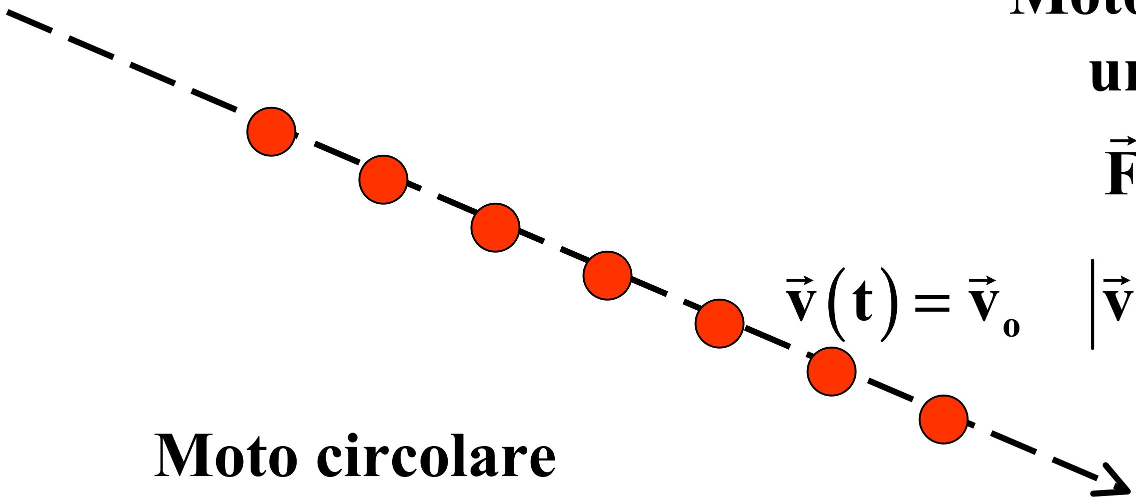
Lavoro ed Energia



Moto rettilineo uniforme

$$\vec{F}(t) = 0$$

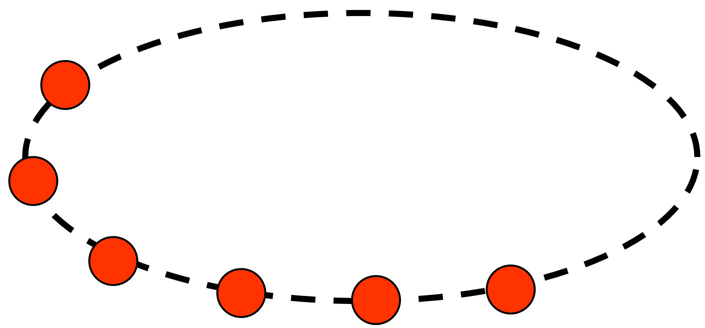
$$\vec{v}(t) = \vec{v}_o \qquad \left|\vec{v}(t)\right|^2 \equiv v^2(t) = \cos t$$

Moto circolare uniforme

$$\vec{F}(t) \neq 0$$

$$\vec{v}(t) = -r_o\omega \mathrm{Sin}(\omega t)\,\hat{i} + r_o\omega \mathrm{Cos}(\omega t)\,\hat{j}$$

$$\left|\vec{v}(t)\right|^2 \equiv v^2(t) = r_o^2\omega^2 = \cos t$$



Quand'è che v^2 cambia?

$$\frac{dv^2}{dt} = \frac{d\left(\vec{v}\cdot\vec{v}\right)}{dt} = \frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt} = 2\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$2\vec{v}\cdot\frac{d\vec{v}}{dt} = 2\vec{v}\cdot\vec{a} = 2\vec{v}\cdot\frac{\vec{F}}{m}$$

$$\frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \vec{F}\cdot\vec{v}$$

$$\frac{1}{2}mv^2 \equiv \text{ Energia Cinetica}$$

Se la forza è nulla o ortogonale alla velocità, l'energia cinetica si conserva

Energia Cinetica: Dimensioni Fisiche

$$[m][v]^2 = [m][l]^2[t]^{-2}\left(=[F][l]\right)$$

Unità di Misura:

$$1\ \mathrm{kg\ m^2 s^{-2}} = 1\ \mathrm{Joule} = 1\ \mathrm{J}$$

$$\text{Treno in corsa} \approx \frac{1}{2}400\times 10^3\ \mathrm{kg}\times\left(50\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 5\times 10^8\,\mathrm{J}$$

$$\text{Molecola di gas a temperatura ambiente} \approx$$

$$\approx \frac{3}{2}k_B T = 1.5\times 1.4\cdot 10^{-23}\frac{\mathrm{J}}{\mathrm{K}}300\mathrm{K} \approx 6.3\times 10^{-21}\,\mathrm{J}$$

Moto rettilineo uniforme

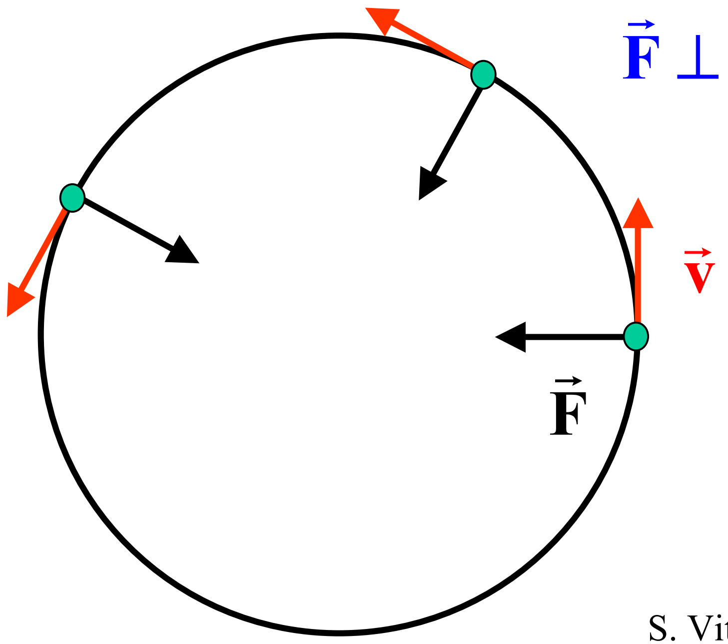
$$\vec{F}=0 \quad \rightarrow \quad \vec{F}\cdot\vec{v}=0 \quad \rightarrow \quad \frac{1}{2}mv^2 = \cos t.$$

Moto circolare uniforme

$$\vec{F}(t) = -mr_o\omega^2\left[\mathrm{Cos}(\omega t)\,\hat{i} + \mathrm{Sin}(\omega t)\,\hat{j}\right]$$

$$\vec{v}(t) = r_o\omega\left[-\mathrm{Sin}(\omega t)\,\hat{i} + \mathrm{Cos}(\omega t)\,\hat{j}\right]$$

$$\vec{F}\perp\vec{v} \quad \rightarrow \quad \vec{F}\cdot\vec{v}=0 \quad \rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}mr_o^2\omega^2$$



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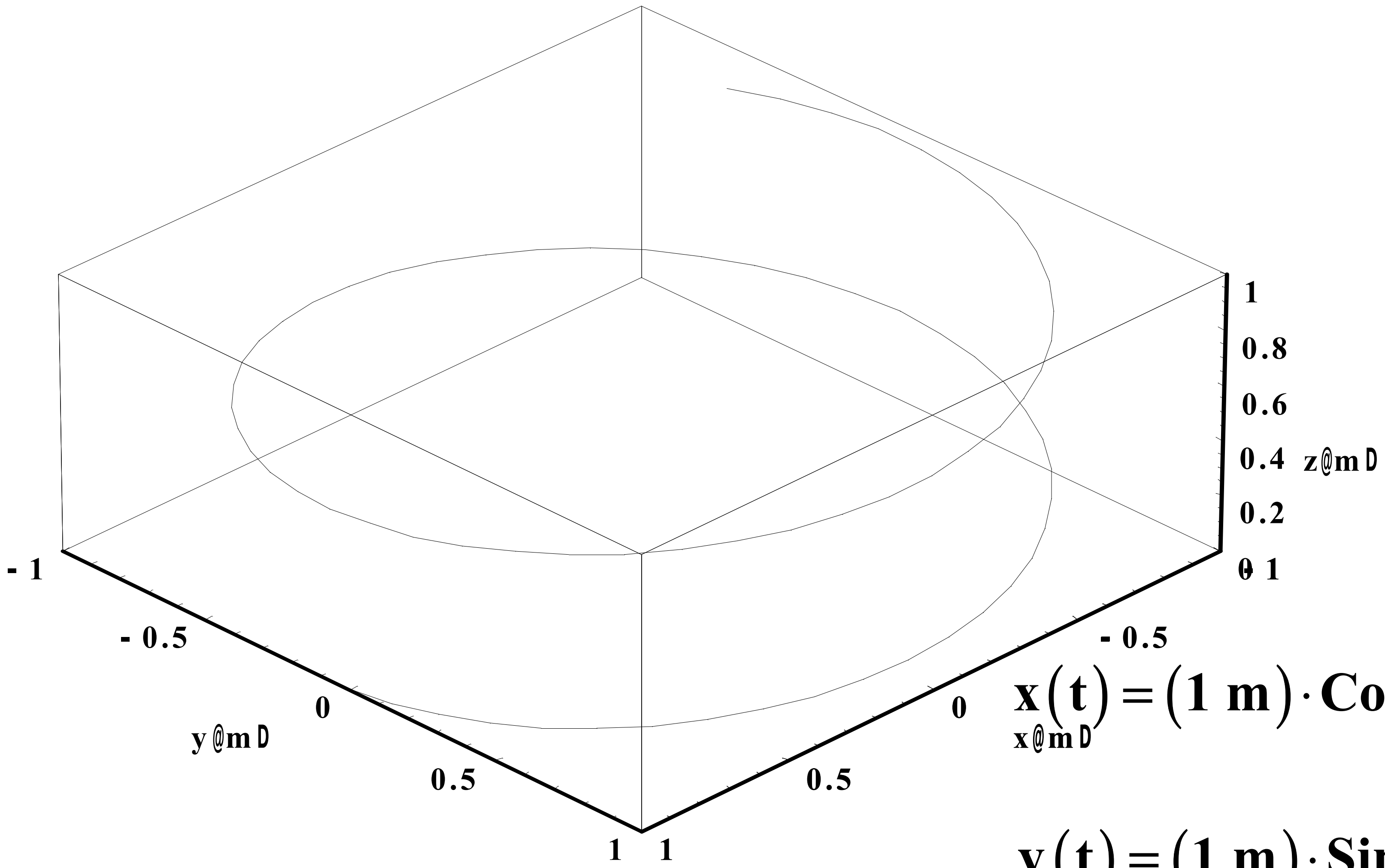
Invertiamo il teorema: la variazione di energia cinetica

$$\frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \vec{F}\cdot\vec{v}$$

$$\frac{1}{2}mv^2(t_B) - \frac{1}{2}mv^2(t_A) = \int_{t_A}^{t_B} \vec{F}(t')\cdot\vec{v}(t')\,dt' =$$

$$= \int_{t_A}^{t_B} \left[F_x(t')v_x(t') + F_y(t')v_y(t') + F_z(t')v_z(t')\right]dt' =$$

$$= \int_{t_A}^{t_B} \left[F_x\frac{dx}{dt} + F_y\frac{dy}{dt} + F_z\frac{dz}{dt}\right]dt'$$



$$x(t) = (1\ m)\cdot \mathrm{Cos}\left(1\frac{rad}{s}\ t\right)$$

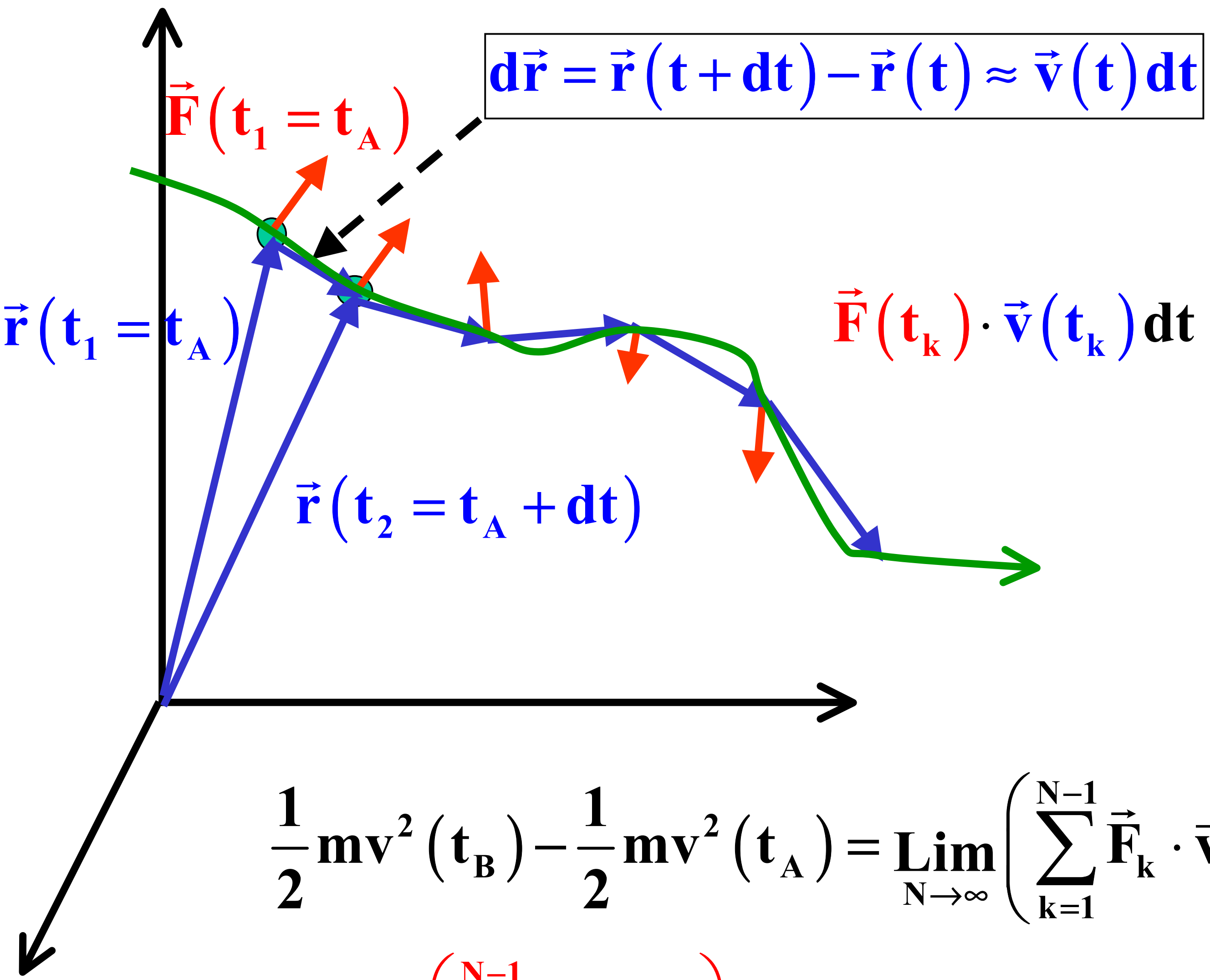
$$y(t) = (1\ m)\cdot \mathrm{Sin}\left(1\frac{rad}{s}\ t\right)$$

$$z(t) = 0.1\frac{m}{s}\ t$$

Equazione parametrica di una curva:

Mentre il parametro “t” scorre x,y e z disegnano una curva: la traiettoria

La variazione di energia come "integrale di linea" della forza



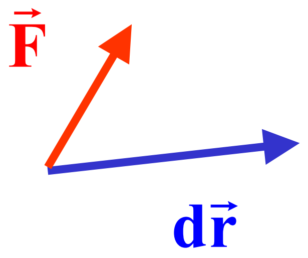
$$\vec{F}(t_k)\cdot\vec{v}(t_k)dt = \vec{F}(t_k)\cdot d\vec{r}(t_k)$$

$$\frac{1}{2}mv^2(t_B) - \frac{1}{2}mv^2(t_A) = \lim_{N\to\infty}\left(\sum_{k=1}^{N-1}\vec{F}_k\cdot\vec{v}_k dt\right) =$$

$$= \lim_{N\to\infty}\left(\sum_{k=1}^{N-1}\vec{F}_k\cdot d\vec{r}_k\right) \equiv \int_{\text{Traiettoria}}\vec{F}\cdot d\vec{r}$$

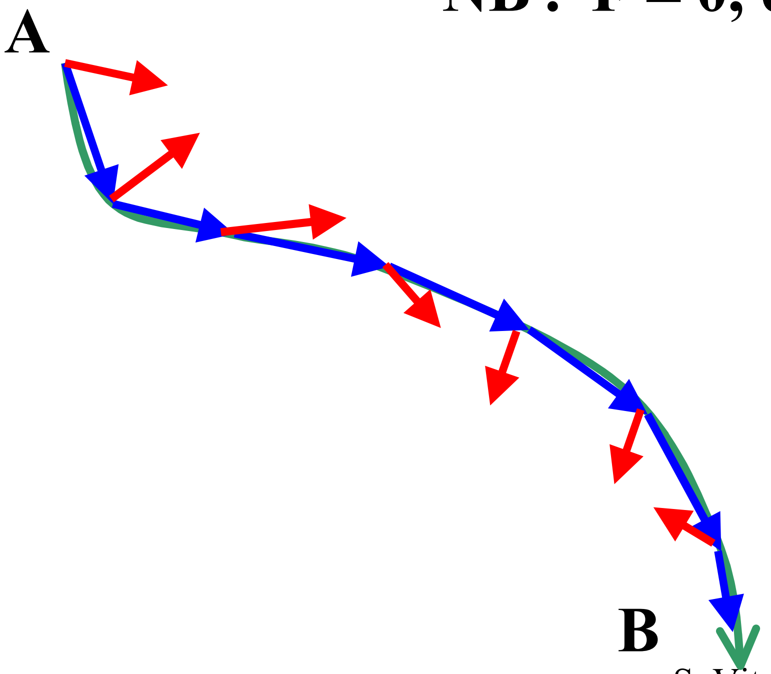
Una definizione: il lavoro fatto da una forza

1 Lavoro elementare



$$dL = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$NB: \ \vec{F} = 0, \ d\vec{r} = 0, \ \vec{F} \perp d\vec{r} \quad \rightarrow \quad dL = 0$$



2 Lavoro "finito":

Somma dei lavori infinitesimi

$$L_{A \to B} = \lim_{N \to \infty} \sum_{k=1}^{N} dL_k = \lim_{N \to \infty} \sum_{k=1}^{N} \vec{F}_k d\vec{r}_k$$

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Se sul punto agisce più di una forza:

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$$

$$L_{tot,A\to B} = \lim_{N\to\infty} \sum_{k=1}^{N} \vec{F}_{tot,k} \cdot dr_k = \lim_{N\to\infty} \sum_{k=1}^{N} \left(\vec{F}_{1,k} + \vec{F}_{2,k} + \ldots + \vec{F}_{n,k} \right) \cdot dr_k =$$

$$\lim_{N\to\infty} \sum_{k=1}^{N} \vec{F}_{1,k} \cdot dr_k + \lim_{N\to\infty} \sum_{k=1}^{N} \vec{F}_{2,k} \cdot dr_k + .. + \lim_{N\to\infty} \sum_{k=1}^{N} \vec{F}_{n,k} \cdot dr_k =$$

$$= L_{1,A\to B} + L_{2,A\to B} + \ldots + L_{N,A\to B}$$

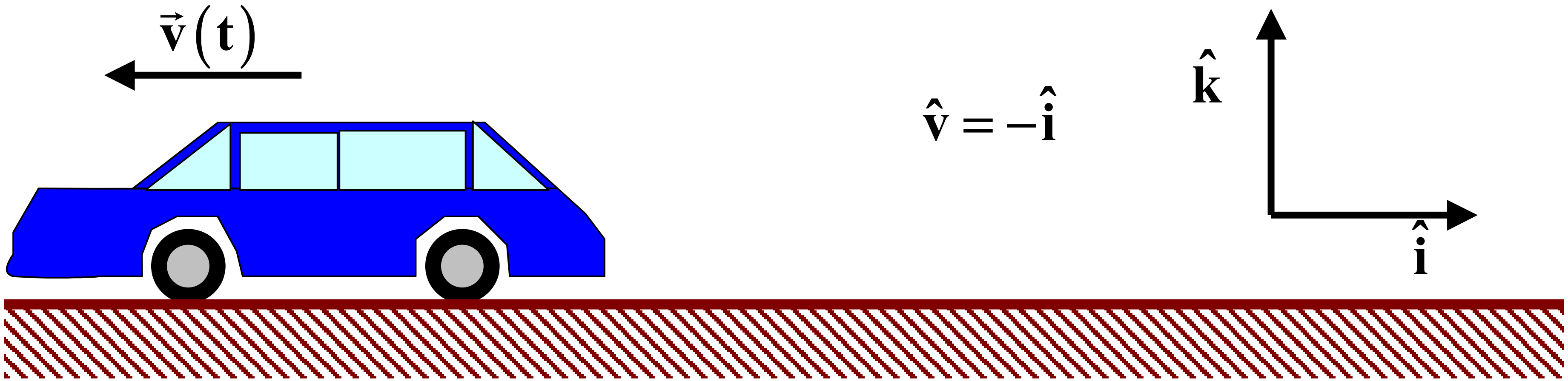
In definitiva: il teorema dell'energia cinetica

$$\frac{1}{2} m v^2 (t_B) - \frac{1}{2} m v^2 (t_A) = L_{tot,A\to B}$$

Esempio: frenata per attrito radente

(auto con ruote bloccate)

$$\vec{F} = -\mu_d mg\hat{v} + \vec{F}_{vincolo} - mg\hat{k} = -\mu_d mg\hat{v}$$



$$\hat{v} = -\hat{i}$$

$$v_x(t) = v_x(0) + \mu_d g t \quad x(t) = v_x(0)t + \frac{1}{2}\mu_d g t^2$$

$$t_A = 0 \;\rightarrow\; x_A = 0 \;\; t_B = -\frac{v_x(0)}{\mu_d g} \;\rightarrow\; v_x(t_B) = 0 \;\rightarrow\; x_B = -\frac{v_x^2(0)}{2\mu_d g}$$

$$L_{attrito,A\rightarrow B} = \int_A^B \mu_d mg\hat{i}\cdot d\vec{r} = \int_{x_A}^{x_B} \mu_d mg\,dx = \mu_d mg(x_B - x_A) = -\frac{1}{2}mv_x^2(0)$$

Secondo metodo

$$\vec{F}(t) = -\mu_d m g \hat{v} \quad \vec{v}(t) = \vec{v}(0) - \mu_d g t \hat{v}$$

$$\vec{F}(t)\cdot\vec{v}(t) = \quad -\mu_d m g \hat{v}\cdot\left[\vec{v}(0) - \mu_d g t \hat{v}\right] \quad = -\mu_d m g\left[-v_x(0) - \mu_d g t\right]$$

$$L_{attrito,A\to B} \quad = \int_{t_A}^{t_B} \mu_d m g\left[v_x(0) + \mu_d g t\right] dt \quad = \mu_d m g\left[v_x(0)\,t_B + \frac{1}{2}\mu_d g t_B^2\right]$$

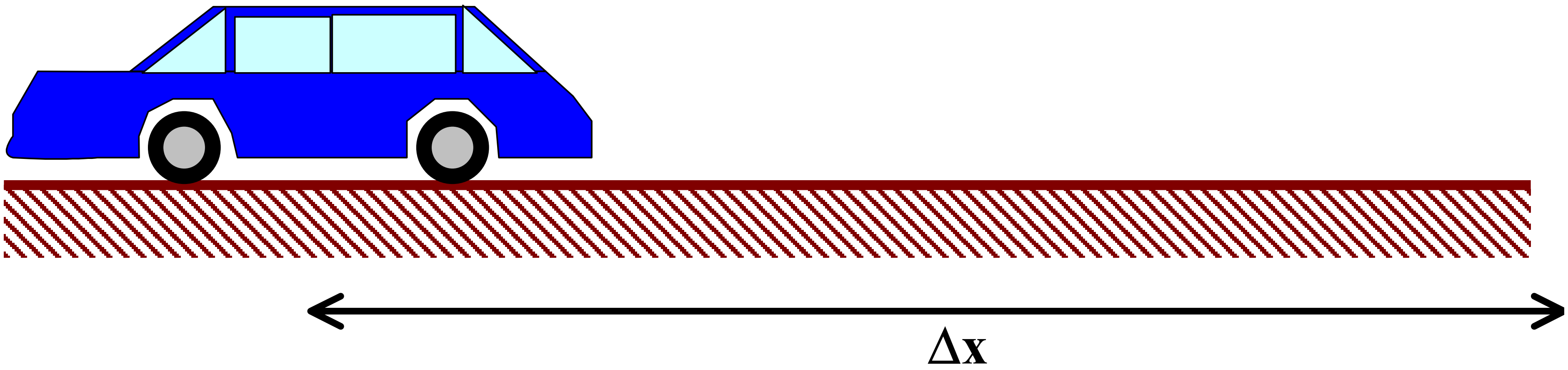
$$t_A = 0 \quad t_B = -\frac{v_x(0)}{\mu_d g}$$

$$L_{attrito,A\to B} = -\mu_d m g\left[\frac{v_x^2(0)}{2\mu_d g}\right] = -\frac{1}{2} m v_x^2(0)$$

Frenata regolare: lo spazio di frenata dipende dall’energia cinetica

$$\vec{F}_{freni}(t) = -\gamma\hat{v}(t) \qquad \vec{F}_{freni}(t)\cdot\vec{v}(t) = -\gamma v(t)$$

(g dipende dalla spinta sul pedale)



$$\frac{1}{2}mv^2(\text{finale}) - \frac{1}{2}mv^2(\text{iniziale}) = L_{attrito} = \int_{t_{iniziale}}^{t_{finale}} -\gamma\frac{dx}{dt}dt = -\gamma\Delta x$$

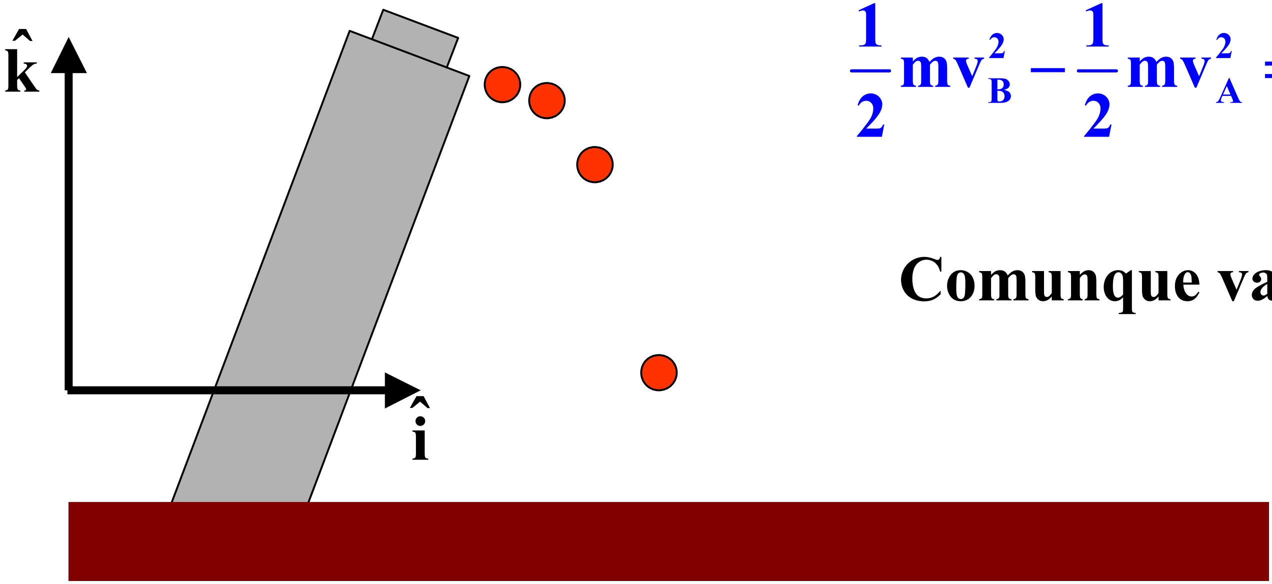
$$\underset{\substack{\|\\0}}{}$$

$$\frac{1}{2\gamma}mv^2(\text{iniziale}) = \Delta x$$

Esempio 2: forza di gravità $\vec{F}(t) = -mg\hat{k}$

$$\vec{F}(t)\cdot\vec{v}(t) = -mg\hat{k}\cdot\vec{v}(t) = -mgv_z(t)$$

$$L_{gravità,A\to B} = \int_{t_A}^{t_B} -mg\frac{dz}{dt}dt = -mg\int_{z_A}^{z_B} dz = -mg(z_B - z_A)$$



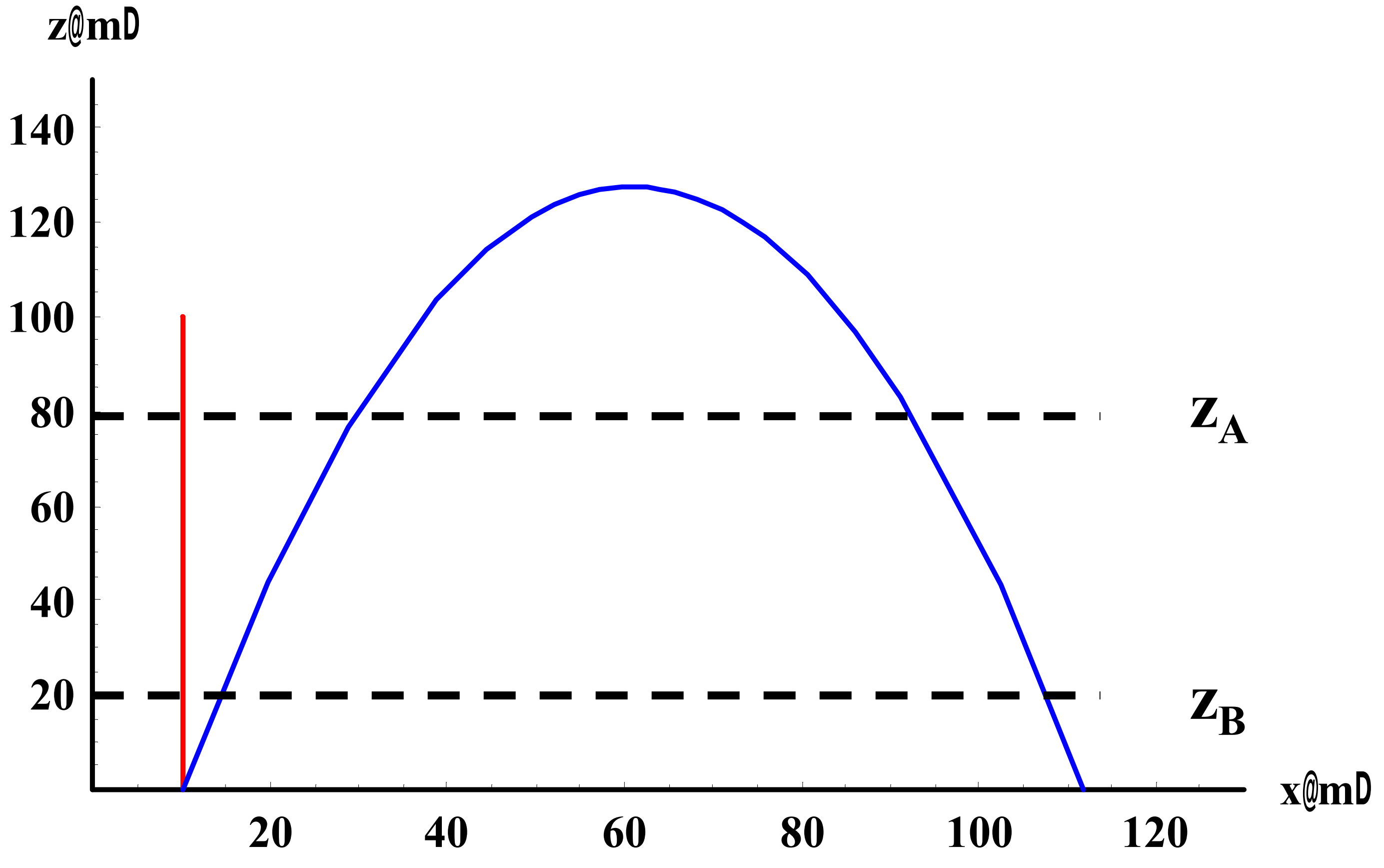
$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mg(z_B - z_A)$$

Comunque vada da A a B !

Controlliamo

$$z(t) = 100\mathrm{m} - \frac{1}{2}gt^2 \quad x(t) = 10\mathrm{m} \quad v_z(t) = -gt \quad v_x(t) = 0$$

$$z(t) = 50\frac{\mathrm{m}}{\mathrm{s}}t - \frac{1}{2}gt^2 \quad x(t) = 10\mathrm{m} + 10\frac{\mathrm{m}}{\mathrm{s}}t \quad v_z(t) = 50\frac{\mathrm{m}}{\mathrm{s}} - gt \quad v_x(t) = 10\frac{\mathrm{m}}{\mathrm{s}}$$



$$\vec{r} = \left\{10\,\mathrm{m},\ 100\,\mathrm{m} - \frac{1}{2}\,9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\,t^2\right\};\ \vec{v} = \partial_t \vec{r} = \left\{0,\ -\frac{9.8\,\mathrm{m}\,t}{\mathrm{s}^2}\right\}$$

$$t_A = t\ /.\ \mathrm{Solve}[\vec{r}[[2]] == 80\,\mathrm{m},\ t] = \{-2.020\,\mathrm{s},\ 2.020\,\mathrm{s}\}$$

$$t_B = t\ /.\ \mathrm{Solve}[\vec{r}[[2]] == 20\,\mathrm{m},\ t] = \{-4.040\,\mathrm{s},\ 4.040\,\mathrm{s}\}$$

$$-9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\,(20\,\mathrm{m} - 80\,\mathrm{m}) = \frac{588.\,\mathrm{m}^2}{\mathrm{s}^2}$$

$$\left(\frac{\vec{v}.\vec{v}}{2}\ /.\ t \to t_B\right) - \left(\frac{\vec{v}.\vec{v}}{2}\ /.\ t \to t_A\right) = \left\{\frac{588\,\mathrm{m}^2}{\mathrm{s}^2}\right.$$

$$\vec{r} = \left\{10\,\mathrm{m} + 10\,\frac{\mathrm{m}}{\mathrm{s}}\,t,\ 50\,\frac{\mathrm{m}}{\mathrm{s}}\,t - \frac{1}{2}\,9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\,t^2\right\};$$

$$\vec{v} = \partial_t \vec{r} = \left\{\frac{10\,\mathrm{m}}{\mathrm{s}},\ \frac{50\,\mathrm{m}}{\mathrm{s}} - \frac{9.8\,\mathrm{m}\,t}{\mathrm{s}^2}\right\}$$

$$t_A = t\ /.\ \mathrm{Solve}[\vec{r}[[2]] == 80\,\mathrm{m},\ t] = \{1.987\,\mathrm{s},\ 8.217\,\mathrm{s}\}$$

$$t_B = t\ /.\ \mathrm{Solve}[\vec{r}[[2]] == 20\,\mathrm{m},\ t] = \{0.4170\,\mathrm{s},\ 9.787\,\mathrm{s}\}$$

$$-9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\,(20\,\mathrm{m} - 80\,\mathrm{m}) = \frac{588.\,\mathrm{m}^2}{\mathrm{s}^2}$$

$$\left(\frac{\vec{v}.\vec{v}}{2}\ /.\ t \to t_B\right) - \left(\frac{\vec{v}.\vec{v}}{2}\ /.\ t \to t_A\right) = \left\{\frac{588.\,\mathrm{m}^2}{\mathrm{s}^2}\right.$$

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Un'importante proprietà:

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mg\left(z_B - z_A\right)$$

$$\Downarrow$$

$$\frac{1}{2}mv_B^2 + mgz_B + C = \frac{1}{2}mv_A^2 + mgz_A + C$$

Definendo: L'energia potenziale $U(z) = mgz(+C)$

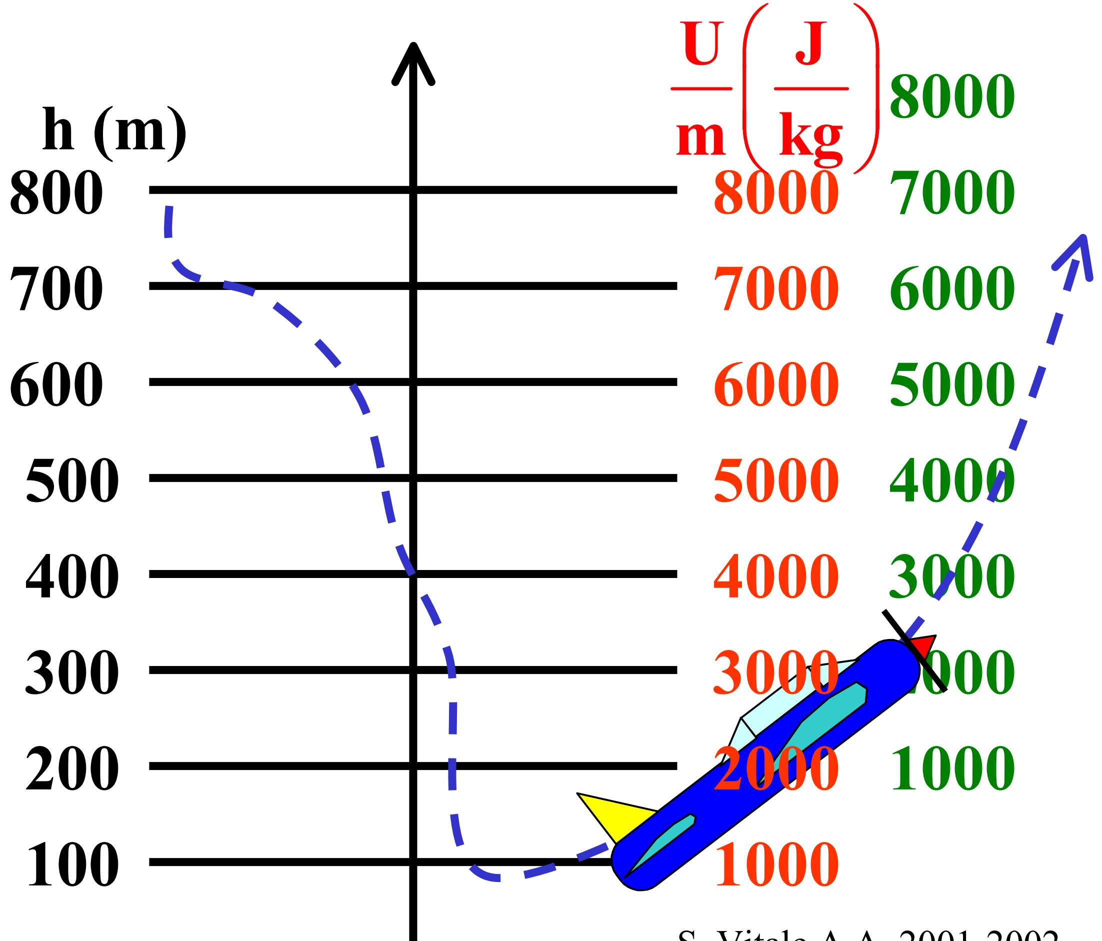
E l'energia meccanica totale $E = U + \frac{1}{2}mv^2$

Teorema di conservazione dell'energia $E_A = E_B$

L'energia potenziale:

1 Solo le differenze U_B-U_A contano

2 Perché potenziale?



Può essere sempre riconvertita in energia cinetica

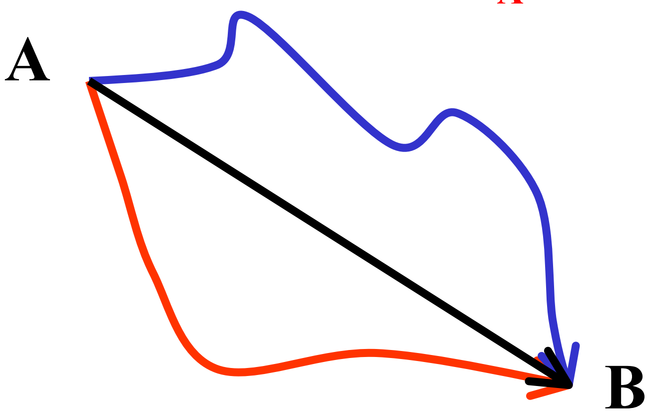
La conservazione dell'energia più in genere. Se:

$$1 \qquad \vec{F} = \vec{F}(x, y, z)$$

(N.B. se: $\vec{F} = \vec{F}(x, y, z, t)$ campo di forze, se

$\vec{F} = \vec{F}(x, y, z)$ campo stazionario)

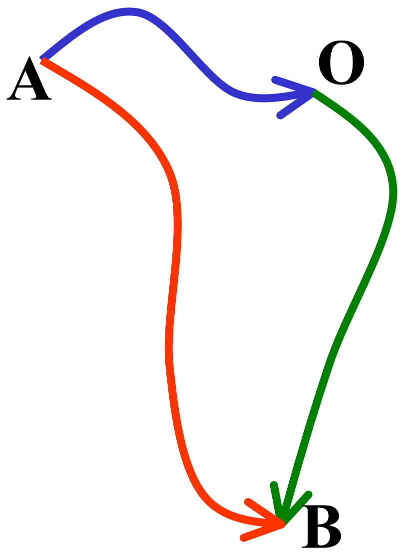
$$2 \qquad L_{A\to B} = \int_A^B \vec{F} \cdot d\vec{r} = f(x_A, y_A, z_A, x_B, y_B, z_B)$$



Cioè se:

$$L_{A\to B} = L_{A\to B} = L_{A\to B}$$

Il campo è conservativo



Se il lavoro non dipende dalla curva effettivamente seguita

$$L_{A\to B} = L_{A\to O} + L_{O\to B}$$

Ma se si inverte il verso di percorrenza della curva

$$\vec{F} \to \vec{F} \quad d\vec{r} \to -d\vec{r}$$

$$\vec{F}\cdot d\vec{r} \to -\vec{F}\cdot d\vec{r}$$

$$L_{A\to O} = -L_{O\to A}$$

$$L_{A\to B} = L_{O\to B} - L_{O\to A} \equiv V_O(B) - V_O(A)$$

Se su un punto materiale agisce solo una forza conservativa

(o una somma di sole forze conservative)

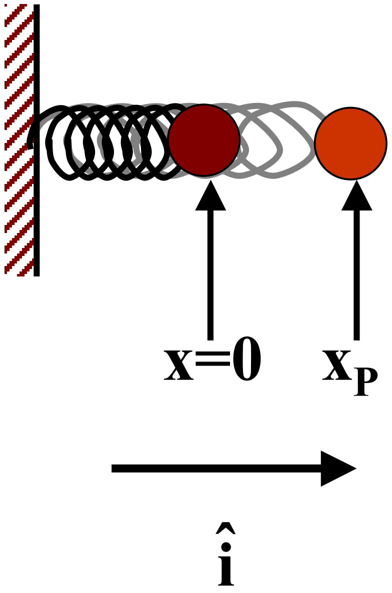
$$\left.\begin{aligned} L_{tot,A\to B} &= V_O(B) - V_O(A) \\ L_{tot,A\to B} &= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 \end{aligned}\right\} \to \frac{1}{2}mv_B^2 - V_O(B) = \frac{1}{2}mv_A^2 - V_O(A)$$

$$E_B = \frac{1}{2}mv_B^2 + U_O(B) = \frac{1}{2}mv_A^2 + U_O(A) = E_A$$

$$\left[\text{Energia potenziale: } U_O(x) = -V_O(x)\right]$$

E l'energia meccanica $E = \frac{1}{2}mv^2 + U$ **si conserva**

Un'esempio semplice: campi unidimensionali:

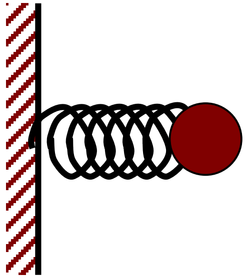


$$\vec{F} = -kx\hat{i}$$

$$L_{0\to P} = \int_0^{x_P} -kx\hat{i}d\vec{r} = \int_0^{x_P} -kxdx = -\frac{1}{2}kx^2\Big|_0^{x_P}$$

$$U_O(P) = -L_{O\to P} = \frac{1}{2}kx_P^2$$

N.B. un campo: $\vec{F} = f(x)\hat{i}$ è sempre conservativo



Ma se il campo è conservativo l'energia meccanica totale si conserva

$$m\frac{d^2x}{dt^2}=-kx \quad \rightarrow \quad m\frac{d^2x}{dt^2}+kx=0$$

$$x(t)=x_c\mathrm{Cos}\left(\sqrt{\frac{k}{m}}t\right)+x_s\mathrm{Sin}\left(\sqrt{\frac{k}{m}}t\right)=x_o\mathrm{Cos}\left(\sqrt{\frac{k}{m}}t+\phi\right)$$

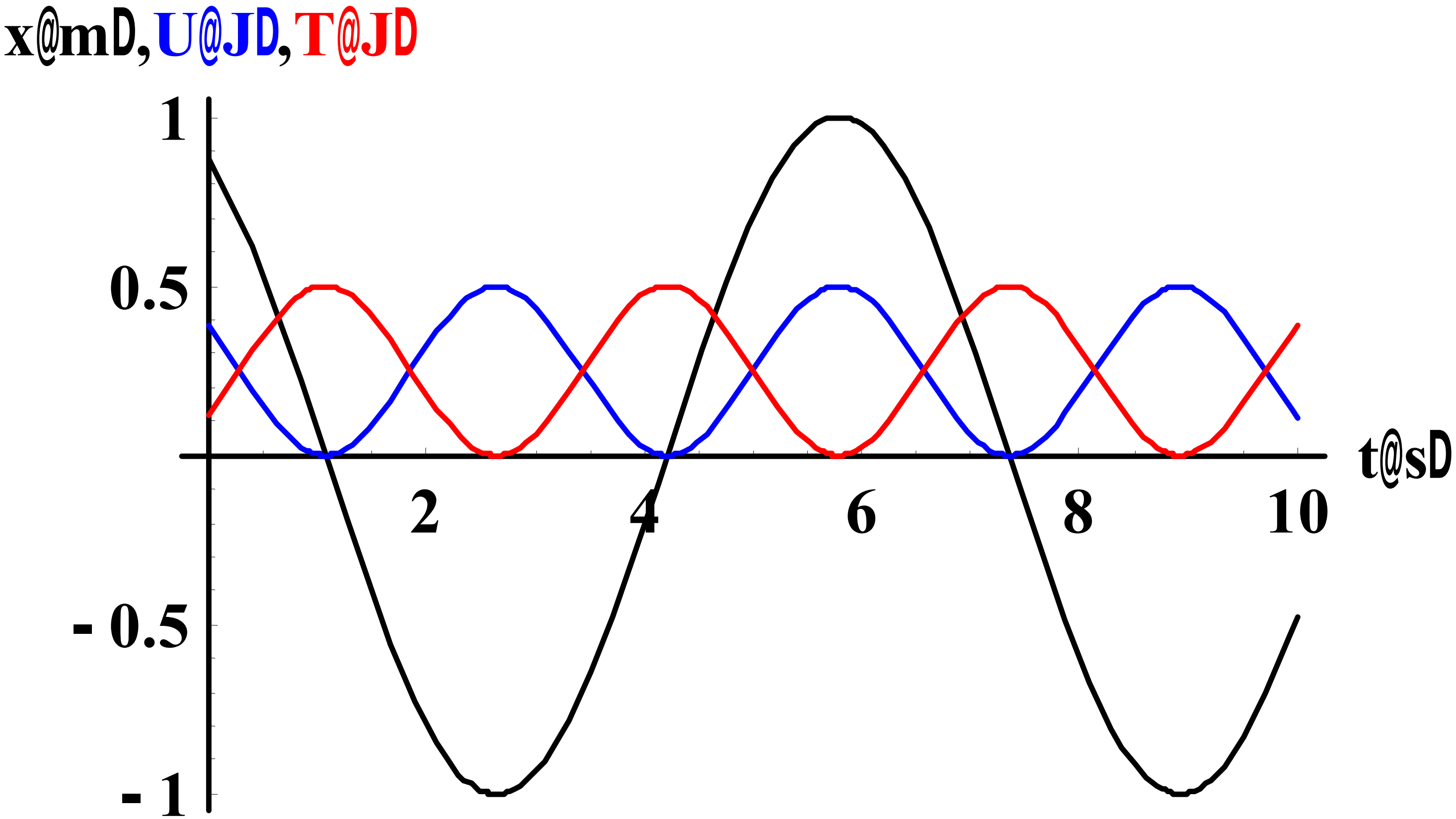
$$\left[x_o=\sqrt{x_c^2+x_s^2} \quad \phi=-\mathrm{Arc}\tan\left(\frac{x_s}{x_c}\right)\right]$$

$$v_x(t)=-x_o\sqrt{\frac{k}{m}}\mathrm{Sin}\left(\sqrt{\frac{k}{m}}t+\phi\right)$$

$$E=\frac{1}{2}mv^2(t)+\frac{1}{2}kx^2(t)=$$

$$=\frac{1}{2}m\frac{k}{m}x_o^2\mathrm{Sin}^2\left(\sqrt{\frac{k}{m}}t+\phi\right)+\frac{1}{2}kx_o^2\mathrm{Cos}^2\left(\sqrt{\frac{k}{m}}t+\phi\right)=$$

$$=\frac{1}{2}kx_o^2$$

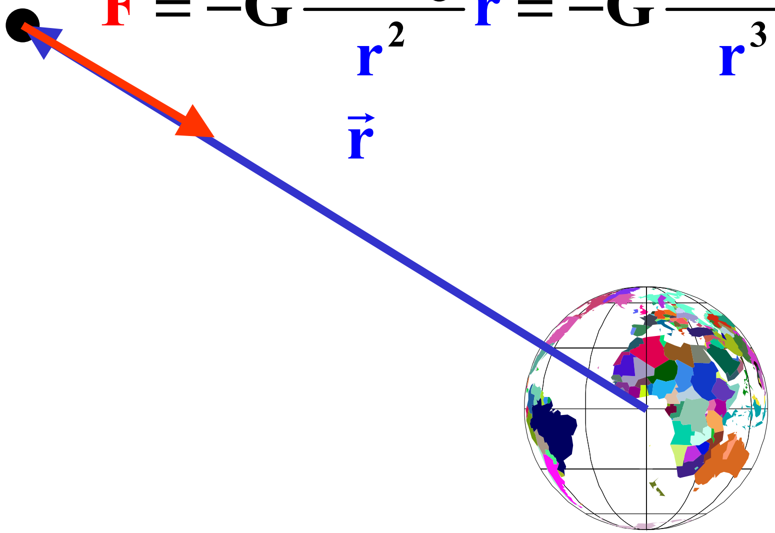


x_o=1m, k=1 N/m, m=1 kg, j=0.5 rad

$$E = \frac{1}{2} k x_o^2 = 0.5\ 1\frac{N}{m}(1m)^2 = 0.5J$$

Un esempio difficile: la gravitazione Newtoniana

$$\vec{F} = -G\frac{mM_{\oplus}}{r^2}\hat{r} = -G\frac{mM_{\oplus}}{r^3}\vec{r}$$



$$G = 6.67\times 10^{-11}\,\frac{m^3}{kg\ s^{-2}}$$

$$\vec{F}\cdot\vec{v} = -\frac{GmM_{\oplus}}{\left(x^2+y^2+z^2\right)^{3/2}}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot$$

$$\cdot\left(\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}+\frac{dz}{dt}\hat{k}\right) =$$

$$= -\frac{GmM_{\oplus}}{\left(x^2+y^2+z^2\right)^{3/2}}\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)$$

$$\frac{d\left(x^2+y^2+z^2\right)}{dt}=2\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)$$

$$\vec{F}\cdot\vec{v}=-\frac{GmM_{\oplus}}{2\left(x^2+y^2+z^2\right)^{3/2}}\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)=$$

$$=-\frac{GmM_{\oplus}}{2\left(x^2+y^2+z^2\right)^{3/2}}\frac{d\left(x^2+y^2+z^2\right)}{dt}=$$

$$=-\frac{GmM_{\oplus}}{2r^3}\frac{dr^2}{dt} \qquad =-\frac{GmM_{\oplus}}{r^2}\frac{dr}{dt} \qquad =GmM_{\oplus}\frac{d\,{}^1\!/_r}{dt}$$

$$\color{red}{U_O\left(P\right)}=-L_{O\to P}=-\int_{t_O}^{t_P}\vec{F}\left(t\right)\cdot\vec{v}\left(t\right)dt=-\int_{t_O}^{t_P}GmM_{\oplus}\frac{d\,{}^1\!/_r}{dt}dt=$$

$$\color{red}{-\frac{GmM_{\oplus}}{r_P}+\frac{GmM_{\oplus}}{r_O}}$$

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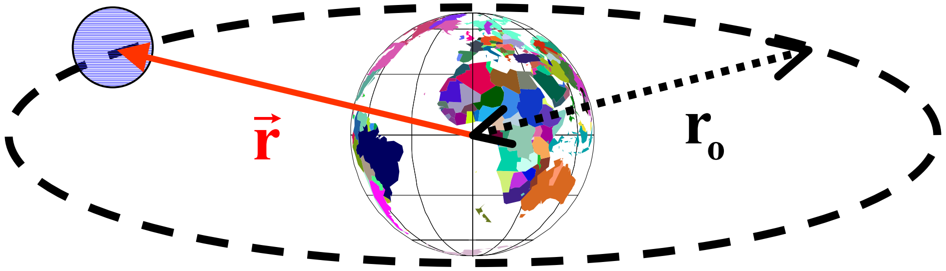
Prendendo il punto O a distanza infinita:

$$U_O(P) = -\frac{GmM_{\oplus}}{r_P} + \frac{GmM_{\oplus}}{r_O = \infty} = -\frac{GmM_{\oplus}}{r_P}$$

L'energia totale si conserva

$$\frac{1}{2}mv^2(t) - \frac{GM_{\oplus}m}{r(t)} = E_o = \mathrm{Costante}$$

Esempio: orbita circolare



Moto circolare uniforme: $\vec{a} = -\omega^2 \vec{r} \quad \vec{F} = m\vec{a} = -\omega^2 m\vec{r}$

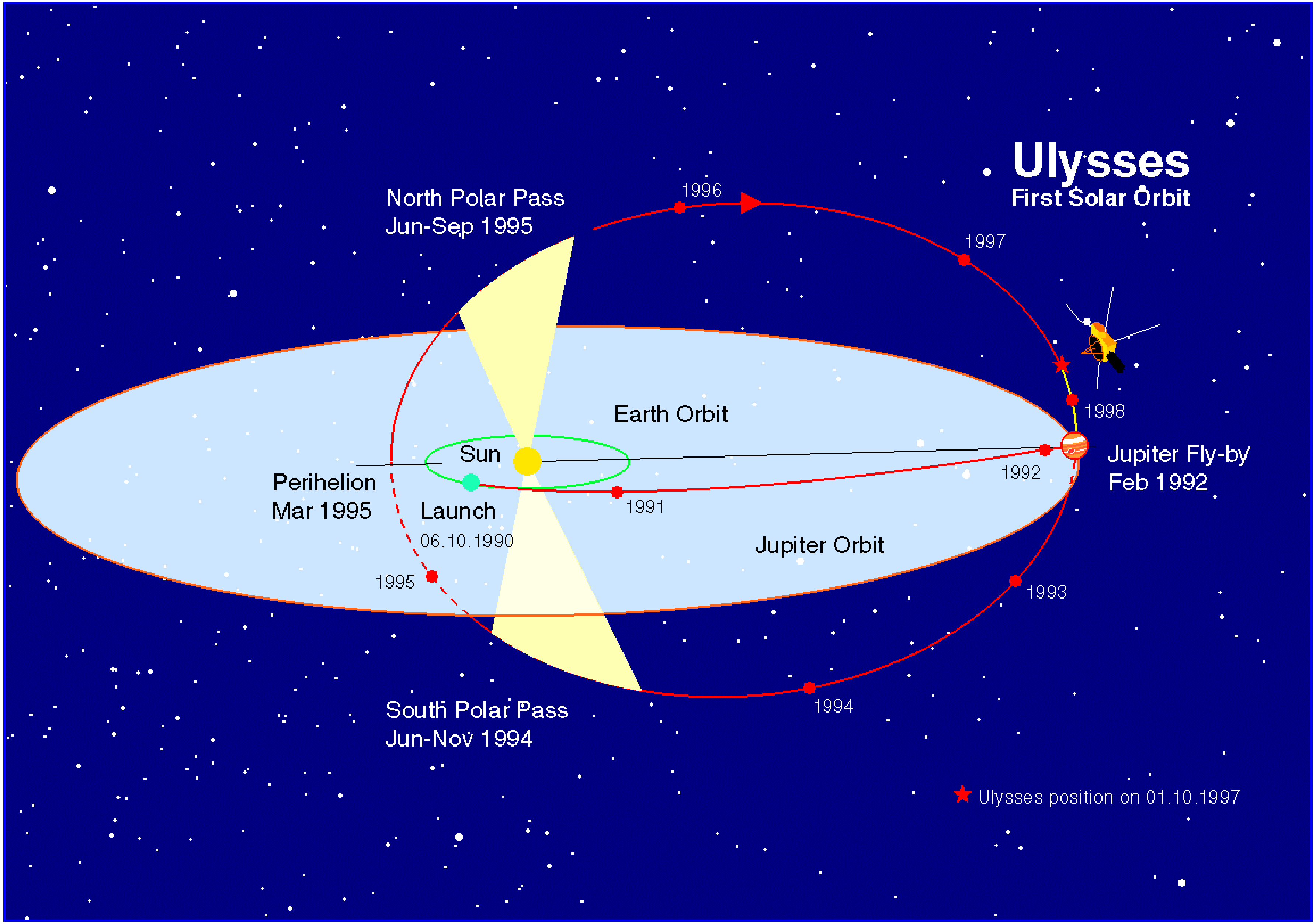
Se la gravità può fornire questa forza il moto circolare uniforme è possibile

$$\cancel{-}\frac{GM_\oplus \cancel{m}}{r^3}\cancel{\vec{r}} = \cancel{-}\cancel{m}\omega^2\cancel{\vec{r}}$$

$$\boxed{\frac{GM_\oplus}{r_o^3} = \omega^2}$$

Keplero: il quadrato del periodo è proporzionale al cubo della distanza

$$\boxed{E = \frac{1}{2} m r_o^2 \omega^2 - \frac{GM_\oplus m}{r_o} = \cos t}$$



Ulysses
First Solar Orbit
North Polar Pass
Jun-Sep 1995
1996
1997
1998
Earth Orbit
Sun
Perihelion
Mar 1995
Launch
06.10.1990
1991
1992
Jupiter Fly-by
Feb 1992
Jupiter Orbit
1995
1993
1994
South Polar Pass
Jun-Nov 1994
Ulysses position on 01.10.1997

Anno	x(AU)	y(AU)	z (AU)	r(AU)
1993.01171875	-4.588	1.614	-1.388	5.058
1993.01989746	-4.58	1.608	-1.4	5.052
1993.02807617	-4.571	1.601	-1.413	5.045
1993.03625488	-4.562	1.595	-1.425	5.039
1993.04443359	-4.553	1.589	-1.437	5.032
1993.0526123	-4.544	1.583	-1.449	5.026
1993.06079102	-4.535	1.576	-1.462	5.019
1993.06896973	-4.526	1.57	-1.474	5.012
1993.07714844	-4.517	1.564	-1.486	5.005
1993.08532715	-4.507	1.557	-1.498	4.999
1993.09350586	-4.498	1.551	-1.51	4.992
1993.10168457	-4.488	1.544	-1.522	4.985
1993.10986328	-4.479	1.538	-1.534	4.978
1993.11804199	-4.469	1.531	-1.546	4.971
1993.1262207	-4.459	1.525	-1.558	4.963

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Anno	x(AU)	y(AU)	z (AU)	r(AU)
1995.25268555	1.227	-0.413	0.459	1.373
1995.26086426	1.219	-0.399	0.512	1.381
1995.26904297	1.211	-0.385	0.564	1.39
1995.27722168	1.201	-0.37	0.617	1.4
1995.28540039	1.19	-0.354	0.668	1.41
1995.2935791	1.178	-0.339	0.719	1.421
1995.30175781	1.164	-0.323	0.769	1.432
1995.30993652	1.15	-0.307	0.818	1.445
1995.31811523	1.135	-0.29	0.867	1.458
1995.32629395	1.119	-0.274	0.915	1.471
1995.33447266	1.101	-0.257	0.962	1.485
1995.34265137	1.083	-0.239	1.009	1.5
1995.35083008	1.064	-0.222	1.055	1.515
1995.35900879	1.045	-0.205	1.099	1.53
1995.3671875	1.024	-0.187	1.143	1.546

Velocità (AU/Anno):

1993			1995		
0.978149	-0.733612	-1.46722	-0.978149	1.71176	6.48024
1.10042	-0.855881	-1.58949	-0.978149	1.71176	6.35797
1.10042	-0.733612	-1.46722	-1.22269	1.83403	6.48024
1.10042	-0.733612	-1.46722	-1.34496	1.9563	6.2357
1.10042	-0.733612	-1.46722	-1.46722	1.83403	6.2357
1.10042	-0.85588	-1.58949	-1.71176	1.9563	6.11343
1.10042	-0.733612	-1.46722	-1.71176	1.9563	5.99116
1.10042	-0.733612	-1.46722	-1.83403	2.07857	5.99116
1.22269	-0.855881	-1.46722	-1.9563	1.9563	5.86889
1.10042	-0.733612	-1.46722	-2.20084	2.07857	5.74663
1.22269	-0.855881	-1.46722	-2.20084	2.20084	5.74663
1.10042	-0.733612	-1.46722	-2.3231	2.07857	5.62436
1.22269	-0.855881	-1.46722	-2.3231	2.07857	5.37982
1.22269	-0.733612	-1.46722	-2.56764	2.20084	5.37982
1.22269	-0.855881	-1.46722	-2.56764	2.20084	5.37982

1993		1995	
$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right)$	$-\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$	$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right)$	$-\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$
8.20862×10^7	-3.50777×10^8	1.03247×10^9	-1.29223×10^9
1.00589×10^8	-3.51194×10^8	9.97146×10^8	-1.28474×10^9
8.78053×10^7	-3.51681×10^8	1.05434×10^9	-1.27642×10^9
8.78053×10^7	-3.521×10^8	1.00186×10^9	-1.26731×10^9
8.78053×10^7	-3.5259×10^8	9.99164×10^8	-1.25832×10^9
1.00589×10^8	-3.5301×10^8	9.93109×10^8	-1.24858×10^9
8.78053×10^7	-3.53503×10^8	9.59803×10^8	-1.23899×10^9
8.78053×10^7	-3.53997×10^8	9.80661×10^8	-1.22784×10^9
9.85707×10^7	-3.54492×10^8	9.47353×10^8	-1.21689×10^9
8.78053×10^7	-3.54917×10^8	9.49374×10^8	-1.20614×10^9
9.85707×10^7	-3.55415×10^8	9.61149×10^8	-1.19477×10^9
8.78053×10^7	-3.55914×10^8	9.30535×10^8	-1.18282×10^9
9.85707×10^7	-3.56414×10^8	8.69979×10^8	-1.17111×10^9
9.41973×10^7	-3.56916×10^8	9.08667×10^8	-1.15963×10^9
9.85707×10^7	-3.57492×10^8	9.08667×10^8	-1.14763×10^9

1993

Valori medi

$$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right) \qquad -\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$$

$$9.24255\times10^{7} \qquad -3.54027\times10^{8}$$

1995

$$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right) \qquad -\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$$

$$9.66285\times10^{8} \qquad -1.22489\times10^{9}$$

Energia Totale: $\frac{1}{2}mv^2-\frac{GM_\odot m}{r}$

1993: -2.62 10^8J/kg 1995: -2.58 10^8 J/kg