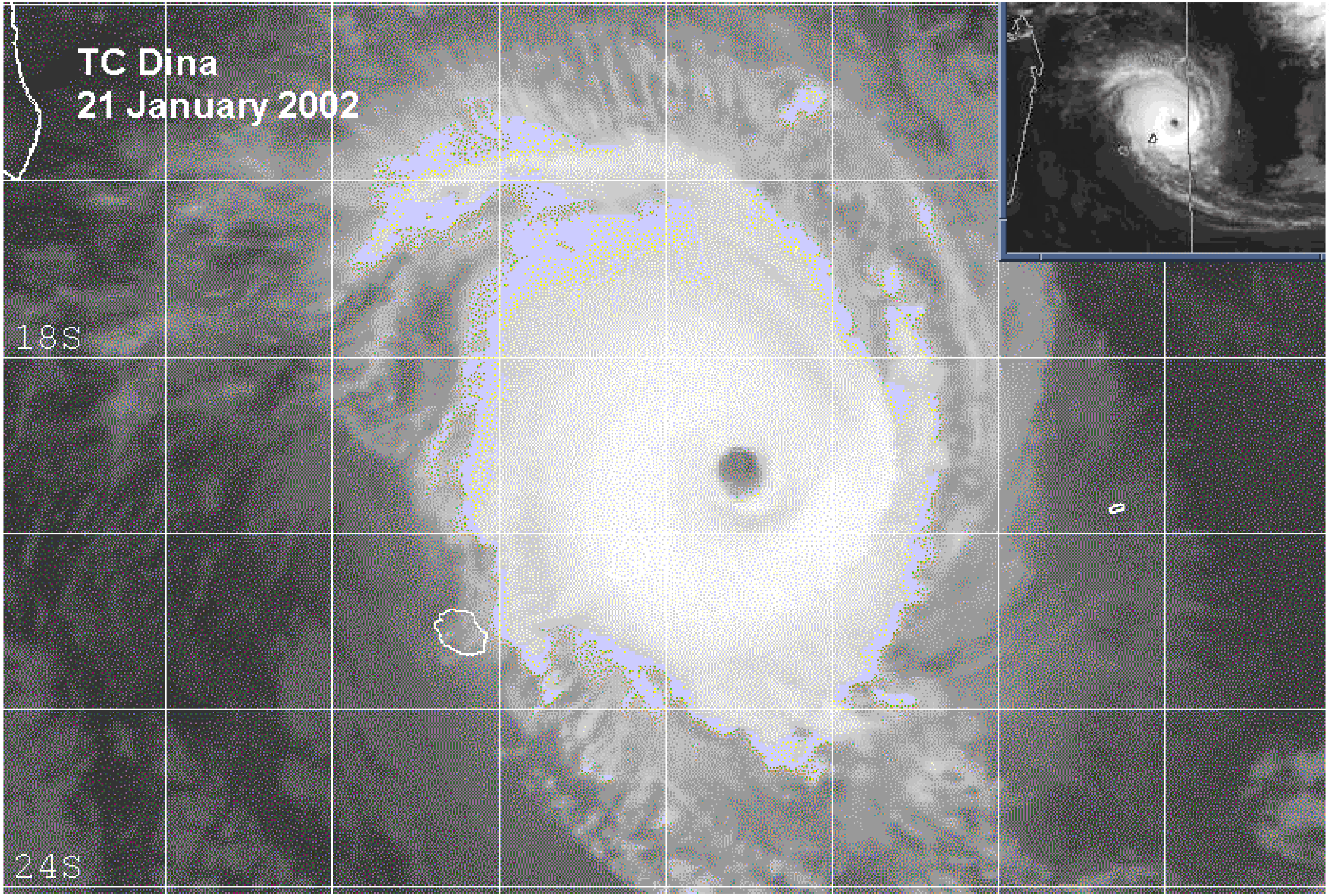




Circolazione antioraria nell'emisfero boreale



970701.21UTC-ir1.mpeg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