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$$= 8\,\underbrace{\frac{n^{2n+1}}{e^{n!}}}_{\to 0}\cdot\underbrace{\frac{1}{1-\frac{3n^{2n}}{e^{n!}}}}_{\to 1}+\underbrace{\frac{1-e^{1/n}}{1/n}}_{\to -1}\xrightarrow[n\to+\infty]{}-1$$

Es 5: Al variare di $a, b \in \mathbb{R}$ studiare la continuità in 1 di

$$f(x) = \begin{cases} \dfrac{ax^2-x}{\log x} & \text{se } 0<x<1 \\ 1 & \text{se } x=1 \\ \dfrac{\arcsin(x-1)-\operatorname{sen}(b(1-x))}{\operatorname{sen}|1-x|}, & \text{se } x>1. \end{cases}$$

Sol:
Debbo determinare $a, b \in \mathbb{R}$ in modo tale che

$$\lim_{x\to1^-} f(x) = 1 = \lim_{x\to1^+} f(x).$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-}\frac{ax^2-x}{\log x} = \lim_{x\to1^-}\frac{(ax-1)x}{\log(1+x-1)}\cdot\frac{(x-1)}{x-1} =$$

$$= \lim_{x\to1^-}\underbrace{\frac{x-1}{\log(1+x-1)}}_{\to 1}\cdot\frac{ax-1}{x-1}\cdot\underbrace{x}_{\to 1} = \lim_{x\to1^-}\frac{ax-1}{x-1}$$

Poiché

$$\frac{ax-1}{\underbrace{x-1}_{\to 0^-}}\xrightarrow[x\to1^-]{}\begin{cases} -\infty & \text{se } a>1 \\ 1 & \text{se } a=1 \\ +\infty & \text{se } a<1 \end{cases}$$

si ha che $\lim_{x\to1^-} f(x) = 1 \iff a=1$

Studiamo ora

$\lim_{x\to1^+} f(x)$. Supponiamo $b\neq0$.

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$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+}\frac{\arcsin(x-1)-\operatorname{sen}(b(1-x))}{\operatorname{sen}|1-x|} \overset{(b\neq0)}{=}$$

$$= \lim_{x\to1^+}\frac{\arcsin(x-1)}{x-1}\cdot\frac{x-1}{\operatorname{sen}|1-x|}-\frac{\operatorname{sen}(b(1-x))}{b(1-x)}\cdot\frac{b(1-x)}{\operatorname{sen}|1-x|}$$

$$= \lim_{x\to1^+}\underbrace{\frac{\arcsin(x-1)}{x-1}}_{\to 1}\cdot\underbrace{\frac{x-1}{\operatorname{sen}(x-1)}}_{\to 1}-\underbrace{\frac{\operatorname{sen} b(1-x)}{b(1-x)}}_{\to 1}\cdot\underbrace{\frac{b(1-x)}{\operatorname{sen}(x-1)}}_{\to -b}$$

Dunque se $b\neq0$: $\lim_{x\to1^+} f(x) = 1 \iff 1+b=1$

$\iff b=0$ Assurdo. (si era supposto $b\neq0$)

Sia $\underline{\underline{b=0}}$. Allora

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+}\frac{\arcsin(x-1)}{\operatorname{sen}|1-x|} = \lim_{x\to1^+}\frac{\arcsin(x-1)}{\operatorname{sen}(x-1)} =$$

$$= \lim_{x\to1^+}\frac{\arcsin(x-1)}{x-1}\cdot\frac{x-1}{\operatorname{sen}(x-1)} = 1$$

Quindi f è continua in 1 $\iff (a=1, b=0)$.

■