Ora:

$$\int_0^{\log(2+\sqrt{3})} \frac{1}{1+\frac{e^t-e^{-t}}{2}}\,dt = \int_0^{\log(2+\sqrt{3})} \frac{2}{2+e^t-e^{-t}}\,dt \qquad (*)$$

Pongo $e^t = z \Rightarrow t = \log z \Rightarrow dt = \frac{1}{z}\,dz$.

Inoltre: se $t=0 \Rightarrow z=1$

se $t=\log(2+\sqrt{3}) \Rightarrow z = 2+\sqrt{3}$.

Pertanto il nostro integrale in $(*)$ diventa

$$\int_0^{2+\sqrt{3}} \frac{2}{2+z-\frac{1}{z}}\cdot\frac{1}{z}\,dz = \int_0^{2+\sqrt{3}} \frac{2z}{2z+z^2-1}\cdot\frac{1}{z}\,dz =$$

$$= 2\int_0^{2+\sqrt{3}} \frac{1}{z^2+2z-1}\,dz.$$

$z^2+2z-1=0: \quad \frac{\Delta}{4} = 1+1 = 2$

$z_1, z_2 = -1 \mp \sqrt{2}$

Cerco $A, B \in \mathbb{R}$ in modo tale che

$$\frac{1}{z^2+2z-1} = \frac{A}{z+1+\sqrt{2}} + \frac{B}{z+1-\sqrt{2}}$$

cioè $1 = A[z+1-\sqrt{2}] + B[z+1+\sqrt{2}] \Leftrightarrow$

$\Leftrightarrow 1 = (A+B)z + A+B - \sqrt{2}(A-B)$

$$\Leftrightarrow \begin{cases} A+B=0 \\ A+B-\sqrt{2}(A-B)=1 \end{cases} \Leftrightarrow \begin{cases} A=-B \\ +2\sqrt{2}B=1 \end{cases} \Leftrightarrow \begin{cases} A=-\frac{1}{2\sqrt{2}} \\ B=\frac{1}{2\sqrt{2}} \end{cases}$$

Pertanto:

$$2\int_0^{2+\sqrt{3}} \frac{1}{z^2+2z-1}\,dz = 2\left[\int_0^{2+\sqrt{3}} -\frac{1}{2\sqrt{2}}\cdot\frac{1}{z+1+\sqrt{2}}\,dz + \int_0^{2+\sqrt{3}} \frac{1}{2\sqrt{2}\,(z+1-\sqrt{2})}\,dz\right] =$$

$$= -\frac{1}{\sqrt{2}}\int_0^{2+\sqrt{3}} \frac{1}{z+1+\sqrt{2}}\,dz + \frac{1}{\sqrt{2}}\int_0^{2+\sqrt{3}} \frac{1}{z+1-\sqrt{2}}\,dz =$$

$$= -\frac{1}{\sqrt{2}}\Big[\log|z+1+\sqrt{2}|\Big]_0^{2+\sqrt{3}} + \frac{1}{\sqrt{2}}\Big[\log|z+1-\sqrt{2}|\Big]_0^{2+\sqrt{3}} =$$

$$= -\frac{1}{\sqrt{2}}\left[\log(3+\sqrt{2}+\sqrt{3}) - \log(1+\sqrt{2})\right] + \frac{1}{\sqrt{2}}\left[\log|3+\sqrt{3}-\sqrt{2}| - \log|1-\sqrt{2}|\right]$$

Conclusione:

$$\int_1^2 \frac{dx}{1+\sqrt{x^2-1}} = \log(2+\sqrt{3}) + \frac{1}{\sqrt{2}}\Big\{\log(3+\sqrt{2}+\sqrt{3}) - \log(1+\sqrt{2}) - \log(3+\sqrt{3}-\sqrt{2}) + \log(\sqrt{2}-1)\Big\}$$