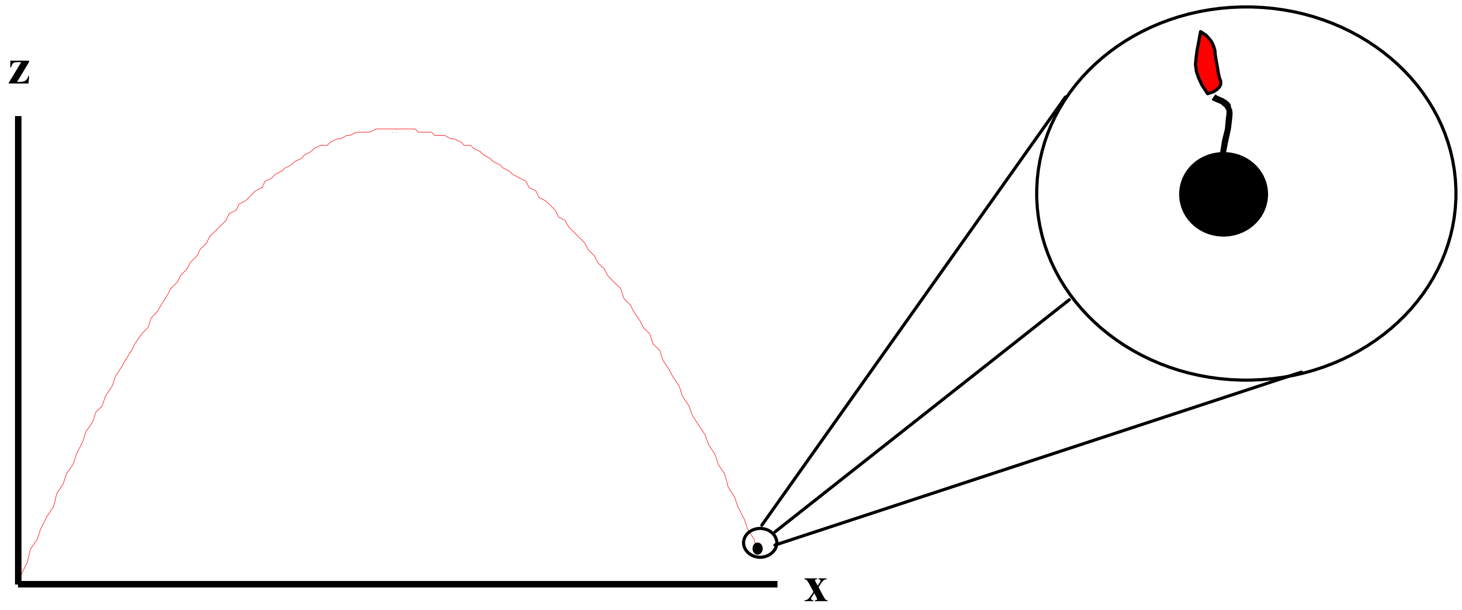
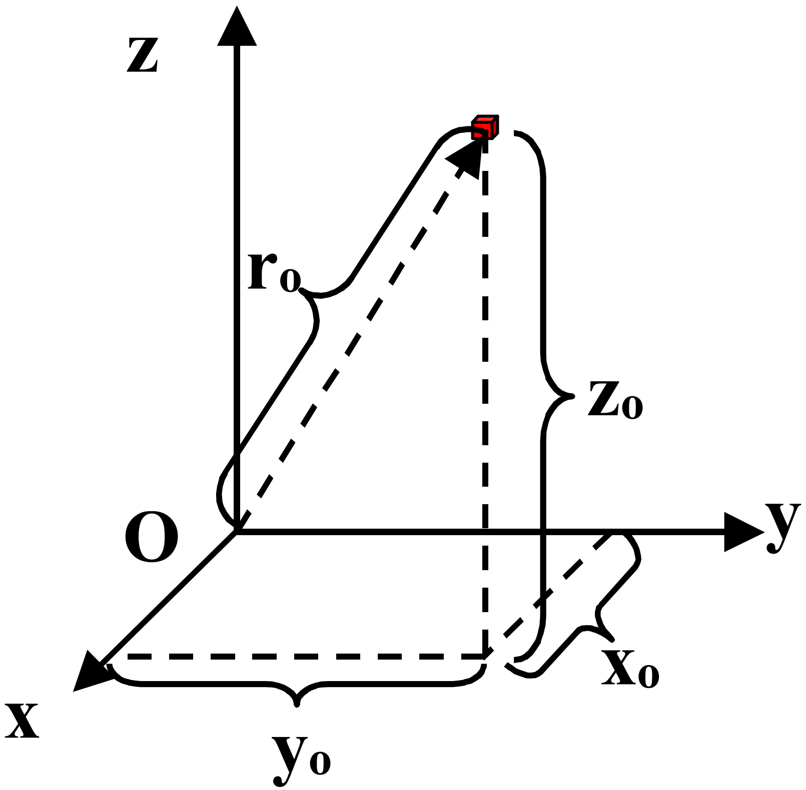
Cinematica del punto materiale

Punto materiale: oggetto di dimensioni lineari trascurabili rispetto alla precisione con cui se ne vuole determinare la posizione



Astronave, atomo, etc…..

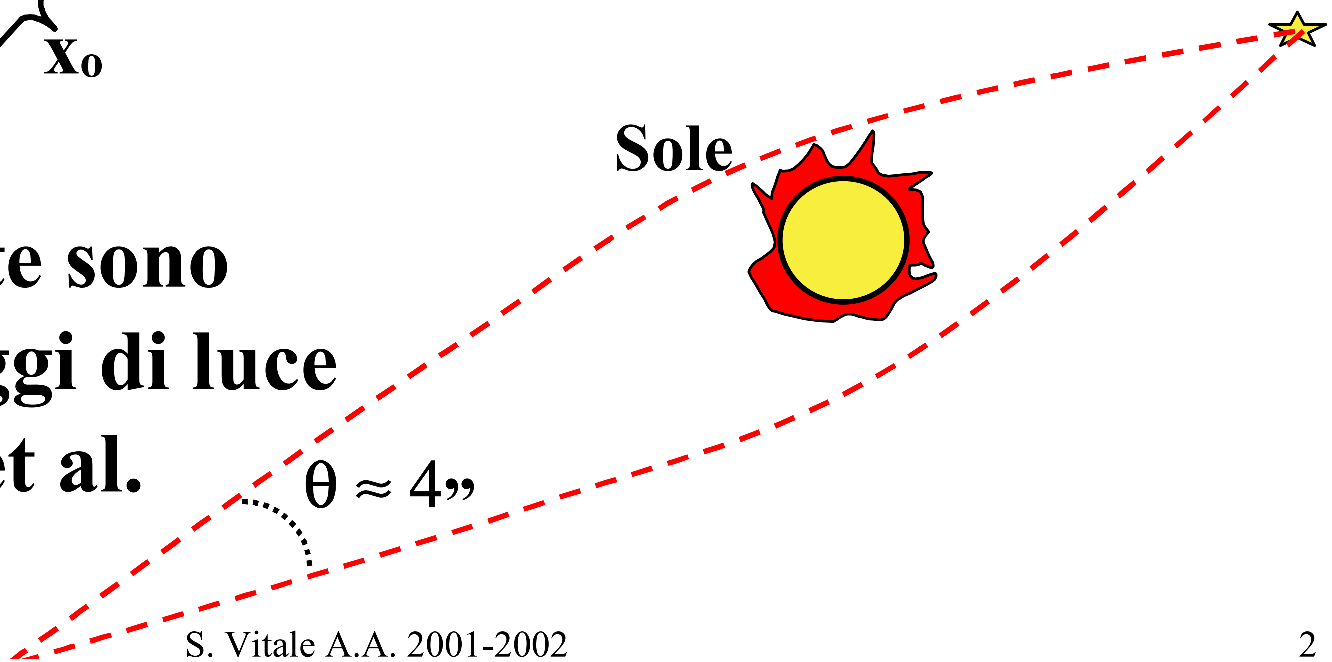
Coordinate nello spazio



Lontano da grandi masse vale sperimentalmente la geometria Euclidea

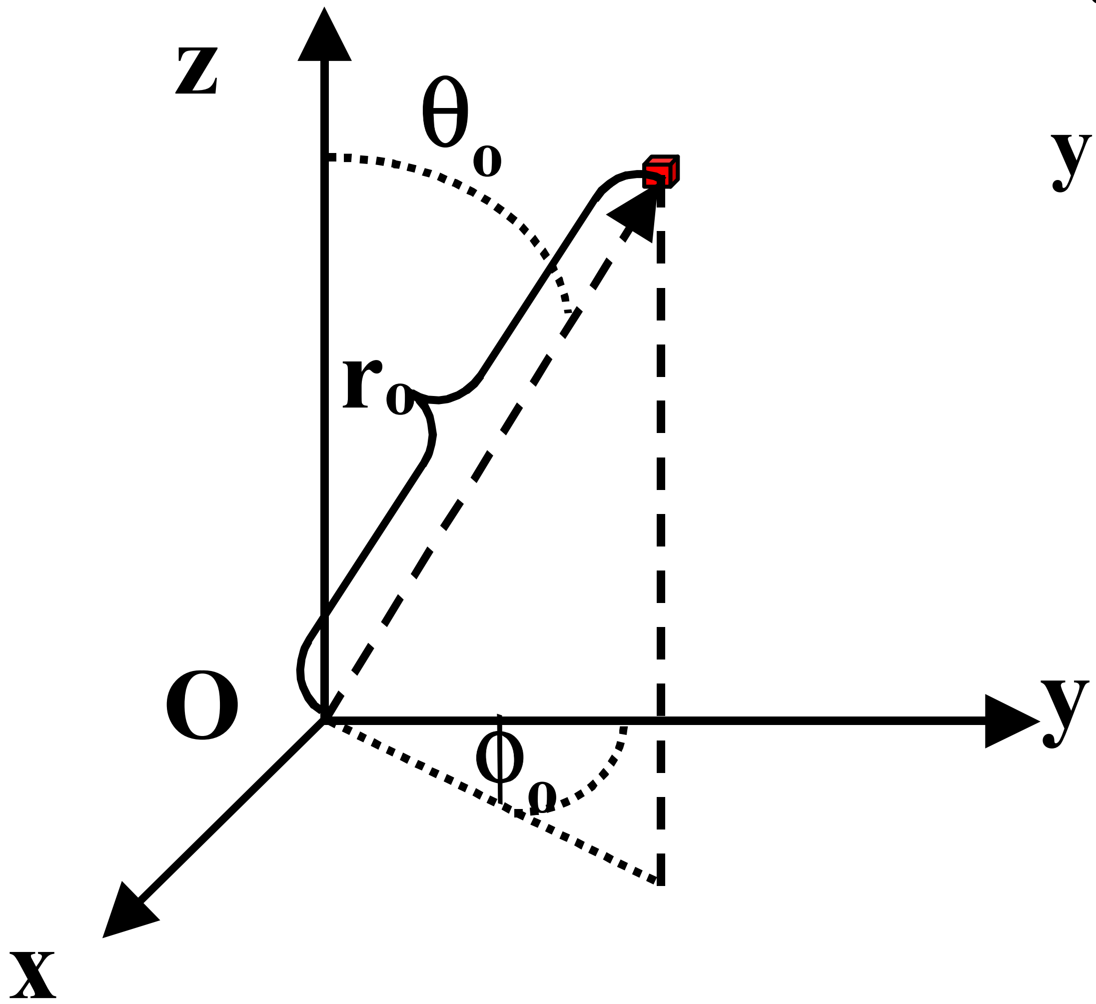
$$r_o = \sqrt{x_o^2 + y_o^2 + z_o^2}$$

Gauss et al., ca 1°



Le linee rette sono definite dai raggi di luce Einstein et al.

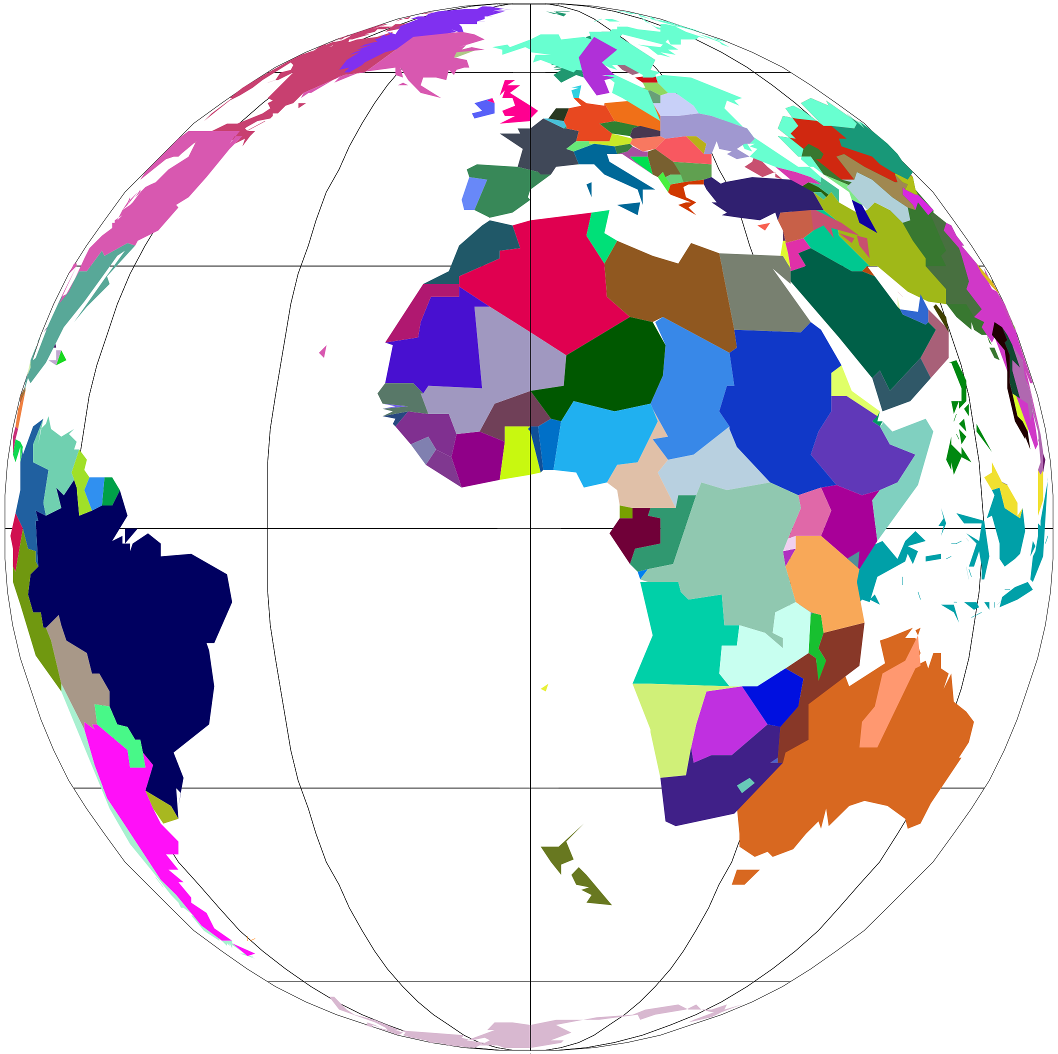
Coordinate Sferiche



$$x_o = r_o \mathrm{Sin}(\theta_o)\mathrm{Cos}(\phi_o)$$

$$y_o = r_o \mathrm{Sin}(\theta_o)\mathrm{Sin}(\phi_o)$$

$$z_o = r_o \mathrm{Cos}(\theta_o)$$

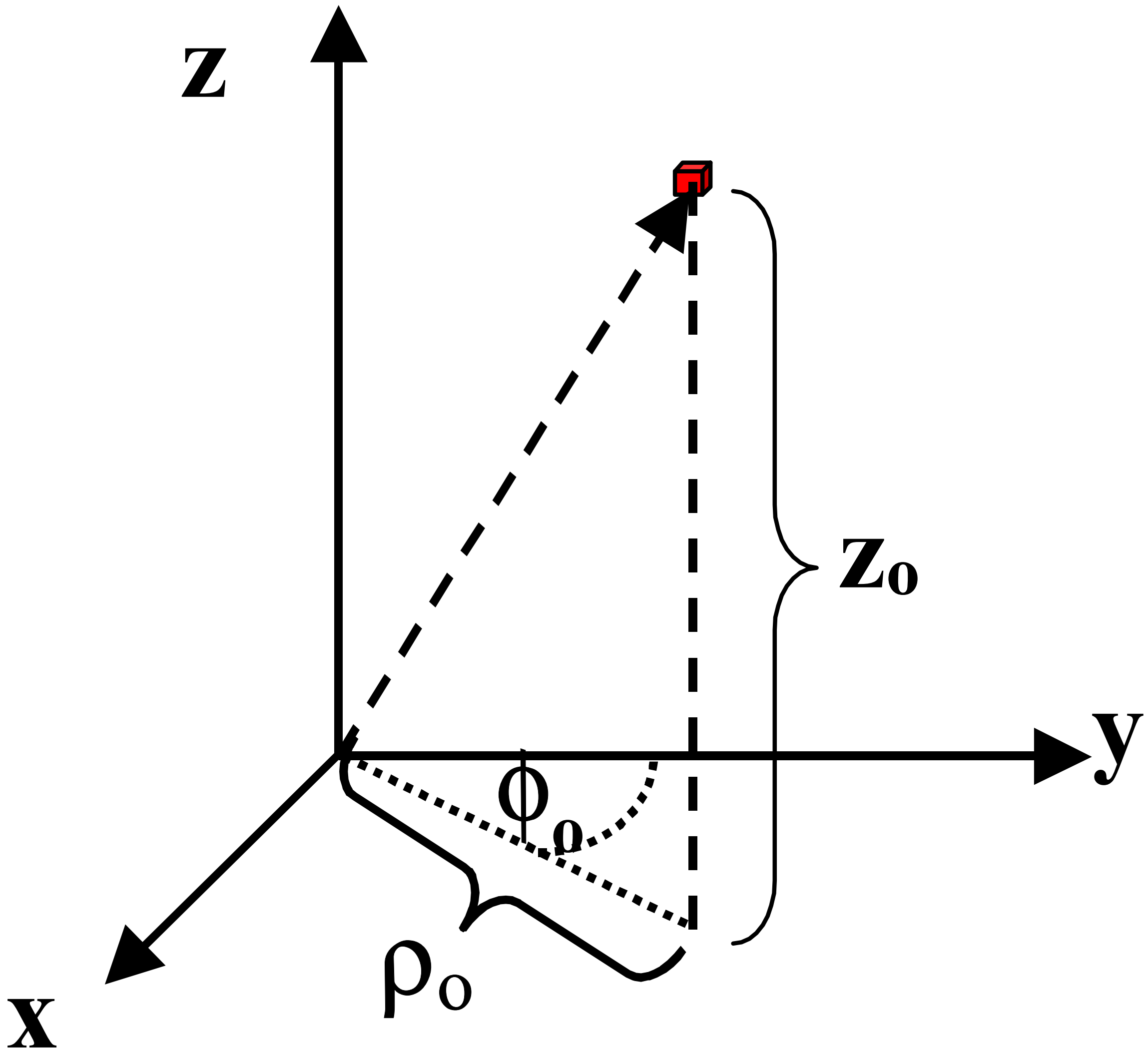


Longitudine = ϕ_o

Latitudine = $90°-\theta_o$

$r_o = R_\oplus$

Coordinate cilindriche



$$x_o = \rho_o \mathrm{Cos}(\phi_o)$$

$$y_o = \rho_o \mathrm{Sin}(\phi_o)$$

$$z_o = z_o$$

Descrizione del moto di un punto materiale

Il moto è interamente noto nell'intervallo di tempo

$t_1 < t < t_2$ se sono note

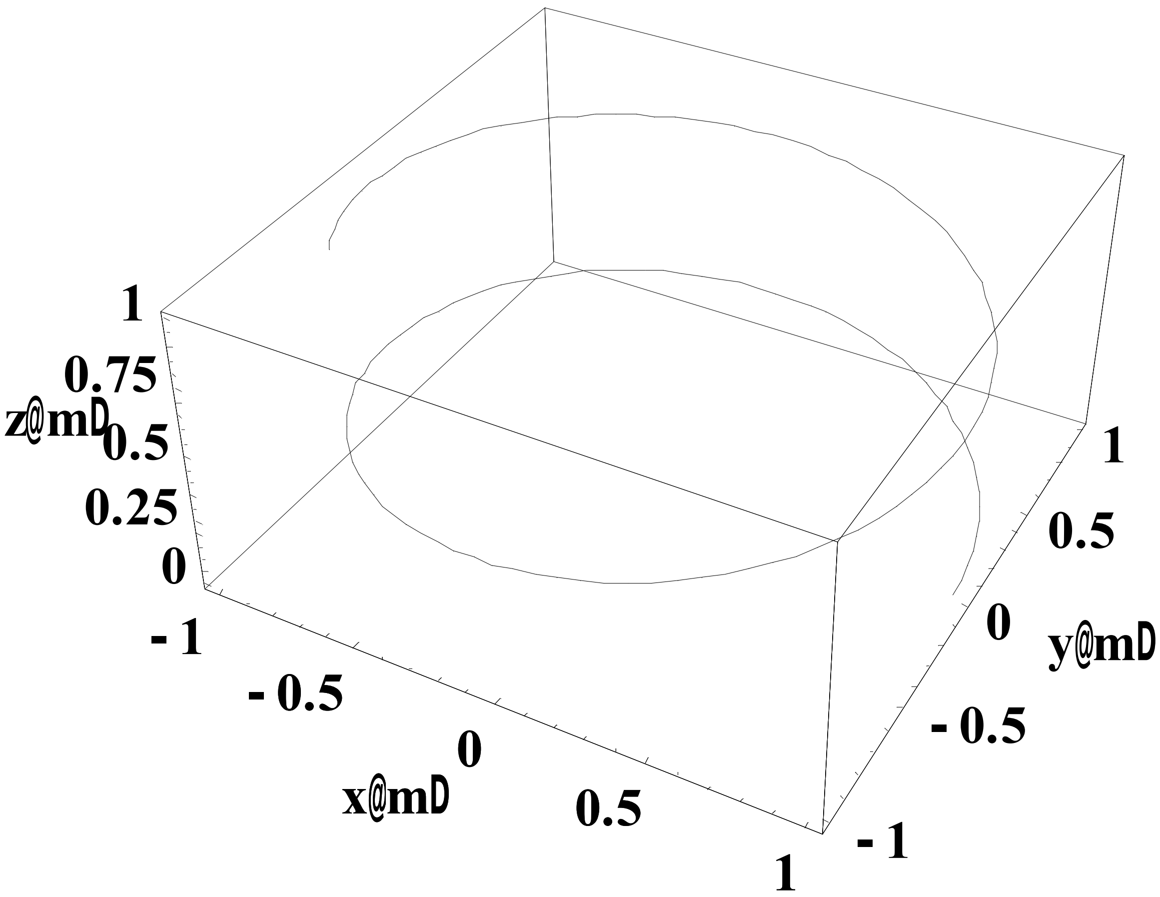
$x_o(t)$, $y_o(t)$ e $z_o(t)$ nello stesso intervallo

(o $r_o(t)$, $\phi_o(t)$ e $z_o(t)$ etc.)

La legge oraria

Al passare del tempo il punto descrive una curva nello spazio: la traiettoria

$$x(t) = r_o \mathrm{Cos}\left(\frac{t}{t_o}\right);\; y(t) = r_o \mathrm{Sin}\left(\frac{t}{t_o}\right);\; z(t) = v_o t;$$

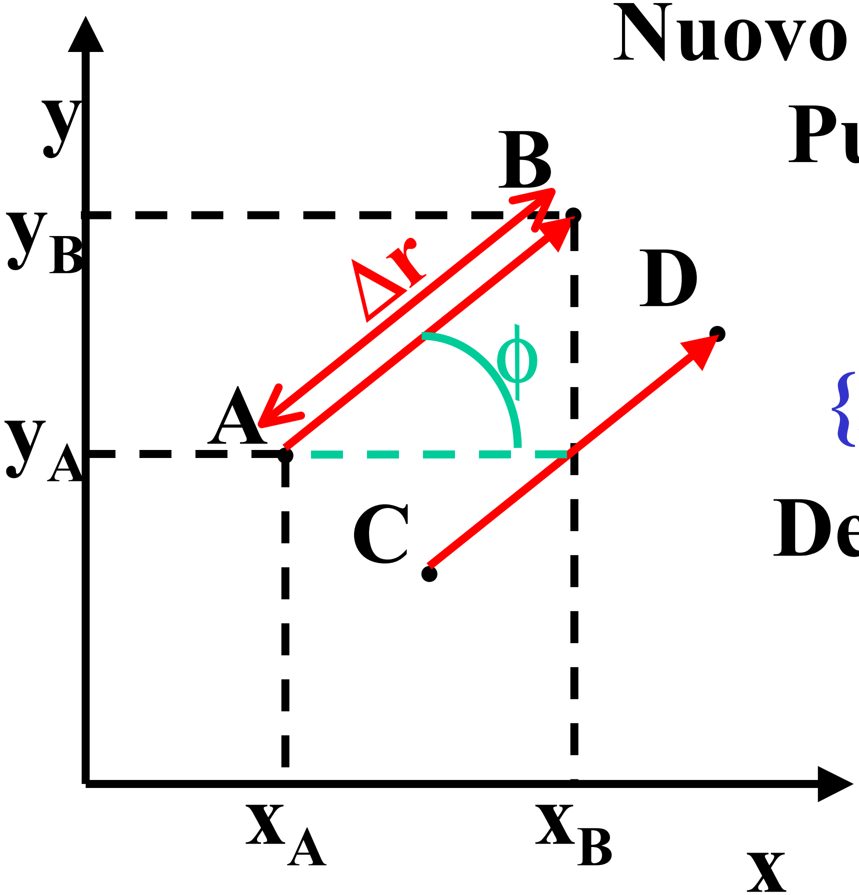


$r_o = 1\ \mathrm{m}$

$t_o = 1\ \mathrm{s}$

$v_o = 0.33\ \dfrac{\mathrm{m}}{\mathrm{s}}$

Nuovo concetto: lo spostamento



Punto che va da A a B

Le due grandezze

$\{\Delta x = x_B - x_A \text{ , } \Delta y = y_B - y_A\}$

Definiscono un nuovo oggetto matematico

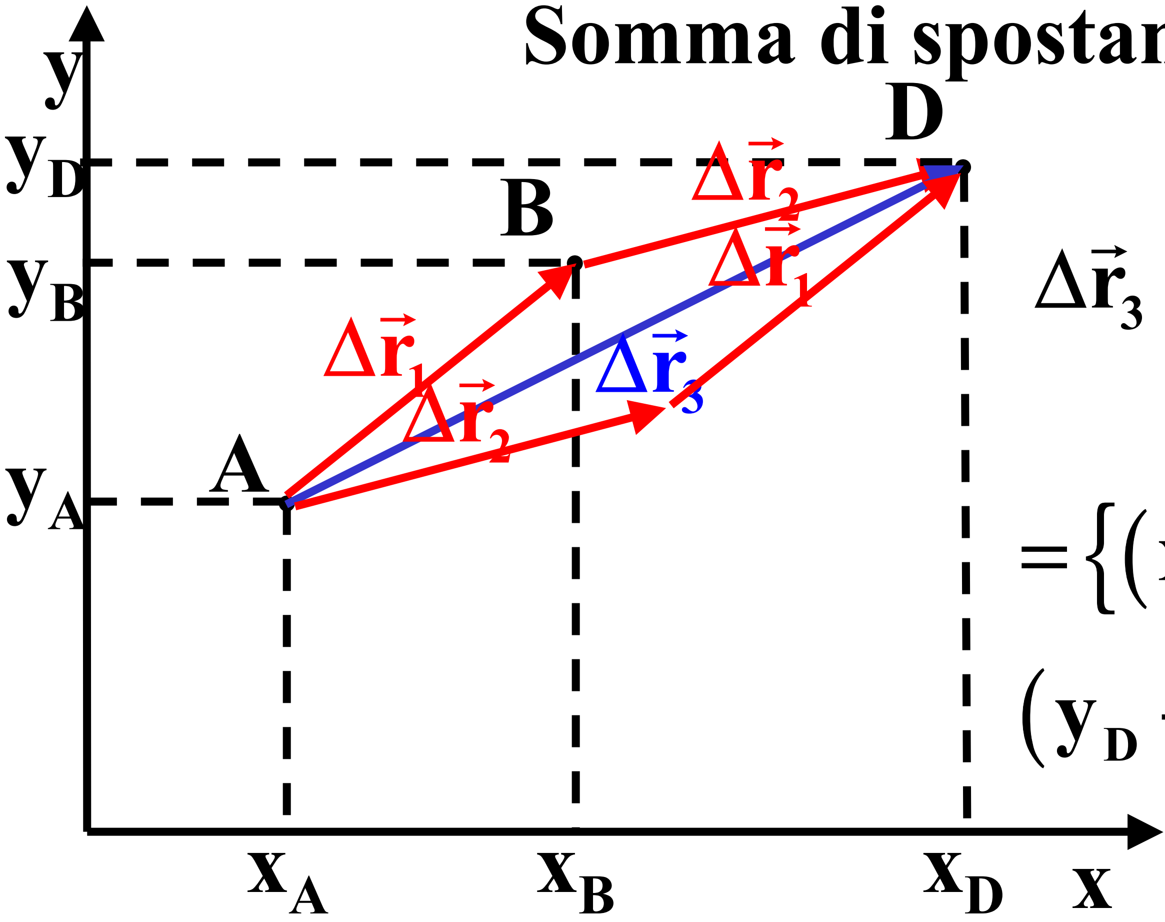
$\Delta\vec{r}$

Nota: lo spostamento A → B è uguale a C→D

$$\Delta r = \sqrt{\left(x_B - x_A\right)^2 + \left(y_B - y_A\right)^2}$$

$$x_B - x_A = \Delta r \ \mathrm{Cos}(\phi); \ y_B - y_A = \Delta r \ \mathrm{Sin}(\phi)$$

Somma di spostamenti



$$\Delta\vec{r}_3 \equiv \left\{x_D - x_A, y_D - y_A\right\}$$

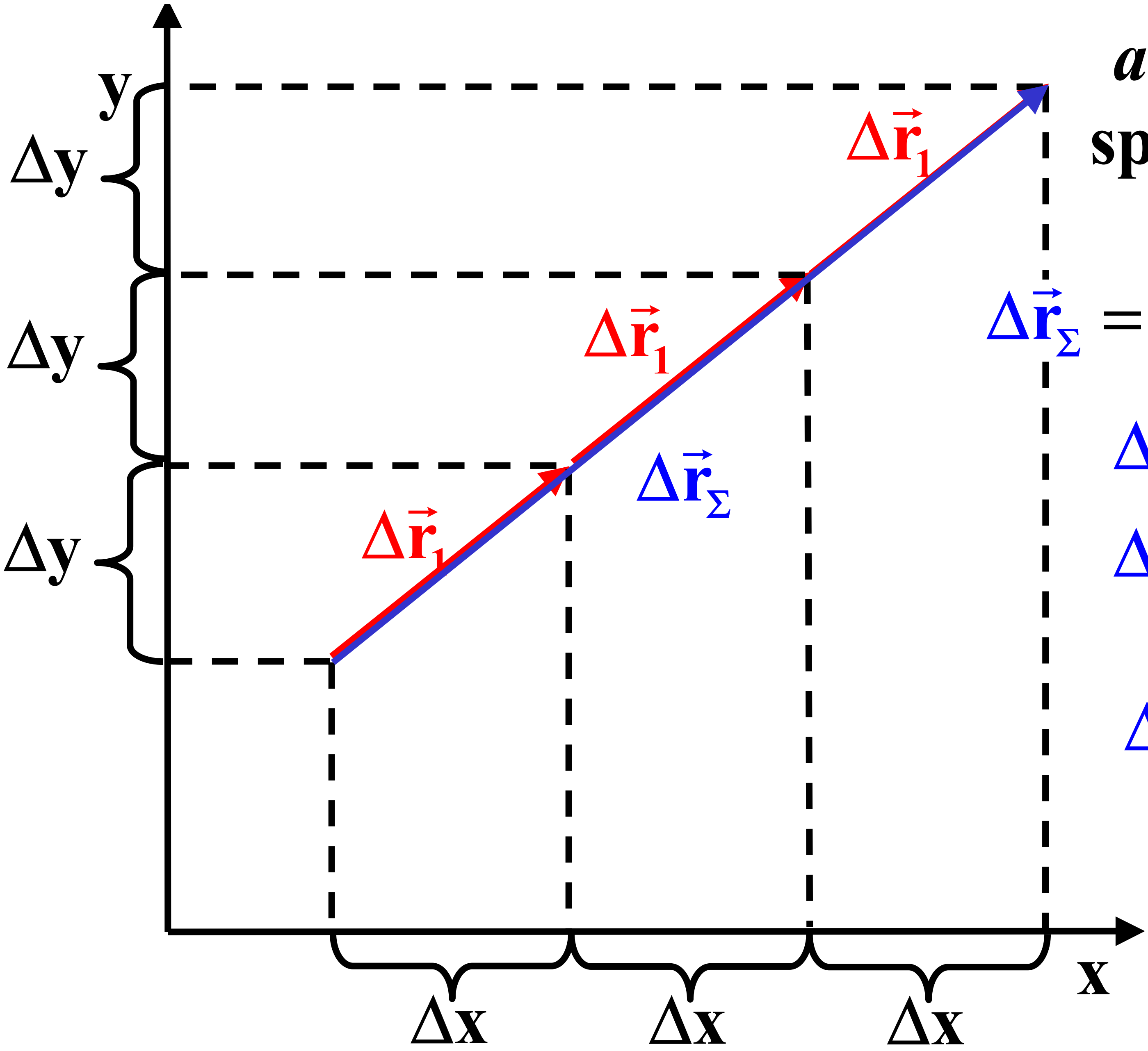
$$=\left\{\left(x_D - x_B\right) + \left(x_B - x_A\right), \left(y_D - y_B\right) + \left(y_B - y_A\right)\right\} =$$

$$=\left\{\Delta x_1 + \Delta x_2, \Delta y_1 + \Delta y_2\right\}$$

$$\Delta\vec{r}_3 \overset{\text{def}}{\equiv} \Delta\vec{r}_1 + \Delta\vec{r}_2$$

Nota: la somma è commutativa

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 = \Delta\vec{r}_2 + \Delta\vec{r}_1$$



a volte uno spostamento

$$\Delta\vec{r}_\Sigma = \Delta\vec{r}_1 + \Delta\vec{r}_1 + \Delta\vec{r}_1$$

$$\Delta x_\Sigma = 3\Delta x_1$$

$$\Delta y_\Sigma = 3\Delta y_1$$

$$\Delta\vec{r}_\Sigma \overset{\text{def}}{\equiv} 3\Delta\vec{r}_1$$

$$\Delta\vec{r}_\Sigma = a\Delta\vec{r}_1 \overset{\text{def}}{\Leftrightarrow} \Delta x_\Sigma = a\Delta x_1, \Delta y_\Sigma = a\Delta y_1$$

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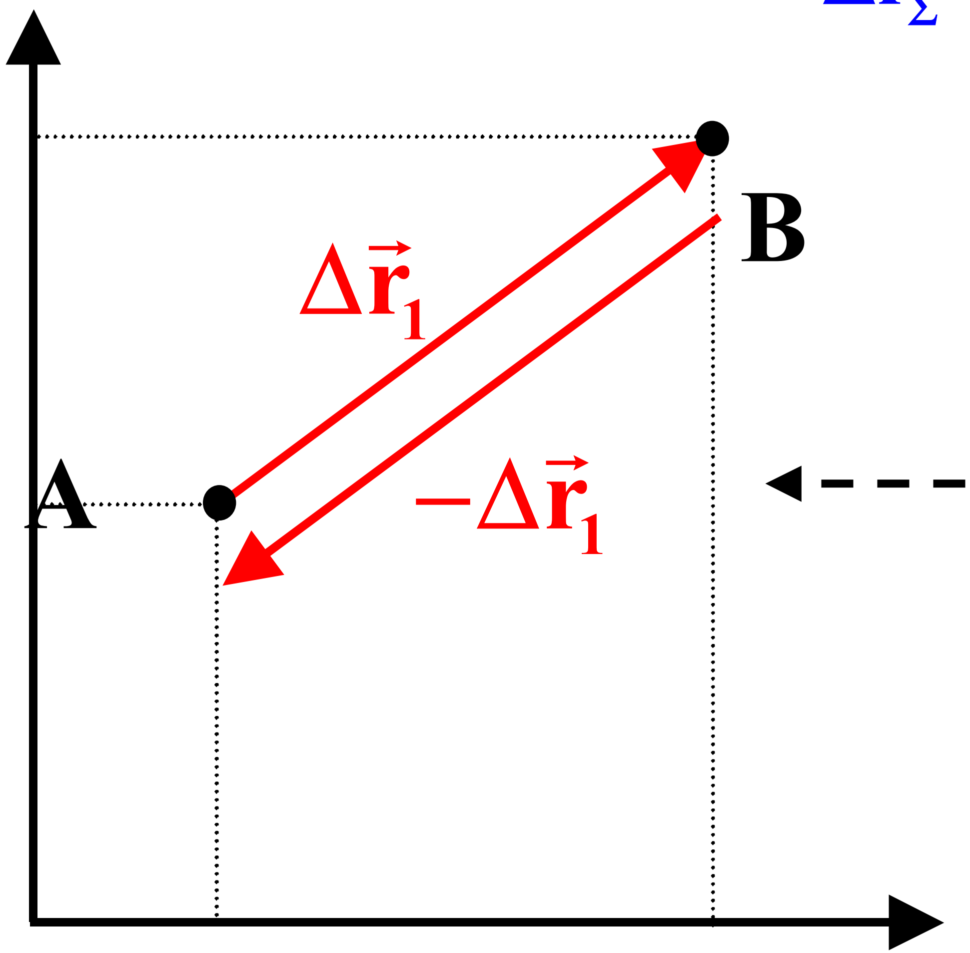
Qualche osservazione

$$\Delta x_{\Sigma} = a\Delta x_1;\ \Delta y_{\Sigma} = a\Delta y_1$$

$$\Delta r_{\Sigma} = \sqrt{a^2\Delta x_1^2 + a^2\Delta y_1^2} = a\Delta r_1$$



Es: $\alpha = -1$

Queste sono le proprietà di un “campo vettoriale”

Gli spostamenti sono dunque vettori e godono di tutte le loro proprietà

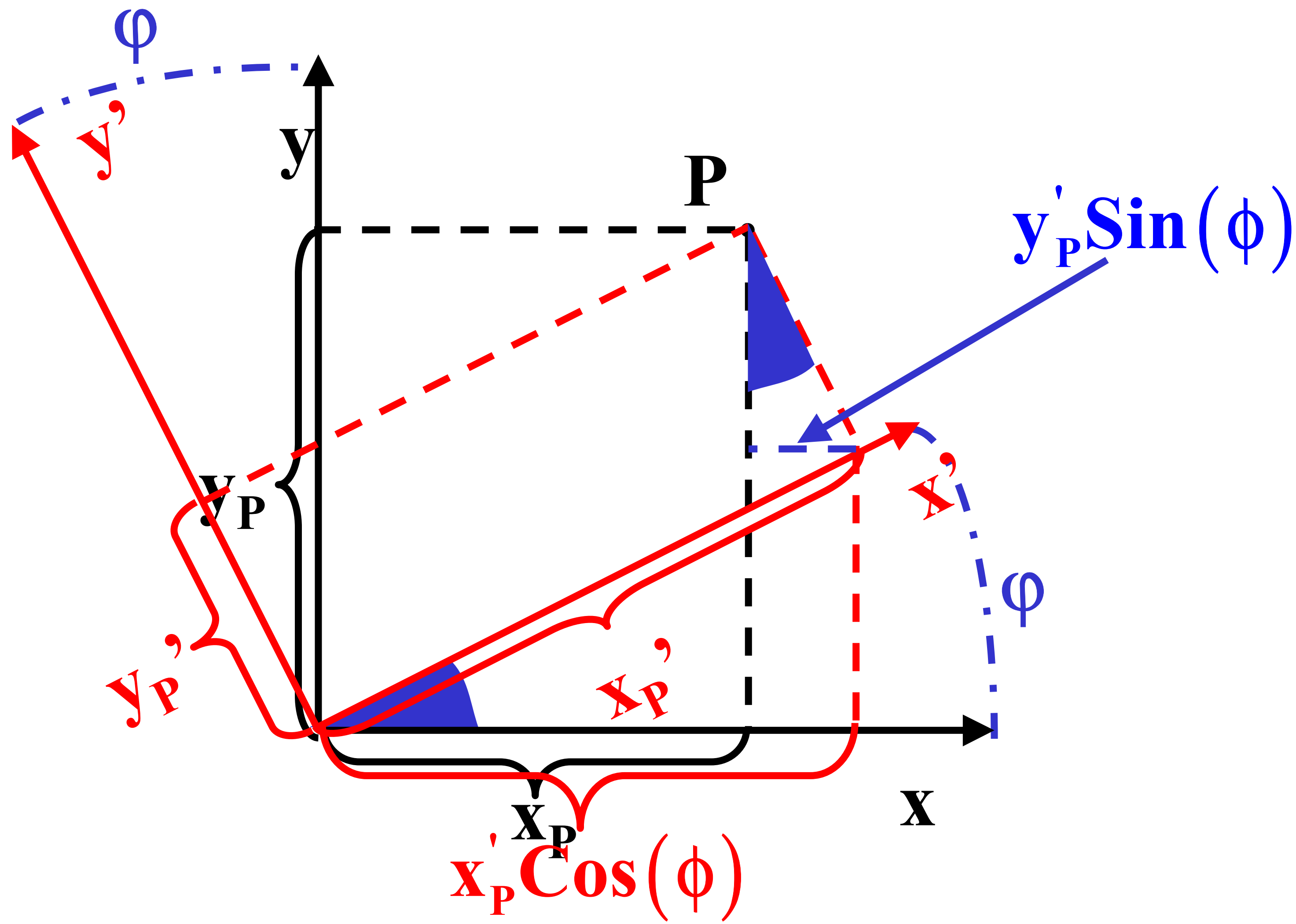
I numeri come a, che non dipendono dalla scelta delle coordinate si chiamano scalari

(Es: misure di tempo, misure di temperatura, misure di massa etc.)

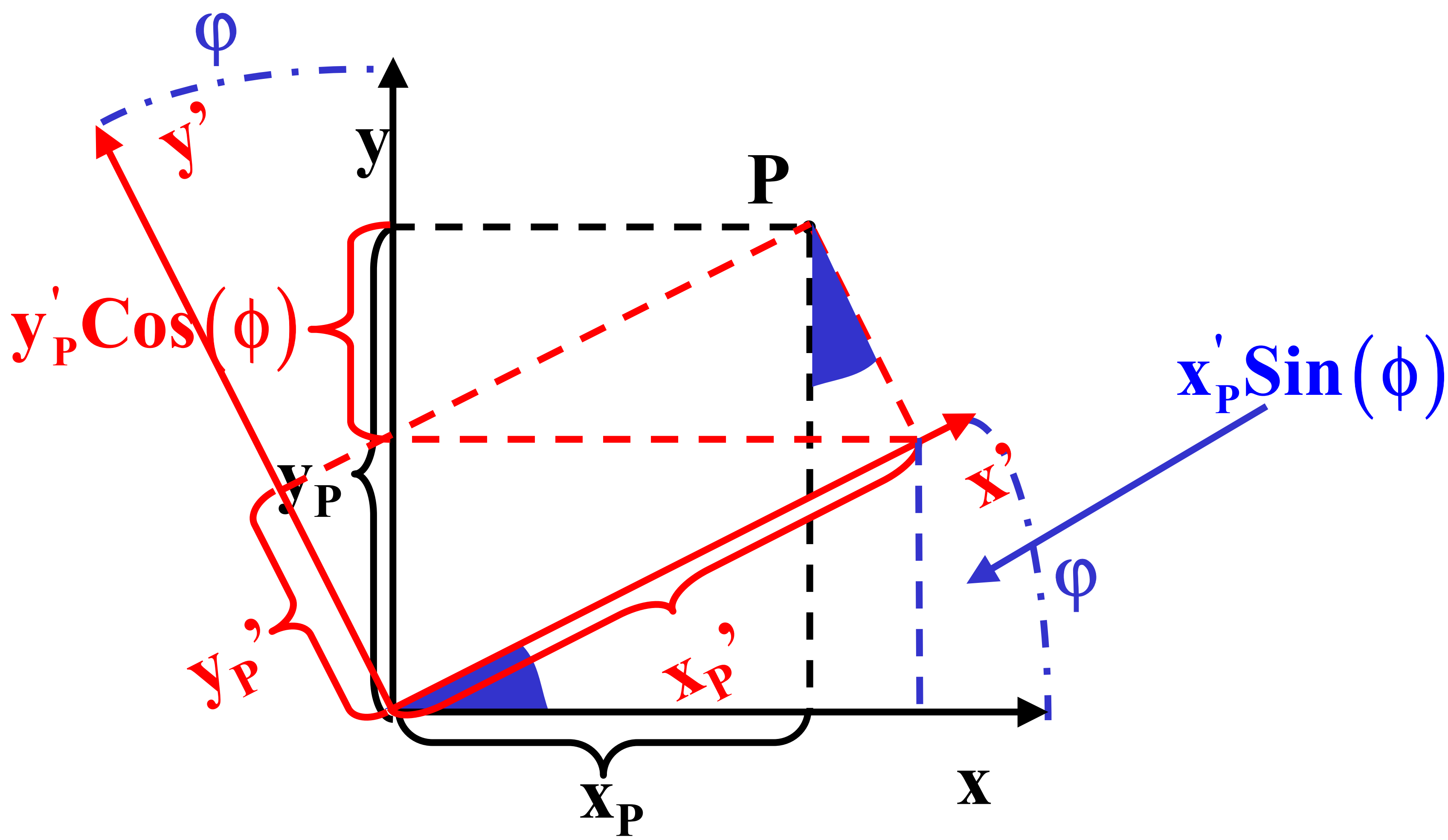
La lunghezza di uno spostamento è uno scalare

(verificare che non dipende dalla scelta delle coordinate)

Trasformando le coordinate



$$x_P = x_P' \mathrm{Cos}(\phi) - y_P' \mathrm{Sin}(\phi)$$



$$y_P = x'_P Sin(\phi) + y'_P Cos(\phi)$$

La legge di trasformazione e la sua inversa:

x’ e y’ sono ruotate di φ rispetto a x e y

x e y sono ruotate di -φ rispetto a x’ e y’

$$x_P = x'_P \mathrm{Cos}(\phi) - y'_P \mathrm{Sin}(\phi) \qquad x'_P = x_P \mathrm{Cos}(\phi) + y_P \mathrm{Sin}(\phi)$$

$$y_P = x'_P \mathrm{Sin}(\phi) + y'_P \mathrm{Cos}(\phi) \qquad y'_P = -x_P \mathrm{Sin}(\phi) + y_P \mathrm{Cos}(\phi)$$

$$\phi \leftrightarrow -\phi$$

Cambiando segno a φ il seno cambia segno ed il coseno no

La trasformazione degli spostamenti

$$x_P = x'_P \mathrm{Cos}(\phi) - y'_P \mathrm{Sin}(\phi) \;\&\; x_Q = x'_Q \mathrm{Cos}(\phi) - y'_Q \mathrm{Sin}(\phi)$$

$$y_P = x'_P \mathrm{Sin}(\phi) + y'_P \mathrm{Cos}(\phi) \;\&\; y_Q = x'_Q \mathrm{Sin}(\phi) + y'_Q \mathrm{Cos}(\phi)$$

$$\underbrace{x_P - x_Q}_{\Delta x} = \underbrace{\left(x'_P - x'_Q\right)}_{\Delta x'} \mathrm{Cos}(\phi) - \underbrace{\left(y'_P - y'_Q\right)}_{\Delta y'} \mathrm{Sin}(\phi)$$

$$\underbrace{y_P - y_Q}_{\Delta y} = \underbrace{\left(x'_P - x'_Q\right)}_{\Delta x'} \mathrm{Sin}(\phi) + \underbrace{\left(y'_P - y'_Q\right)}_{\Delta y'} \mathrm{Cos}(\phi)$$

Le componenti dello spostamento si trasformano come le coordinate dei punti

Il modulo di uno spostamento

$$\Delta x = \Delta x' \mathrm{Cos}(\phi) - \Delta y' \mathrm{Sin}(\phi)$$

$$\Delta y = \Delta x' \mathrm{Sin}(\phi) + \Delta y' \mathrm{Cos}(\phi)$$

$$\Delta x^2 = \Delta x'^2 \mathrm{Cos}^2(\phi) + \Delta y'^2 \mathrm{Sin}^2(\phi) - 2\Delta x' \Delta y' \mathrm{Cos}(\phi)\mathrm{Sin}(\phi)$$

$$+ \qquad + \qquad + \qquad +$$

$$\Delta y^2 = \Delta x'^2 \mathrm{Sin}^2(\phi) + \Delta y'^2 \mathrm{Cos}^2(\phi) + 2\Delta x' \Delta y' \mathrm{Sin}(\phi)\mathrm{Cos}(\phi)$$

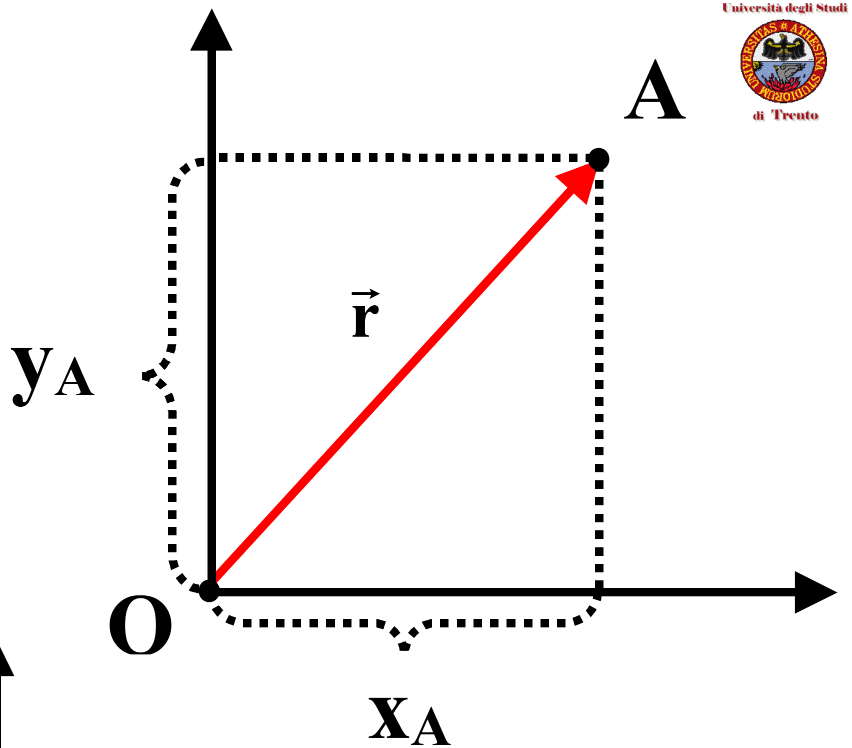
$$\Delta r^2 = \Delta x^2 + \Delta y^2 = \Delta x'^2 \underbrace{\left[\mathrm{Cos}^2(\phi) + \mathrm{Sin}^2(\phi)\right]}_{=1} +$$

$$+\Delta y'^2 \underbrace{\left[\mathrm{Sin}^2(\phi) + \mathrm{Cos}^2(\phi)\right]}_{=1} = \Delta r'^2$$

E' uno scalare

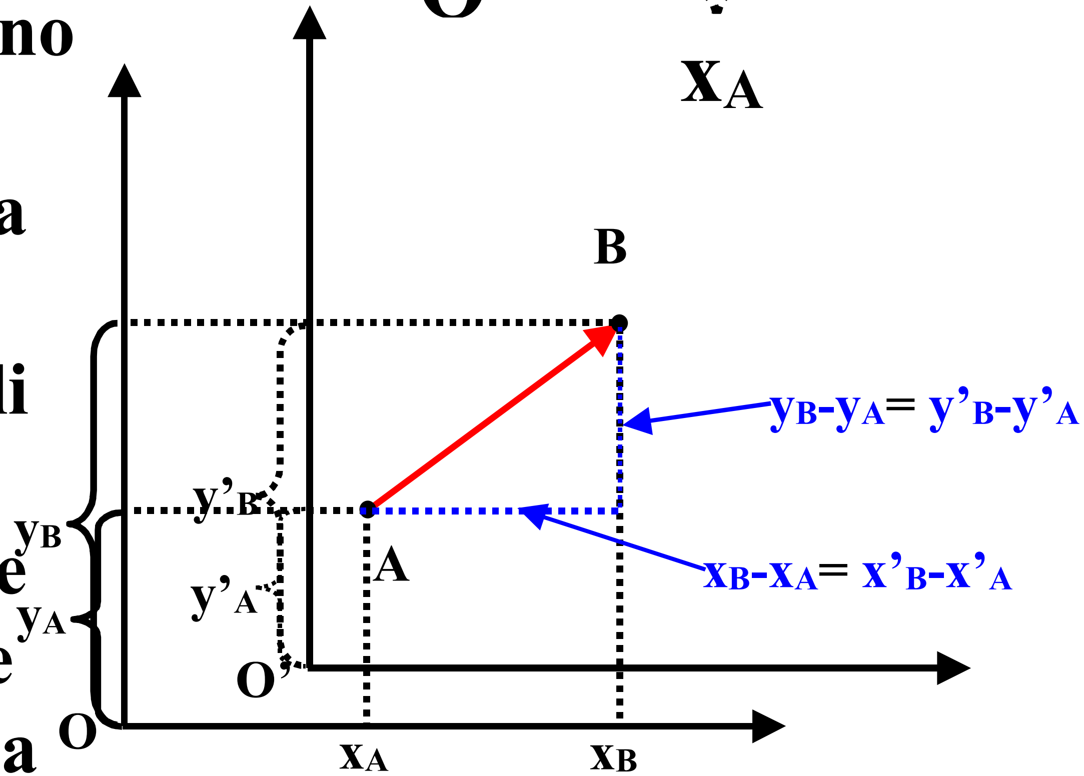
Note: Le tre coordinate cartesiane di un punto sono le componenti dello spostamento che porta dall'origine a quel punto:

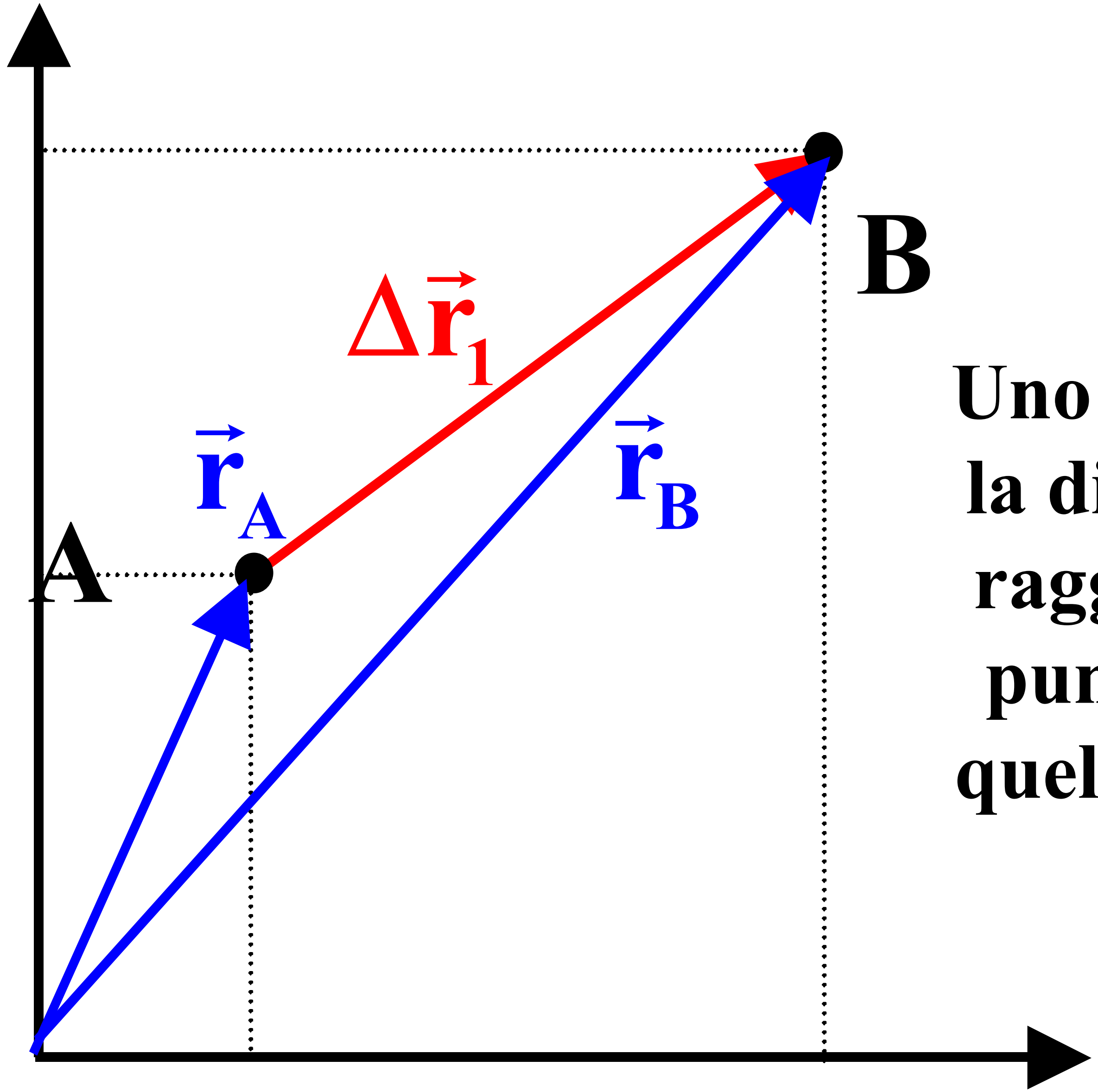
Il raggio vettore $\vec{r}$



Le tre componenti di uno spostamento non dipendono dalla scelta dell'origine ma solo dall'orientazione degli assi

Se si cambia origine le coordinate cartesiane cambiano ed $\vec{r}$ cambia





Uno spostamento è la differenza fra il raggio vettore del punto di arrivo e quello del punto di partenza

Un utile esercizio: la legge oraria della Terra
1 AU=distanza media Sole-Terra=1.496×10^{11}m

EARTH coordinates:

					$90°-\theta$		φ
YYYY	DDD	AU	ELAT	ELON	HLAT	HLON	HILON
2000	1	0.983	0.00	99.86	-2.95	8.01	23.92
2000	21	0.984	0.00	120.24	-5.06	104.63	44.24
2000	41	0.987	0.00	140.54	-6.55	201.31	64.60
2000	61	0.991	0.00	160.71	-7.21	297.92	84.90
2000	81	0.996	0.00	180.68	-6.99	34.35	105.02
2000	101	1.002	0.00	200.43	-5.95	130.51	124.87
2000	121	1.007	0.00	219.95	-4.23	226.35	144.40
2000	141	1.012	0.00	239.28	-2.05	321.90	163.63
2000	161	1.015	0.00	258.46	0.34	57.25	182.67

In coordinate cartesiane

t	x	y	z
86400 s	1.34246 ¥ 10^{11} m	5.95459 ¥ 10^{10} m	- 7.56809 ¥ 10^{9} m
1814400 s	1.0505 ¥ 10^{11} m	1.02299 ¥ 10^{11} m	- 1.29833 ¥ 10^{10} m
3542400 s	6.29202 ¥ 10^{10} m	1.3251 ¥ 10^{11} m	- 1.68428 ¥ 10^{10} m
5270400 s	1.30745 ¥ 10^{10} m	1.46497 ¥ 10^{11} m	- 1.86065 ¥ 10^{10} m
6998400 s	- 3.83271 ¥ 10^{10} m	1.42839 ¥ 10^{11} m	- 1.81327 ¥ 10^{10} m
8726400 s	- 8.52369 ¥ 10^{10} m	1.22321 ¥ 10^{11} m	- 1.55384 ¥ 10^{10} m
10454400 s	- 1.22156 ¥ 10^{11} m	8.74551 ¥ 10^{10} m	- 1.11116 ¥ 10^{10} m
12182400 s	- 1.45163 ¥ 10^{11} m	4.26412 ¥ 10^{10} m	- 5.41557 ¥ 10^{9} m
13910400 s	- 1.51674 ¥ 10^{11} m	- 7.07319 ¥ 10^{9} m	9.01042 ¥ 10^{8} m
15638400 s	- 1.41283 ¥ 10^{11} m	- 5.59949 ¥ 10^{10} m	7.11378 ¥ 10^{9} m
17366400 s	- 1.14958 ¥ 10^{11} m	- 9.86355 ¥ 10^{10} m	1.25333 ¥ 10^{10} m
19094400 s	- 7.58947 ¥ 10^{10} m	- 1.30296 ¥ 10^{11} m	1.65406 ¥ 10^{10} m
20822400 s	- 2.82744 ¥ 10^{10} m	- 1.47242 ¥ 10^{11} m	1.87016 ¥ 10^{10} m
22550400 s	2.25384 ¥ 10^{10} m	- 1.47461 ¥ 10^{11} m	1.87392 ¥ 10^{10} m
24278400 s	7.07336 ¥ 10^{10} m	- 1.30601 ¥ 10^{11} m	1.65811 ¥ 10^{10} m
26006400 s	1.10665 ¥ 10^{11} m	- 9.853 ¥ 10^{10} m	1.25205 ¥ 10^{10} m
27734400 s	1.37325 ¥ 10^{11} m	- 5.46206 ¥ 10^{10} m	6.94371 ¥ 10^{9} m
29462400 s	1.47296 ¥ 10^{11} m	- 4.11434 ¥ 10^{9} m	5.14361 ¥ 10^{8} m
31190400 s	1.39268 ¥ 10^{11} m	4.68417 ¥ 10^{10} m	- 5.95286 ¥ 10^{9} m

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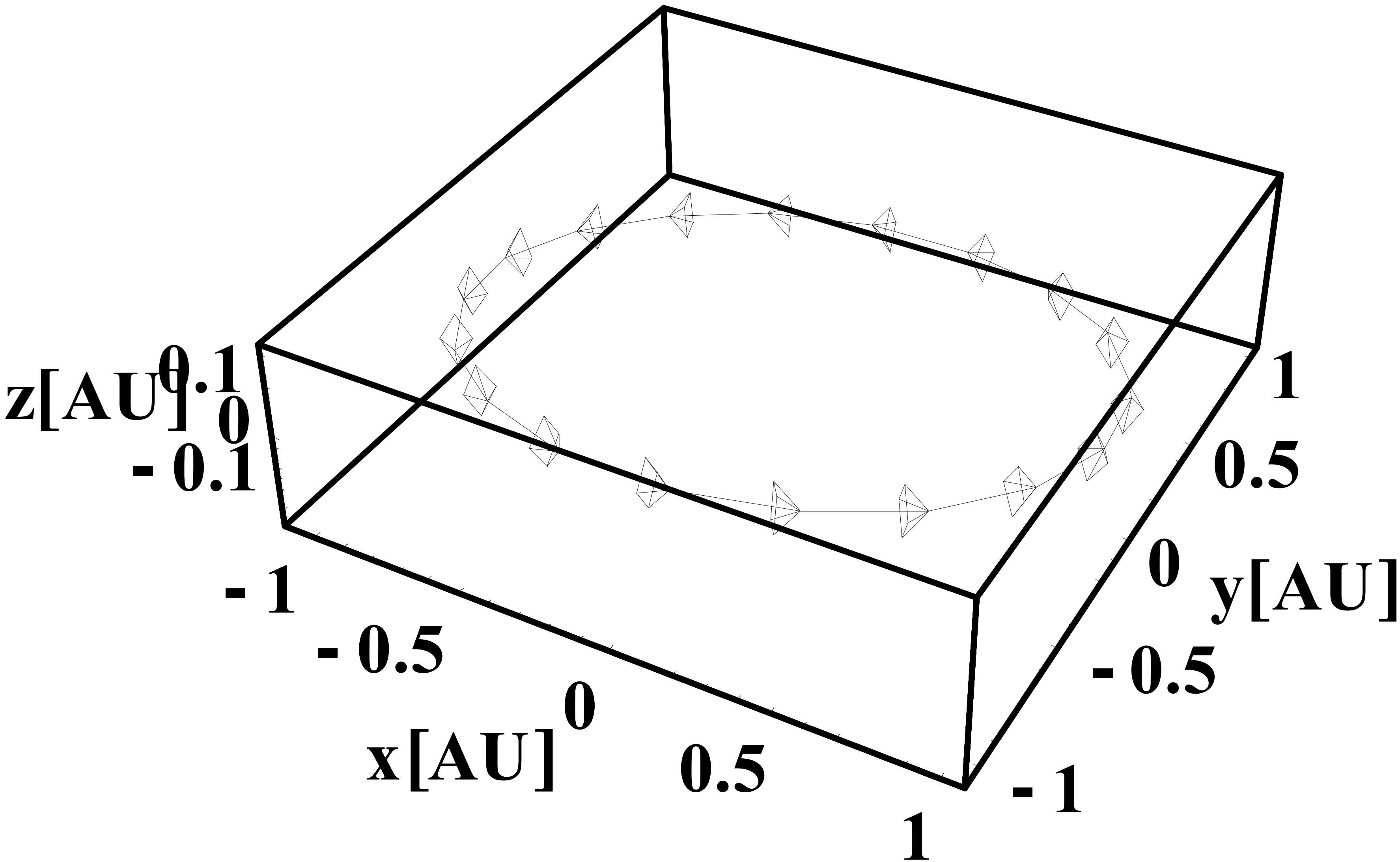
Coordinate

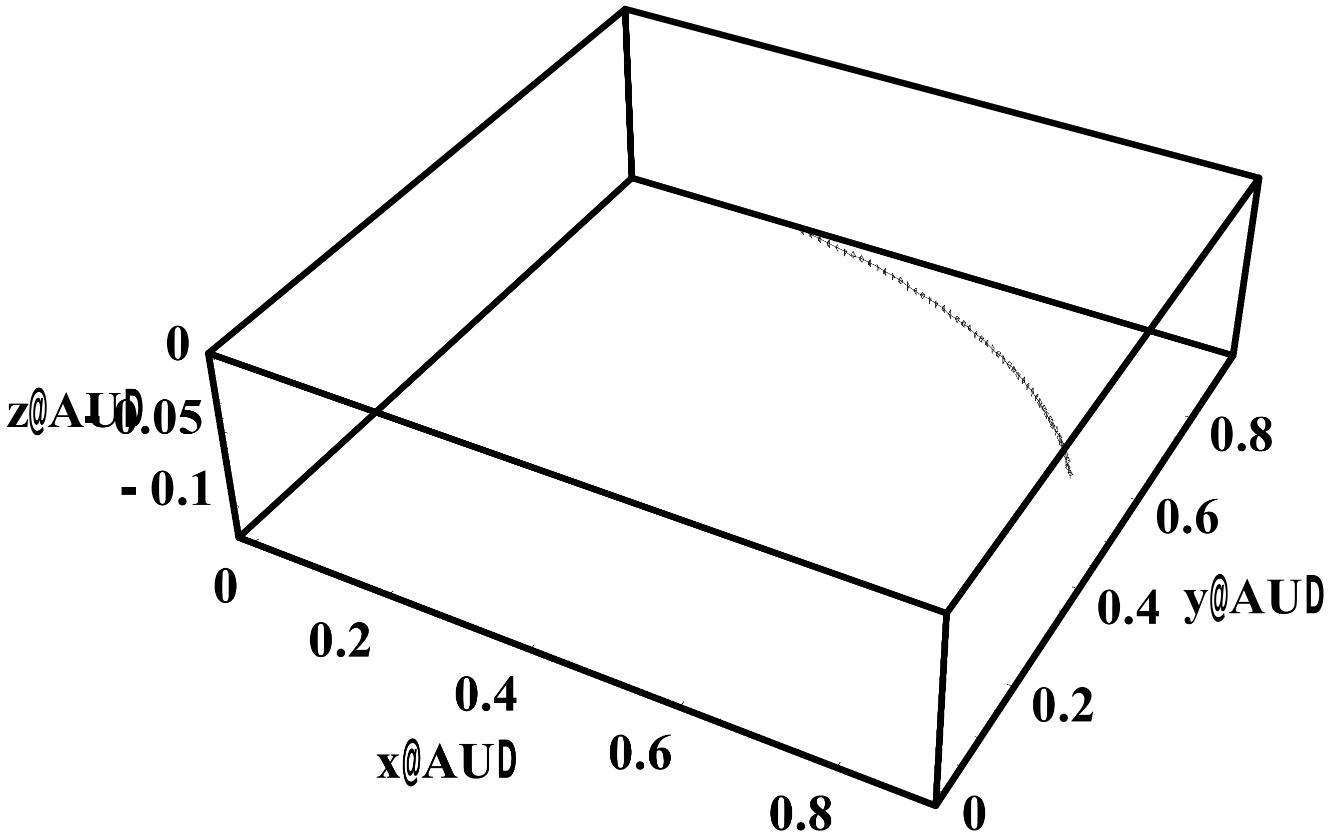
x	y	z
0.134246 Tm	0.0595459 Tm	−0.00756809 Tm
0.10505 Tm	0.102299 Tm	−0.0129833 Tm
0.0629202 Tm	0.13251 Tm	−0.0168428 Tm
0.0130745 Tm	0.146497 Tm	−0.0186065 Tm
−0.0383271 Tm	0.142839 Tm	−0.0181327 Tm
−0.0852369 Tm	0.122321 Tm	−0.0155384 Tm

Spostamenti

Δx	Δ y	Δ z
−0.0291967 Tm	0.0427532 Tm	−0.00541517 Tm
−0.0421295 Tm	0.0302105 Tm	−0.00385956 Tm
−0.0498457 Tm	0.0139873 Tm	−0.00176369 Tm
−0.0514016 Tm	−0.0036576 Tm	0.000473852 Tm
−0.0469098 Tm	−0.0205187 Tm	0.00259425 Tm

Spostamenti





Un tratto molto più piccolo(50 gg)
Punti molto ravvicinati
Costruiamo gli spostamenti e "dividiamoli" per il tempo impiegato ad effettuarli

YYYY	DDD	AU	ELAT	ELON	HLAT	HLON	HILON
2000	1	0.983	0.00	99.86	-2.95	8.01	23.92
2000	2	0.983	0.00	100.88	-3.07	354.83	24.93
2000	3	0.983	0.00	101.90	-3.18	341.66	25.95
2000	4	0.983	0.00	102.92	-3.30	328.49	26.96
2000	5	0.983	0.00	103.94	-3.41	315.32	27.98
2000	6	0.983	0.00	104.96	-3.52	302.16	28.99
2000	7	0.983	0.00	105.98	-3.63	288.99	30.01
2000	8	0.983	0.00	106.99	-3.75	275.82	31.02
2000	9	0.983	0.00	108.01	-3.85	262.65	32.04
2000	10	0.983	0.00	109.03	-3.96	249.48	33.05
2000	11	0.983	0.00	110.05	-4.07	236.31	34.07
2000	12	0.983	0.00	111.07	-4.18	223.14	35.09
2000	13	0.983	0.00	112.09	-4.28	209.97	36.10
2000	14	0.984	0.00	113.11	-4.38	196.81	37.12
2000	15	0.984	0.00	114.13	-4.48	183.64	38.13
2000	16	0.984	0.00	115.15	-4.58	170.47	39.15
2000	17	0.984	0.00	116.16	-4.68	157.30	40.17
2000	18	0.984	0.00	117.18	-4.78	144.14	41.19
2000	19	0.984	0.00	118.20	-4.88	130.97	42.20
2000	20	0.984	0.00	119.22	-4.97	117.80	43.22
2000	21	0.984	0.00	120.24	-5.06	104.63	44.24
2000	22	0.984	0.00	121.25	-5.15	91.47	45.26
2000	23	0.984	0.00	122.27	-5.24	78.30	46.27
2000	24	0.984	0.00	123.29	-5.33	65.13	47.29
2000	25	0.984	0.00	124.31	-5.42	51.97	48.31
2000	26	0.985	0.00	125.32	-5.50	38.80	49.33
2000	27	0.985	0.00	126.34	-5.59	25.64	50.34
2000	28	0.985	0.00	127.36	-5.67	12.47	51.36
2000	29	0.985	0.00	128.37	-5.74	359.30	52.38
2000	30	0.985	0.00	129.39	-5.82	346.14	53.40
2000	31	0.985	0.00	130.40	-5.90	332.97	54.42
2000	32	0.985	0.00	131.42	-5.97	319.80	55.44
2000	33	0.985	0.00	132.43	-6.04	306.64	56.45
2000	34	0.986	0.00	133.45	-6.11	293.47	57.47
2000	35	0.986	0.00	134.46	-6.18	280.30	58.49
2000	36	0.986	0.00	135.48	-6.25	267.14	59.51
2000	37	0.986	0.00	136.49	-6.31	253.97	60.53
2000	38	0.986	0.00	137.51	-6.37	240.81	61.54
2000	39	0.986	0.00	138.52	-6.43	227.64	62.56
2000	40	0.986	0.00	139.53	-6.49	214.47	63.58
2000	41	0.987	0.00	140.54	-6.55	201.31	64.60
2000	42	0.987	0.00	141.56	-6.60	188.14	65.61
2000	43	0.987	0.00	142.57	-6.65	174.97	66.63
2000	44	0.987	0.00	143.58	-6.70	161.80	67.65
2000	45	0.987	0.00	144.59	-6.75	148.64	68.66
2000	46	0.988	0.00	145.60	-6.79	135.47	69.68
2000	47	0.988	0.00	146.61	-6.84	122.30	70.70
2000	48	0.988	0.00	147.62	-6.88	109.13	71.71
2000	49	0.988	0.00	148.63	-6.92	95.96	72.73
2000	50	0.988	0.00	149.64	-6.95	82.80	73.75

86400 s	1.34246×10^{11} m	5.95459×10^{10} m	-7.56809×10^{9} m
172800 s	1.33161×10^{11} m	6.18962×10^{10} m	-7.87566×10^{9} m
259200 s	1.32024×10^{11} m	6.42501×10^{10} m	-8.15756×10^{9} m
345600 s	1.30856×10^{11} m	6.65594×10^{10} m	-8.46506×10^{9} m
432000 s	1.29636×10^{11} m	6.88705×10^{10} m	-8.7469×10^{9} m
518400 s	1.28387×10^{11} m	7.11366×10^{10} m	-9.02871×10^{9} m
604800 s	1.27085×10^{11} m	7.3402×10^{10} m	-9.31049×10^{9} m
691200 s	1.25754×10^{11} m	7.56205×10^{10} m	-9.61784×10^{9} m
777600 s	1.24374×10^{11} m	7.78381×10^{10} m	-9.87393×10^{9} m
864000 s	1.22966×10^{11} m	8.00079×10^{10} m	-1.01556×10^{10} m
950400 s	1.21506×10^{11} m	8.21731×10^{10} m	-1.04372×10^{10} m
1036800 s	1.20007×10^{11} m	8.43114×10^{10} m	-1.07188×10^{10} m
1123200 s	1.18487×10^{11} m	8.64025×10^{10} m	-1.09748×10^{10} m
1209600 s	1.17034×10^{11} m	8.85763×10^{10} m	-1.12421×10^{10} m
1296000 s	1.15439×10^{11} m	9.06133×10^{10} m	-1.14983×10^{10} m
1382400 s	1.13792×10^{11} m	9.26411×10^{10} m	-1.17544×10^{10} m
1468800 s	1.12109×10^{11} m	9.46387×10^{10} m	-1.20105×10^{10} m
1555200 s	1.1039×10^{11} m	9.66054×10^{10} m	-1.22665×10^{10} m
1641600 s	1.08654×10^{11} m	9.85217×10^{10} m	-1.25225×10^{10} m
1728000 s	1.06869×10^{11} m	1.00427×10^{11} m	-1.27529×10^{10} m
1814400 s	1.0505×10^{11} m	1.02299×10^{11} m	-1.29833×10^{10} m
1900800 s	1.03197×10^{11} m	1.04138×10^{11} m	-1.32136×10^{10} m
1987200 s	1.01331×10^{11} m	1.05926×10^{11} m	-1.34438×10^{10} m
2073600 s	9.94152×10^{10} m	1.07697×10^{11} m	-1.36741×10^{10} m
2160000 s	9.74679×10^{10} m	1.09434×10^{11} m	-1.39043×10^{10} m

2246400 s	9.55886×10^{10} m	1.1125×10^{11} m	-1.41232×10^{10} m
2332800 s	9.35985×10^{10} m	1.129×10^{11} m	-1.43536×10^{10} m
2419200 s	9.15613×10^{10} m	1.14533×10^{11} m	-1.45584×10^{10} m
2505600 s	8.9497×10^{10} m	1.1613×10^{11} m	-1.47375×10^{10} m
2592000 s	8.74032×10^{10} m	1.17689×10^{11} m	-1.49422×10^{10} m
2678400 s	8.52821×10^{10} m	1.19209×10^{11} m	-1.51469×10^{10} m
2764800 s	8.31359×10^{10} m	1.20693×10^{11} m	-1.53259×10^{10} m
2851200 s	8.09852×10^{10} m	1.22124×10^{11} m	-1.5505×10^{10} m
2937600 s	7.88681×10^{10} m	1.23655×10^{11} m	-1.56999×10^{10} m
3024000 s	7.66443×10^{10} m	1.25023×10^{11} m	-1.58791×10^{10} m
3110400 s	7.43967×10^{10} m	1.26351×10^{11} m	-1.60582×10^{10} m
3196800 s	7.21273×10^{10} m	1.27641×10^{11} m	-1.62118×10^{10} m
3283200 s	6.98581×10^{10} m	1.28877×10^{11} m	-1.63653×10^{10} m
3369600 s	6.75449×10^{10} m	1.30085×10^{11} m	-1.65188×10^{10} m
3456000 s	6.52108×10^{10} m	1.31251×10^{11} m	-1.66723×10^{10} m
3542400 s	6.29202×10^{10} m	1.3251×10^{11} m	-1.68428×10^{10} m
3628800 s	6.05686×10^{10} m	1.33585×10^{11} m	-1.69708×10^{10} m
3715200 s	5.81751×10^{10} m	1.34628×10^{11} m	-1.70988×10^{10} m
3801600 s	5.57636×10^{10} m	1.35629×10^{11} m	-1.72268×10^{10} m
3888000 s	5.33588×10^{10} m	1.36576×10^{11} m	-1.73548×10^{10} m
3974400 s	5.09664×10^{10} m	1.37632×10^{11} m	-1.74748×10^{10} m
4060800 s	4.85032×10^{10} m	1.38503×10^{11} m	-1.76029×10^{10} m
4147200 s	4.60504×10^{10} m	1.39325×10^{11} m	-1.77053×10^{10} m
4233600 s	4.35593×10^{10} m	1.40111×10^{11} m	-1.78078×10^{10} m
4320000 s	4.10556×10^{10} m	1.40855×10^{11} m	-1.78846×10^{10} m

-9.31908×10^{10} m	4233600 s	-22012.2	$\frac{m}{s}$
-8.328×10^{10} m	3888000 s	-21419.8	$\frac{m}{s}$
-7.36778×10^{10} m	3542400 s	-20798.8	$\frac{m}{s}$
-6.43883×10^{10} m	3196800 s	-20141.5	$\frac{m}{s}$
-5.53783×10^{10} m	2851200 s	-19422.8	$\frac{m}{s}$
-4.68432×10^{10} m	2505600 s	-18695.4	$\frac{m}{s}$
-3.86578×10^{10} m	2160000 s	-17897.1	$\frac{m}{s}$
-3.10489×10^{10} m	1814400 s	-17112.5	$\frac{m}{s}$
-2.3856×10^{10} m	1468800 s	-16241.9	$\frac{m}{s}$
-1.72124×10^{10} m	1123200 s	-15324.5	$\frac{m}{s}$
-1.12802×10^{10} m	777600 s	-14506.4	$\frac{m}{s}$
-5.85969×10^{9} m	432000 s	-13564.1	$\frac{m}{s}$
-1.08513×10^{9} m	86400 s	-12559.4	$\frac{m}{s}$

x

8.13094×10^{10} m	4233600 s	$19205.7\ \frac{m}{s}$
7.80866×10^{10} m	3888000 s	$20084.\ \frac{m}{s}$
7.40388×10^{10} m	3542400 s	$20900.7\ \frac{m}{s}$
6.93312×10^{10} m	3196800 s	$21687.7\ \frac{m}{s}$
6.41093×10^{10} m	2851200 s	$22485.\ \frac{m}{s}$
5.81426×10^{10} m	2505600 s	$23205.1\ \frac{m}{s}$
5.17039×10^{10} m	2160000 s	$23937.\ \frac{m}{s}$
4.45924×10^{10} m	1814400 s	$24577.\ \frac{m}{s}$
3.70595×10^{10} m	1468800 s	$25231.2\ \frac{m}{s}$
2.90305×10^{10} m	1123200 s	$25846.2\ \frac{m}{s}$
2.0462×10^{10} m	777600 s	$26314.3\ \frac{m}{s}$
1.15907×10^{10} m	432000 s	$26830.4\ \frac{m}{s}$
2.35028×10^{9} m	86400 s	$27202.3\ \frac{m}{s}$

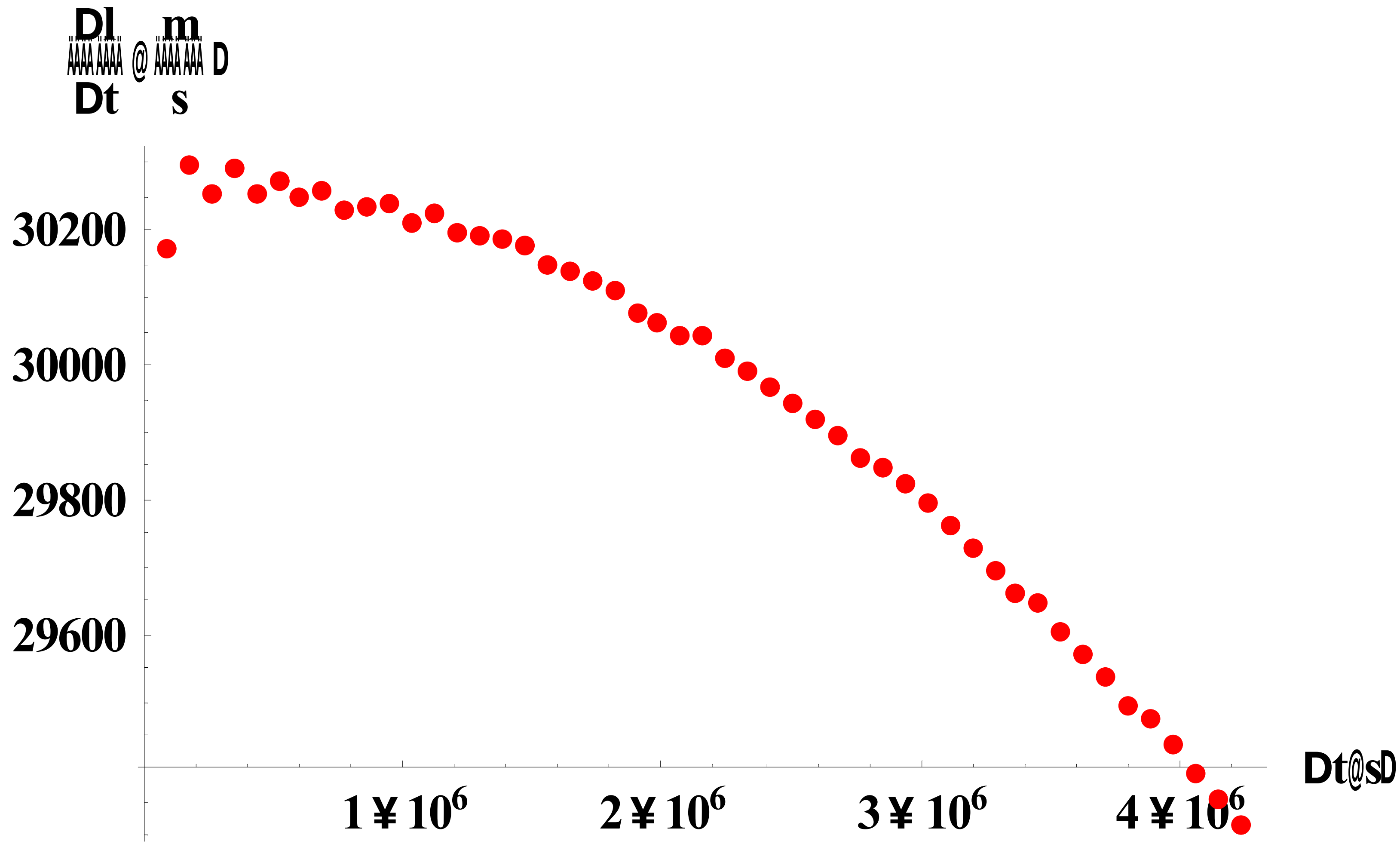
y

-1.03165×10^{10} m	4233600 s	$-2436.81\ \frac{\mathrm{m}}{\mathrm{s}}$
-9.90672×10^{9} m	3888000 s	$-2548.02\ \frac{\mathrm{m}}{\mathrm{s}}$
-9.40273×10^{9} m	3542400 s	$-2654.34\ \frac{\mathrm{m}}{\mathrm{s}}$
-8.79721×10^{9} m	3196800 s	$-2751.88\ \frac{\mathrm{m}}{\mathrm{s}}$
-8.13183×10^{9} m	2851200 s	$-2852.07\ \frac{\mathrm{m}}{\mathrm{s}}$
-7.37412×10^{9} m	2505600 s	$-2943.06\ \frac{\mathrm{m}}{\mathrm{s}}$
-6.55515×10^{9} m	2160000 s	$-3034.79\ \frac{\mathrm{m}}{\mathrm{s}}$
-5.64548×10^{9} m	1814400 s	$-3111.48\ \frac{\mathrm{m}}{\mathrm{s}}$
-4.69844×10^{9} m	1468800 s	$-3198.83\ \frac{\mathrm{m}}{\mathrm{s}}$
-3.67405×10^{9} m	1123200 s	$-3271.05\ \frac{\mathrm{m}}{\mathrm{s}}$
-2.58751×10^{9} m	777600 s	$-3327.56\ \frac{\mathrm{m}}{\mathrm{s}}$
-1.46062×10^{9} m	432000 s	$-3381.06\ \frac{\mathrm{m}}{\mathrm{s}}$
-3.07566×10^{8} m	86400 s	$-3559.79\ \frac{\mathrm{m}}{\mathrm{s}}$

z

Il modulo

$$\sqrt{\left[\frac{x(t+\Delta t)-x(t)}{\Delta t}\right]^2+\left[\frac{y(t+\Delta t)-y(t)}{\Delta t}\right]^2+\left[\frac{z(t+\Delta t)-z(t)}{\Delta t}\right]^2}$$



Alcune conclusioni

Dividendo le tre componenti del vettore spostamento per lo scalare tempo si ottiene ancora un vettore: la velocità media

$$\vec{\bar{v}} = \left\{ \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \frac{y(t_2) - y(t_1)}{t_2 - t_1}, \frac{z(t_2) - z(t_1)}{t_2 - t_1} \right\}$$

$$\vec{\bar{v}}(t_1, t_2) \equiv \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

$$\text{Se } t_2 \to t_1 \text{ allora } \frac{x(t_2)-x(t_1)}{t_2-t_1} \to v_x(t_1) \text{ indip da } t_2$$

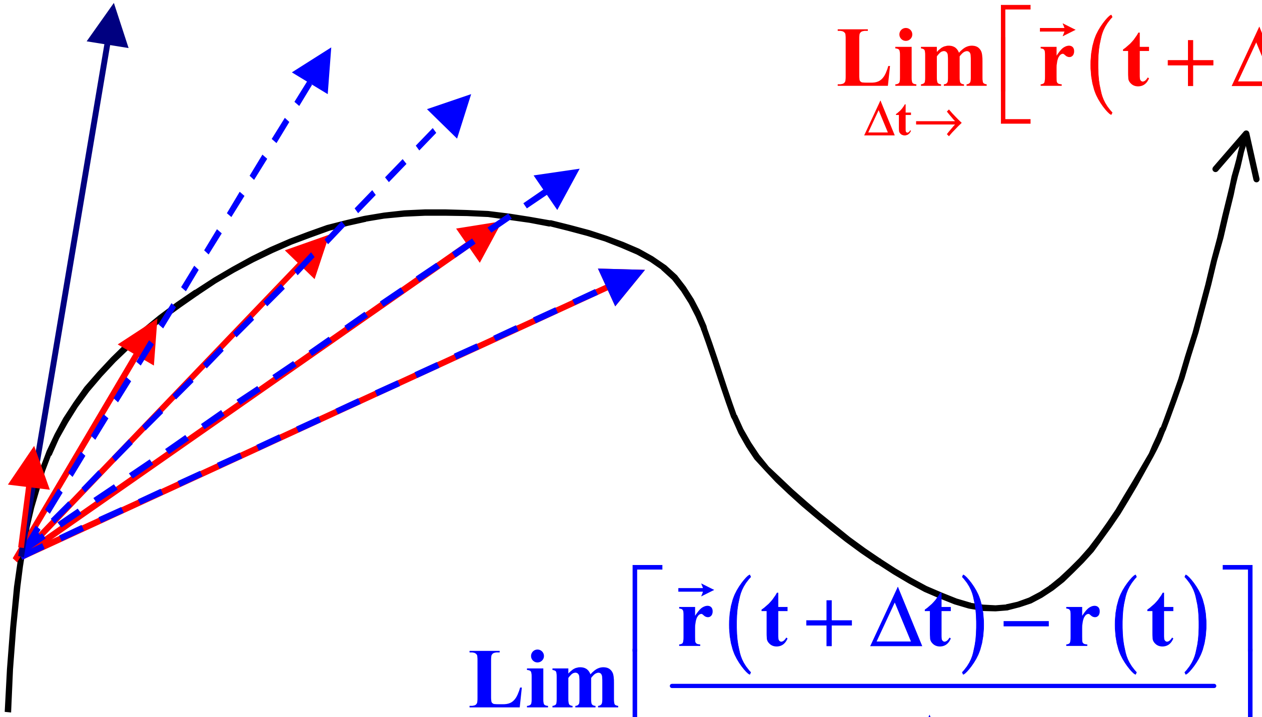
Vettore velocità istantanea

$$\vec{v}(t) \equiv$$

$$\left\{ \lim_{\Delta t \to 0} \frac{x(t+\Delta t)-x(t)}{\Delta t}, \lim_{\Delta t \to 0} \frac{y(t+\Delta t)-y(t)}{\Delta t}, \lim_{\Delta t \to 0} \frac{z(t+\Delta t)-z(t)}{\Delta t} \right\}$$

$$\equiv \left\{ v_x(t), v_y(t), v_z(t) \right\}$$

$$\lim_{\Delta t \to}\left[\vec{r}(t+\Delta t)-r(t)\right]=0$$



$$\lim_{\Delta t \to}\left[\frac{\vec{r}(t+\Delta t)-r(t)}{\Delta t}\right]=\vec{v}(t)$$

Modulo: $\dfrac{\textbf{lunghezza di traiettoria}}{\textbf{tempo impiegato}}$

Direzione: tangente alla traiettoria

Verso: stesso verso di percorrenza della traiettoria

Moto rettilineo uniforme

$$x(t) = v_{ox}t + x_o \quad y(t) = v_{oy}t + y_o \quad z(t) = v_{oz}t + z_o$$

$$\vec{r}(t) = \{v_{ox}t + x_o, v_{oy}t + y_o, v_{oz}t + z_o\}$$

$$\vec{v}(t) \overset{\text{def}}{\equiv} \frac{d\vec{r}(t)}{dt}$$

$$= \left\{ \frac{d(v_{ox}t + x_o)}{dt}, \frac{d(v_{oy}t + y_o)}{dt}, \frac{d(v_{oz}t + z_o)}{dt} \right\}$$

Un vettore costante $\longleftarrow$ $= \{v_{ox}, v_{oy}, v_{oz}\}$

$$|\vec{v}(t)| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} = \sqrt{v_{ox}^2 + v_{oy}^2 + v_{oz}^2}$$

$$x[t] = v_{0x}\, t + x_0;$$

$$y[t] = v_{0y}\, t + y_0;\ z[t] = v_{0z}\, t + z_0;$$

$$\vec{r}[t] =$$

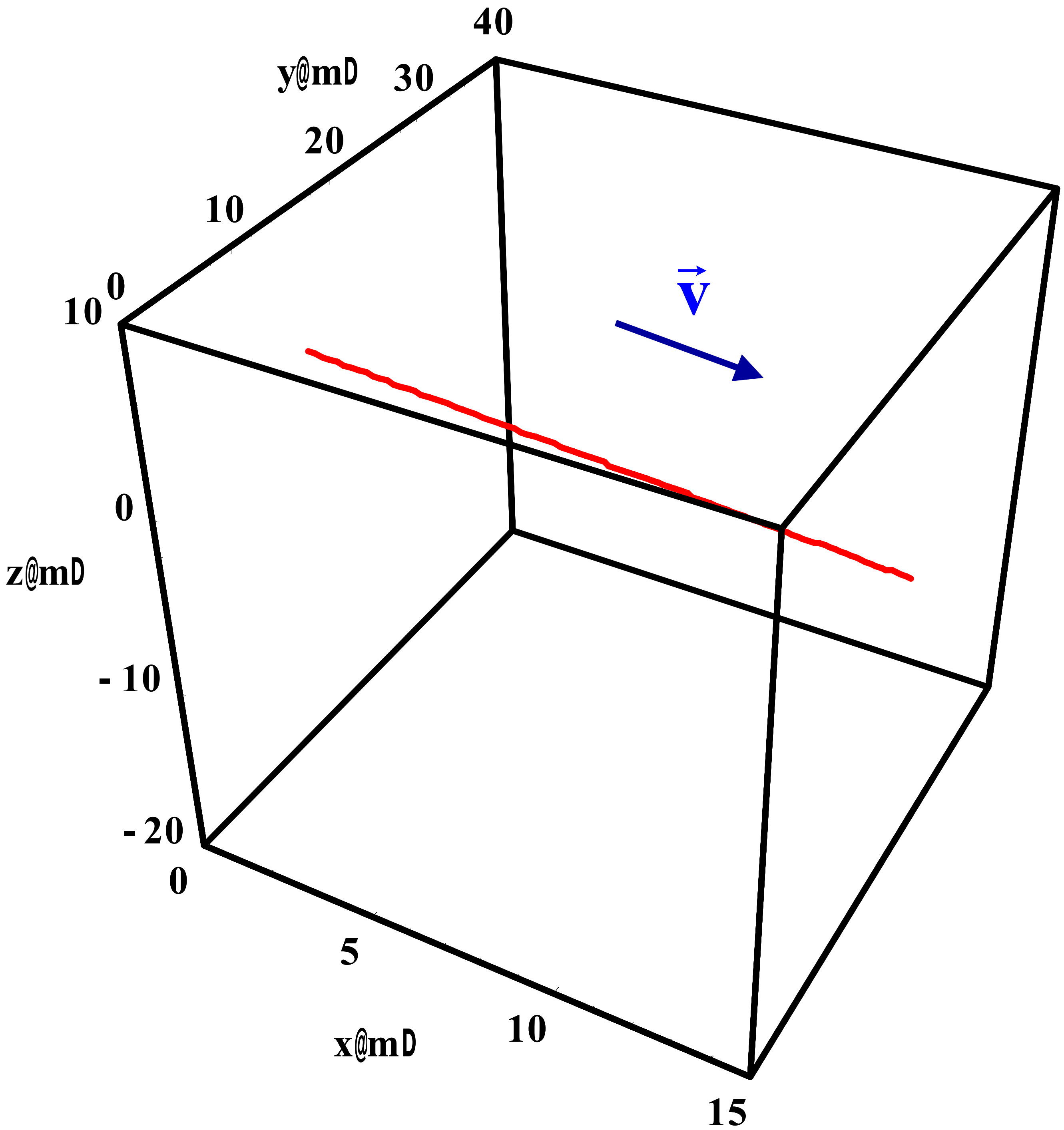
$$\{x[t], y[t], z[t]\} /.$$

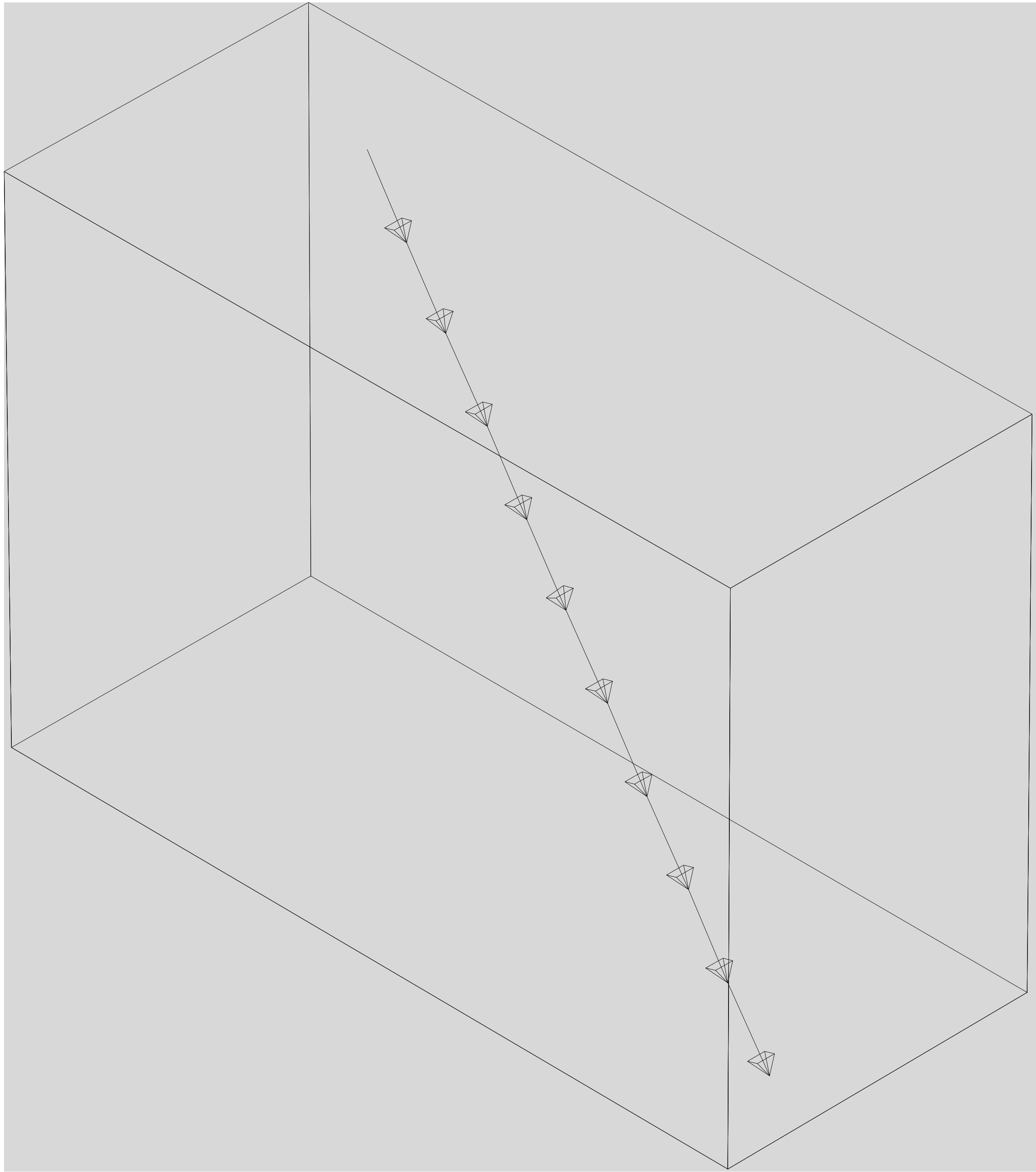
$$\left\{v_{0x} \to 1\,\frac{m}{s},\ v_{0y} \to 3\,\frac{m}{s},\ v_{0z} \to -2\,\frac{m}{s},\ x_0 \to 3\,m,\ y_0 \to 6\,m,\ z_0 \to 8\,m\right\} =$$

$$= \left\{3\,m + \frac{m\,t}{s},\ 6\,m + \frac{3\,m\,t}{s},\ 8\,m - \frac{2\,m\,t}{s}\right\}$$

$$\vec{v}[t] = \partial_t \vec{r}[t] = \left\{\frac{m}{s},\ \frac{3\,m}{s},\ -\frac{2\,m}{s}\right\}$$

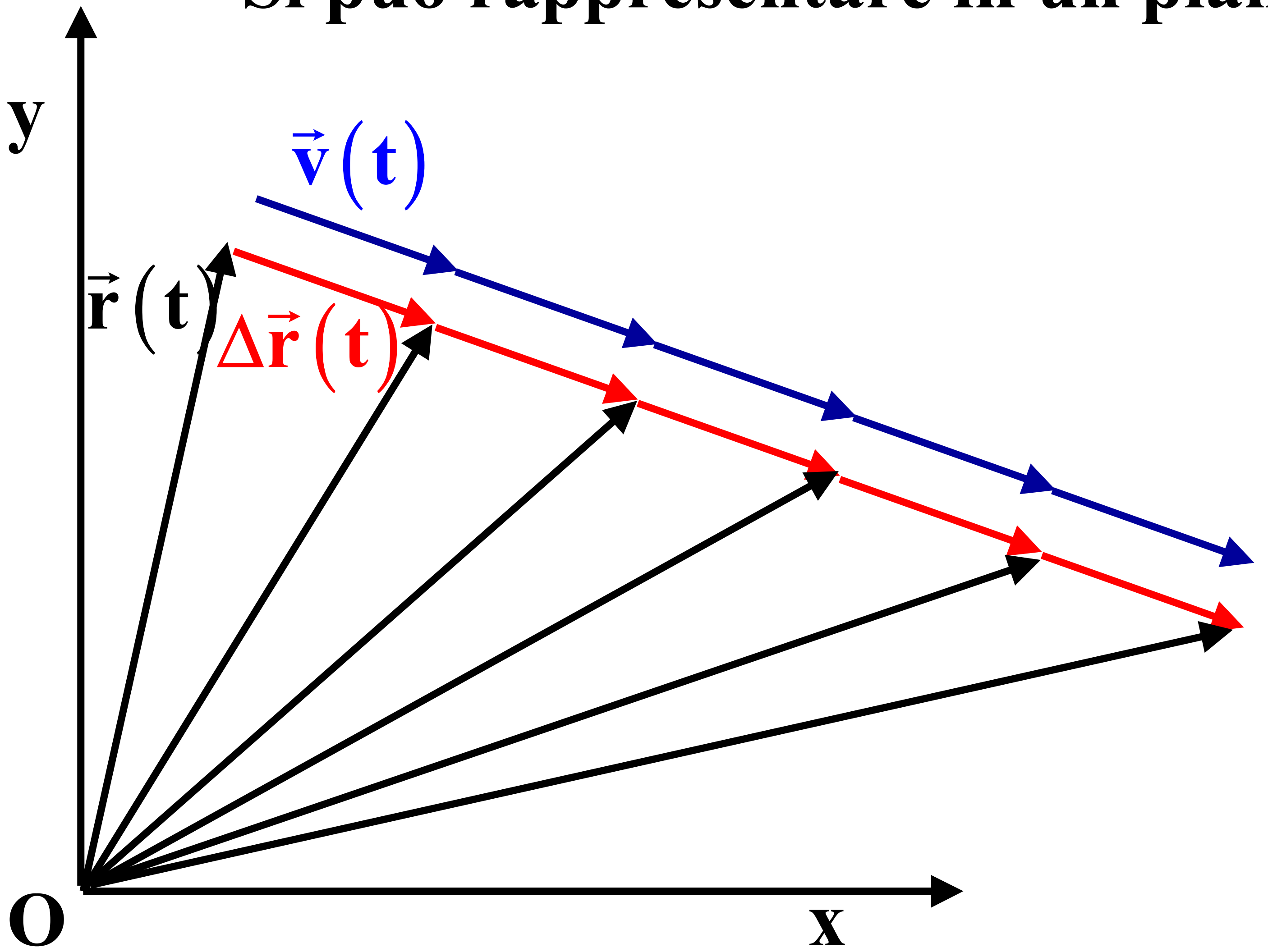
Traiettoria

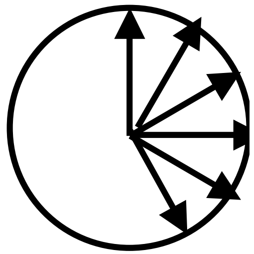




Velocità

Si può rappresentare in un piano





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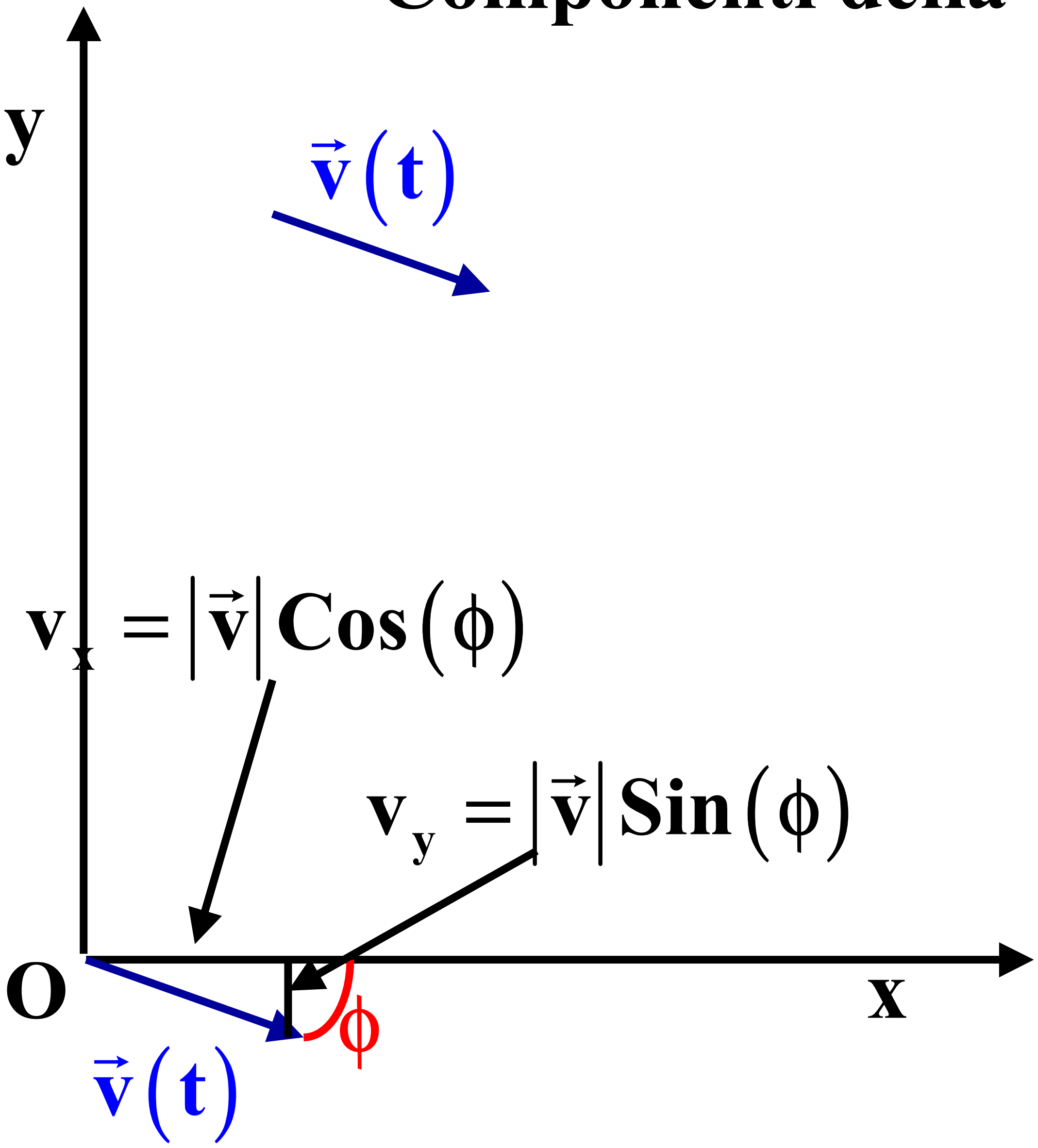
Componenti della velocità



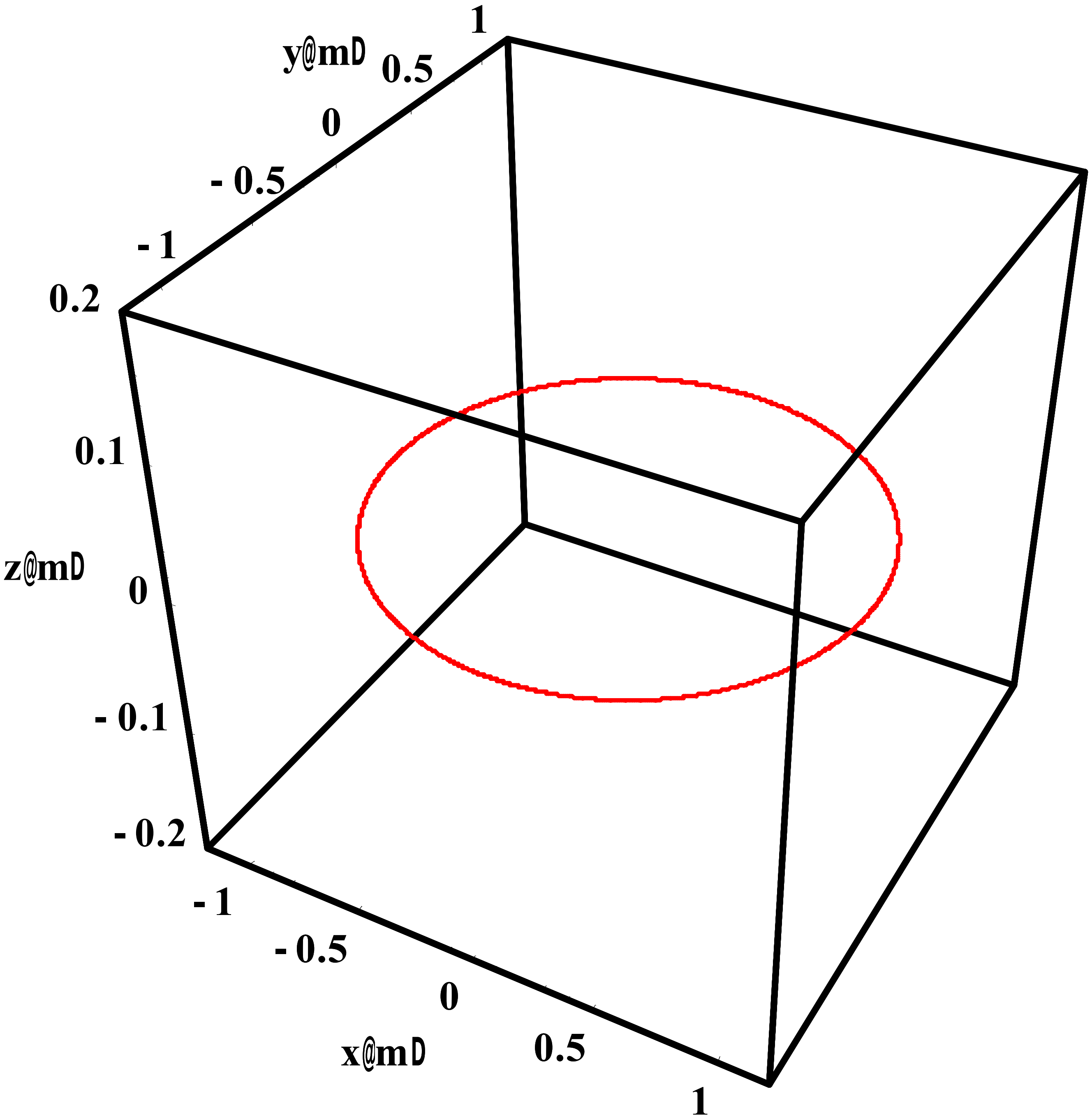
Moto circolare uniforme

$$x(t) = r_o \mathrm{Cos}(\omega_o t) \qquad y(t) = r_o \mathrm{Sin}(\omega_o t) \qquad z(t) = 0$$

$$[r_o] = l \qquad [\omega_o] = t^{-1}$$

$$\left|\vec{r}(t)\right| = \sqrt{x^2(t) + y^2(t) + z^2(t)} =$$

$$\sqrt{r_o^2 \mathrm{Cos}^2(\omega t) + r_o^2 \mathrm{Sin}^2(\omega t)} = \quad r_o$$



y@mD
1
0.5
0
- 0.5
- 1
0.2
0.1
z@mD
0
- 0.1
- 0.2
- 1
- 0.5
0
x@mD
0.5
1

Velocità

$$v_x(t) = \frac{dx(t)}{dt} = \frac{dr_o Cos(\omega_o t)}{dt} = r_o\omega_o\left[-Sin(\omega_o t)\right]$$

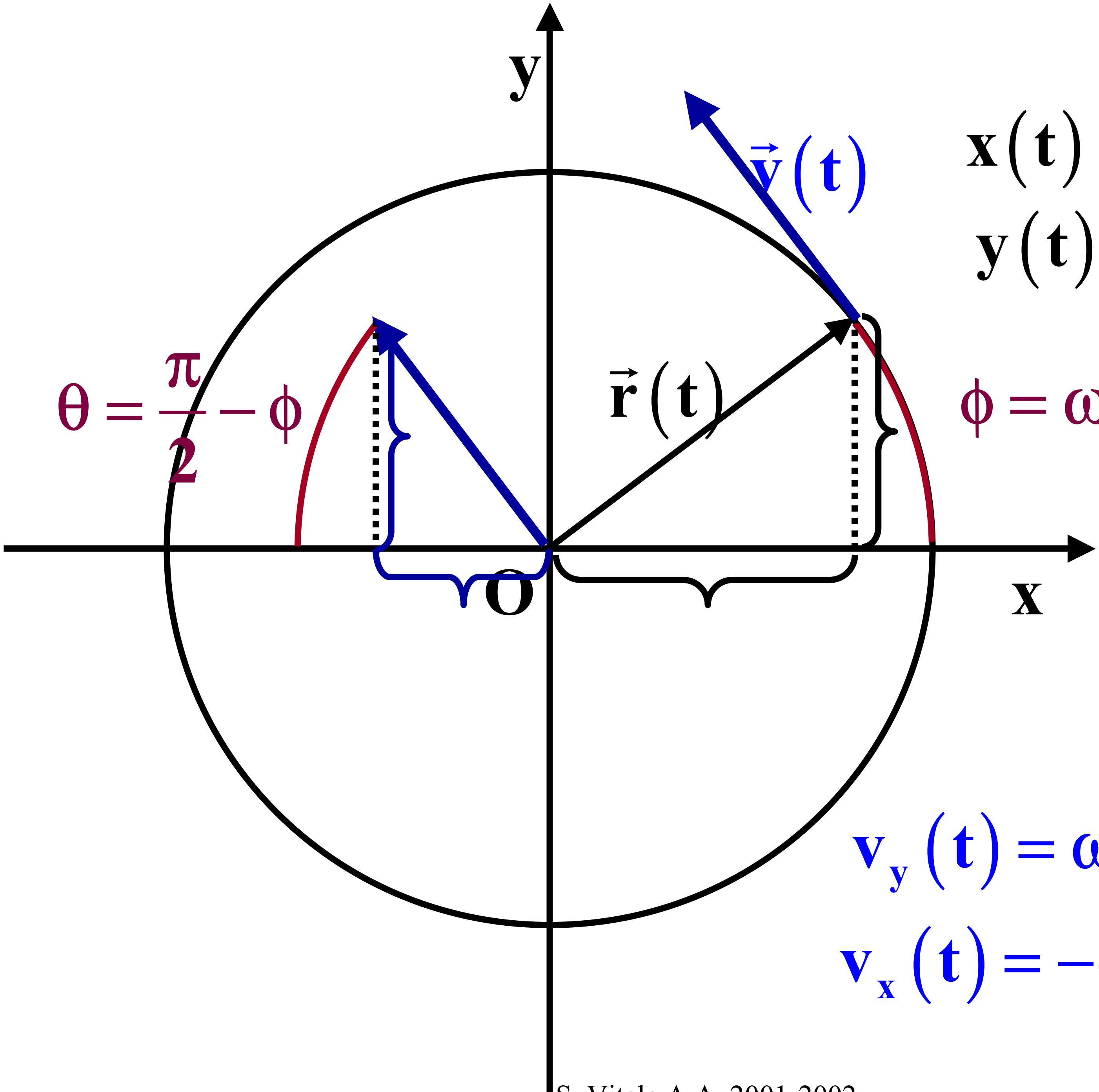
$$v_y(t) = \frac{dy(t)}{dt} = \frac{dr_o Sin(\omega_o t)}{dt} = r_o\omega_o Cos(\omega_o t)$$

$$v_z(t) = \frac{dz(t)}{dt} = 0$$

$$\vec{v}(t) = r_o\omega_o\left\{-Sin(\omega_o t), Cos(\omega_o t)\right\}$$

$$\left|\vec{v}(t)\right| = r_o\omega_o\sqrt{Sin^2(\omega_o t) + Cos^2(\omega_o t)} = r_o\omega_o$$

$$x(t) = r_o \mathrm{Cos}(\omega_o t)$$
$$y(t) = r_o \mathrm{Sin}(\omega_o t)$$



$$v_y(t) = \omega_o r_o \mathrm{Cos}(\omega_o t)$$
$$v_x(t) = -\omega_o r_o \mathrm{Sin}(\omega_o t)$$

$$\vec{r}(t) \perp \vec{v}(t)$$

$$\vec{r}(t)\cdot\vec{v}(t) \equiv x(t)\,v_x(t) + y(t)\,v_y(t) + z(t)\,v_z(t)$$

$$= r_o \mathrm{Cos}(\omega_o t) \times \left[-\omega_o r_o \mathrm{Sin}(\omega_o t)\right]$$

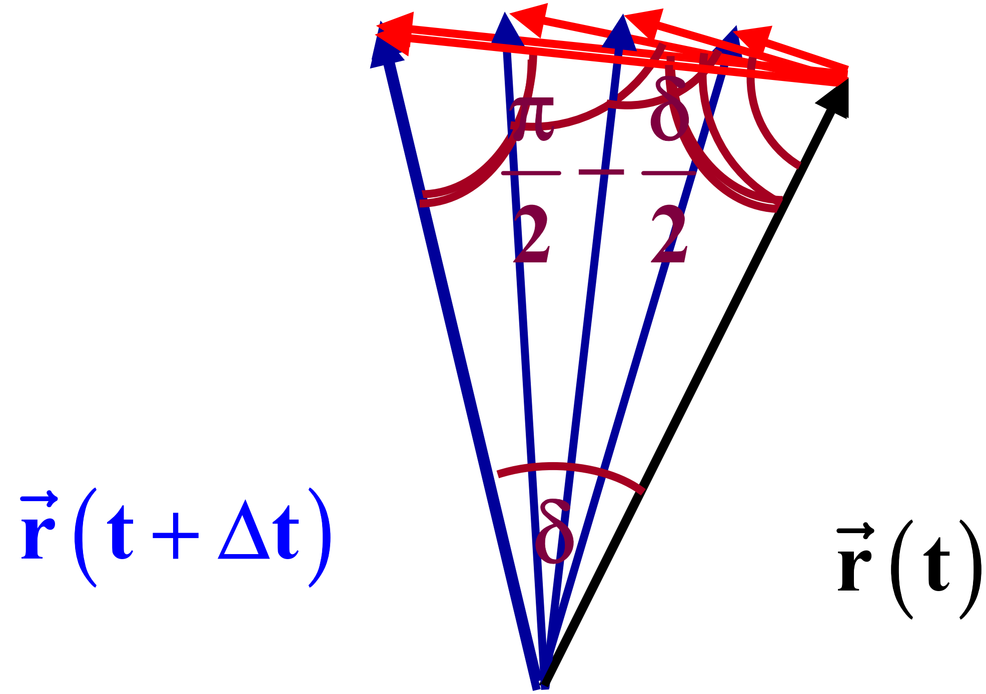
$$+ r_o \mathrm{Sin}(\omega_o t)\,\omega_o r_o \mathrm{Cos}(\omega_o t) \quad = 0$$

Ma anche

$$\vec{r}(t)\cdot\vec{v}(t) = \left|\vec{r}(t)\right|\left|\vec{v}(t)\right| \mathrm{Cos}\left[\widehat{\vec{r}(t)\cdot\vec{v}(t)}\right]$$

$$\widehat{\vec{r}(t)\cdot\vec{v}(t)} = \pm\frac{\pi}{2}$$

$\vec{r}(t+\Delta t)-\vec{r}(t)$



Al tendere di $\Delta t \rightarrow 0$, $\delta \rightarrow 0$ e $\pi/2-\delta/2 \rightarrow \pi/2$

$$\vec{r}(t+\Delta t)-\vec{r}(t)\perp\vec{r}(t)$$

La derivata di un vettore di modulo costante è ortogonale al vettore derivato

$$x[t] = r_\circ \, \mathrm{Cos}[\omega\, t];$$

$$y[t] = r_\circ \, \mathrm{Sin}[\omega\, t]; \; z[t] = 0;$$

$$\vec{r}[t] =$$

$$\{x[t], y[t], z[t]\} \,/.\, \{\omega \to 4\, s^{-1}, r_\circ \to 2\, m\} =$$

$$= \left\{2\, m\, \mathrm{Cos}\left[\frac{4\, t}{s}\right], 2\, m\, \mathrm{Sin}\left[\frac{4\, t}{s}\right], 0\right\}$$

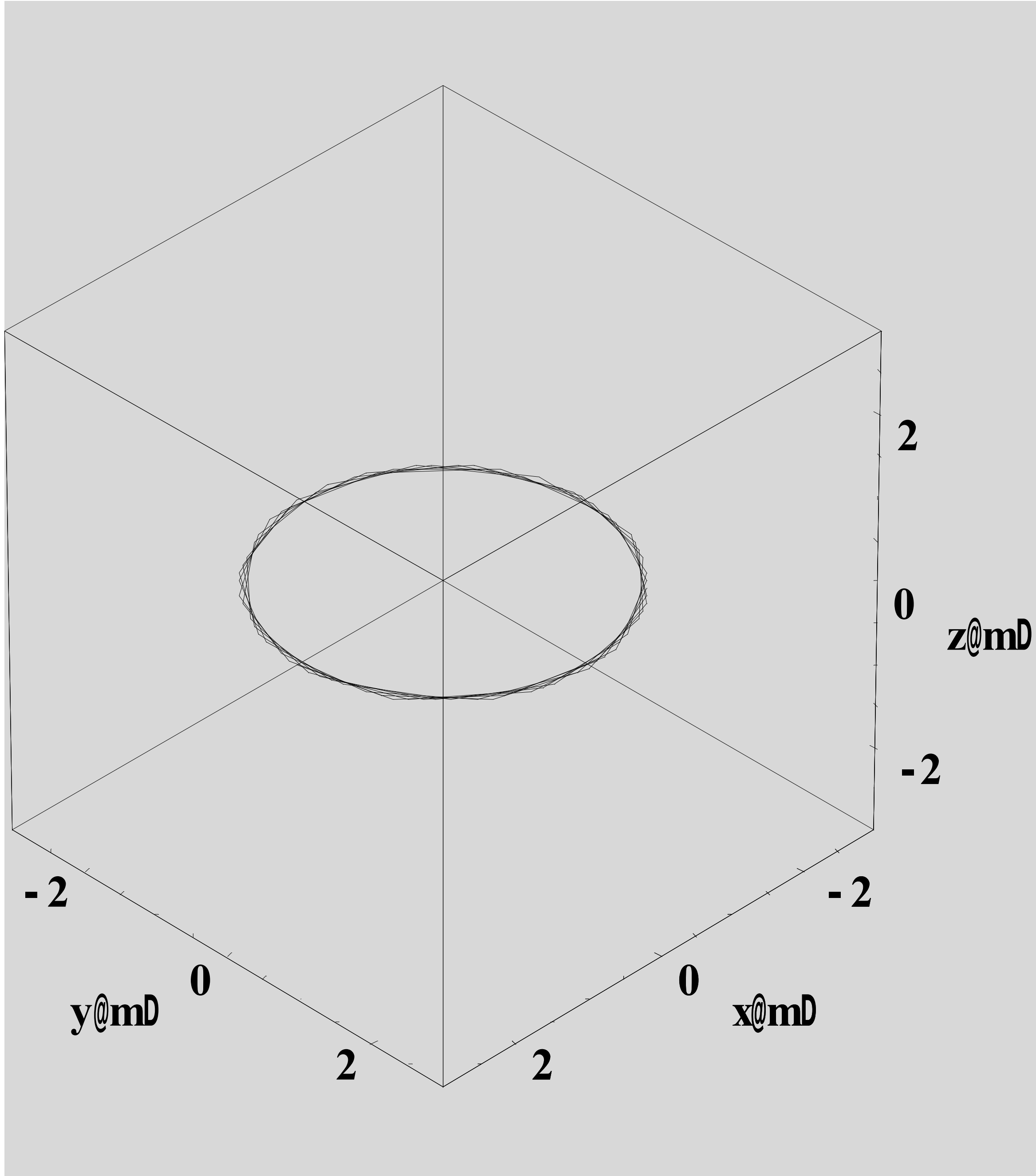
$$\mathrm{Simplify}\left[\sqrt{\vec{r}[t].\vec{r}[t]}\right] = 2\, m$$

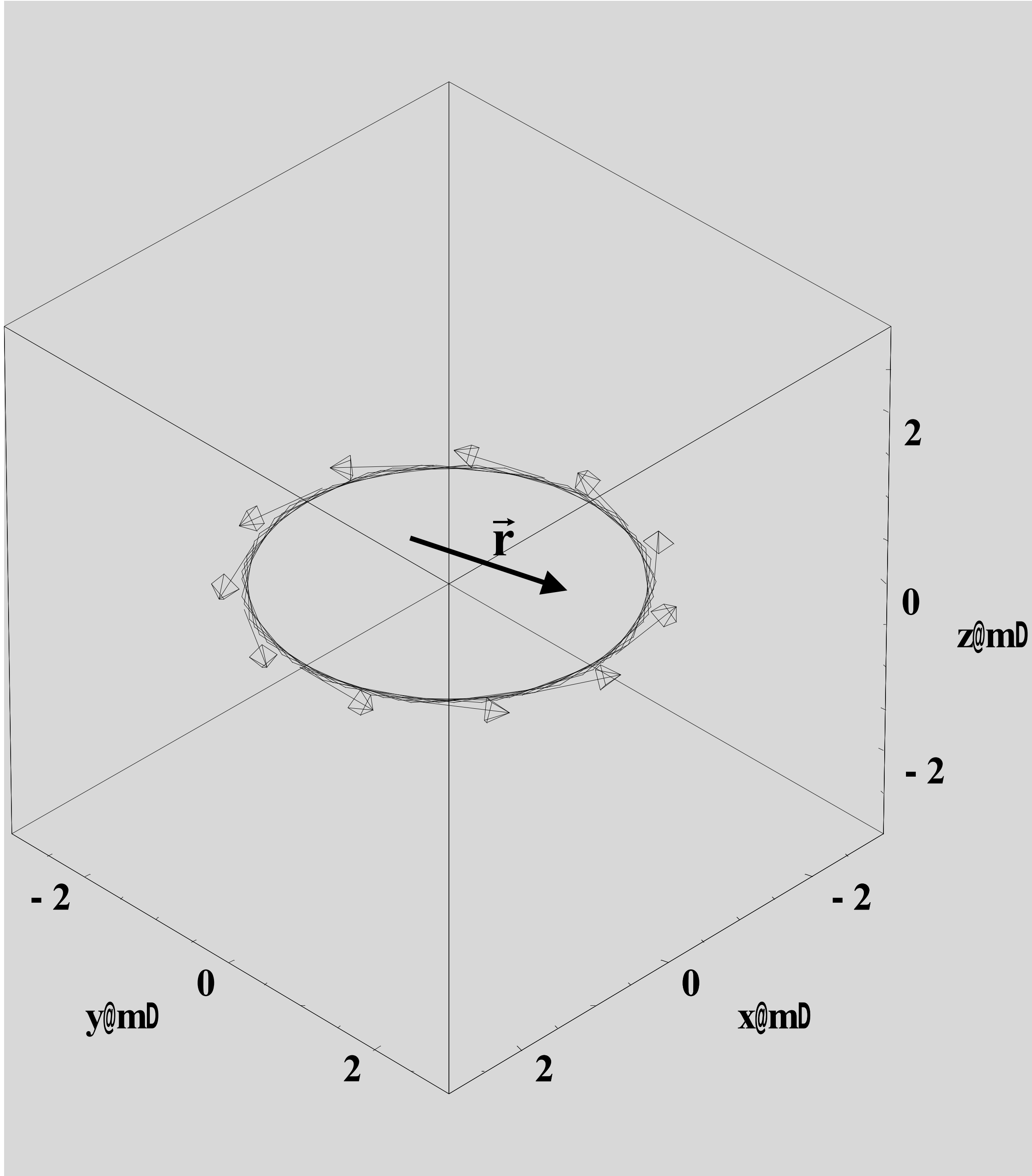
$$\vec{v}[t] = \partial_t \vec{r}[t] = \left\{-\frac{8\, m\, \mathrm{Sin}\left[\frac{4\, t}{s}\right]}{s}, \frac{8\, m\, \mathrm{Cos}\left[\frac{4\, t}{s}\right]}{s}, 0\right\}$$

$$\mathrm{modulo} = \mathrm{Simplify}\left[\sqrt{\vec{v}[t].\vec{v}[t]}\right] = 8\, \frac{m}{s}$$

$$\vec{v}[t].\vec{r}[t] = 0$$

Traiettoria





velocità

Moto parabolico

$$\text{x@tD} = v_o\,\text{Cos@}\theta\text{D}\;t;$$

$$\text{y@tD} = v_o\,\text{Sin@}\theta\text{D}\;t + \frac{1}{2}\,a\,t^2;\;\; \text{z@tD} = 0;$$

$$\overset{„}{\text{r}}\text{@tD} = \text{8x@tD, y@tD, z@tD<}\;\hat{\text{e}}.$$

$$\text{9}v_o \to 1800\,\frac{\text{m}}{\text{s}},\; a \to -9.8\,\frac{\text{m}}{\text{s}^2},\;\; \theta \text{ -> } \frac{\pi}{6}\;\text{=}$$

$$\text{9}\,\frac{900\;\text{è—}3\;\text{m t}}{\text{s}},\;\frac{900\;\text{m t}}{\text{s}} - \frac{4.9\;\text{m t}^2}{\text{s}^2},\; \text{0=}$$

$$\overset{„}{\text{v}}\text{@tD} = \partial_t\,\overset{„}{\text{r}}\text{@tD}$$

$$\text{9}\,\frac{900\;\text{è—}3\;\text{m}}{\text{s}},\;\frac{900\;\text{m}}{\text{s}} - \frac{9.8\;\text{m t}}{\text{s}^2},\; \text{0=}$$

Traiettoria e velocità