Accelerazione

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$$

$$a_x(t) = \frac{dv_x(t)}{dt} = \frac{d^2x(t)}{dt^2}$$

$$a_y(t) = \frac{dv_y(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

$$a_z(t) = \frac{dv_z(t)}{dt} = \frac{d^2z(t)}{dt^2}$$

Dimensioni fisiche

$$[a] = \frac{[v]}{[t]} = \frac{[l]}{[t]^2} \rightarrow \text{S.I. } \frac{m}{s^2}$$

Valori tipici

Accelerazione di gravità sulla superficie terrestre:

$$g = 9.8\ m/s^2$$

Accelerazione automobile:

"da 0 a 100 km/h in 10 s"

$$\frac{100\ km/h}{10\ s} = \frac{\frac{100}{3.6}\ \frac{m}{s}}{10} \approx 2.8\ \frac{m}{s^2}$$

Accelerazione di un razzo alla partenza:

$$\approx 5\text{-}8\ g \quad \approx 50\text{-}80\ m/s^2$$

Ultracentrifuga : $\approx 10^5\ g \quad \approx 10^6\ m/s^2$

Es: moto rettilineo uniforme

$$\vec{r}(t) = \{v_{ox}t + x_o, v_{yo}t + y_o, v_{zo}t + z_o\}$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \{v_{ox}, v_{yo}, v_{zo}\}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt} = \{0,0,0\} \equiv \vec{0}\ (0)$$

Velocità costante = accelerazione nulla

Un altro esempio molto importante:

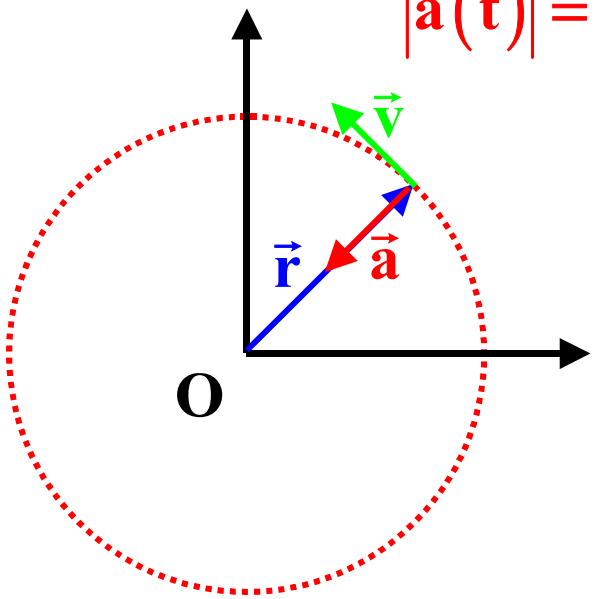
Il moto circolare (uniforme)

$$\vec{r}(t) = \{r_o \mathrm{Cos}(\omega t),\ r_o \mathrm{Sin}(\omega t),\ 0\} \quad |\vec{r}(t)| = r_o$$

$$\vec{v}(t) = \{-\omega\, r_o \mathrm{Sin}(\omega t),\ \omega\, r_o \mathrm{Cos}(\omega t),\ 0\} \quad |\vec{v}(t)| = \omega r_o$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \{-\omega^2\ r_o \mathrm{Cos}(\omega t),\ -\omega^2\ r_o \mathrm{Sin}(\omega t),\ 0\} = -\omega^2 \vec{r}(t)$$

$$|\vec{a}(t)| = \sqrt{(\omega^2\ r_o)^2 \mathrm{Cos}^2(\omega t) + (\omega^2\ r_o)^2 \mathrm{Sin}^2(\omega t)} = \omega^2 r_o$$



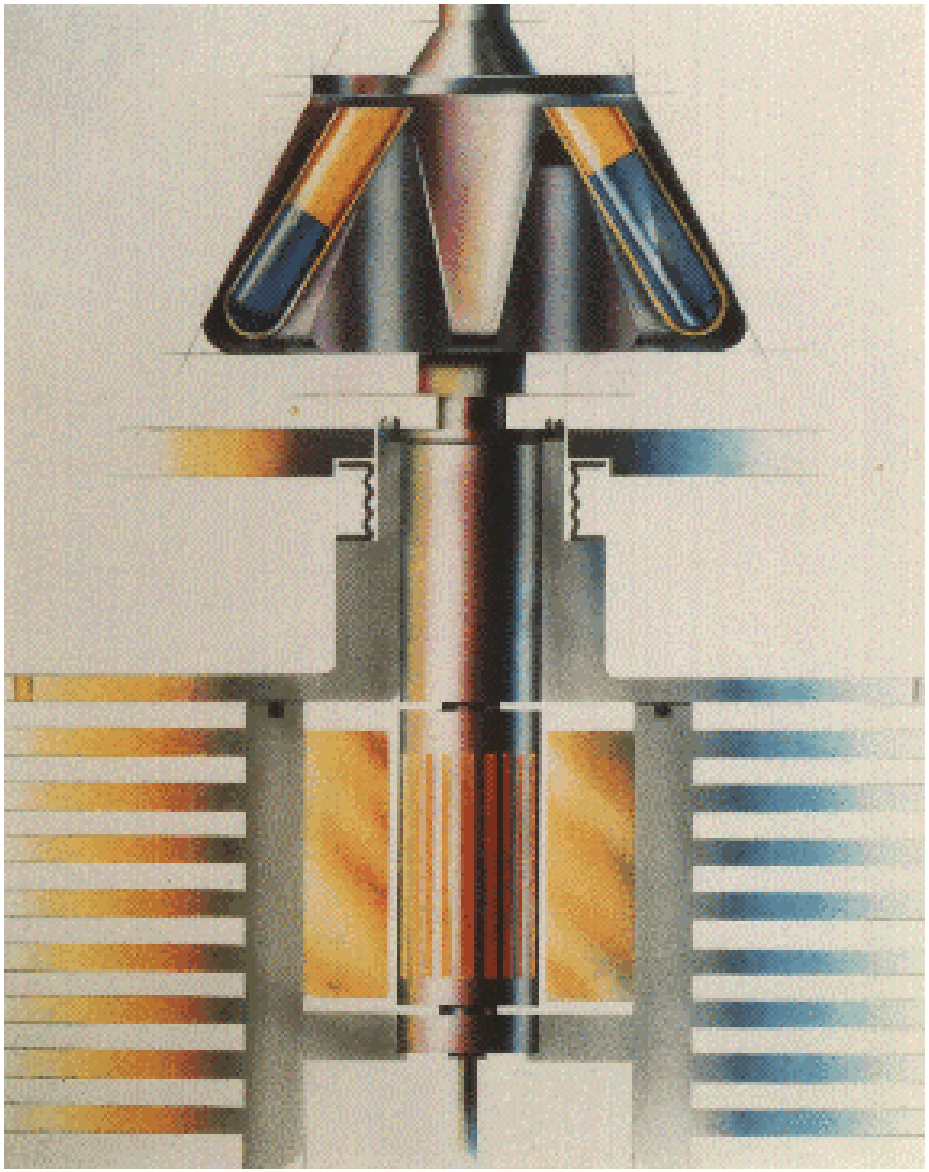
Accelerazione "centripeta"

$$\vec{r} \cdot \vec{v} = 0 \quad \vec{v} \cdot \vec{a} = 0$$

Ultracentrifuga : $\omega \approx 2\pi 1000\,\mathrm{rad/s}$;

$$r_o \approx 0.1\mathrm{m};\ \omega^2 r_o \approx 4\times 10^6\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Ultracentrifuga Preparativa (Beckmann Coulter)



Spins up to 8 x 6.5 mL tubes up to 802,400 x g @ 100,000 rpm in the XL-100K

Es: moto uniformemente accelerato

$$\vec{r}(t) =$$

$$\left\{ x_o + v_{xo} t + \frac{1}{2} a_{xo} t^2, y_o + v_{yo} t + \frac{1}{2} a_{yo} t^2, z_o + v_{zo} t + \frac{1}{2} a_{zo} t^2 \right\}$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \left\{ v_{xo} + a_{xo} t, v_{yo} + a_{yo} t, v_{zo} + a_{zo} t \right\}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \left\{ a_{xo}, a_{yo}, a_{zo} \right\} = \vec{a}_o$$

Es: moto rettilineo uniformemente accelerato

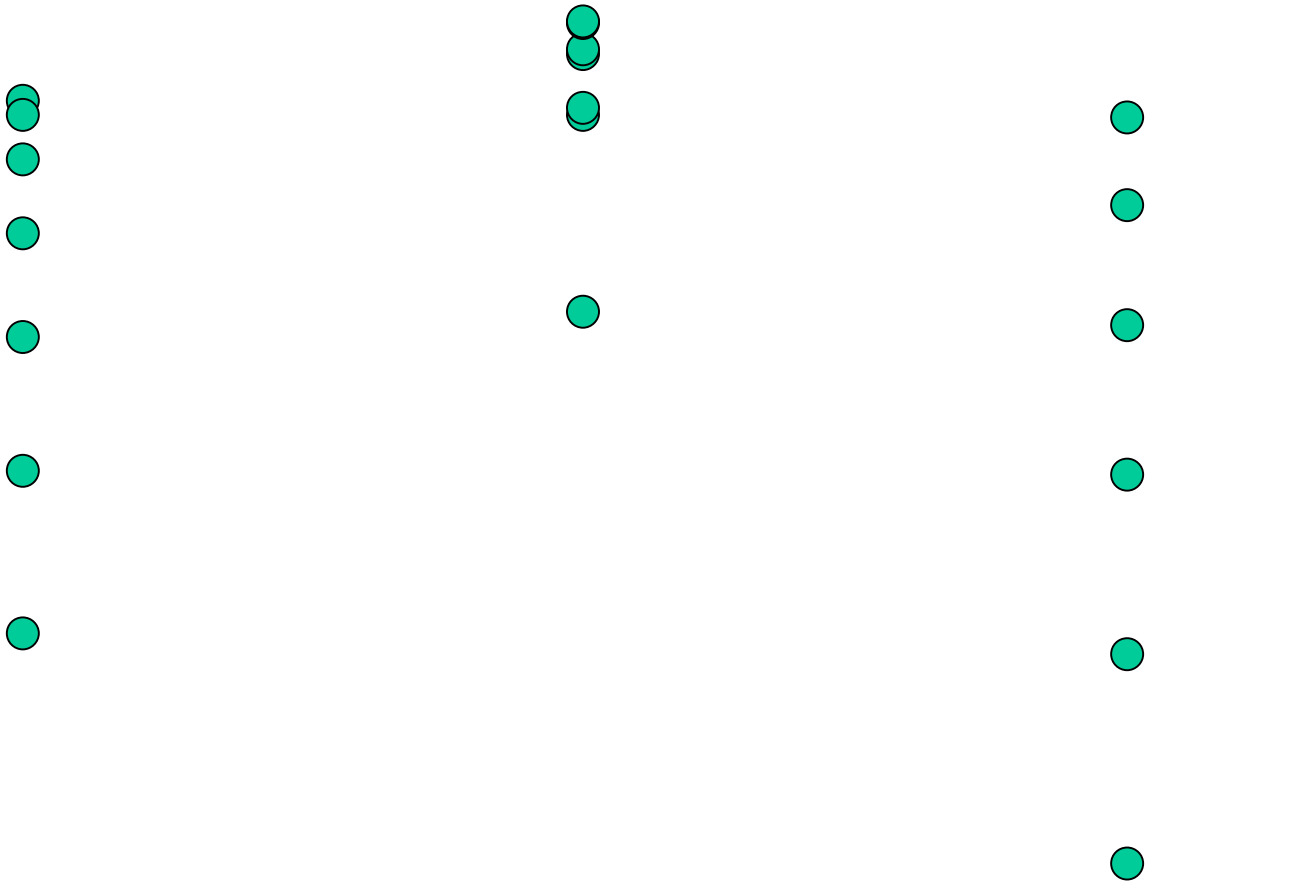
$$z(t) = z_o + v_o t + \frac{1}{2} a_o t^2$$

$$x(t) = y(t) = 0 \qquad v_x(t) = v_y(t) = 0 \qquad a_x(t) = a_y(t) = 0$$

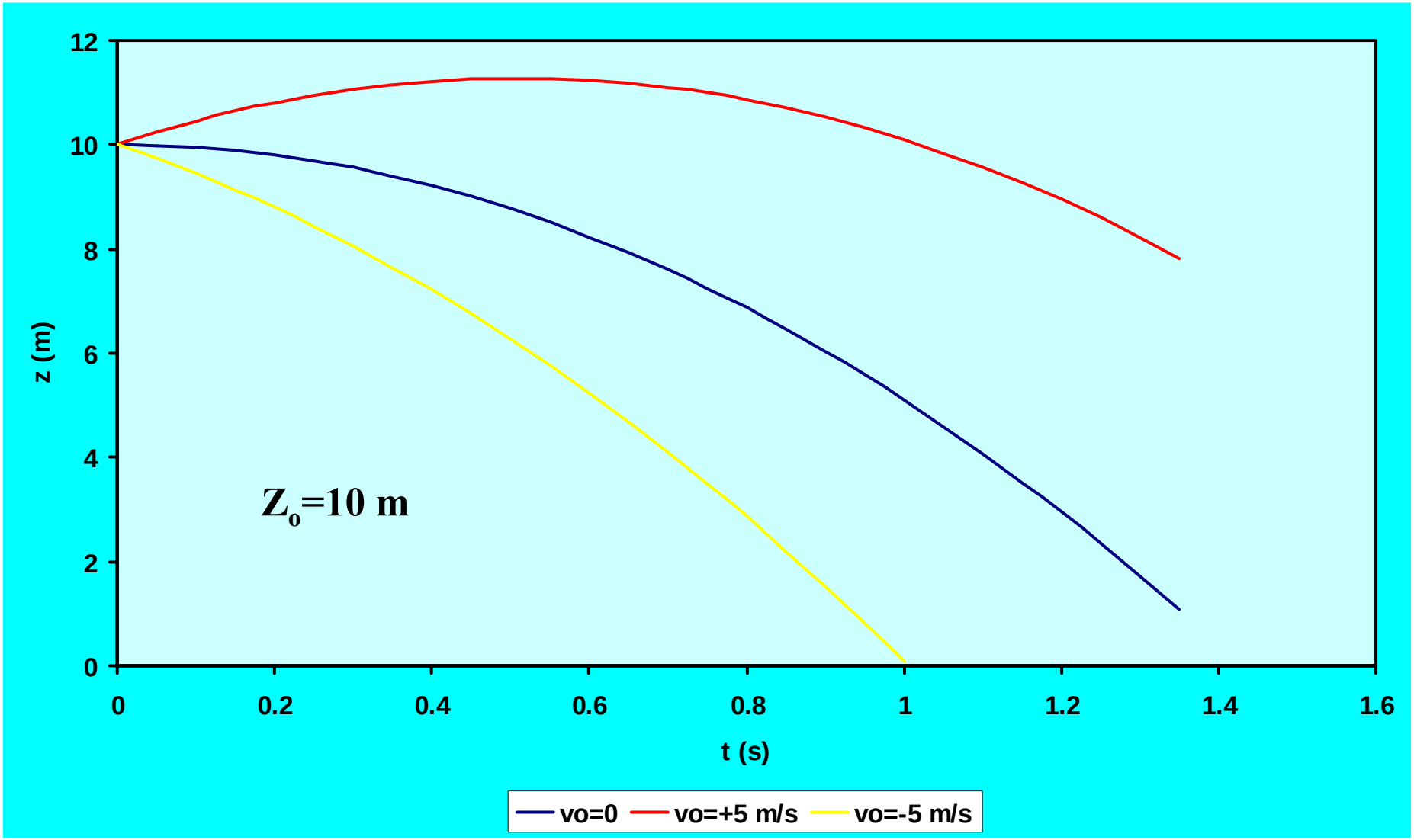
$$v_z(t) = v_o + a_o t$$

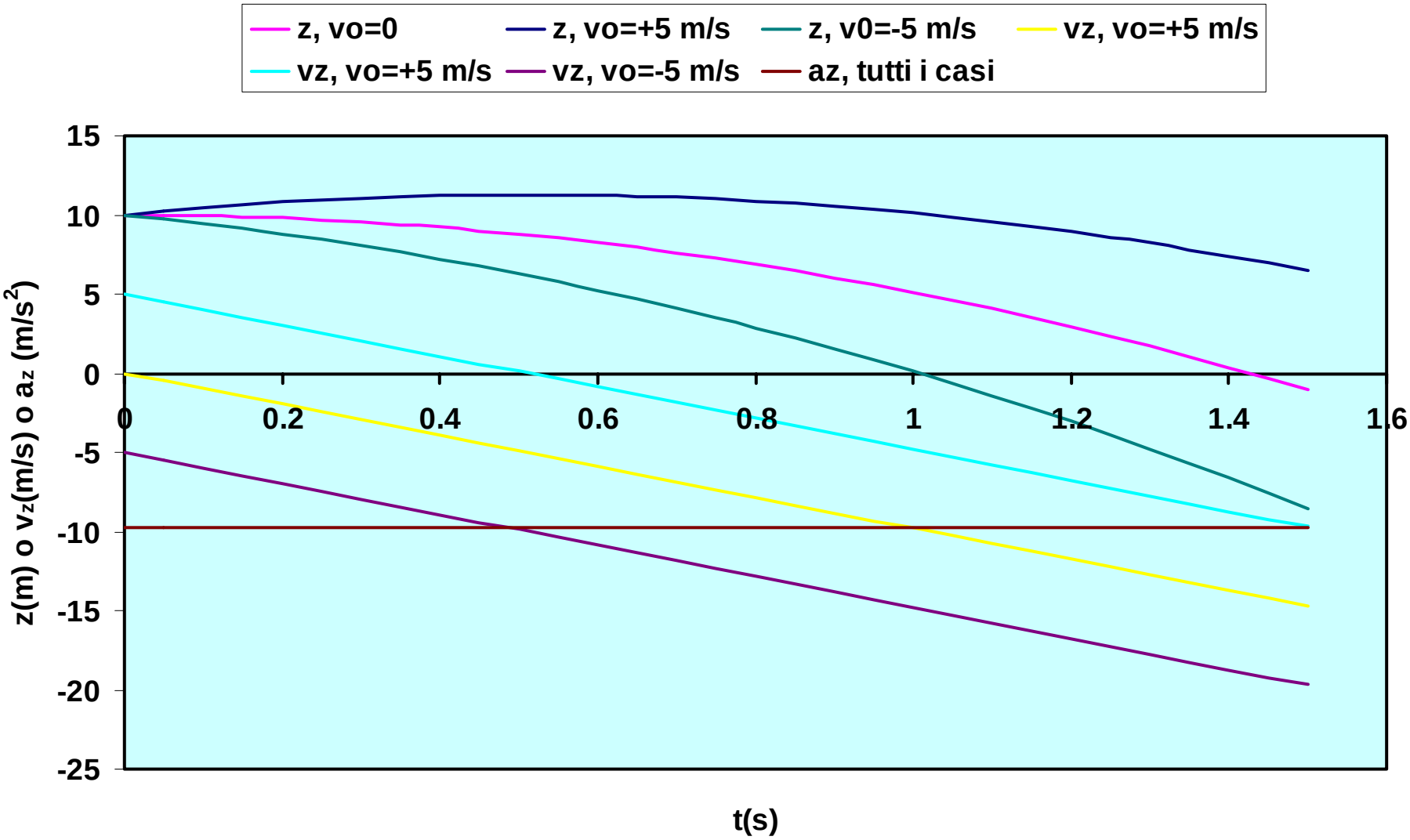
$$a_z(t) = a_o$$

$$a_o = -9.8 \frac{m}{s^2}$$



v_o=0 v_o=+ 5 m/s v_o=- 5 m/s


12
10
8
6
4
2
0
z (m)
Z_o=10 m
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
t (s)
vo=0
vo=+5 m/s
vo=-5 m/s


z, vo=0
z, vo=+5 m/s
z, v0=-5 m/s
vz, vo=+5 m/s
vz, vo=+5 m/s
vz, vo=-5 m/s
az, tutti i casi
15
10
5
0
-5
-10
-15
-20
-25
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
z(m) o vz(m/s) o az (m/s2)
t(s)

La velocità e l'accelerazione hanno versi indipendenti

La velocità può essere verso l'alto e l'accelerazione verso il basso o viceversa

2° caso: moto anche lungo x

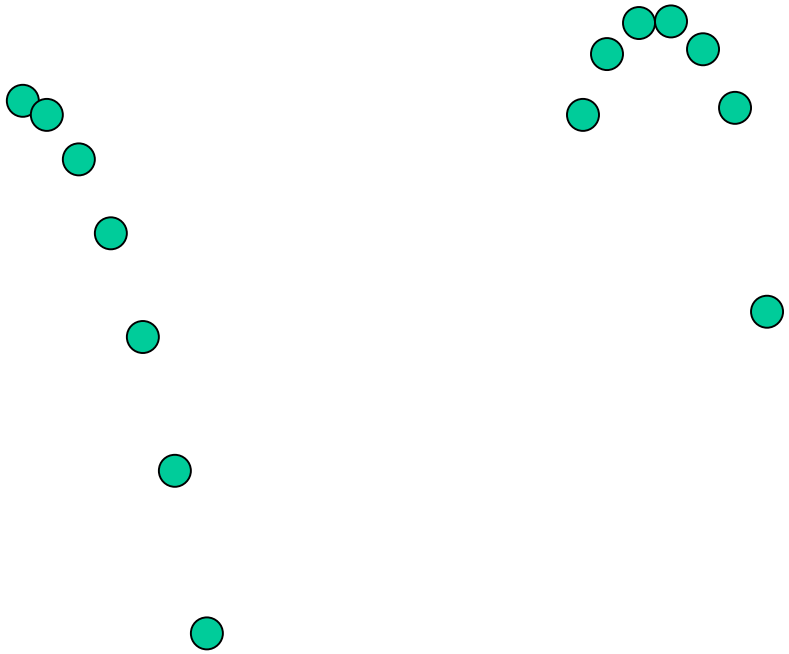
$$z(t) = z_o + v_{zo}t + \frac{1}{2}a_o t^2$$

$$x(t) = x_o + v_{xo}t$$

$$v_z(t) = v_{zo} + a_o t$$

$$v_x(t) = v_{xo}$$

Composizione dei moti

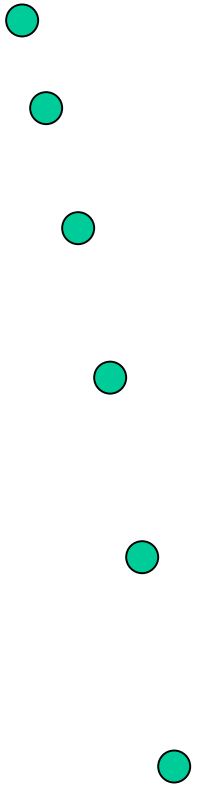


v_{oz}=0

v_{ox}= 2m/s

v_{oz}=+ 5 m/s

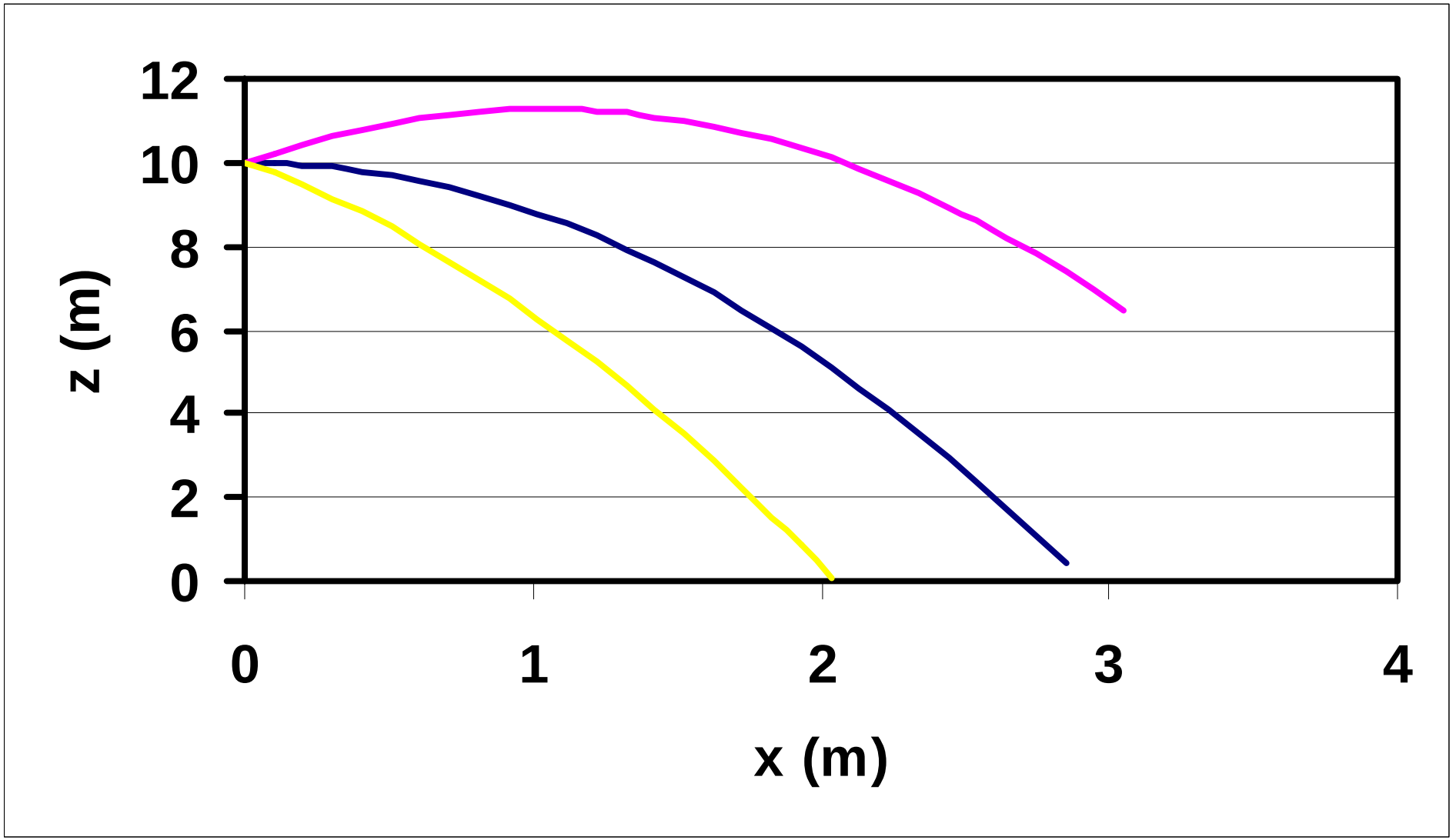
v_{ox}= 2m/s

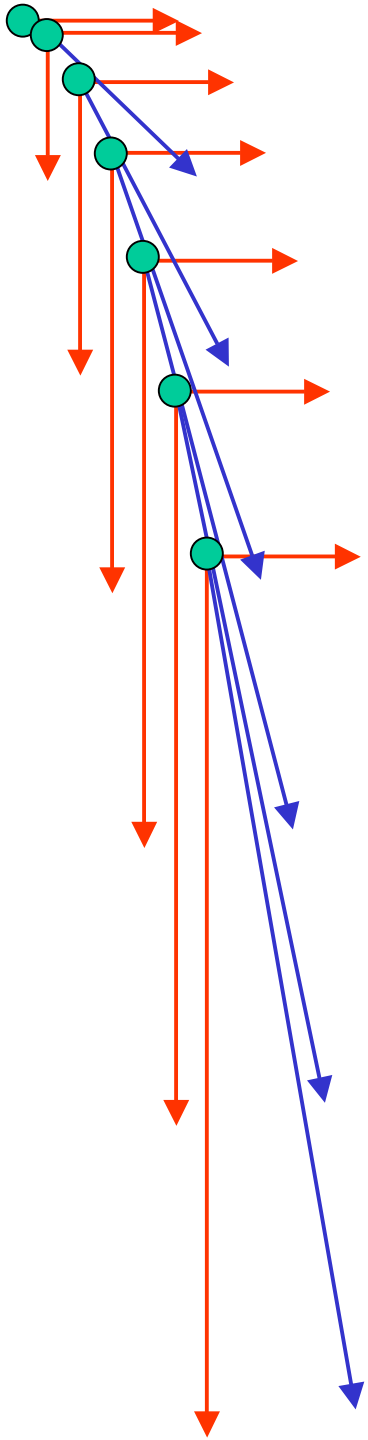


v_{oz}=- 5 m/s

v_{ox}= 2m/s

La traiettoria



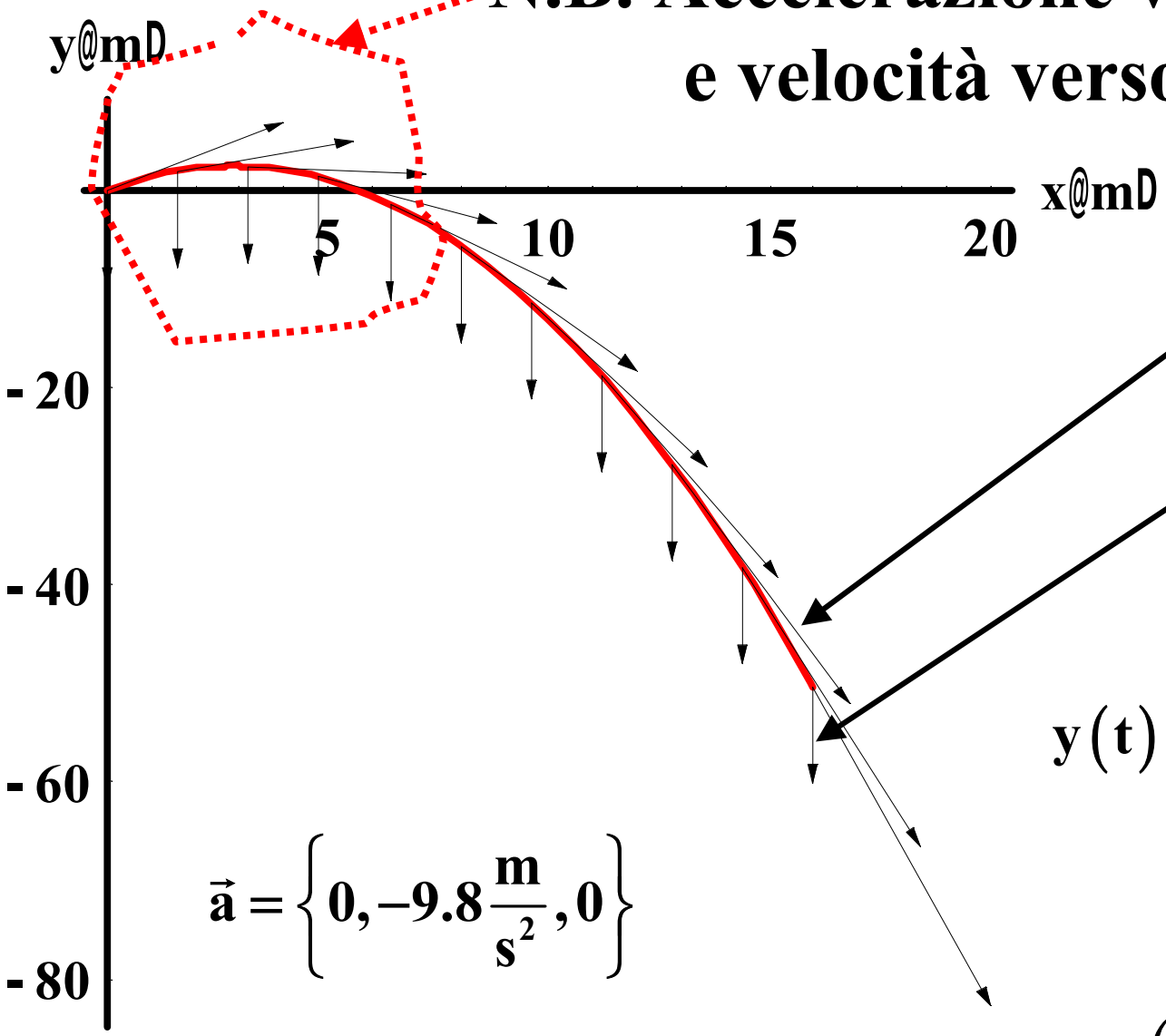


La composizione delle velocità

$$v_z(t) = -9.8\left(\frac{m}{s^2}\right)t$$

$$v_x(t) = 2\frac{m}{s}$$

N.B. Accelerazione verso il basso e velocità verso l'alto



Velocità

Accelerazione

$$\vec{a} = \left\{0, -9.8\frac{m}{s^2}, 0\right\}$$

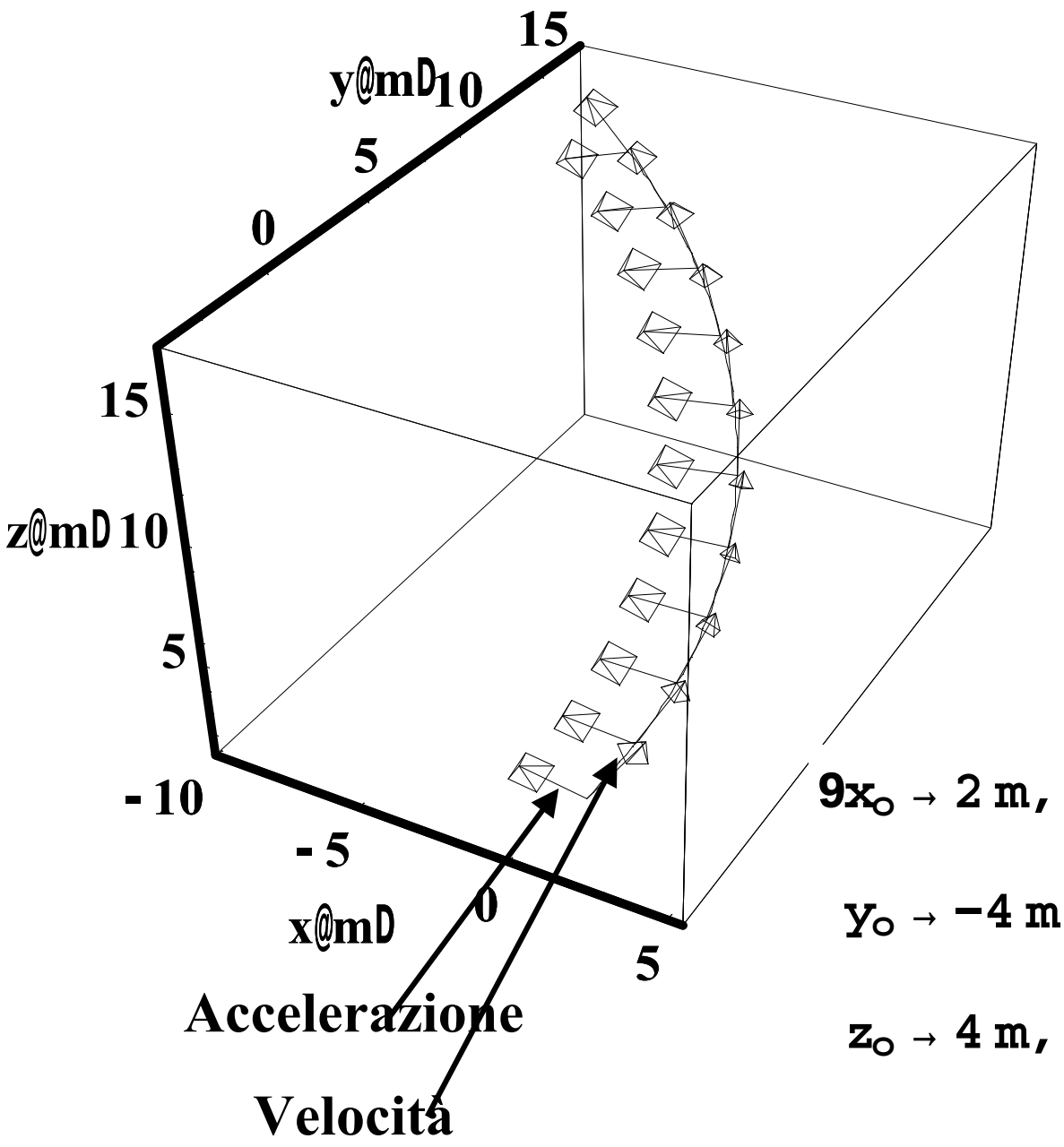
$$x(t) = 4\frac{m}{s}t;$$

$$y(t) = 7\frac{m}{s}t - \frac{1}{2}9.8\frac{m}{s^2}t^2$$

$$v_x(t) = 4\frac{m}{s};$$

$$v_y(t) = 7\frac{m}{s} - 9.8\frac{m}{s^2}t$$

Un altro esempio



Un esempio in tre dimensioni

$$9x_o \to 2\,m,\ v_{xo} \to 4\,\frac{m}{s},\ a_{xo} \to -3\,\frac{m}{s^2},$$

$$y_o \to -4\,m,\ v_{yo} \to .3\,\frac{m}{s},\ a_{yo} \to 2\,\frac{m}{s^2},$$

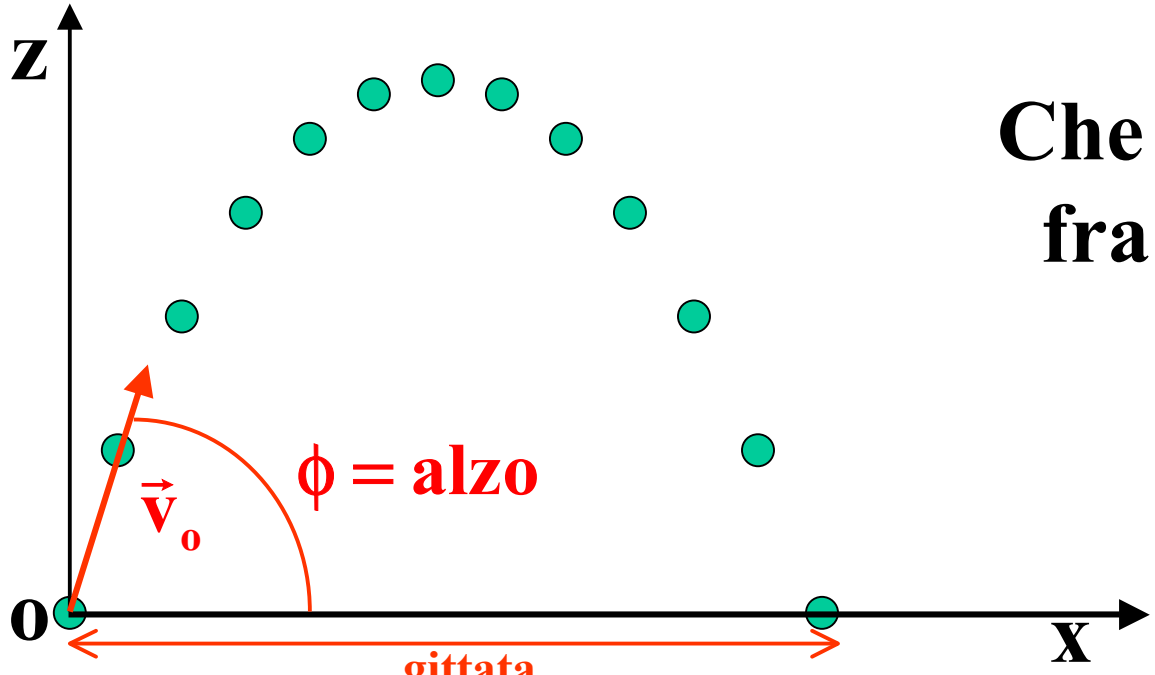
$$z_o \to 4\,m,\ v_{zo} \to 7\,\frac{m}{s},\ a_{zo} \to -2\,\frac{m}{s^2} =$$

Un'esempio importante:

Il lancio di un proiettile: partenza dall'origine posta al suolo

$$z(t) = v_{zo}t - \frac{1}{2}gt^2 \qquad g = 9.8\frac{m}{s^2} \qquad x(t) = v_{xo}t$$

$$v_z(t) = v_{zo} - gt \qquad v_x(t) = v_{xo}$$

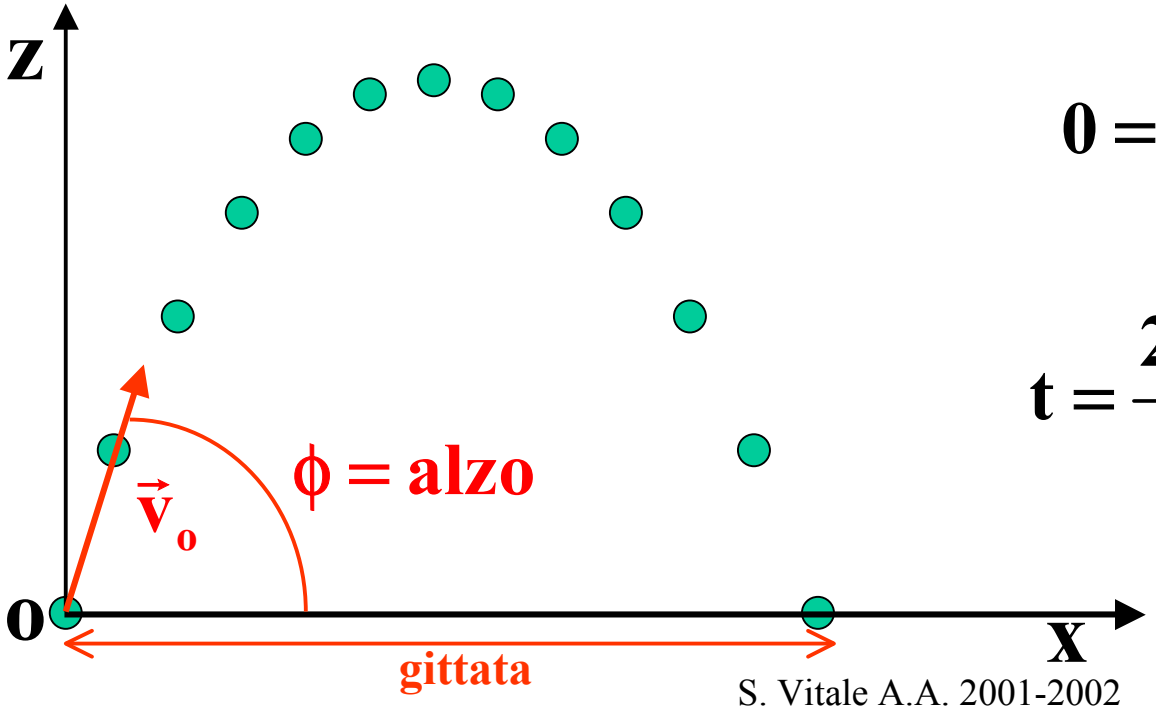


Che relazione c'è fra alzo, v_o e la gittata?

$$v_{ox} = |\vec{v}_o| \mathrm{Cos}(\phi) \qquad v_{oz} = |\vec{v}_o| \mathrm{Sin}(\phi)$$

$$z(t) = |\vec{v}_o| \mathrm{Sin}(\phi) t - \frac{1}{2} g t^2 \qquad x(t) = |\vec{v}_o| \mathrm{Cos}(\phi) t$$

$$v_z(t) = |\vec{v}_o| \mathrm{Sin}(\phi) - gt \qquad v_x(t) = |\vec{v}_o| \mathrm{Cos}(\phi)$$



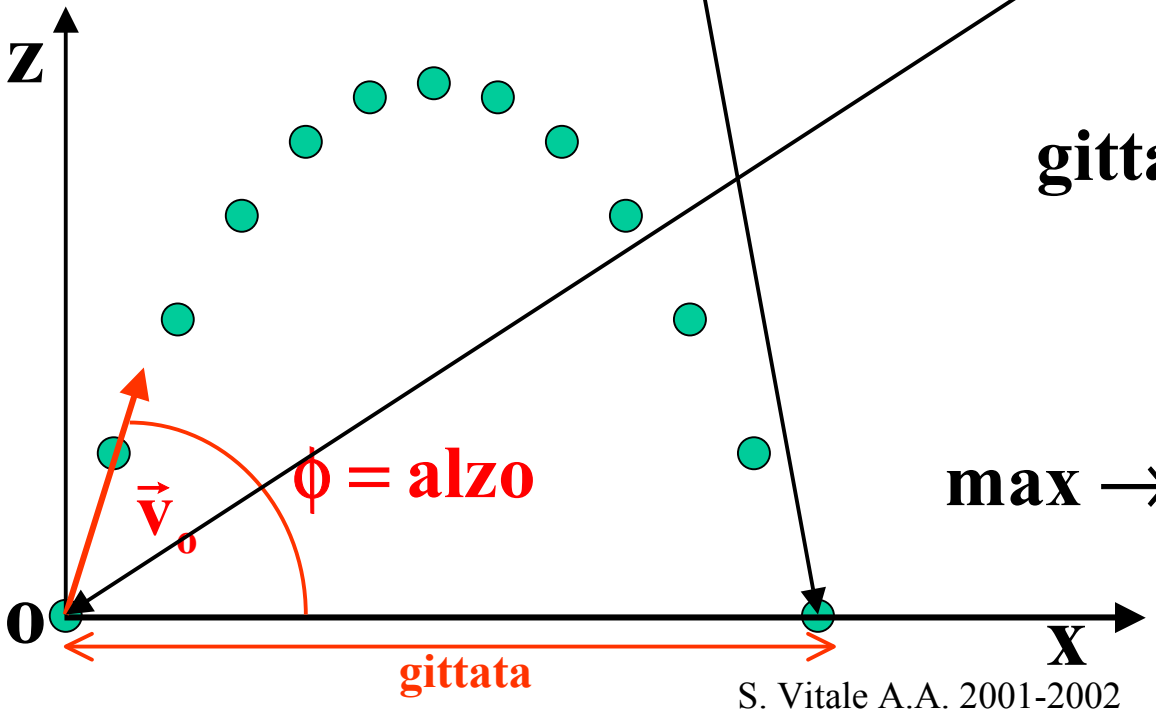
Impatto: z(t)=0

$$0 = \left[|\vec{v}_o| \mathrm{Sin}(\phi) - \frac{1}{2} g t \right] t$$

$$t = \frac{2 |\vec{v}_o| \mathrm{Sin}(\phi)}{g} \leftrightarrow t = 0$$

$$x(t) = |\vec{v}_o| \mathrm{Cos}(\phi) t \qquad t = \frac{2|\vec{v}_o| \mathrm{Sin}(\phi)}{g} \leftrightarrow t = 0$$

$$x = \frac{2|\vec{v}_o|^2 \mathrm{Sin}(\phi)\mathrm{Cos}(\phi)}{g} \leftrightarrow x = 0$$



$$\text{gittata} = \frac{|\vec{v}_o|^2 \mathrm{Sin}(2\phi)}{g}$$

$$\text{max} \rightarrow \frac{|\vec{v}_o|^2 \mathrm{Sin}\left(2\frac{\pi}{4}\right)}{g} = \frac{|\vec{v}_o|^2}{g}$$

Perchè l'accelerazione è importante?

$$\vec{F} = m\vec{a}$$